

## Work and Energy Unit: Quantitative Bar Graphs and Problems

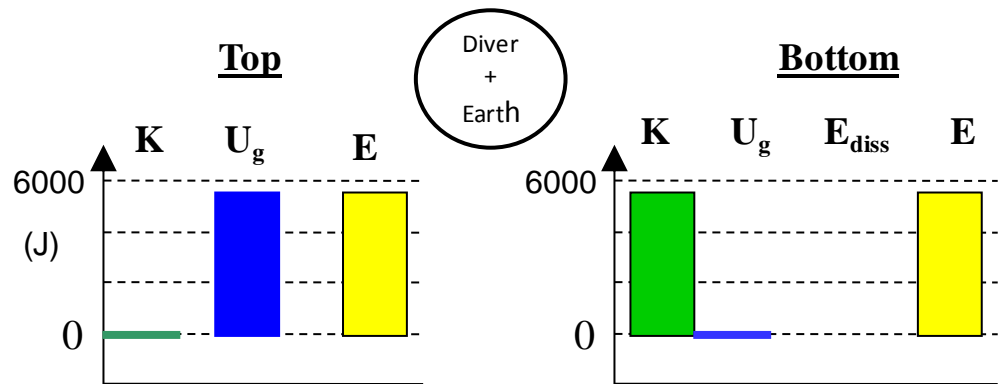
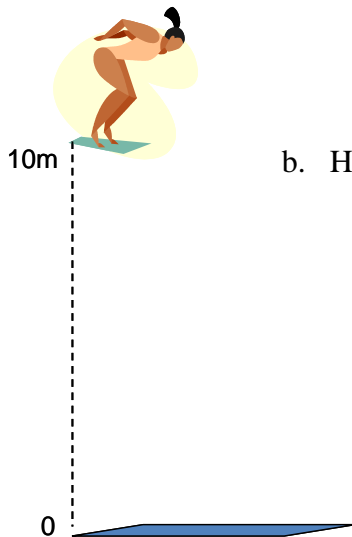
For each situation shown below: Assume the systems to be frictionless, unless stated otherwise.

- In the energy flow diagram show the system you choose to analyze; indicate what is included in the system in the circle. If the system gains or loses energy via work by nonconservative forces, indicate that energy flow into or out of circle
- Complete the energy bar graph. Include numbers in the vertical axes.
- In the space below each diagram use conservation of energy equations to solve for the quantity called for in the question.

- A 60. kg student jumps from a 10.0 m platform into a pool below.

System of diver and earth is isolated; mechanical energy is conserved. No nonconservative forces are doing work to change the mechanical energy of the system.

- Determine her  $U_g$  at the top of the platform.



Energy Flow Diagram

$$\begin{aligned}
 E_T &= K_T + U_{gT} \\
 &= 0 + mgh_T = (60)(9.8)(10) = 5880J = U_{gT}
 \end{aligned}$$

- How much K does she possess at impact? What is her speed at impact?

$$\begin{aligned}
 E_T &= E_B \\
 5880 &= K_B + U_{gB} \\
 5880 &= \frac{1}{2}mv_B^2 + 0 \\
 v_B &= \sqrt{2(60)(5880)} \\
 v_B &= 14m/s
 \end{aligned}$$

- Do you think a heavier diver of 75 kg would have more speed and kinetic energy on impact? Repeat steps a and b for a 75 kg diver.

$$\begin{aligned}
 E_T &= E_B \\
 mgh_T &= \frac{1}{2}mv_B^2 \\
 v_B &= \sqrt{2gh} = 14m/s \\
 K_B &= \frac{1}{2}mv_B^2 \\
 &= 7350J
 \end{aligned}$$

The speed is independent of the mass of the diver

The kinetic energy depends on the mass of the diver

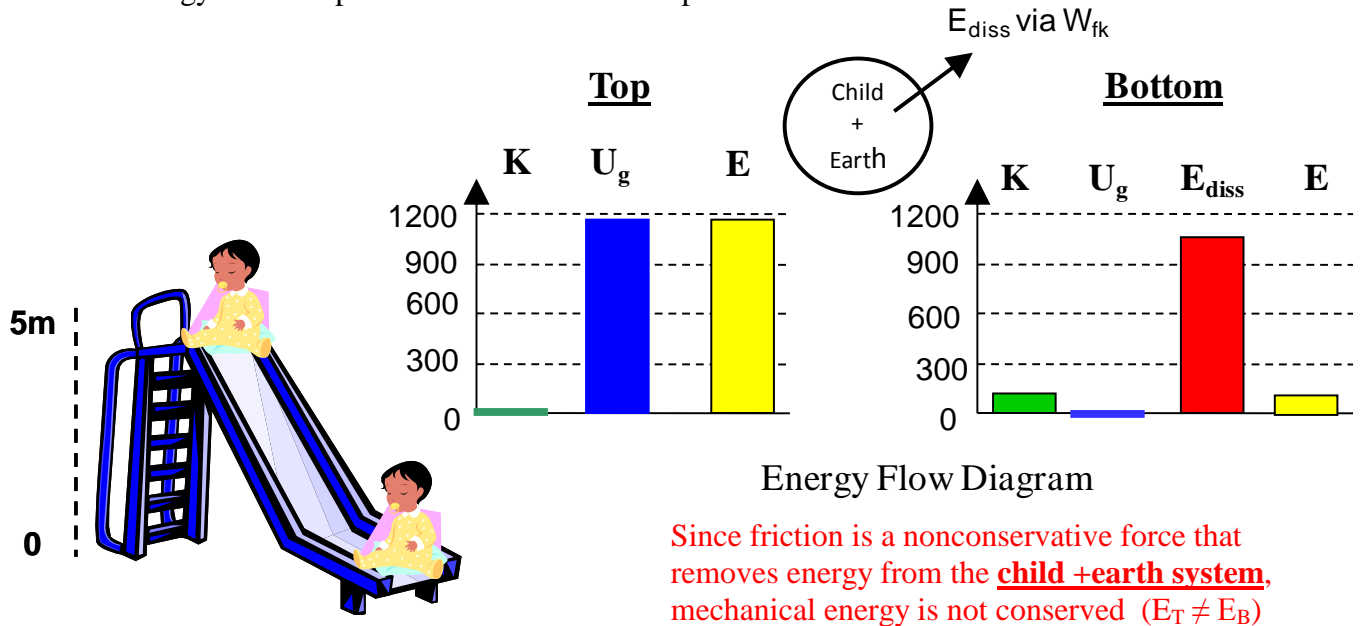
- d. If she jumped from a platform that was twice as high, how many times greater would be her speed at impact?

$$v_B = \sqrt{2gh} \quad \text{Double the height,} \quad v_B = \sqrt{2xs} = 1.4xs$$

- e. How much higher would the platform have to be in order for her speed to be twice as great?

**4 xs**

2. A 24 kg child descends a 5.0 m high slide and reaches the ground with a speed of 2.8 m/s. How much energy was dissipated due to friction in the process?



$$W_{NC} + E_i = E_f$$

$$W_f + E_T = E_B$$

$$\begin{aligned} E_T &= K_T + U_{gT} \\ &= 0 + mgh_T = (24)(9.8)(5) \end{aligned}$$

$$E_T = 1176J = U_{gT} \quad \text{All mechanical energy at top is in the form of } U_g$$

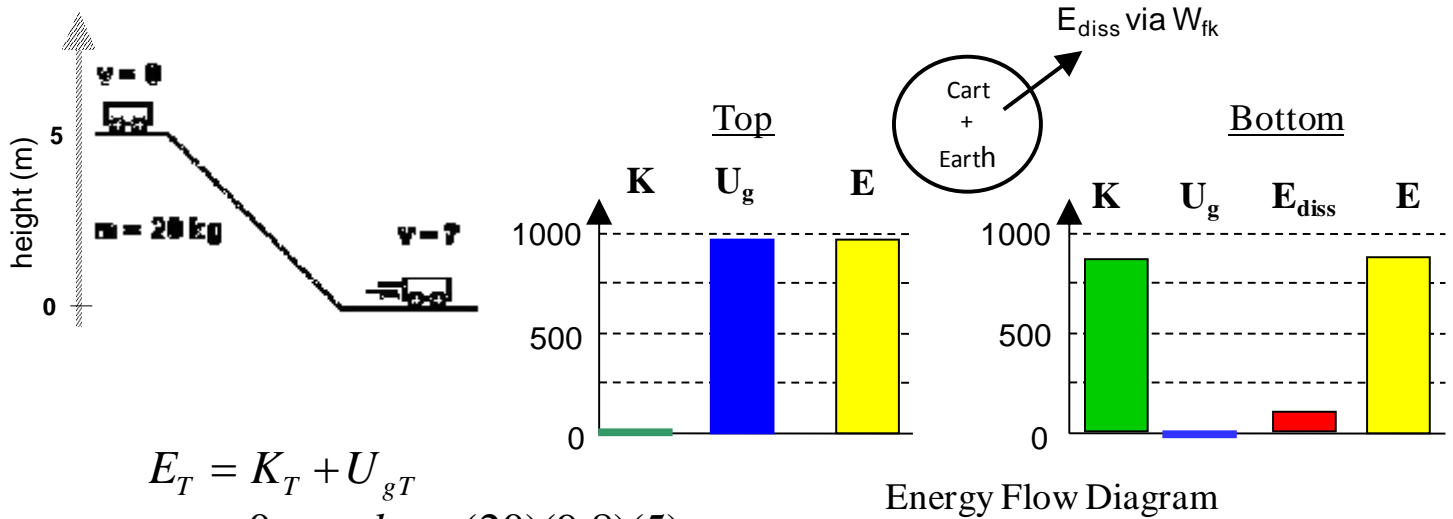
$$\begin{aligned} E_B &= K_B + U_{gB} \\ &= \frac{1}{2}mv_B^2 + 0 = \frac{1}{2}(24)2.8^2 \end{aligned}$$

$$E_B = 94.1J = K_B \quad \text{All mechanical energy at bottom is in the form of } K$$

Energy lost is due to friction

$$W_f = E_B - E_T = 94.1 - 1176 = -1082J = -E_{diss}$$

3. Determine final velocity of the cart, assuming that 10% of the energy is dissipated by friction.



$$E_T = K_T + U_{gT}$$

$$= 0 + mgh_T = (20)(9.8)(5)$$

$$E_T = 980 \text{ J} = U_{gT}$$

Since friction is a nonconservative force that removes energy, mechanical energy is not conserved:

$$E_B = 0.9E_T = 882 = K_B + U_{gB}$$

$$882 = K_B + U_{gB}$$

$$= \frac{1}{2}mv_B^2 + 0 = \frac{1}{2}(20)v_B^2$$

$$v_B = 9.39 \text{ m/s}$$

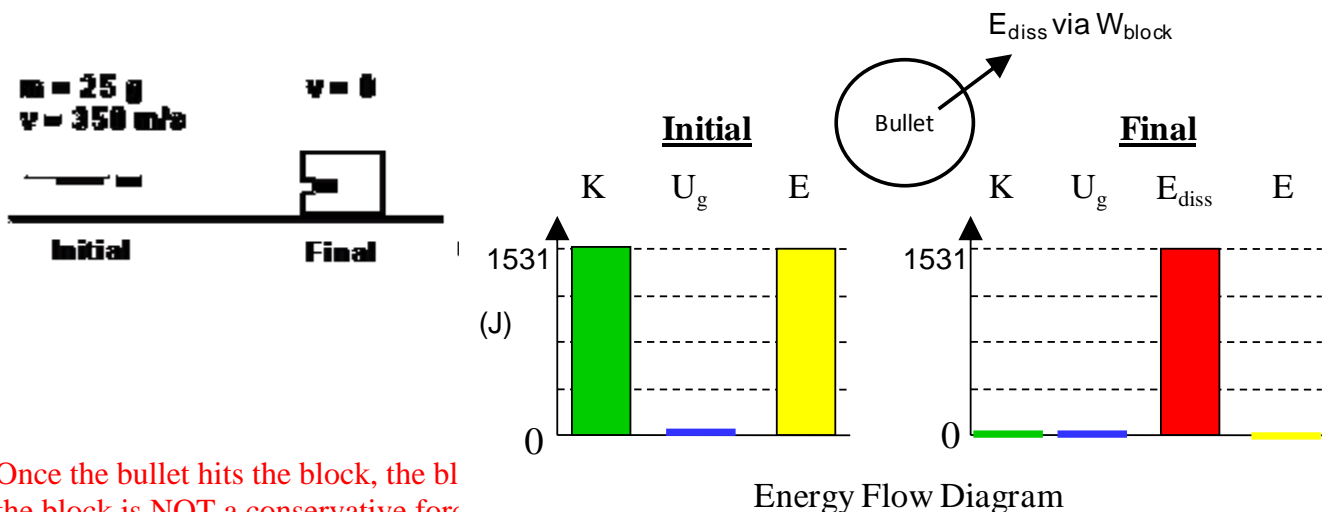
$$W_{NC} + E_i = E_f$$

$$W_f + E_T = E_B$$

$$W_f = 882 - 980$$

$$= -108 \text{ J} = -E_{\text{diss}}$$

4. The bullet strikes a stationary block of wood which exerts, on average, a force of 50,000N opposing the motion of the bullet. How far does the bullet penetrate? Assume that the block is mounted to the surface and does not move after the bullet is embedded in the block. Assume also that the gravitational potential energy of the bullet does not change between right before the bullet penetrates the block and when it comes to rest. **System is bullet (don't need to include Earth since we are neglecting the gravitational interaction betw bullet and Earth that would change  $U_g$ ).**



Once the bullet hits the block, the bl  
the block is NOT a conservative force

conserved. It gets dissipated by the block. The energy dissipated is the work done by the block in stopping the bullet

$$W_{block} + E_i = E_f$$

$$F_{block} \Delta x \cos 180 + 1531.25 = 0$$

$$-50000 \Delta x = -1531.25$$

$$\Delta x = 0.031m$$

$$E_i = K_i + U_{gi}$$

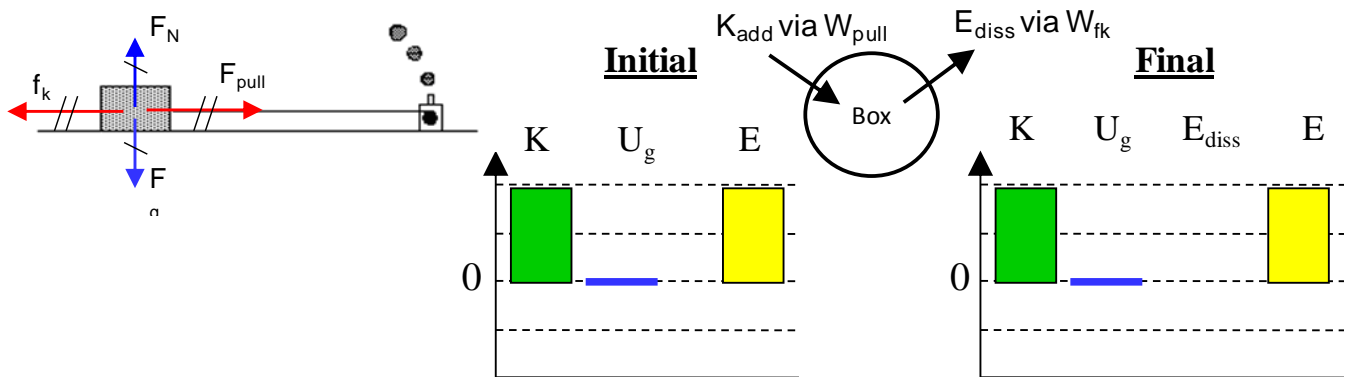
$$= \frac{1}{2} m v_i^2 + 0$$

$$= \frac{1}{2} (0.025) 350^2$$

$$= 1531.25 J$$

$$E_f = 0$$

5. A 200. kg box is pulled at constant speed by the little engine pictured below. The box moves a distance of 2.5 m across a horizontal surface. The force of friction acting on the box is 400 N. System is box (don't need to include Earth since the gravitational interaction between box and Earth does not change  $U_g$ ). This system is not isolated since the non conservative forces of the pull and friction do work to add and remove energy to the system, respectively. Since these forces add and remove the same amount of energy, the mechanical energy of the system remains the same.



- a) Draw a force diagram of all relevant forces acting on the box.
- b) How much energy is transferred by the engine?

$$\text{Energy Transferred} = W_{pull}$$

$$W_{pull} = F_{pull} \Delta x \cos 0 = (400)(2.5) = 1000 J$$

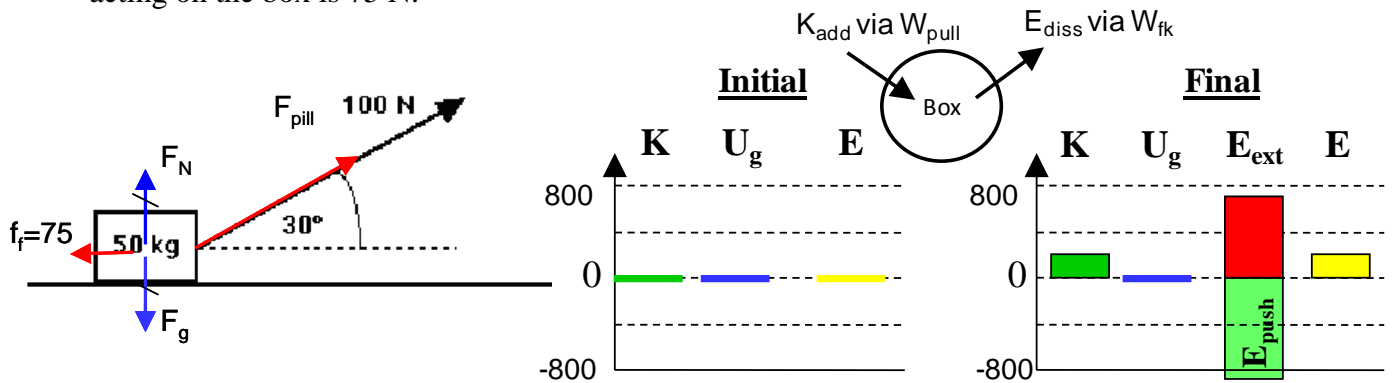
- c) What type of motion would occur if the engine pulled with a force of 500 N?  
There would be a net force to the right and so the box would accelerate to the right and the kinetic energy and total mechanical energy would increase. the pulling force would do more work and add more energy than friction would remove.

- d) How far could the box be pulled *at constant velocity* with the expenditure of 8,000 J of energy?

$$W_{pull} = 8000 = F_{pull} \Delta x \cos 0 = (400)(\Delta x)$$

$$\Delta x = 20m$$

6. A person pulls a 50 kg box pictured below with a force of 100 N. The force of friction acting on the box is 75 N.



Energy Flow Diagram

- a. Draw all forces acting on the box above. **System is box (don't need to include Earth since the gravitational interaction between box and Earth does not change  $U_g$ ). This system is not isolated since the non conservative forces of the pull and friction do work to add and remove energy to the system, respectively. Since the pulling force adds more energy than friction removes, the result is an addition of energy that appears as kinetic energy. E<sub>ext</sub> shows the energy lost (green) and gained (red) by the environment, external to the system..**
- b. How much energy is transferred (via work) by the person who pulls the box a distance of 10. m?

$$W_{pull} = F_{pull} \Delta x \cos \theta = (100)(10) \cos 30 = 866J$$

- c. Is the box moving at constant speed? Explain how you know. What does this tell you about the kinetic energy  $E_k$  of the system?

**No the box is accelerating since  $F_{pullx} > f_k = F_{netx}$ . Since there is a net force in the x-direction, there is positive, net work done on the box. Therefore, the kinetic energy of the box increases (work energy theorem)**

- d. How much energy is dissipated by friction in the pulling process? Where does this energy "go"?

$$W_f = f_{fl} \bullet \Delta x = -(75)(10) = -750J$$

**The energy lost in friction doing negative work is dissipated as heat and sound**

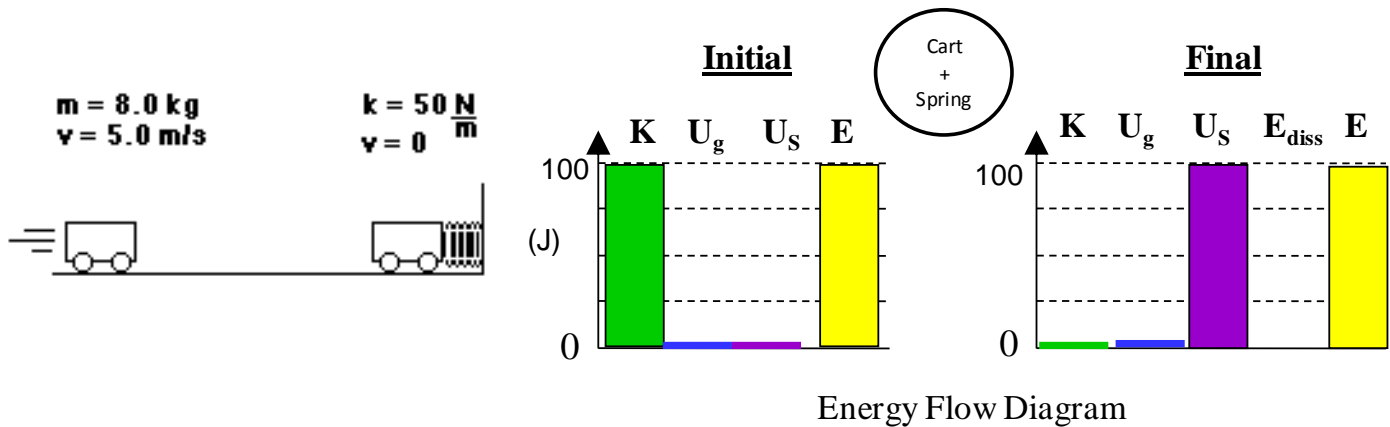
- e. How much energy is "left over," and what does it "do"?

$$W_{net} = W_{pull} - W_f = 866 - 750 = 116J$$

**This net positive energy transfer increases the kinetic energy of the block.**

- f. Show that energy is conserved in the system using a bar graph, accounting for all the energy stored and transferred in the process.

7. A moving cart hits a spring, traveling at 5.0 m/s at the time of contact. At the instant the cart is motionless, by how much is the spring compressed?



$$E_1 = E_2$$

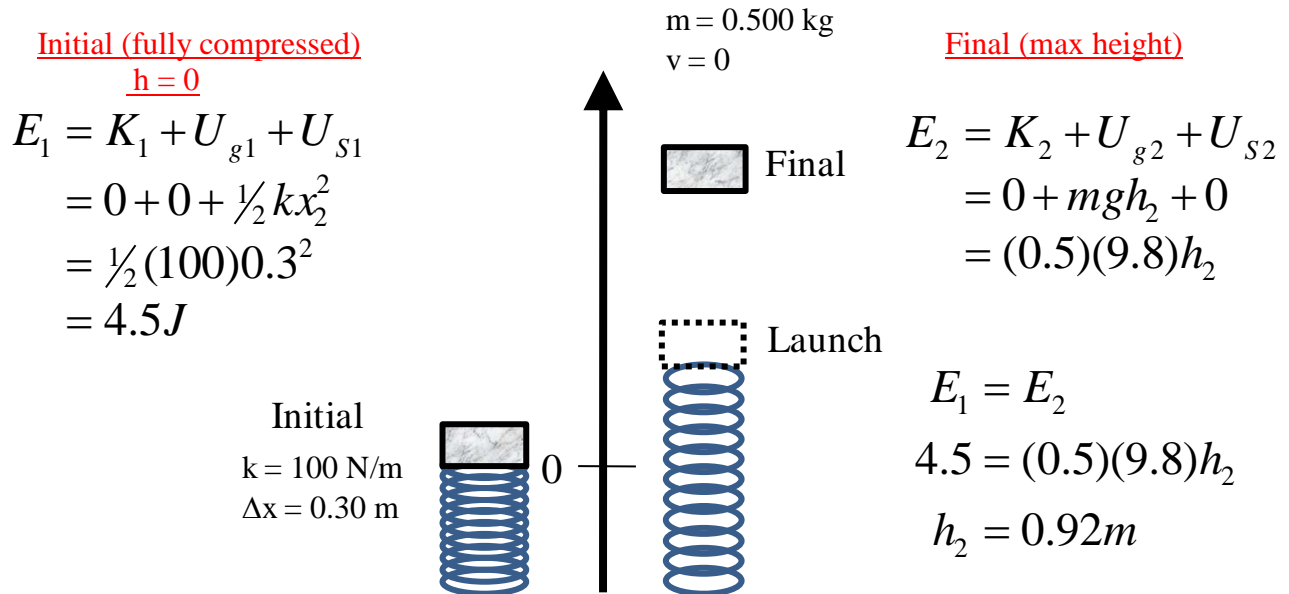
$$K_1 + U_{g1} + U_{s1} = K_2 + U_{g2} + U_{s2}$$

$$\frac{1}{2}mv_1^2 + 0 + 0 = 100 = 0 + 0 + \frac{1}{2}kx_2^2$$

$$x_2 = \sqrt{\frac{mv_1^2}{k}} = \sqrt{\frac{(8)5^2}{50}}$$

$$x_2 = 2\text{m}$$

8. A block is placed on a spring, compressing it 0.30m. What height does the block reach when launched by the spring? What is the launch velocity of the block (once the spring is back at equilibrium)? **System of block, spring and earth are isolated ; mechanical energy is conserved. No nonconservative forces are doing work to change the mechanical energy of the system.**



Launch (spring uncompressed)  
 $h = 0.3\text{m}$

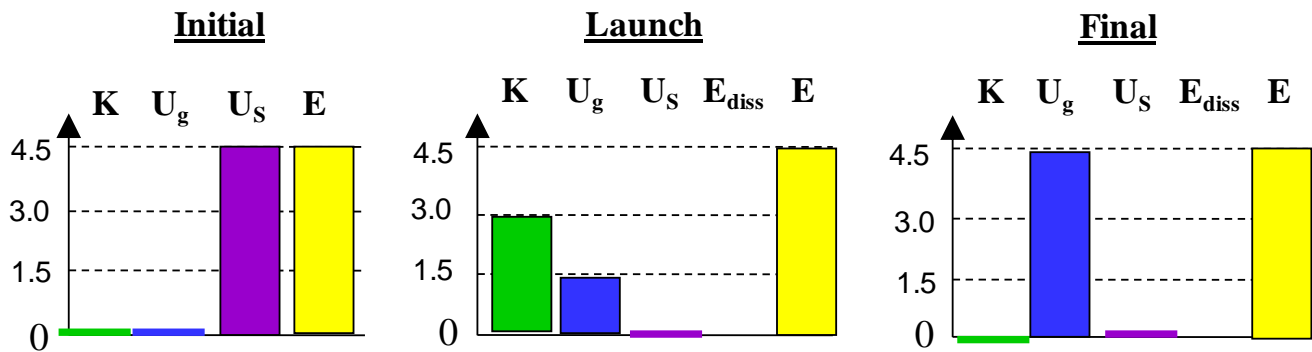
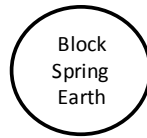
$$E_L = K_L + U_{gL} + U_{SL}$$

$$4.5 = \frac{1}{2}mv_L^2 + mg(0.3) + 0$$

$$= 0.25v_L^2 + 1.47$$

$$0.25v_L^2 = K = 3.03\text{J}$$

$$v_L = 3.48\text{m/s}$$



Energy Flow Diagram