

Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications

Author(s): Susan N. Friel, Frances R. Curcio, George W. Bright

Reviewed work(s):

Source: *Journal for Research in Mathematics Education*, Vol. 32, No. 2 (Mar., 2001), pp. 124-158

Published by: [National Council of Teachers of Mathematics](#)

Stable URL: <http://www.jstor.org/stable/749671>

Accessed: 16/01/2012 14:32

---

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at  
<http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



National Council of Teachers of Mathematics is collaborating with JSTOR to digitize, preserve and extend access to *Journal for Research in Mathematics Education*.

# Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications

Susan N. Friel, *University of North Carolina at Chapel Hill*  
Frances R. Curcio, *Queens College of the City University of New York*  
George W. Bright, *University of North Carolina at Greensboro*

Our purpose is to bring together perspectives concerning the processing and use of statistical graphs to identify critical factors that appear to influence graph comprehension and to suggest instructional implications. After providing a synthesis of information about the nature and structure of graphs, we define *graph comprehension*. We consider 4 critical factors that appear to affect graph comprehension: the purposes for using graphs, task characteristics, discipline characteristics, and reader characteristics. A construct called *graph sense* is defined. A sequence for ordering the introduction of graphs is proposed. We conclude with a discussion of issues involved in making sense of quantitative information using graphs and ways instruction may be modified to promote such sense making.

*Key Words:* All school levels; Content knowledge; Research issues; Review of research; Statistics; Stochastics

Statistics and data analysis emerged as a major component of the school mathematics curriculum during the 1990s (National Council of Teachers of Mathematics [NCTM], 1989, 2000). The current meaning of *data analysis* includes a heavy reliance on graphical representations (Shaughnessy, Garfield, & Greer, 1996), reflecting that the use of visual displays of quantitative data is pervasive in our highly technological society. Creation of such displays often is as easy as the click of a button. Clearly, to be functionally literate, one needs the ability to read and understand statistical graphs and tables. Yet educators have much to learn about the processes involved in reading, analyzing, and interpreting information presented in data graphs and tables.

As early as 1915, efforts to standardize graphics methods appeared in the form of preliminary recommendations from the Joint Committee on Standards for Graphic Presentation with the premise that

if simple and convenient standards can be found and made generally known, there will be possible a more universal use of graphic methods with a consequent gain to mankind because of the greater speed and accuracy with which complex information may be imparted and interpreted. (McCall, 1939, p. 475)

Several graphics handbooks (e.g., Brinton, 1914; Haskell, 1922; Modley & Lowenstein, 1952; Schmid & Schmid, 1979; Tufte, 1983) provided sets of design

guidelines that were based primarily on authors' intuitions drawn from the wisdom of practice. Similar handbooks focused on the design of tables (e.g., Hall, 1943; Walker & Durost, 1936). In the late 1970s, researchers interested in information processing and the psychology of graphics began to study graph perception. Later graphics handbooks (e.g., Cleveland, 1985; Kosslyn, 1994) reflected integration of the results of this research.

Research concerning the processing and use of graphs has received and continues to receive attention in several fields. Research in information systems and decision sciences reflects efforts to evaluate the effectiveness of business graphics as an aid in decision making. Several researchers have considered graphical information as part of investigations related to audiovisual communications. Psychology and human-factors researchers focus attention on various features of human interaction with a display, including pattern perception, memory of images, spatial reasoning, and vision. Information processing models of graphical perception help educators make sense of how people process visual information displayed in graphs. In addition, research in the fields of reading, document literacy, statistics, and, more recently, mathematics education and statistics education offers further relevant information. However, there appears to have been little communication among researchers holding these different perspectives. Overall, no one has proposed a coherent framework that addresses the domain of graph comprehension. Researchers need to synthesize what is known and to consider how this knowledge informs both practice and research.

Our purpose in this article is to bring together key ideas from various perspectives, going beyond several earlier reviews of the literature (DeSanctis, 1984; Jarvenpaa & Dickson, 1988; MacDonald-Ross, 1977; Malter, 1952), to identify critical factors that appear to influence comprehension of graphs and to suggest instructional implications. In the first part of the article, we define what we mean by *graphs* and provide an analysis of the structure of graphs and tables. Next, we define *graph comprehension*. We then address several of the critical factors: purposes for using graphs, task characteristics, discipline characteristics, and reader characteristics.

In the second part of the article, we suggest instructional implications that reflect consideration of these critical factors. We propose a construct called *graph sense* and identify associated behaviors that may provide evidence of graph comprehension. We suggest a sequence for the introduction of different types of graphs and identify considerations about the nature of representations not addressed by earlier research. We conclude with a discussion of what is involved in creating and adapting displays for purposes of making sense of quantitative information, as related to the development of graph comprehension.

## PART I: GRAPHS, COMPREHENSION, AND CRITICAL FACTORS

### Defining Graphs and the Structural Components of Graphs and Tables

Exactly what constitutes a graph has been the subject of various papers (e.g., Bertin, 1980; Doblin, 1980; Fry, 1984; Guthrie, Weber, & Kimmerly, 1993;

Twyman, 1979). Fry's definition of a graph was generic: "*A graph is information transmitted by position of point, line or area on a two-dimensional surface*" (p. 5), including all spatial designs and excluding displays that incorporate the use of symbols such as words and numerals (e.g., tables). Wainer (1992), however, characterized graphs in a way that includes statistical graphs used to convey information in a variety of fields but excludes many other kinds of visualizations authors of earlier work had included. Unlike plans, maps, or geometric drawings that use spatial characteristics (e.g., shape or distance) to represent spatial relations, graphs use spatial characteristics (e.g., height or length) to represent quantity (e.g., the number of cars sold or the cost of living) (Gillan & Lewis, 1994).

William Playfair (late 1700s) has been credited with inventing most of the currently used statistical graphs, including picture graphs, line plots, bar graphs, pie graphs, and histograms. General use of graphs in scientific reporting did not occur until the 19th century (Spence & Lewandowsky, 1990). Tukey (1977) introduced displays that are now considered important in the school curriculum: stem-and-leaf plots (or stem plots) and box-and-whisker plots (or box plots). Several authors (e.g., Fry, 1984; Tukey, 1977) have implicitly or explicitly developed taxonomies of graphs that are similar and have relevance for the school curriculum. For this article, we reviewed only literature related to statistical graphs that dominate the school curriculum, namely, standard graphs and tables (as a type of display linked to graphs) of univariate data and line graphs. On the basis of our consideration of even this limited range of graphs, we were able to lay out critical factors that seem to influence graph comprehension.

Graphs share similar structural components (Kosslyn, 1989, 1994). The *framework* of a graph (e.g., axes, scales, grids, reference markings) gives information about the kinds of measurements being used and the data being measured. The simplest framework has an L shape, with one leg (*x*-axis) standing for the data being measured and the other (*y*-axis) providing information about the measurements being used. Picture graphs, line plots, bar graphs, histograms, and line graphs are examples of those with implicit or explicit L-shaped frameworks. Box plots use a variation of an L-shaped framework. Other graphs such as stem plots and tables have T-shaped frameworks. Still other graphs such as pie graphs have a framework based on polar coordinates (Fry, 1984).

Visual dimensions, called *specifiers*, are used to represent data values. For example, specifiers may be the lines on a line graph, the bars on a bar graph, or other marks that specify particular relations among the data represented within the framework. Graphs also include *labels*. In an L-shaped framework, each leg of the framework has a label naming the type of measurement being made or the data to which the measurement applies. The title of the graph itself may be considered a kind of label. The *background* of a graph includes any coloring, grid, and pictures over which the graph may be superimposed.

Although every graph has these four components, each kind of graph also has its own "language" associated with these structural components; that language may be used to discuss the data displayed. For example, in a line plot, distribution of

Xs (specifiers) across several data values as marked on a horizontal scale (framework) indicates that varied measures are being reported. Interpreting the graph in Figure 1, for example, one might ask, “Do all the boxes of raisins have the same number of raisins?” An explanation might provide evidence of the reader’s knowledge of the structure of the graph: “No, because if they did, all the Xs would be on the same number.”

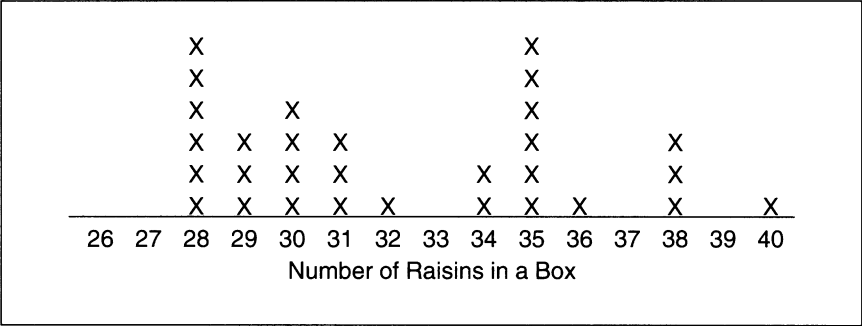


Figure 1. Number of raisins in a half-ounce box.

The structure of tables can be linked to the structure of graphs. Mosenthal and Kirsch (1990a, 1990b) have explored graphs from the perspective of their relation to organized lists or tables, which are types of document structures. “Simple lists are made up of a set of items that share a common feature that can be represented by a label” (1990a, p. 372), for example, using the label *vehicles* to describe a simple list with items: cars, vans, and motor homes. Given the frequency with which the people owning vehicles in each of these categories wash their vehicles (i.e., weekly, biweekly, monthly, or never), one may combine a variety of simple lists and organize them as intersecting lists (e.g., as a table in which the rows represent different vehicles and the columns represent frequencies of washing these vehicles). If the data are recorded in percentages, three pie graphs, one for each vehicle type, showing frequency of washing can be constructed. Similarly, such information can be represented in various bar graphs (and can be reported in forms other than percentages). Because one can move back and forth between such list structures and types of graphs, visually displaying a table of information has inherent advantages.

Tables appear to be used in two ways. One way is as a type of data display. Recommendations made by Ehrenberg (as cited in MacDonald-Ross, 1977) for the design of tables as a display type included several principles, such as rounding numbers to two significant digits to facilitate mental arithmetic and providing row or column averages or both as conceptual reference points. Mosenthal and Kirsch’s

work (1990a, 1990b) highlighted the links between well-structured tables and graphical representations.

Tables may also be used for organizing information as an intermediate step to creating graphical representations. The graph maker may need to organize data in tables (e.g., frequency tables) before graphs can be made. Computer graphics programs and graphing calculators often require data to be entered in tables. To represent these data graphically, one must decide how to set up the table. For example, in some spreadsheet programs, if one's goal is to display data in a bar graph, the data must be entered already organized as frequencies. However, with other software, raw data may be entered in a table without regard for order or frequency; the software organizes the data to reflect the desired graphical representation. Apparently attention to the use of tables as transition tools for organizing information to be represented graphically is needed.

Bright and Friel (1996, 1998) have outlined possible benefits of focusing on particular transitions among graphs (in addition to previously noted transitions between tables and graphs) to promote understanding. One can use these transitions to highlight the structural relationships between graphs. For example, transitions to using bar graphs showing data grouped by frequencies may be made easier for students if instruction includes opportunities to transform a line plot into a bar graph and to highlight similarities and differences between these two representations. Similarly, stem plots and histograms<sup>1</sup> are closely linked, with stem plots providing a natural transitional device for one to use for grouping data into equal-width intervals when constructing a histogram. In fact, by turning a stem plot on its side, one can easily imagine a histogram superimposed on top of the leaves of a stem plot.

Other types of graphs may be related to but are not necessarily developed from one another. Data represented in a circle graph can be displayed in a bar graph, but the reverse is not always the case. Box plots may be related to histograms; both graphs involve the use of scaled intervals to characterize data distributions.<sup>2</sup> Box plots provide information about variation and center but do not give a sense of the overall shape of a distribution. Histograms may provide insights into the overall shape of a distribution if one chooses appropriate intervals for scaling. Line graphs typically reflect functional relationships or time-series data. Time-series data can be presented using bar graphs, but bar graphs, by convention, are not used to convey functional relationships (Follettie, 1980).

---

<sup>1</sup> Generally, histograms, as used in the school curriculum and discussed here, involve the use of equal-width class intervals; the data are shown in blocks (i.e., bars) that highlight frequencies (determined by reference to a vertical scale) of values within each interval. In any interval, the left endpoint is included in the class interval; the right endpoint is excluded. In statistics (Freedman, Pisani, & Purves, 1998), histograms often function as area graphs; the blocks in a graph are drawn so that the area of each block is proportional to the number of data values found in that respective class interval. The areas of the blocks represent percentages; class intervals may not all be the same width. Although a vertical scale is not necessary, it may be drawn as a density scale to show percentages.

<sup>2</sup> In histograms, equal-width intervals are used; the number of data values in an interval varies across intervals. In box plots, variable-width intervals (i.e., quartiles) are used; the number of data values is the same in each interval.

### A Definition of Graph Comprehension

Many researchers have focused on graph comprehension as reading and interpreting graphs. Very few have addressed other possible aspects of graph comprehension, including graph construction or invention or graph choice. In general, comprehension of information in written or symbolic form involves three kinds of behaviors (Jolliffe, 1991; Wood, 1968) that seem to be related to graph comprehension, namely, translation, interpretation, and extrapolation/interpolation. *Translation* requires a change in the form of a communication. To translate between graphs and tables, one could describe the contents of a table of data in words or interpret a graph at a descriptive level, commenting on the specific structure of the graph (Jolliffe, 1991; Wood, 1968). *Interpretation* requires rearranging material and sorting the important from the less important factors (Wood, 1968). To interpret graphs, one can look for relationships among specifiers in a graph or between a specifier and a labeled axis. *Extrapolation* and *interpolation*, considered to be extensions of interpretation, require stating not only the essence of the communication but also identifying some of the consequences. In working with graphs, one could extrapolate or interpolate by noting trends perceived in data or by specifying implications (Wood, 1968).

These three kinds of behaviors seem related to comprehension considered in the context of literacy. In a recent international survey of adult literacy (Murray, Kirsch, & Jenkins, 1997; Organization for Economic Co-operation and Development [OECD], 1995), literacy was equated with the ability to use written information to function in society. Among the three domains of literacy identified (Murray et al., 1997; OECD, 1995)—prose literacy, document literacy, and quantitative literacy—only document literacy includes graphs. *Document literacy* is “the knowledge and skills required to locate and use information contained in various formats, including ... tables, and graphics” (Murray et al., 1997, p. 17). Three major aspects of processing information need to be considered: locating, integrating, and generating information (OECD, 1995). For *locating tasks* (i.e., translation), one finds information based on specific conditions or features. For *integrating tasks* (i.e., interpretation), the reader “pulls together” two or more pieces of information. For *generating tasks* (i.e., extrapolation/interpolation), one must not only process information in the document but also make document-based inferences or draw on personal background knowledge.

Questioning (i.e., question asking and question posing) is an important aspect of comprehension. Researchers have proposed that question-asking is a fundamental component of cognition and plays a central role in the comprehension of text (Graesser, Swamer, Baggett, & Sell, 1996). Low-level questions “address the content and interpretation of explicit material whereas deep questions involve inference, application, synthesis, and evaluation” (p. 23). In comprehending text, readers need to be able to ask questions that help them identify gaps, contradictions, incongruities, anomalies, and ambiguities in their knowledge bases and in the text itself. Teachers need to develop a framework within which to think about

which questions to ask. Such a framework for question-asking is relevant for considering comprehension of graphs.

Several authors (Bertin, 1967/1983; Carswell, 1992; Curcio, 1981a, 1981b, 1987; McKnight, 1990; Wainer, 1992) have characterized the kinds of questions that graphs can be used to answer (see Table 1). Three levels of graph comprehension have emerged: an elementary level focused on extracting data from a graph (i.e., locating, translating); an intermediate level characterized by interpolating and finding relationships in the data as shown on a graph (i.e., integrating, interpreting), and an advanced level that requires extrapolating from the data and analyzing the relationships implicit in a graph (i.e., generating, predicting). At the third level, questions provoke students' understanding of the deep structure of the data presented. We use Curcio's (1981a, 1981b, 1987) terminology when referring to these three levels, that is, read the data, read between the data, and read beyond the data.

We found a somewhat surprising consensus about the need to consider all three types of questions. Such questions can provide cues that activate the process of graph comprehension. Students experience few difficulties with "read the data" questions, but they make errors when they encounter "read between the data" ques-

Table 1  
*Taxonomy of Skills Required for Answering Questions at Each Level*

Author	Question level		
	Elementary (extract information from the data)	Intermediate (find relationships in the data)	Overall (move beyond the data)
Bertin (1983)	Extraction of elementary information (e.g., What was the value of Stock X on June 15?)	Reduction in the number of data categories through combining and compiling data to discover or create fewer categories (e.g., Over the first five days, how did the value of Stock X change?)	Reduction of all the data to a single statement or relationship about the data (e.g., For the period of June 15 to June 30, what was the trend for the value of Stock X?)
Curcio (1987)	(Reading the data) Lifting information from the graph to answer explicit questions for which the obvious answer is in the graph (e.g., How many boxes of raisins have 30 raisins in them?)	(Reading between the data) Interpretation and integration of information that is presented in a graph—the reader completes at least one step of logical or pragmatic inferring to get from the question to the answer (e.g., How many boxes of raisins have more than 34 raisins in them?)	(Reading beyond the data) Extending, predicting, or inferring from the representation to answer questions—the reader gives an answer that requires prior knowledge about a question that is related to the graph (e.g., If students opened one more box of raisins, how many raisins might they expect to find?)

*Table 1 continues*



Table 1 *continued*

Author	Question level		
	Elementary (extract information from the data)	Intermediate (find relationships in the data)	Overall (move beyond the data)
McKnight (1990)	Observing single facts and relationships in graphically presented data or interpreting relationships when responses involve paraphrasing or restating the facts (e.g., "What is the projected food production in 1985 for the developed countries?" [p. 174])	Observing relationships within graphs and interpreting graphs as visual displays without reference to the meaning of graphical elements in context (e.g., "Considering the two curves of the graph only as marks on a piece of paper, how do the changes in these two curves compare?" [p. 175]) Interpreting relationships either by stating that a relationship exists without describing the relationship or by making straightforward statements of relationships (e.g., "Given your understanding of the relationship between changes in the two curves and given your understanding of what each curve represents, what interpretation can be made about the relationship between the two curves?" [p. 176])	Interpreting relationships when responses require making statements that go beyond the statement of relationships to draw inferences or to recast interpretations in more technical terms Determining values of the data conveyed in the graph as evidence to support or reject a proposition (e.g., "If this graph was offered as a piece of evidence to prove true the statement 'Storks bring babies,' how would you describe the connection between the graph and the attempt to prove the statement true?" [p. 178]) Assessing one's own evaluation of evidence provided by quantitative data
Wainer (1992)	Data extraction (e.g., "What was petroleum use in 1980?" [p. 16])	Identification of trends seen in parts of the data (e.g., "Between 1970 and 1985 how has the use of petroleum changed?" [p. 16])	Understanding of the deep structure of the data in their totality, usually through comparing trends and seeing groups (e.g., "Which fuel is predicted to show the most dramatic increase in use?" or "Which fuels show the same pattern of growth?" [p. 16])
Carswell (1992)	Point reading or attention to a single specifier (e.g., "What is the value of [the pie-slice] B?" [p. 541])	Local or global visual comparison of actual graph features and attention to more than a single specifier (e.g., "Is [the pie-slice] D greater than pie-slice C?" or "Is [the pie-slice] A + pie-slice B equal to [the pie-slice] C + pie-slice D?" [p. 541])	Synthesis or integration of most or all the graphed values (e.g., "Is the variability of the data points large?" [p. 541])

tions (e.g., Dossey, Mullis, & Jones, 1993; Pereira-Mendoza & Mellor, 1991; Wainer, 1980; Zawojewski & Heckman, 1997). Such errors may be related to mathematics knowledge, reading/language errors, scale errors, or reading-the-axes errors<sup>3</sup> (see, e.g., Bright & Friel, 1998; Curcio, 1987; McKnight & Fisher, 1991; Pereira-Mendoza & Mellor, 1991). “Read beyond the data” questions seem to be even more challenging. Students must make inferences from the representation in order to interpret the data, for example, to compare and contrast data sets, to make a prediction about an unknown case, to generalize to a population, or to identify a trend. Gal (1998) collapsed the three types of questions to two types: literal-reading questions that involve reading the data or reading between the data and opinion questions that focus on reading beyond the data. He, too, highlighted the challenge of the latter type of question because it requires eliciting and evaluating opinions (rather than facts) about information presented in representations.

By *graph comprehension*, we mean graph readers’ abilities to derive meaning from graphs created by others or by themselves. Different levels of questioning provoke different levels of comprehension. In addition, several critical factors influence graph comprehension: the purposes for using graphs, task characteristics, discipline characteristics, and reader characteristics.

### Critical Factors Influencing Graph Comprehension

#### *Purposes for Using Graphs*

The reasons for using graphs are commonly divided into two classes: analysis and communication (Kosslyn, 1985). Graphs used for data analysis function as discovery tools at the early stages of data analysis when the student is expected to make sense of the data; often alternative plots for the same data set are explored. Graphs used for purposes of analysis at this stage “are predominantly tools for the detection of important or unusual features in the data” (Spence & Lewandowsky, 1990, p. 20). A good pictorial display of data “*forces us to notice what we never expected to see*” (Tukey, 1977, p. vi).

This aspect of graph use appears to be related to the school curriculum. The instructional focus is on students’ construction of various graphs. Traditionally, such instruction has been didactic in nature; prescriptions on ways to create different kinds of graphs are offered with little attention given to the analysis of reasons the graphs were constructed in the first place. As Lehrer and Romberg (1996) noted, textbook examples of graphs often are too preprocessed. More recently, attention to students’ construction of graphs has been addressed within a broader context of statistical investigations that focus on the use of graphs for purposes of making sense of data (e.g., Cobb, 1999).

Very little is known about the relationship between the development of graph comprehension and the practice of creating graphs within the context of statistical

---

<sup>3</sup> See later sections of this article for more detail.

investigations. Most researchers have investigated students' invention or reinvention of graphical representations. Curcio and Folkson (1996; Folkson, 1996) conducted an exploratory study to examine how kindergarten children invent visual displays to communicate data they have collected to answer questions of interest to them. The activities were designed to allow children to formulate their own questions, conduct a survey, collect and organize data, and communicate their findings to their peers in ways that were meaningful to them. Children demonstrated that without formal instruction they were able to represent data in four distinct ways: (a) by writing specific responses repeatedly, (b) by writing numerals to record their counting of data items, (c) by using tally marks, and (d) by writing one number to represent the total number of data items for each category.

DiSessa, Hammer, Sherin, and Kolpakowski (1991), in their work with a sixth-grade class involved in inventing graphing using explorations related to motion, suggested that

one of the difficulties with conventional instruction ... is that students' meta-knowledge is often not engaged, and so they come to know "how to graph" without understanding what graphs are for or why the conventions make sense.... Particular representations may not be at the core of what we should teach so much as the uses they serve, criteria they meet, and resources they build on. (p. 157)

Graph instruction within a context of data analysis may promote a high level of graph comprehension that includes flexible, fluid, and generalizable understanding of graphs and their uses. Note that many researchers who study inventing or reinventing graphs work from the perspective of designing computer environments to analyze data (e.g., Berg & Smith, 1994; diSessa et al., 1991; Hancock, Kaput, & Goldsmith, 1992; Jackson, Edwards, & Berger, 1993; Lehrer & Romberg, 1996; Pratt, 1995).

Graphs used for communication are defined as pictures intended to convey information about numbers and relationships among numbers; "a good graph forces the reader to see the information the designer wanted to convey" (Kosslyn, 1994, p. 271). Such graphs usually contain summary statistics rather than the original data, are simple in form and content, and are intended to display patterns (Spence & Lewandowsky, 1990).

Because graphs are pervasive in our society and are found in such media as magazines, newspapers, and television, individuals must use graphs to make sense of information structured by and communicated from external sources. Within the school curriculum, students encounter graphs from external sources in applied situations related to disciplines such as science and social studies—contexts in which already-designed graphs are presented for purposes of communication. Clearly, students' graph comprehension is often tested (e.g., on standardized tests) with graphs used to communicate.

Much graph-comprehension research focuses on graphs used as tools for communication; this research is reported in the next three sections on critical factors related to characteristics of tasks, discipline, and readers. However, both purposes for graph use identified by Kosslyn (1985) are relevant to school instruction, which seems

to fall short in enabling students to comprehend graphs well enough to respond comfortably and easily to tasks requiring them to read between the data and read beyond the data, that is, to attain high levels of graph comprehension.

### *Characteristics of Tasks*

*Graph perception* “refers to the part played by visual perception in analyzing graphs” (Legge, Gu, & Luebker, 1989, p. 365). To understand perceptual processes, one must identify mental processes that (a) affect early vision and establish a mental representation, (b) operate on the representation to enable one to identify or to make inferences about nonobvious properties, and (c) integrate one’s understanding of context with the mental representation to generate a task-appropriate response (Simkin & Hastie, 1987). In the first point, we address the syntax of graph perception (i.e., visual decoding); in the second point, we acknowledge the importance of operations that involve use of the syntactic properties of graphs (i.e., judgment tasks); and in the third point, we take into account the semantic content of a graph (i.e., context).

*Visually decoding graphs.* Researchers developing theory about graph perception have addressed the most fundamental issue: Which of the many physical dimensions associated with graphs (e.g., line length, circular area, dot position) should be employed to represent data values to facilitate graph use? Initially, such research focused primarily on the visual processing of graphical material, because in an early phase in cognitive processing, one attends to the graph and forms a perceptual image. Context-free graphs (see Figure 2), the type used in most of this early research, generally have unlabeled specifiers; the information presented in the graph cannot be interpreted as data. The graph has no obvious context, and, without labels, units of measure cannot be determined. If labels are used, they often are letters of the alphabet, with frequency axes shown with a numerical scale and no other labels.

There are two important contributions to early understanding of graph perception. Tufte (1983, 1990) emphasized the distinction between data-ink and nondata-ink. Data-ink is the nonerasable core of a graphic, the nonredundant ink arranged in response to variation in the numbers represented. Tufte recommended eliminating all ink that does not convey information. However, this design principle has been shown to lack experimental validation (e.g., Carswell, 1992; Kosslyn, 1994; Spence & Lewandowsky, 1990; Stock & Behrens, 1991). As a general rule, additional ink is considered helpful if it completes a form so that the reader has fewer perceptual units to distinguish (e.g., Kosslyn, 1994).

Cleveland and McGill (1984, 1985; Cleveland, 1985), responsible for formulating one of the first theories of graphical perception, identified 10 elementary graphical-perception tasks that characterize the basic perceptual judgments a person performs to decode visually presented quantitative information encoded on graphs. The 10 tasks are ordered from most accurately judged to least accurately judged on the basis of what is known about the accuracy with which a person performs these tasks. Some tasks are identified as being at the same difficulty level, for example, making

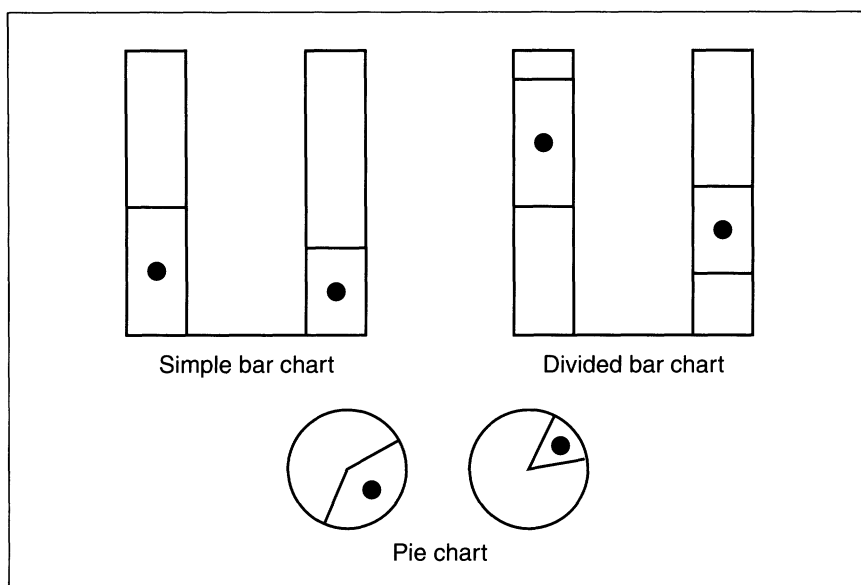


Figure 2. Examples of context-free graphs. *Note.* From "An information-processing analysis of graph perception," by D. Simkin and R. Hastie, 1987, *Journal of the American Statistical Association*, 82, p. 455. Copyright ©1987 by the American Statistical Association. Reprinted with permission.

angle judgments or slope judgments (see Table 2). The task of judging the lengths of bars in a bar graph is considered to be performed more accurately than the task of comparing proportions in a pie graph because the former requires judgments of length or of position on a common scale whereas the latter requires judgments of angle and possibly area. However, the ordering, based on a theory of visual perception, on experiments in graphical perception, and on informal experimentation, applies to the visual decoding of quantitative variables and not of categorical variables.

"The basic principle of data display that arises from ordering the graphical-perception tasks ... is the following: *encode data on a graph so that the visual decoding involves tasks as high in the ordering [in accuracy of judgment] as possible*" (Cleveland, 1985, p. 255). For example, data from a pie graph always can be shown by a bar graph; in reading a bar graph, one can make judgments of position along a common scale instead of having to make less accurate angle judgments.

The taxonomy (see Table 2) has been shown to have several limitations as a stand-alone description of what makes a good graph (e.g., Carswell, 1992; Spence & Lewandowsky, 1990). This taxonomy is limited to a single parameter of graphical-display design, that is, the choice of the physical, predominantly geometric dimensions that are used to convey quantitative information. Cleveland and McGill

Table 2  
*Cleveland and McGill's Taxonomy of Specifiers Ordered  
From Most to Least Accurately Used (Carswell, 1992; Cleveland, 1985)*

Specifier	Representative graphical forms
Position on common aligned scale All data values are referenced using a single scale (e.g., four box plots with a single scale).	Line graphs, bar charts (horizontal and vertical), univariate dot charts and point plots, many types of pictographs, histograms, profiles, bars with decorative depth, stem plots, box plots (last two added by authors)
Position on common nonaligned scales All data values are referenced using more than one scale (e.g., two box plots shown with one scale and two box plots shown with a second, identical scale).	Polygon displays (stars, polar plots) with reference axes, bivariate point plots, scatter plots, statistical maps with framed rectangles
Length Some lengths are easier to compare than others; there is a need for a fixed percentage increase in line length for detection of a difference.	Polygon displays (stars, polar plots) without reference axes, hanging histograms, segmented bar charts, trees, castles, cosmographs
Angle/slope Angle judgments are subject to bias; acute angles are underestimated and obtuse angles are overestimated. Further, angles with horizontal bisectors tend to be seen as larger than those with vertical bisectors. Angles of line segments contaminate judgments of slopes.	Pie charts, disks, meters
Area Volume/density/color saturation	Circles, blobs, some pictographs Cubes, some pictographs, statistical maps with shading (choropleth charts), luminance-coded displays
Color hue	Statistical maps with color coding

emphasized the early stages of perceptual processing of these dimensions, and these stages affect only the initial event of registering a display. Others (Carswell, 1992; Simkin & Hastie, 1987; Spence & Lewandowsky, 1990) have found that the ordering of these visual dimensions may not be as distinct as Cleveland and McGill proposed. Carswell found little difference in accuracy for position, length, or angle judgments, but she found that area and volume judgments were less accurate than judgments of the other dimensions. Kosslyn (1994) suggested that these two categories (i.e., position/length/angle and area/volume) for ordering of judgment difficulty are more appropriate than the five individual categories proposed by Cleveland and McGill.

Supplementary to these theories are more detailed concerns about perception arising from research on how the human mind organizes visual information. Relevant factors, primarily syntactic in nature, include visual principles of salience and orientation sensitivity; processing priorities involving line weight, orientation, length, and so on; and perceptual distortion of such things as area, intensity, and volume (see Kosslyn, 1985, 1994, for additional detail).

*Taxonomy of judgment tasks.* Models of graph comprehension discussed so far have focused primarily on perceptual processing, and graph design is emphasized in these models. Simkin and Hastie (1987) broadened the focus by noting that display design and judgment tasks interact to determine graph-comprehension performance. They found that when presented with a bar chart (type of graph) and asked to provide a summary of the information in the display (judgment task), graph readers spontaneously made comparisons between the absolute lengths of the bars (referred to as *comparison judgments*). With pie graphs, most respondents compared individual slices with the whole (referred to as *proportion judgments*). When making a comparison judgment, research participants were most accurate in decoding position (simple bar graph); thus, Cleveland and McGill's theory was supported. However, when making a proportion judgment, participants were most accurate in decoding angle (pie graphs), a finding that ran counter to Cleveland and McGill's proposed order of accuracy. Simkin and Hastie concluded that the codes originally proposed by Cleveland and McGill interacted with the judgment tasks. Apparently, although visual decoding is a necessary component of graph comprehension, it is not sufficient (Dibble & Shaklee, 1992).

Using the results of research, we have detailed a taxonomy of the kinds of judgment tasks that are most often used when one reads graphs and tables and have described the interaction of the tasks with different kinds of graphs and tables. In the Mixed Arithmetic-Perceptual (MA-P) Model, Gillan and Lewis (1994) found that when people interacted with displays, they did so to answer common questions that required them to complete a number of arithmetic operations. On the basis of Gillan and Lewis's list and several discussions of the nature of judgment tasks (e.g., Dibble & Shaklee, 1992; Feliciano, 1962/1963; Follettie, 1980; Hollands & Spence, 1992; Lohse, 1993; Maichle, 1994; Simkin & Hastie, 1987; Vessey, 1991; Washburne, 1927a, 1927b), we suggest the following taxonomy of judgment tasks:

1. The focus of attention is on one quantity. Tasks require point reading, that is, identifying the value of a single specifier or extracting an absolute point value.
2. The focus of attention is on integrating information across data points. Tasks involve the use of two or more values in the data. The graph reader uses the information to (a) perform computations such as determining the sum of a set of values, a mean among values in the data, or the ratio of two values; (b) make comparisons, either part-to-part comparisons among values or part-to-whole comparisons carried out quantitatively or qualitatively (the reader may identify exact values in order to state a numerical difference, make estimations to determine relative differences, or determine proportions); or (c) identify trends on the basis of qualitative trend information or compare trends qualitatively or quantitatively (to determine trends, one may identify increases, decreases, or fluctuations).

According to Gillan and Lewis (1994), to carry out the judgment tasks, people apply a set of component processes—searching for spatial locations of specifiers, encoding the values of specifiers (e.g., using an axis and associated labels), performing arithmetic operations on the encoded values, making spatial compar-

isons among specifiers (e.g., relative heights or lengths), and responding with an answer. The interaction between the type of display and the task determines the combination and order of these processing steps. Further, “when there is compatibility between the task and the type of display, perception of the judged characteristic is direct, requiring simpler or fewer mental operations” (Hollands & Spence, 1992, p. 315).

Vessey (1991) classified tasks as spatial or symbolic. *Spatial tasks*, those that lead to assessing the problem area as a whole, are facilitated by the use of graphs. *Symbolic tasks*, those that lead to precise data values, are facilitated by the use of tables. She noted that graphs may also include table information; for example, bar graphs may have numerical values at the end of each bar, in which case these graphs include both spatial and symbolic information. Like Hollands and Spence (1992), Vessey pointed to a need for compatibility of display type and task, characterizing this compatibility as *cognitive fit* when the match of display type and task led to use of similar consistent problem-solving processes.

Feliciano (1962/1963) provided evidence of levels of difficulty with respect to judgment tasks that do not appear to be tied to the type of display. For example, she found that locating an absolute value or deriving a total was easier than making a comparison. Lohse (1993) noted that of the three types of tasks (i.e., point reading, making comparisons, and identifying trends), point-reading questions were answered the fastest, followed by trend questions, and finally by comparison questions.

From the field of statistics, Graham (1987) suggested that the use of graphs may be related to one’s purposes for data analysis in a statistical investigation: (a) describing data, (b) summarizing data, (c) comparing and contrasting two or more data sets, or (d) generalizing about a population or predicting the next case. He recommended that line plots, bar graphs, pie graphs, line graphs, stem plots, and histograms be used for the first and third purposes and, with the exception of the pie graph, for the fourth purpose as well. Box plots are most useful for the second, third, and fourth purposes. He also included the use of tables as a representational tool for the first, third, and fourth purposes. Summary statistics related to center, spread, and variation are most appropriate for the second purpose.

*Contextual setting.* In the work reviewed to this point, researchers have used primarily context-free graphs and focused on discriminability of symbols and perceptual processes. Carpenter and Shah (1998; Shah & Carpenter, 1995) were among the first graph-perception researchers we found to use graphs that show data from real-world contexts (e.g., axes are labeled and the graphs titled) in their tasks (see Figure 3). We refer to these graphs as *within-context* graphs. Carpenter and Shah proposed that graph comprehension of line graphs emerged from an integrated sequence of several types of processes: (a) perceptual processes of pattern recognition that encode graphic patterns; (b) perceptual processes that operate on those patterns to retrieve or construct qualitative or quantitative meanings (e.g., judgment tasks); and (c) conceptual processes that translate the visual features into conceptual relations when one interprets titles, labels, and scales as well as any other keys



or symbols that are part of the display. One uses this last set of processes only when completing tasks set within real-world contexts. Researchers must consider the effect of the graph's visual characteristics (i.e., syntax) and the graph's context (i.e., semantics) on one's comprehension.

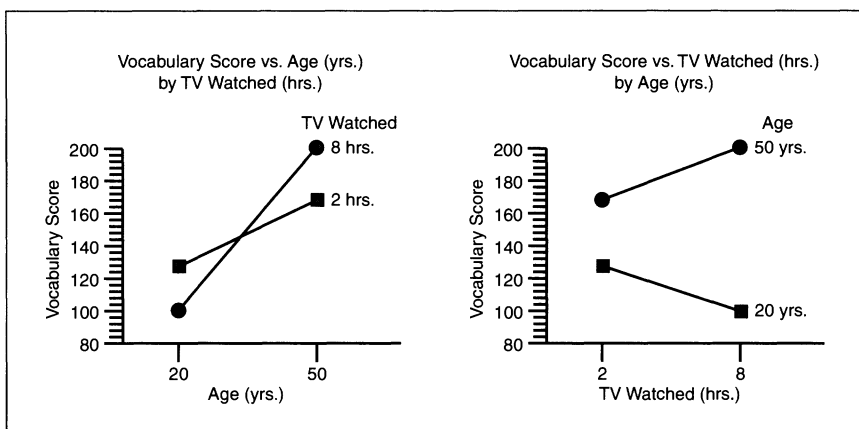


Figure 3. Examples of within-context graphs. *Note.* From “A model of the perceptual and conceptual processes in graph comprehension,” by P. A. Carpenter and P. Shah, 1998, *Journal of Experimental Psychology: Applied*, 4, p. 78. Copyright ©1998 by the American Psychological Association. Reprinted with permission.

A major component of the graph reader's interpretation process is relating graph features to their referents. “Indeed, the majority of the time spent in graph comprehension involves reading and rereading information from the axes and label regions of the graph ... and less time is spent solely on the pattern of lines on the graph” (Shah & Carpenter, 1998, p. 96). Carpenter and Shah's (1998) work focused on processing of line graphs for data sets involving three continuous variables. Tasks on such data sets are not commonly encountered; the data and the resulting graphs reflect complex analyses. Even so, having situated their consideration of graphs within real-world settings illustrates the need for researchers to address the interaction of graphs not only with judgment tasks but also with the contexts in which data are situated. A graph user must sort through the “situation” of the graph within the context of his or her own frame of reference and focus interpretation on what is presented by the data in the graph, regardless of preconceived notions about the situation.

Others have raised the issue of context. Peterson and Schramm (1954), in discussing the generalizability of their results, acknowledged that although “there is no evidence in the literature that subject matter makes any difference in the accu-

racy with which ... graphs are read, ... it is intuitively convincing that subject matter should have some effect on the choice of graphic form" (p. 187). They recommended that the relationship between subject matter and choice of graph form be further investigated.

In some earlier studies, the real-world contexts in which the data were situated were controlled (e.g., Culbertson & Powers, 1959; Feliciano, 1962/1963; Feliciano, Powers, & Kearl, 1963; Washburne, 1927a, 1927b) so that the same contextual situation was used with multiple display types and tasks. However, Follettie (1980) argued that, all things considered, semantic content could not usefully be held constant in investigations that involved several kinds of displays. He noted that even when the given semantic content could be presented using each of several forms, using different displays usually would not do justice to each form if the semantic content were held constant. For example, numerical tables are useful for conveying precise numerical values. Although this information can be conveyed using bar graphs, a bar graph highlights relative magnitudes, which are considered analog information; numerical values can be approximated only through scale interpolation.

More recently, Mooney (1999) defined a construct called *statistical intuition* as "the ability to apply statistical skills within various contexts or situations suited to what is needed" (p. 64). Because data are grounded in real-world contexts, a graph reader must be able to describe, organize, represent, and analyze and interpret data, taking into account the contextual frame of the data. Part of one's statistical intuition is a sense of *reasonableness*, that is, "the use of logic or sensibility in connecting statistical thinking to the context" (p. 127). *Balance* is another component in statistical intuition. The graph reader must balance statistical application and context. For example, if context outweighs statistical application in the student's mind, a student asked to argue for the "best allowance" using data represented on a histogram showing allowances for 30 students, with a cluster of allowances around \$5.50, might ignore the data and argue that \$3.00 makes sense because that is the allowance he received.

McKnight, Kallman, and Fisher (1990) have discussed the nature of graph-reading processing errors. The translations to the "messy" world of everyday reality in which knowledge has links to one's other knowledge as well as to personal beliefs and emotional reactions introduce yet another level of complexity. The graph reader's situational knowledge may interrupt her work on the cognitive, information-processing tasks performed in interpreting the graph. Such situational knowledge is diverse (Janvier, 1981; McKnight et al., 1990). Janvier commented that readers of most graphs used in his study showed "remarkable diversity of personal perceptions and/or conceptions" (p. 120) related to the context provided by the situation. Consideration of the role of context increases the number of elements to which the graph reader must attend and, in effect, possibly provides for a different kind of abstraction that may distract from the original purposes for reading a graph (Janvier, 1981). Clearly, the literature related to situated cognition (e.g., Kirshner & Whitson, 1997) is relevant here, though a comprehensive review is beyond the scope of this article.

### *Characteristics of the Discipline*

Statistics involves the systematic study of data, specifically, collecting data, describing and presenting data, and drawing conclusions from data (Moore, 1991). Associated with this discipline are various tools and concepts, some of which affect graph comprehension. The spread and variation within a data set, the type of data, the size of a data set, and the way a representation provides structure for data (i.e., graph complexity) can influence graph comprehension.

*Spread and variation.* In structuring information, one should consider data reduction and scaling. The transition from tabular and graphical representations that display raw data to those that present grouped data or other aggregate summary representations is called *data reduction*. In this process, one first considers how to reduce data to meaningful summaries (Ehrenberg, 1975). *Scaling* is a tool for data reduction.

When considering graphs with L-shaped frameworks as tools for data reduction, one should note that the axes have different meanings. For example, in bar graphs of ungrouped data, the vertical axis displays the value for each observation whereas the vertical axis for bar graphs of grouped data or for histograms (as considered here) provides the frequency of occurrence of each observation or group of observations. To display ungrouped data from a bar graph as grouped data on a bar graph, one must redefine the *x*- and *y*-axes. In a display of reduced data, the *y*-axis provides information about the frequencies of repeated data values; the frequencies are designated by the heights of the bars rather than by individual plot elements. Readers find distinguishing the two axes problematic (Bright & Friel, 1998). Researchers have considered other graph types in a similar manner, highlighting possible graph-reading difficulties that reflect issues related to data reduction (Friel, 1998; Friel & Bright, 1996).

Both Fry (1984) and Rangecroft (1994) highlighted the use of scale (i.e., the lack of one-to-one correspondence between the data and a square on the chart) as an important component of graph structure. Fry distinguished between the kinds of scaling (e.g., nominal, ratio) and the kinds of scaling units (e.g., arithmetical, percentage, standard score). Rangecroft (1994) noted that often students are able to draw or read a given scale but have little idea how to choose an appropriate scale for a given data set.

Graph scale affects one's reading of the frequency of values. For example, in bar graphs or histograms, the frequency axis will often be scaled to accommodate increased sizes in the counts of data values. Similarly, the idea of scale may be implicit in the axis that provides information about the data. Beeby and Taylor (as cited in MacDonald-Ross, 1977) found that in reading data from line graphs, people persistently misread the scale on the vertical axis; when only alternate lines were numbered (e.g., 0, 2, 4, 6, 8), the unnumbered lines were read as halves (e.g., the line between 6 and 8 was read as 6.5). Dunham and Osborne (1991) found that if students do not attend to scale when they use line graphs for laboratory or statistical data, they may have problems in interpreting asymmetric scales and in

choosing appropriate scales to make good use of the graphing space. Leinhardt, Zaslavsky, and Stein (1990), in their review of research on function graphs, noted that the shape of a graph changes depending on the scale; this change may create a “conceptual demand” (p. 17) that affects the mental image a graph user is able to construct. We believe that these issues related to scale and line graphs are applicable to other graphs as well.

*Data type and size of data set.* One should consider both data type and the size of the data set in determining which graph to use (Landwehr & Watkins, 1986). Picture graphs, line plots, and bar graphs are useful for summarizing data that include repeated measures (Landwehr & Watkins, 1986) and are appropriate for studying nominal, ordinal, and interval data involving counts. In contrast, histograms are used most often to organize continuous data in which there may be few repeated measures (Moore, 1991). Because one can scale both the frequency and the data-value axes, histograms are useful for work with large data sets. One can use pie or circle graphs to efficiently compare percentage or proportion data as they relate to a single characteristic (Mosenthal & Kirsch, 1990b). Stem plots may be used to present large numbers of data values; for comparison of two data sets, back-to-back stem plots are useful. Box plots are useful for highlighting comparisons across two or more data sets (Landwehr & Watkins, 1986), regardless of the size of those data sets.

*Graph complexity.* We found no empirical studies that addressed the relative order of complexity in graphs related to issues that arise from data reduction. Rangecroft (1991a, 1991b, 1994) posited the need for a well-thought-out and detailed treatment of what students need to know to use and understand graphs in various subject areas and across stages or grade levels of schooling.

During the early school years, teachers should promote the fundamental notion that one needs a common baseline when comparing frequencies or measures. The teacher creates a gradual transition from objects themselves to the more abstract bar graph (Rangecroft, 1994). Moritz and Watson (1997) noted that using pictographs is “particularly important to establish links between actual objects and one-to-one [correspondence in the] representation of data, prior to [introducing] more symbolic forms of scaled representation” (p. 222). This sequence applies to the use of line plots.

At the upper levels, the progression of graph work is much less clear-cut. Scaling (a next level of data reduction) emerges as another fundamental notion students must develop if they are to understand bar graphs and other types of graphs (Rangecroft, 1994). Stem plots and histograms have few repeated measures, have a large spread in the data, and necessitate the use of scaling of both frequency and data values for purposes of data reduction. Histograms are more difficult for students to understand conceptually and cause major problems for many pupils (Rangecroft, 1994). Box plots, although not difficult to construct once the concept of median has been developed, offer minimal information about the shape of a distribution and the size of the data set and seem to be relatively abstract (Friel, 1998).

Line graphs may be more difficult to comprehend than other graphs; to realize that the relationship between two variables can be shown by a Cartesian graph is a big step for students (Bell, Brekke, & Swan, 1987).

### *Characteristics of Graph Readers*

Several researchers (e.g., Carpenter & Shah, 1998; Meyer, Shinar, & Leiser, 1997; Peterson & Schramm, 1954) have acknowledged the importance of graph readers' characteristics. For example, Meyer et al. stated, "The relative efficiency of a display may depend partly on the characteristics of the user population" (p. 269). Carpenter and Shah noted that "individual differences in graphic knowledge should play as large a role in the comprehension process as does variation in the properties of the graph itself" (p. 97).

Users differ on a number of variables, only a few of which have been studied in the context of displays (Meyer et al., 1997). Cognitive ability as it relates to Piagetian development has been considered. For example, Berg and Phillips (1994) investigated the relationship between 7th, 9th, and 11th graders' logical-thinking structures and their abilities to construct and interpret line graphs. Results indicated a significant positive relationship among logical thinking, proportional reasoning, and graphing ability. Wavering (1989) indicated that a logical progression from simple to complex reasoning in graphing needs to be developed at the middle and high school levels. Others (Dillashaw & Okey, 1980; Padilla, McKenzie, & Shaw, 1986) have found that to interpret line graphs, one needs abstract-reasoning ability.

Roth and McGinn (1997) suggested an alternative to researchers' studying graphing ability from a cognitive perspective, namely, studying graphing as practice. This perspective "focuses on participation in meaningful practice and experience; lack of competence is then explained in terms of experience and degree of participation rather than exclusively in terms of cognitive ability" (p. 92). They argued that, in school, students make graphs for the purpose of making graphs, whereas, outside schools, people use graphs to achieve certain ends. Like students learning a second language with few opportunities to practice it, students with few opportunities to engage in graphing as practice show less competence than those for whom it is routine.

MacDonald-Ross (1977) raised the idea of a *master performer* (p. 403) as a critical construct, reflecting a difference between a reader familiar with graph format and one who is not. Maichle (1994), in investigating good line-graph readers, found, for example, that they experienced an "orientation phase" to the graph before they responded, in the "verification phase," to specific questions about the graph. Roth (1998) suggested that expertise was complex; for graphs used by scientists in different fields, not only experience but also knowledge of the phenomenon depicted affected graph comprehension.<sup>4</sup> Roth, in his attempts to understand

---

<sup>4</sup> The novice/expert literature is relevant. Space limits prohibit a comprehensive review here.

graph-interpretation practices, moved beyond thinking about the user's *familiarity* with the context in which the data are situated to address the effect of well-developed domains of knowledge or experience that can color such familiarity in ways that are more complex than originally may have been imagined. Researchers need to learn how a graph user sorts through both the situation of a graph and his or her own preconceived notions with regard to this situation to focus on interpreting the data in the graph.

Examining general learner characteristics (e.g., general intelligence) may be important for researchers who are trying to understand how the learner interprets information displayed in graphs. For example, Vernon (1946) noted that levels of education and general intelligence were confounded in her data; she suggested that general intelligence might be the more influential variable. Winn (1991) noted that in the context of dual-coding theory, "the additional support provided by redundant imaginal encoding would be helpful to low-ability students but would not be required by high-ability students" (p. 232). Yet, there is no evidence that measures of general intelligence are effective in explaining differences in interpretations of information presented in graphs.

Mathematics knowledge and experience have been identified as other characteristics necessary for graph comprehension (e.g., Curcio, 1987; Eells, 1926; Fisher, 1992; Gal, 1993, 1998; Maichle, 1994; McKnight & Fisher, 1991; Russell, 1991; Thomas, 1933). Gillan and Lewis (1994), in the MA-P model described earlier, separated processing steps into nonarithmetic and arithmetic components. They suggested that graph users often read graphs for quantitative purposes and perform a variety of arithmetic operations (included earlier as part of the taxonomy of judgment tasks) when reading graphs. They suggested that the time to apply calculation procedures may vary depending on the procedure being used.

Those studying graph comprehension may need to consider the development of number knowledge. Russell (1991) commented that "data analysis activities are closely related to key mathematical ideas involved in the processes of counting, measuring, and classifying" (p. 164) and that younger students need to deal with smaller sets of data and smaller numbers. Curcio (1987) reported that the mathematical content of a graph, that is, the "number concepts, relationships, and fundamental operations contained in it" (p. 383), was a factor in which prior knowledge seemed necessary for graph comprehension. Gal (1993) stated that "many [high school] students have difficulty comprehending basic proportional concepts, such as 'percent' or 'ratio,' and applying them to numerical data presented in *statistical* contexts" (p. 199).

More generally, McKnight and Fisher (1991) found a relationship among mathematics experience, identifying typicality of types of bar graphs, and reading bar graphs. Maichle (1994) found that medical-school applicants whose majors were in mathematics normally attained the highest graph-comprehension scores on the admission test.

## PART II: INSTRUCTIONAL IMPLICATIONS

*Graph comprehension* was defined earlier as the abilities of graph readers to derive meaning from graphs created by others or by themselves. What does this review contribute in the way of instructional implications for developing graph comprehension? We address this question by defining a construct called *graph sense* and associated behaviors that may be used to characterize the nature of graph comprehension that we want to see developed in the school environment. We use what we have learned about critical factors, complemented with our intuition and experiences, to propose a progression for sequencing development of traditional types of graphs for the K–8 grade levels and discuss the importance of emerging graph work carried out in more dynamic environments provided by technology. Finally, we reflect on the use of graphs as tools for making sense of information. What might be the nature of the instructional environment that would support this purpose? We wonder whether it is possible to view data representation from a constructivist perspective, such that teachers seek to let learners struggle with organizing and making sense of the information before introducing formal work with the traditional types of displays that are so commonly used.

### Graph Sense

Graph comprehension involves being able to read and make sense of already constructed graphs such as those often encountered daily in the popular press. It also includes a consideration of what is involved in constructing graphs as tools for structuring data and, more important, what is the optimal choice (e.g., Meyer et al., 1997) for a graph in a given situation. Central to graph comprehension is the interaction among the three task characteristics discussed earlier: the process of visual decoding, the nature of the judgment tasks, and the effect of contextual setting.

We build on earlier work on defining number sense (NCTM, 1989; Sowder, 1992) and symbol sense (Fey, 1990; NCTM, 1989). Number sense and symbol sense can be considered as representing certain ways of thinking rather than as bodies of knowledge that can be transmitted to others. A similar approach seems to be a profitable way to think about graph sense: *Graph sense* develops gradually as a result of one's creating graphs and using already designed graphs in a variety of problem contexts that require making sense of data. Like others who have worked on number sense (Sowder, 1992) and symbol sense (Fey, 1990), we provide a suggested list of behaviors that seem to demonstrate a presence of graph sense. In Table 3, we list both these behaviors and areas of attention attributable to each behavior.

We are uncertain how the purposes for using graphs (i.e., analysis of data and communication) interact to support the development of graph comprehension. We need to think carefully about the kinds of examples that will help us understand their interactions and the ways that the behaviors identified here might play out as demonstrations of graph sense.

Table 3  
Behaviors Associated With Graph Sense

Ability	Focus of attention
1. To recognize the components of graphs, the interrelationships among these components, and the effect of these components on the presentation of information in graphs	Graphs are used to make visible quantitative and categorical information at a variety of levels of detail. Data reduction involves moving from tables and graphs that display raw data to those that present data that are grouped.
2. To speak the language of specific graphs when reasoning about information displayed in graphical form	Through their language related to communicating statistical ideas (Gal, 1993), students build awareness of the structural components of a graph and their interactions with contextual information. Each kind of graph has its own language, that is, the identified structural components and their interrelationships that may be used to discuss the data that are displayed.
3. To understand the relationships among a table, a graph, and the data being analyzed	Graph readers need to be aware of both symbolic and spatial tasks and the ways in which tables and graphs help address these tasks.
4. To respond to different levels of questions associated with graph comprehension or, more generally, to interpret information displayed in graphs	The three levels of questioning involve extracting data from a graph, interpolating and finding relationships in the data as shown on a graph, and extrapolating from the data and interpreting the relationships identified from a graph.
5. To recognize when one graph is more useful than another on the basis of the judgment tasks involved and the kind(s) of data being represented	Making decisions about which graph is most useful for representing a set of data includes consideration of both the nature of the data (Landwehr & Watkins, 1986) and the purposes for analysis (Graham, 1987). Some graph formats are more appropriate for specific types of data and specific purposes than others.
6. To be aware of one's relationship to the context of the graph, with the goal of interpretation to make sense of what is presented by the data in the graph and avoid personalization of the data	Although context may help students use prior knowledge, such prior knowledge also may cause misinterpretations of the information in the graph. Personalization of the context can bring in various interpretations of the goals of a task and a range of strategies, increase the number of elements to which one must attend, and possibly provide for a different kind of abstraction that may distract from the original learning goals (Janvier, 1981). Thus, understanding the constraints imposed by a context is an important factor in making a sensible interpretation (Mooney, 1999).

### Progression of Graphs for Instruction

The critical factors discussed earlier indicate some guidelines for creating a progression for sequencing the development of traditional types of graphs for the K–8 grade levels. Figure 4 is a visual organizer, showing a suggested sequencing of graphs related to grade levels and accompanying themes that need to be considered when addressing this sequence. We propose a number of guidelines; although much of the research reported focused on reading and interpreting graphs, in these



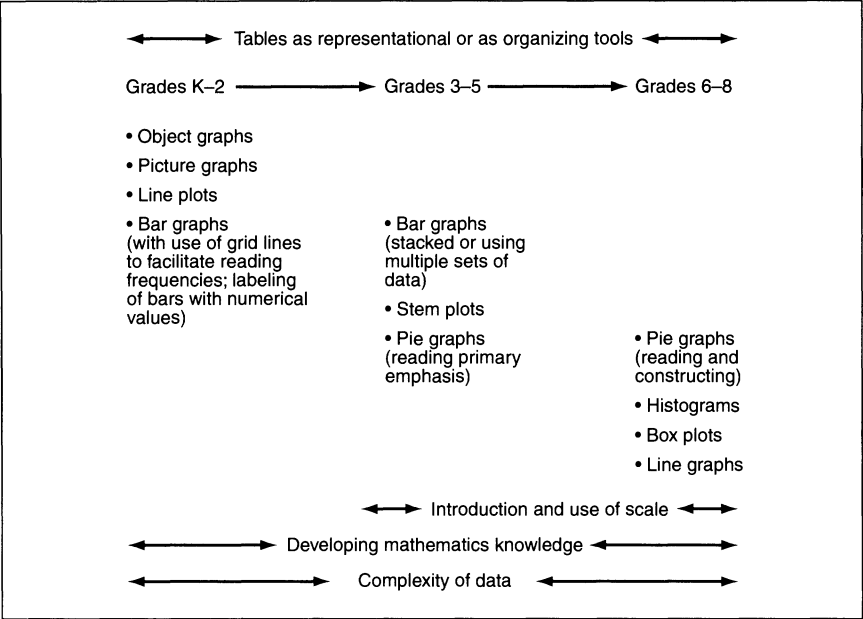


Figure 4. Graph/display complexity: Suggested progression for introduction of types of graphs (includes both reading and constructing displays).

guidelines we address both constructing and reading graphs. We assume that constructions are within context, using either students' own data or data provided (e.g., data sets from Web sites that relate to students' lives) for students to represent and describe.

1. Tables serve as effective tools for data representation and organization. How consistently tables have been incorporated into the work of organizing and describing data is unclear. The two distinctions for use of tables (i.e., as a display type or as an organizational tool) need to be more deliberately considered as part of any increased attention to students' use of tables in exploring data representations.
2. Two other components that need to be considered are children's mathematical knowledge and the complexity of the data being explored. What is known about how children's mathematical knowledge develops is relevant to considering such things as the numbers of data items, the numbers of categories for comparison, the use of additive versus multiplicative reasoning, and so on. The complexity of the data refers not only to the number of data items or categories but also to the kinds of data types (e.g., discrete vs. continuous), the spread and variation within the data set, and so on.
3. At Grades K-2, we would emphasize display types that can function in ways to help students tally responses; these types include simple tables, object graphs,

line plots, and bar graphs. We recommend that students initially use physical objects and then use pictures or materials such as linking cubes to represent numbers of objects before using the more abstract representations provided by line plots or bar graphs. At this abstract level, students first consider frequencies of occurrences of measures. Line plots maintain the evidence of the presence of individual data values; a child can point to his or her *X*. In traditional bar graphs, the individual data values may seem to “disappear,” so that initially shading bars with alternating white and black colors (or the like) to parallel the structure of line plots and labeling the bars with the frequencies are strategies that may help students develop understanding of the concept of frequencies. Often, in the early grades, students make bar graphs in which each bar represents an individual student’s datum; the vertical axis becomes the axis that marks the measure of the data values. This version of a bar graph is often used for data that have considerable variation, such as height data or circumference-of-pumpkins-measured-using-strings data. The move from this kind of bar graph (i.e., having no data reduction) to a bar graph that displays frequencies (i.e., showing simple data reduction with the horizontal axis marking the measures of data values) may be confusing for students if they are not given opportunities to thoughtfully explore the transition.

4. At Grades 3–5, students continue to use graphs that were introduced in the earlier grades. In addition, they may encounter more complex data sets. Bar graphs continue to be used and can include stacked bar graphs and multiple bars to show sets of data displayed within the same graph framework. In addition, if the data sets are large, students may begin to use scaling to label the frequency axis (e.g., a class of students surveying pets finds that the students own a total of 75 dogs, 24 cats, 8 cows, and so on; they wish to represent those data with a bar graph). In creating picture graphs in which one picture represents some number (other than 1) of data items, one is beginning to use notions of scale. Data with considerable spread and variation can be represented using stem plots. Pie graphs can be used; although students at this age generally do not do mathematics related to percentages, they can begin to read pie graphs included in applied contexts.

5. In Grades 6–8, paralleling students’ increased mathematical sophistication (e.g., moving from additive to multiplicative reasoning) and greater abstract-reasoning capabilities (e.g., extrapolating or predicting from data), more complex data sets with greater spread and variation than those seen in earlier grades and with data that are continuous provoke the need for graphs with scaled intervals as well as scaled frequencies. Students will continue to use graphs introduced at earlier grades but will augment their work with use of histograms, box plots, and line graphs. Students may be asked to compare data sets, necessitating the use of the graph types noted.

Such a progression need not be considered “hard and fast.” On the one hand, researchers have shown that stem plots may be introduced to young children (Pereira-Mendoza & Dunkels, 1989). Also, pie graphs may be introduced informally to students well before they are able to carry out the mathematics needed to

make pie graphs (e.g., Joyner, Pfieffer, & Friel, 1997). On the other hand, middle-grades students may experience difficulties working with histograms (Friel & Bright, 1996) and with box plots (Friel, 1998), apparently in part because data reduction results in a “disappearance” of the actual data. Considering relationships among variables, instead of simply constructing and reading line graphs, also reveals increased levels of complexity.

Given the extensive research focused on graph perception, we believe that the perceptual demands related to graph design affect graph comprehension. Because researchers know much about perceptual processing, graph comprehension may be addressed, in part, by controlling perceptual demands by following Kosslyn’s (1994) recommendations with respect to graph design as a current “standard” against which to assess graphs used as part of research, found in the popular press, or created by available technology.

Although several researchers indicated that mathematics knowledge may be related to graph comprehension, we found no analyses in which researchers considered the development of mathematics (and arithmetic) knowledge and specifically related this development to the kinds of displays being used or judgment tasks required for reading displays. For example, young children demonstrate their early understanding of relationships among numbers by directly modeling, using objects; young children may not identify differences between two or among three quantities even when the quantities are modeled with cube towers (similar to bars on a bar graph) that are placed adjacent to one another (Richardson, 1990). These and other aspects of number development seem to have implications for the ways data are explored and represented in the early grades; for example, representing data organized initially in no more than two or three classification categories may make sense.

Such analysis can be extended to expected growth and development of mathematics knowledge for Grades K–12 students, and this analysis has implications for their work with graphs and tables. As one example, the use of scale may depend on the students’ being able to count by 2s, 5s, 10s, and so on, as well as on their thinking about certain numbers as units themselves (e.g., 10 as a unit). Also, for children by ages 12–15, multiplicative reasoning as opposed to additive reasoning emerges as a central mathematical process (Harel & Confrey, 1994). Relative frequencies and percentages are both ratios that require multiplicative reasoning.

In addition to providing guidelines for the introduction and use of traditional graphs, we consider emerging graph work being carried out in dynamic technological environments. This work may lead to new directions in exploring and representing data. For example, Hancock et al. (1992) discussed how in using *Tabletop*<sup>TM</sup> (TERC, 1995), a computer-based data-analysis tool, students have opportunities to explore animated visual representations that include nontraditional representations such as Venn diagrams and variations of traditional representations that focus the user on the structure of the data being explored rather than on the characteristics of the graph being used. In work with microcomputer-based labs (MBL) (e.g., Mokros, 1985, 1986), researchers have shown ways students can understand change-over-time graphs through such dynamic, interactive investi-

gations. Cobb (1999) discussed the use of a new kind of computer-based minitool designed to provide students with a means of ordering, partitioning, and otherwise organizing sets of data points in ways that seem to provoke discussion about comparing data sets through the use of multiplicative reasoning strategies rather than additive reasoning strategies. These data-representation tools focus attention on the underlying logico-mathematical structures related to the structure of data rather than on how to make a particular kind of graph.

### Creating and Adapting Graphs to Make Sense of Data

The goal in using displays is to be able to make sense of information with as much ease as possible. We briefly examined literature that is focused on students' inventing or reinventing representations as tools for use in understanding data, an area we believe needs further study.

How do educators help children become inventors of displays that convey their own messages about the meanings of the data they are using? Are there explicit tasks that provoke such opportunities? Exploring the ways children (in particular), when not limited to standard representations, choose to represent data may be worthwhile. Invented or reinvented representations may better convey explicit understandings about data and the relationship between analyzing data and answering questions that have been posed.

Inventing or reinventing representations often does not require one to create nonstandard displays. Making sense of data is the purpose for inventing or reinventing representations. When students explore representations from the context of data sets, teachers can gain information about their understandings of complex ideas. For example, one of the authors is reminded of an exploration carried out with Grade 8 science students unfamiliar with scatter plots but familiar with the use of bar and line graphs in various problem situations. The students carried out an activity that involved estimating and counting the number of raisins in several half-ounce boxes of a specific brand of raisins. Boxes of a different brand of raisins were also available; one student wondered whether they would find a similar distribution for that brand. In fact, students found two different distributions; this finding motivated them to weigh the raisins in each box for each of the two brands (sample size was 24 boxes of raisins). In the end, they had two sets of 24 data pairs, that is, number of raisins and mass in grams. For homework, their teacher asked them to represent the data in some way that would permit them to *compare* these two brands of raisins. Students devised interesting displays, many of which they recognized as unsuitable when they tried to explain them to the class. One student returned with a scatter plot; whether she invented this representation or was helped by a parent we do not know. However, her classmates found her presentation exciting and certainly understood it more clearly than if it had been introduced through direct instruction without the motivation provoked by need.

The use of technology may serve as a tool to provoke curiosity about various display formats. Technology-rich environments can foster a dynamic process of

data analysis that includes exploration and experimentation with graphs (e.g., Berg & Smith, 1994; diSessa et al., 1991; Hancock et al., 1992; Jackson et al., 1993; Lehrer & Romberg, 1996; Pratt, 1995). Such environments may prove helpful in developing the kind of flexible thinking about the interaction of data and graphs that supports the development of graph comprehension. A “protocol” may be needed for using such tools. Perhaps, educators want users to be able to anticipate how information will be displayed and to think about modifications that may be needed to make the technology do what they know they want done.

Often, when students move to use technology such as computer graphing programs, they demonstrate a lack of understanding of the relationships among the graph, the type of data, the purpose of analysis, and the judgment task. The process of statistical investigation may need to be set within the broader context of problem solving (diSessa et al., 1991; Hancock et al., 1992; Janvier, 1981; Lajoie, Jacobs, & Lavigne, 1996). In this broader context, other complex issues may surface. For example, Hancock et al. noted the dilemmas students face in addressing the differences among explaining the logic of a graph and

(a) using that graph to characterize group trends; (b) constructing the graph to generate, confirm, or disconfirm a hypothesis; (c) connecting the graph with the data structures necessary to produce it; and (d) embedding the graph in the context of a purposeful, convergent project. (pp. 361–362)

To resolve this dilemma, educators should attend to the three task characteristics of visual decoding, judgment task, and context. When students are making graphs to represent their own data, teachers should provide access to Kosslyn’s (1994) guidelines for graph design. By applying the guidelines, students can account for the task characteristic of visual decoding. Students also need to be clear about the questions they are asking about data. Teachers need to be aware of the kinds of judgment tasks that arise from the questions students ask and the ways judgment tasks interact with graph choice. Clearly, when students explore and collect their own data, they may become familiar with the context; however, a teacher needs to help students understand the richness of possible questions to be explored. Similarly, in reading displays found in the popular press, students might be asked to assess the perceptual demands of the displays and to relate graph choices to judgment tasks. By listening to one another’s interpretations, students will better understand the ways prior knowledge affects understanding of data displays.

### Conclusion

Making sense of graphs appears to be more complex than once thought. In this review we have sought to build some connections and coherence among a variety of perspectives. In doing so, clearly we have introduced more questions than conclusions.

Being explicit about the types of difficulties that may be encountered in reading graphs will help educators to interpret students’ thinking. In particular, both clar-

ification of three critical areas of graph perception (i.e., visual decoding, judgment task, and context) and detailing of three levels of questions with the associated depths of understanding necessary in reading a graph are important in graph comprehension.

Although the researchers cited throughout this article are from different disciplines and used different terminology, similarities in their views on understanding graph comprehension exist. We identified three main components of graph comprehension; these components show a progression of attention from local to global features of a graph: (a) To read information directly from a graph, one must understand the conventions of graph design (e.g., Kosslyn, 1994); (b) to manipulate the information read from a graph, one makes comparisons and performs computations; and (c) to generalize, predict, or identify trends, one must relate the information in the graph to the context of the situation. Research on understanding what makes these three main components difficult for graph readers is needed. For example, what is it about the nature of the reasoning and the understanding of necessary information that makes comparing data sets a challenging task (e.g., Friel, 1998; Gal, 1998)?

Although in this article we have emphasized the use of univariate data, study of bivariate data is important for upper-level mathematics curricula. Two major contexts in which bivariate data appear are (a) change over time and (b) cause/effect and correlation. Cause/effect and correlational situations occur in various real-world scenarios. One must, however, interpret correlational data carefully to avoid the common mistake of imposing a cause/effect relationship that may not be present. How the study of graphs might affect ability to distinguish between cause and effect and correlational relationships is unclear.

Many directions may be pursued in the design of research. For example, a learner's understanding of the task seems to affect both the types of information sought and the strategies for seeking that information. How does a learner determine the purpose of a task? How does understanding of that purpose relate to one's interpretation of data represented in a graph? Can a teacher pose questions to increase a learner's understanding of purpose (see Table 1)?

Context is important for graph comprehension, as it is for most learning. How does the learner's understanding of the context contribute to his or her interpretation of data represented in a graph? Can one interpret data accurately without having a significant level of understanding of the context? How do the characteristics of the information (e.g., similarity or difference in magnitudes of data values and frequencies) affect the interpretation?

For instruction on graphs, one needs to consider several elements: for example, sequencing of types of graphs, developing understanding of data reduction, and developing various aspects of graph sense. For students to gain deep knowledge about graphs and to make and use graphs effectively, they need instructional materials that are carefully constructed. What are characteristics of effective instructional material? How is sequencing of graphs in instruction related to the development of graph sense? What are the characteristics of effective questioning and discussion techniques?

To provide effective instruction, teachers need to increase their knowledge of graphs and of how to teach graphs. Because of the recent emphasis on statistics and data analysis, graphs have only recently become an important part of the elementary and middle school mathematics curriculum. Consequently, teachers may not have had adequate opportunities to learn about graphs. More materials (e.g., Friel & Joyner, 1997) need to be developed to fill this gap. But beyond the materials, how should professional-development experiences be structured so that teachers learn not only how to better interpret data presented in graphs but also how to help students develop similar skills?

The design of future research needs to be directed at sorting out causes of difficulties that influence graph comprehension. An agenda for future research should take into account the ideas and issues discussed here.

#### REFERENCES

- Bell, A., Brekke, G., & Swan, M. (1987). Misconceptions, conflict and discussion in the teaching of graphical interpretation. In J. D. Novak (Ed.), *Proceedings of the second international seminar: Misconceptions and educational strategies in science and mathematics* (Vol. 1, pp. 46–58). Ithaca, NY: Cornell University.
- Berg, C., & Phillips, D. G. (1994). An investigation of the relationship between logical thinking structures and the ability to construct and interpret line graphs. *Journal of Research in Science Teaching*, 31, 323–344.
- Berg, C. A., & Smith, P. (1994). Assessing students' abilities to construct and interpret line graphs: Disparities between multiple-choice and free-response instruments. *Science Education*, 78, 527–554.
- Bertin, J. (1980). The basic test of the graph: A matrix theory of graph construction and cartography. In P. A. Kolers, M. E. Wrolstad, & H. Bouma (Eds.), *Processing of visible language* (Vol. 2, pp. 585–604). New York: Plenum Press.
- Bertin, J. (1983). *Semiology of graphics* (2nd ed., W. J. Berg, Trans.). Madison: University of Wisconsin Press. (Original work published 1967)
- Bright, G. W., & Friel, S. N. (1996). Connecting stem plots to histograms. In E. Jakubowski, D. Watkins, & H. Biske (Eds.), *Proceedings of the eighteenth annual meeting, North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 235–240). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Bright, G. W., & Friel, S. N. (1998). Graphical representations: Helping students interpret data. In S. P. Lajoie (Ed.), *Reflections on statistics: Agendas for learning, teaching, and assessment in K–12* (pp. 63–88). Mahwah, NJ: Erlbaum.
- Brinton, W. C. (1914). *Graphic methods for presenting facts*. New York: Engineering Magazine.
- Carpenter, P. A., & Shah, P. (1998). A model of the perceptual and conceptual processes in graph comprehension. *Journal of Experimental Psychology: Applied*, 4, 75–100.
- Carswell, C. M. (1992). Choosing specifiers: An evaluation of the basic tasks model of graphical perception. *Human Factors*, 34, 535–554.
- Cleveland, W. S. (1985). *The elements of graphing data*. Monterey, CA: Wadsworth.
- Cleveland, W. S., & McGill, R. (1984). Graphical perception: Theory, experimentation, and application to the development of graphical methods. *Journal of the American Statistical Association*, 79, 531–554.
- Cleveland, W. S., & McGill, R. (1985). Graphical perception and graphical methods for analyzing scientific data. *Science*, 229, 828–833.
- Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning*, 1, 4–43.
- Culbertson, H. M., & Powers, R. D. (1959). A study of graph comprehension difficulties. *AV Communication Review*, 7, 97–110.

- Curcio, F. R. (1981a). The effect of prior knowledge, reading and mathematics achievement, and sex on comprehending mathematical relationships expressed in graphs (Doctoral dissertation, New York University, 1981). *Dissertation Abstracts International*, 42, 3047A–3048A.
- Curcio, F. R. (1981b). *The effect of prior knowledge, reading and mathematics achievement, and sex on comprehending mathematical relationships expressed in graphs* (Final Report). Brooklyn, NY: St. Francis College. (ERIC Document Reproduction Service No. ED 210 185)
- Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education*, 18, 382–393.
- Curcio, F. R., & Folkson, S. (1996). Exploring data: Kindergarten children do it their way. *Teaching Children Mathematics*, 2, 382–385.
- DeSanctis, G. (1984). Computer graphics as decision aids: Directions for research. *Decision Sciences*, 15, 463–487.
- Dibble, E., & Shaklee, H. (1992, April). *Graph interpretation: A translation problem*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 346 150)
- Dillashaw, F. G., & Okey, J. R. (1980). Test of the integrated science process skills for secondary science students. *Science Education*, 64, 601–608.
- diSessa, A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: Meta-representational expertise in children. *Journal of Mathematical Behavior*, 10, 117–160.
- Doblin, J. (1980). A structure for nontextual communications. In P. A. Kolars, M. E. Wroldstad, & H. Bouma (Eds.), *Processing of visible language* (Vol. 2, pp. 89–111). New York: Plenum Press.
- Dossey, J. A., Mullis, I. V. S., & Jones, C. O. (1993). *Can students do mathematical problem solving?* Washington, DC: U.S. Department of Education.
- Dunham, P. H., & Osborne, A. (1991). Learning how to see: Students' graphing difficulties. *Focus on Learning Problems in Mathematics*, 13(4), 35–49.
- Eells, W. C. (1926). The relative merits of circles and bars for representing component parts. *Journal of the American Statistical Association*, 21, 119–132.
- Ehrenberg, A. S. C. (1975). *Data reduction: Analysing and interpreting statistical data*. New York: Wiley.
- Feliciano, G. D. (1963). The relative effectiveness of seven methods of presenting statistical information to a general audience (Doctoral dissertation, University of Wisconsin, 1962). *Dissertation Abstracts International*, 23, 2515.
- Feliciano, G. D., Powers, R. D., & Kearl, B. E. (1963). The presentation of statistical information. *AV Communication Review*, 11, 32–39.
- Fey, J. T. (1990). Quantity. In L. A. Steen (Ed.), *On the shoulders of giants* (pp. 61–99). Washington, DC: National Academy Press.
- Fisher, M. A. (1992, April). *Categorization, or schema selection in graph comprehension*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco. (ERIC Document Reproduction Service No. ED 347 176)
- Folkson, S. (1996). Meaningful communication among children: Data collection. In P. C. Elliott & M. J. Kenney (Eds.), *Communication in mathematics, K–12 and beyond* (pp. 29–34). Reston, VA: National Council of Teachers of Mathematics.
- Follett, J. F. (1980). *Bar graph-using operations and response time* (Technical Report). Los Alamitos, CA: Southwest Regional Laboratory for Educational Research and Development. (ERIC Document Reproduction Service No. ED 250 381)
- Freedman, D., Pisani, R., & Purves, R. (1998). *Statistics* (3rd ed.). New York: Norton.
- Friel, S. N. (1998). Comparing data sets: How do students interpret information displayed using box plots? In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the twentieth annual meeting, North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 365–370). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Friel, S. N., & Bright, G. W. (1996, April). *Building a theory of graphicacy: How do students read graphs?* Paper presented at the annual meeting of the American Educational Research Association, New York. (ERIC Document Reproduction Service No. ED 395 277)



- Friel, S. N., & Joyner, J. M. (Eds.). (1997). *Teach-stat for teachers: Professional development manual*. Palo Alto, CA: Dale Seymour.
- Fry, E. (1984). *A theory of graphs for reading comprehension and writing communication*. New Brunswick, NJ: Rutgers University. (ERIC Document Reproduction Service No. ED 240 528)
- Gal, I. (1993). Reaching out: Some issues and dilemmas in expanding statistics education. In L. Pereira-Mendoza (Ed.), *Introducing data-analysis in the schools: Who should teach it and how?* (pp. 189–203). Voorburg, The Netherlands: International Statistics Institute.
- Gal, I. (1998). Assessing statistical knowledge as it relates to students' interpretation of data. In S. P. Lajoie (Ed.), *Reflections on statistics: Agendas for learning, teaching, and assessment in K–12* (pp. 275–295). Mahwah, NJ: Erlbaum.
- Gillan, D. J., & Lewis, R. (1994). A componential model of human interaction with graphs: 1. Linear regression modeling. *Human Factors*, 36, 419–440.
- Graesser, A. C., Swamer, S. S., Baggett, W. B., & Sell, M. A. (1996). New models of deep comprehension. In B. K. Britton & A. C. Graesser (Eds.), *Models of understanding text* (pp. 1–32). Mahwah, NJ: Erlbaum.
- Graham, A. (1987). *Statistical investigations in the secondary school*. New York: Cambridge University Press.
- Guthrie, J. T., Weber, S., & Kimmerly, N. (1993). Searching documents: Cognitive processes and deficits in understanding graphs, tables, and illustrations. *Contemporary Educational Psychology*, 18, 186–221.
- Hall, R. O. (1943). *Handbook of tabular presentation: How to design and edit statistical tables*. New York: Ronald Press.
- Hancock, C., Kaput, J. J., & Goldsmith, L. T. (1992). Authentic inquiry with data: Critical barriers to classroom implementation. *Educational Psychologist*, 27, 337–364.
- Harel, G., & Confrey, J. (Eds.). (1994). *The development of multiplicative reasoning in the learning of mathematics*. Albany: State University of New York Press.
- Haskell, A. C. (1922). *Graphic charts in business: How to make and use them*. New York: Codex.
- Hollands, J. G., & Spence, I. (1992). Judgments of change and proportion in graphical perception. *Human Factors*, 34, 313–334.
- Jackson, D. F., Edwards, B. J., & Berger, C. F. (1993). Teaching the design and interpretation of graphs through computer-aided graphical data analysis. *Journal of Research in Science Teaching*, 30, 483–501.
- Janvier, C. (1981). Use of situations in mathematics education. *Educational Studies in Mathematics*, 12, 113–122.
- Jarvenpaa, S., & Dickson, G. W. (1988). Graphics and managerial decision making: Research based guidelines. *Communications of the ACM*, 31, 764–774.
- Jolliffe, F. R. (1991). Assessment of the understanding of statistical concepts. In D. Vere-Jones (Ed.), *Proceedings of the third international conference on teaching statistics* (Vol. 1, pp. 461–466). Voorburg, The Netherlands: International Statistical Institute.
- Joyner, J. M., Pfeiffer, S. M., & Friel, S. N. (1997). *Teach-Stat activities: Statistics investigations for grades 1–3*. Palo Alto, CA: Dale Seymour.
- Kirshner, D., & Whitson, J. A. (Eds.). (1997). *Situated cognition*. Mahwah, NJ: Erlbaum.
- Kosslyn, S. M. (1985). Graphics and human information processing: A review of five books. *Journal of the American Statistical Association*, 80, 499–512.
- Kosslyn, S. M. (1989). Understanding charts and graphs. *Applied Cognitive Psychology*, 3, 185–226.
- Kosslyn, S. M. (1994). *Elements of graph design*. New York: W. H. Freeman.
- Lajoie, S. P., Jacobs, V. R., & Lavigne, N. C. (1996). Empowering children in the use of statistics. *Journal of Mathematical Behavior*, 14, 401–425.
- Landwehr, J. M., & Watkins, A. E. (1986). *Exploring data*. Palo Alto, CA: Dale Seymour.
- Legge, G. E., Gu, Y., & Luebker, A. (1989). Efficiency of graphical perception. *Perception and Psychophysics*, 46, 365–374.
- Lehrer, R., & Romberg, T. (1996). Exploring children's data modeling. *Cognition and Instruction*, 14, 69–108.

- Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60, 1–64.
- Lohse, G. L. (1993). A cognitive model for understanding graphical perception. *Human-Computer Interaction*, 8, 353–388.
- MacDonald-Ross, M. (1977). How numbers are shown: A review of research on the presentation of quantitative data in texts. *AV Communication Review*, 25, 359–409.
- Maichle, U. (1994). Cognitive processes in understanding line graphs. In W. Schnotz & R. W. Kulhavy (Eds.), *Comprehension of graphics* (pp. 207–226). New York: Elsevier Science.
- Malter, M. S. (1952). Studies of the effectiveness of graphic materials. *Journal of Educational Research*, 46, 263–272.
- McCall, W. A. (1939). *Measurement*. New York: Macmillan.
- McKnight, C. C. (1990). Critical evaluation of quantitative arguments. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics* (pp. 169–185). Washington, DC: American Association for the Advancement of Science.
- McKnight, C. C., & Fisher, M. A. (1991, April). *Features of knowledge representation for data presented graphically in populations with diverse mathematical experience*. Paper presented at the annual meeting of the American Educational Research Association, Chicago. (ERIC Document Reproduction Service No. ED 334 227)
- McKnight, C. C., Kallman, C., & Fisher, M. A. (1990, April). *Task analyses of critical evaluations of quantitative arguments: First steps in critical interpretations of graphically presented data*. Paper presented at the annual meeting of the American Educational Research Association, Boston. (ERIC Document Reproduction Service No. ED 319 604)
- Meyer, J., Shinar, D., & Leiser, D. (1997). Multiple factors that determine performance with tables and graphs. *Human Factors*, 39, 268–286.
- Modley, R., & Lowenstein, D. (1952). *Pictographs and graphs: How to make and use them*. New York: Harper.
- Mokros, J. R. (1985). *The impact of microcomputer-based labs* (Technical Report 85-2). Cambridge, MA: Technical Education Research Centers.
- Mokros, J. R. (1986, April). *The impact of MBL on children's use of symbol systems*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco.
- Mooney, E. S. (1999). Development of a middle school statistical thinking framework (Doctoral dissertation, Illinois State University, 1999). *Dissertation Abstracts International*, 60, 1056A.
- Moore, D. S. (1991). *Statistics: Concepts and controversies* (3rd ed.). New York: W. H. Freeman.
- Moritz, J. B., & Watson, J. M. (1997). Pictograph representation: Telling the story. In N. Scott & H. Hollingsworth (Eds.), *Mathematics creating the future* (pp. 222–231). Adelaide: Australian Association of Mathematics Teachers.
- Mosenthal, P. B., & Kirsch, I. S. (1990a). Understanding graphs and charts, Part I. *Journal of Reading*, 33, 371–373.
- Mosenthal, P. B., & Kirsch, I. S. (1990b). Understanding graphs and charts, Part II. *Journal of Reading*, 33, 454–457.
- Murray, T. S., Kirsch, I. S., & Jenkins, L. B. (1997). *Adult literacy in OECD countries: A technical report for the first international adult literacy survey*. Washington, DC: National Center for Education Statistics, U.S. Government Printing Office.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Organization for Economic Co-operation and Development. (1995). *Literacy, economy, and society: Results of the first international adult literacy survey*. Ottawa: Statistics Canada.
- Padilla, M. J., McKenzie, D. L., & Shaw, E. L. (1986). An examination of the line graphing ability of students in grades seven through twelve. *School Science and Mathematics*, 86, 20–26.
- Pereira-Mendoza, L., & Dunkels, A. (1989). Stem-and-leaf plots in the primary grades. *Teaching Statistics*, 13, 34–37.

- Pereira-Mendoza, L., & Mellor, J. (1991). Students' concepts of bar graphs—Some preliminary findings. In D. Vere-Jones (Ed.), *Proceedings of the third international conference on teaching statistics* (Vol. 1, pp. 150–157). Voorburg, The Netherlands: International Statistical Institute.
- Peterson, L. V., & Schramm, W. (1954). How accurately are different kinds of graphs read? *AV Communication Review*, 11, 178–189.
- Pratt, D. (1995). Young children's active and passive graphing. *Journal of Computer Assisted Learning*, 11, 157–169.
- Rangecroft, M. (1991a). Graphwork—Developing a progression: Part 1—The early stages. *Teaching Statistics*, 13, 44–46.
- Rangecroft, M. (1991b). Graphwork—Developing a progression: Part 2—A diversity of graphs. *Teaching Statistics*, 13, 90–92.
- Rangecroft, M. (1994). Graphwork—Developing a progression. In D. Green (Ed.), *The best of teaching statistics* (pp. 7–12). Sheffield, England: The Teaching Statistics Trust.
- Richardson, K. (1990). *A look at children's thinking: Study guide*. Norman, OK: Educational Enrichment.
- Roth, W. -M. (1998). Unspecified things, signs, and 'natural objects': Towards a phenomenological hermeneutic of graphing. In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the twentieth annual meeting, North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 291–297). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Roth, W. -M., & McGinn, M. (1997). Graphing: Cognitive ability or practice? *Science Education*, 81, 91–106.
- Russell, S. J. (1991). Counting noses and scary things: Children construct their ideas about data. In D. Vere-Jones (Ed.), *Proceedings of the third international conference on teaching statistics* (Vol. 1, pp. 158–164). Voorburg, The Netherlands: International Statistical Institute.
- Schmid, C. F., & Schmid, S. E. (1979). *Handbook of graphic presentation* (2nd ed.). New York: Wiley.
- Shah, P., & Carpenter, P. A. (1995). Conceptual limitations in comprehending line graphs. *Journal of Experimental Psychology: General*, 124, 43–61.
- Shaughnessy, J. M., Garfield, J., & Greer, B. (1996). Data handling. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 205–237). Boston, MA: Kluwer.
- Simkin, D., & Hastie, R. (1987). An information-processing analysis of graph perception. *Journal of the American Statistical Association*, 82, 454–465.
- Sowder, J. T. (1992). Making sense of numbers in school mathematics. In G. Leinhardt, R. Putnam, & R. A. Hattrop (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 1–51). Hillsdale, NJ: Erlbaum.
- Spence, I., & Lewandowsky, S. (1990). Graphical perception. In J. Fox & J. S. Long (Eds.), *Modern methods of data analysis* (pp. 13–57). Newbury Park, CA: Sage.
- Stock, W. A., & Behrens, J. T. (1991). Box, line, and midgap displays: The effects of display characteristics on the accuracy and bias of estimates of whisker length. *Journal of Educational Statistics*, 16, 1–20.
- TERC. (1995). *Tabletop™* [Computer software]. Novato, CA: Brøderbund.
- Thomas, K. C. (1933). The ability of children to interpret graphs. In G. M. Whipple (Ed.), *The teaching of geography*. 32nd Yearbook of the National Society for the Study of Education (pp. 492–494). Bloomington, IL: Public School.
- Tufte, E. R. (1983). *The visual display of quantitative information*. Cheshire, CT: Graphics Press.
- Tufte, E. R. (1990). *Envisioning information*. Cheshire, CT: Graphics Press.
- Tukey, J. W. (1977). *Exploratory data analysis*. Reading, MA: Addison-Wesley.
- Twyman, M. (1979). A schema for the study of graphic language. In P. A. Kolers, M. E. Wrolstad, & H. Bouma (Eds.), *Processing of visible language* (Vol. 1, pp. 117–150). New York: Plenum Press.
- Vernon, M. D. (1946). Learning from graphical material. *British Journal of Psychology*, 36, 145–158.
- Vessey, I. (1991). Cognitive fit: A theory-based analysis of the graphs versus tables literature. *Decision Sciences*, 22, 219–240.

- Wainer, H. (1980). A test of graphicacy in children. *Applied Psychological Measurement*, 4, 331–340.
- Wainer, H. (1992). Understanding graphs and tables. *Educational Researcher*, 21(1), 14–23.
- Walker, H. M., & Durost, W. N. (1936). *Statistical tables: Their structure and use*. New York: Bureau of Publications, Teachers College.
- Washburne, J. N. (1927a). An experimental study of various graphic, tabular and textual methods of presenting quantitative material, Part 1. *Journal of Educational Psychology*, 28, 361–376.
- Washburne, J. N. (1927b). An experimental study of various graphic, tabular and textual methods of presenting quantitative material, Part 2. *Journal of Educational Psychology*, 28, 465–476.
- Wavering, M. J. (1989). Logical reasoning necessary to make line graphs. *Journal of Research in Science Teaching*, 26, 373–379.
- Winn, W. (1991). Learning from maps and diagrams. *Educational Psychology Review*, 3, 211–247.
- Wood, R. (1968). Objectives in the teaching of mathematics. *Educational Research*, 10, 83–98.
- Zawojewski, J. S., & Heckman, D. J. (1997). What do students know about data analysis, statistics, and probability? In P. A. Kenney & E. A. Silver (Eds.), *Results from the sixth mathematics assessment of the National Assessment of Educational Progress* (pp. 195–223). Reston, VA: National Council of Teachers of Mathematics.

## Authors

---

**Susan N. Friel**, Associate Professor, Education, University of North Carolina at Chapel Hill, School of Education, CB #3500 Peabody Hall, Chapel Hill, NC 27599-3500; sfriel@email.unc.edu

**Frances R. Curcio**, Professor, Mathematics Education, Queens College of the City University of New York, Department of Secondary Education and Youth Services, 65-30 Kissena Boulevard, Flushing, NY 11367; Frances\_Curcio@qc.edu

**George W. Bright**, Professor, Department of Curriculum and Instruction, University of North Carolina at Greensboro, School of Education, Curry Building, Greensboro, NC 27412-5001; gbright@uncg.edu