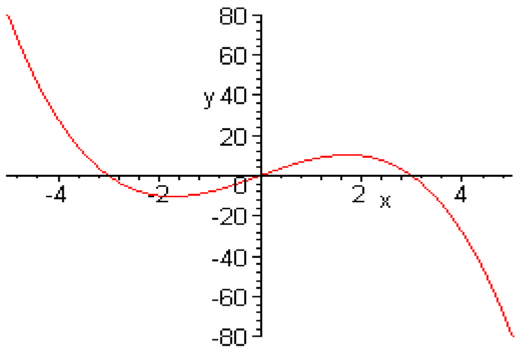
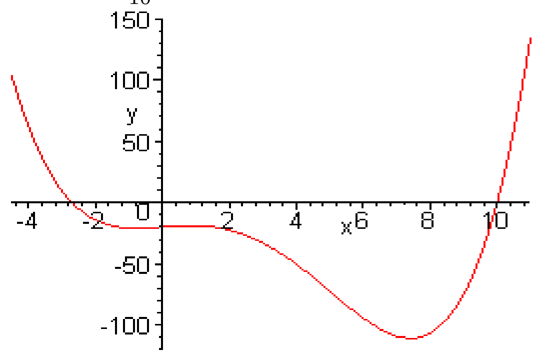
Graphs of Polynomial Functions

A polynomial function has the general form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, where n is a nonnegative integer and the "coefficients" $a_n,\ a_{n-1},\ \ldots,\ a_0$ are real numbers. We'll always assume that polynomials are written in descending powers of the variable. Here are some graphs of typical polynomials:

$y = -x^3 + 9x$



$y = \frac{1}{10}x^4 - x^3 + 2x - 20$



With these as examples, you should familiarize yourself with the following features of graphs of polynomials:

1. The graph of a polynomial function is a continuous, smooth curve.
2. The graph has two "arms" with y approaching $+\infty$ or $-\infty$ as x approaches $+\infty$ or $-\infty$.
3. The graph rises to the right if the leading coefficient a_n is positive, and falls to the right if a_n is negative.
4. The left arm goes in the same direction as the right arm if the degree n of the polynomial is even, and in the opposite direction if the degree n is odd.
5. The graph has no more than $(n-1)$ "peaks" and "valleys"; it may have fewer than this.
6. By writing the polynomial in factored form (when this is easy to do), the x-intercepts of the graph may be identified as those real values of x for which a factor is 0.
7. If a factor $(x-a)$ appears to an odd power, the graph crosses the x-axis at that point, but if the factor appears to an even power, the graph just touches the x-axis at that point and then curves back in the opposite direction. (see Figure 1 below)
8. If all powers of x in the polynomial are even, then $f(x)$ is an "even" function in the sense described in class, and the graph is symmetric with respect to the y-axis (see Fig. 2). Similarly, if only odd powers of x appear, then the function is an "odd" function and its graph is symmetric with respect to the origin (see Fig.3).

Fig.1 $y = (x-1)(x+2)^2$

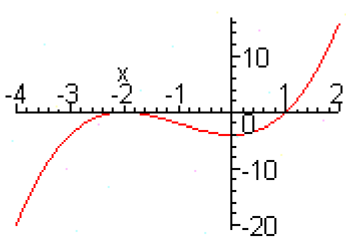


Fig.2. $y = x^4 - 3x^2 - 1$

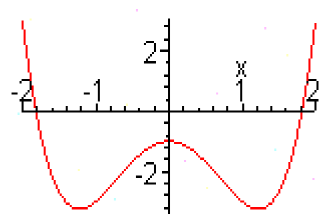


Fig.3 $y = x^5 - 5x^3 + 10x$