

MAT103: Fundamentals of Mathematics I
Using Venn Diagrams to Solve Survey-Type Problems

Consider a situation in which a poll of a group of college students was conducted. From such polls, results are reported to the public in many ways. One of the most descriptive ways is to list numbers of people surveyed in all combinations of positive responses as discussed below. The challenge is interpreting results in a mathematically accurate fashion. Venn diagrams can often be used to represent such situations, and from those illustrations, interpreting the results becomes relatively simple, although some basic arithmetic is often necessary to determine desired results.

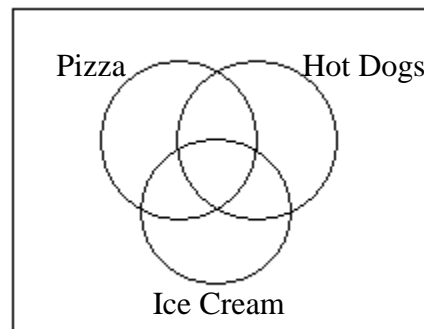
Background: A poll of 360 students yielded the following results:

- 190 enjoy eating pizza
- 170 enjoy eating hot dogs
- 210 enjoy eating ice cream
- 90 enjoy eating both pizza and hot dogs
- 110 enjoy eating both pizza and ice cream
- 60 enjoy eating both hot dogs and ice cream
- 40 enjoy eating all three foods

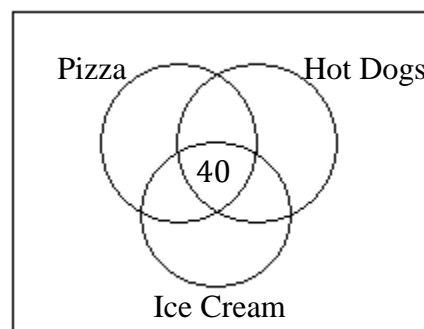
- Questions:
- 1) How many students enjoy eating only pizza?
 - 2) How many students do not enjoy eating any of the three food items discussed?
 - 3) How many students enjoy eating hot dogs?
 - 4) How many students enjoy eating pizza but not ice cream?
 - 5) How many students enjoy eating hot dogs or ice cream?
 - 6) How many students enjoy eating pizza and hot dogs?
 - 7) How many students enjoy eating exactly one of the three food items discussed?
 - 8) How many students enjoy eating exactly two of the three food items discussed?
 - 9) How many students enjoy eating pizza or hot dogs but not ice cream?
 - 10) How many students enjoy eating at least two of the three food items discussed?
 - 11) How many students do not enjoy eating ice cream?
 - 12) How many students do not enjoy eating hot dogs or pizza?

Solution:

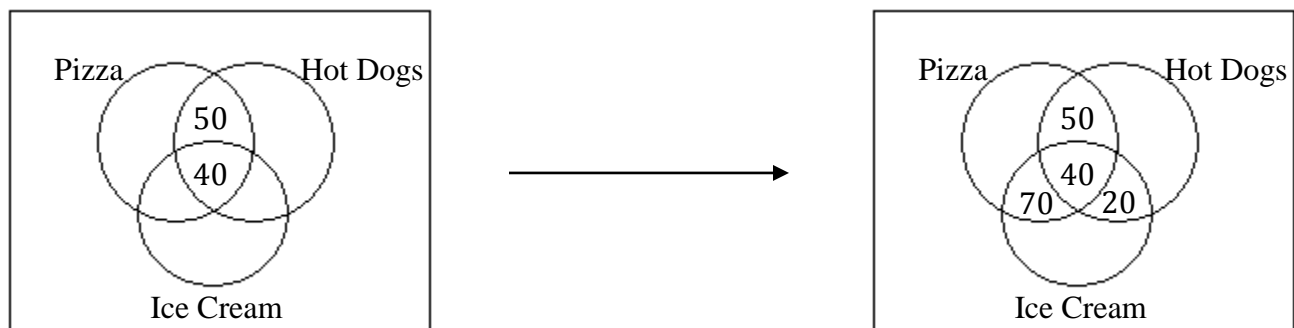
Constructing a Venn diagram is a viable and appropriate first step to answering the questions above. Begin with a Venn diagram consisting of one circle to represent each of the food items discussed.



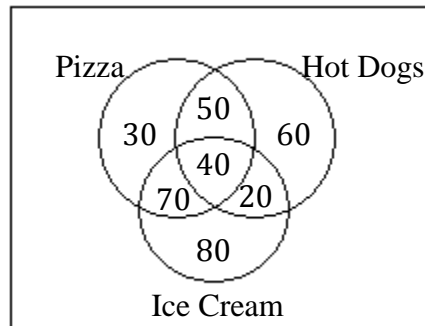
After that initial setup, begin filling in the regions of the Venn diagram from the innermost region outward (the area showing the intersection of the greatest number of sets first). In this case, begin with the number of students enjoying all three food items.



Next, move outward to any regions showing the overlap of as many remaining sets as possible (in this case, the intersection of two sets). Note that the value in the center of the diagram counts as part of the total for each of the next regions, so that value must be subtracted from the total for each region to obtain the value for the missing piece.

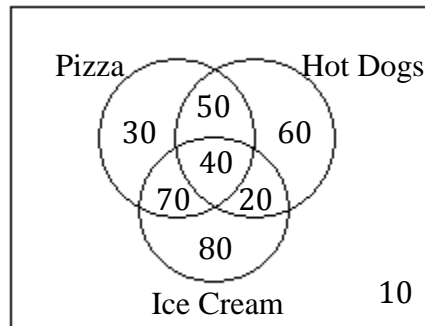


Then, move outward one more level to fill in the remaining regions in the Venn diagram. Keep in mind that it is now necessary to subtract multiple values from the reported total for a region illustrating the intersection of multiple sets to obtain the quantity to be stated in the area being calculated.



This process can be repeated as many times as necessary to accommodate Venn diagrams representing larger numbers of sets. Regardless of the number of iterations of this process, at some point it will be reduced to one remaining section of the circle representing each set to be completed.

The final step in the process is to determine what quantity must be calculated to represent the area of the Venn diagram outside all sets. Again, a subtraction process is required. The total of all values inside all regions of all sets in the Venn diagram is subtracted from the total number of objects in the original sample, and the difference obtained is the value represented by the region inside the Venn diagram but outside all sets.



Solutions to Questions:

- 1) This is represented by the region inside the Pizza circle but outside all other circles.
Answer: $\boxed{30}$
- 2) This is represented by the region inside the Venn diagram but outside all circles.
Answer: $\boxed{10}$
- 3) This is represented by the sum of all regions inside the Hot Dogs circle.
Answer: $50 + 60 + 40 + 20 = \boxed{170}$
- 4) This is represented by the sum of all regions inside the Pizza circle but outside the Ice Cream circle.
Answer: $30 + 50 = \boxed{80}$
- 5) This is represented by the sum of all regions inside the Hot Dogs circle, the Ice Cream circle, or both circles simultaneously.
Answer: $50 + 60 + 70 + 40 + 20 + 80 = \boxed{320}$
- 6) This is represented by the sum of all regions inside both the Pizza and Hot Dogs circles simultaneously.
Answer: $50 + 40 = \boxed{90}$
- 7) This is represented by the sum of all regions inside exactly one circle.
Answer: $30 + 60 + 80 = \boxed{170}$
- 8) This is represented by the sum of all regions inside exactly two circles.
Answer: $50 + 70 + 20 = \boxed{140}$
- 9) This is represented by the sum of all regions inside the Pizza circle, the Hot Dogs circle, or both circles simultaneously, but outside the Ice Cream circle.
Answer: $30 + 50 + 60 = \boxed{140}$
- 10) This is represented by the sum of all regions inside more than one circle.
Answer: $50 + 70 + 40 + 20 = \boxed{180}$
- 11) This is represented by the sum of all regions outside the Ice Cream circle.
Answer: $30 + 50 + 60 + 10 = \boxed{150}$
- 12) This is represented by the sum of all regions outside both the Hot Dogs circle and the Pizza circle.
Answer: $80 + 10 = \boxed{90}$