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MODIFIED CRITICAL PATH METHOD TO SOLVE NETWORKING PROBLEMS UNDER AN INTUITIONISTIC FUZZY ENVIRONMENT

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ABSTRACT

In this paper, we propose a new method to solve networking problems under an intuitionistic fuzzy environment. We use triangular Intuitionistic fuzzy numbers to represent activity duration in the project network. We obtain the intuitionistic fuzzy critical path for the project network using a new type of arithmetic operations and a ranking function on triangular intuitionistic fuzzy numbers. Numerical example is provided to show the efficiency of the proposed algorithm.

Keywords: triangular intuitionistic fuzzy number, left fuzziness index function, right fuzziness index function, intuitionistic fuzzy project network, ranking method.

1. INTRODUCTION

A project is combination of interrelated activities which must be executed in a certain order before the entire task can be completed. An activity in a project is usually viewed as a job requiring time and resources for its completion. The Critical Path Method (CPM) plays a vital role in planning, scheduling and controlling the complex projects consisting of number of work contents. The successful implementation of CPM requires the availability of clear determined parameters (time duration, cost etc) for each activity. But in reality, due to uncertainty of information as well as the variation of management scenario, it is often difficult to obtain the exact activity time estimates. Under such situations it is highly impossible to formulate the mathematical model through the classical traditional methods. Therefore the fuzzy set theory proposed by Zadeh [7] can play a significant role in this kind of problems to handle the ambiguity about the time duration of deeds in a project network. Atanassov [5], [6] extended the fuzzy sets to the theory intuitionistic fuzzy sets in which both the degree of belonging and degree of non-belonging are considered. Several authors such as Takahashi *et al* [8], Sophia Porchelvi and Sudha [14] to [16]**Error! Reference source not found.**, Kiran Yadav *et al* [1] and Nagoor Gani *et al* [9] have studied fuzzy Critical Path Method in Intuitionistic Fuzzy Environment. Sophia Porchelvi and Sudha have studied Intuitionistic Fuzzy Critical Path in a Network using a new ranking method. De. P.K and Amita Bhinchar [12], [13] discussed fuzzy critical path analysis by a ranking method. Jayagowri and Geetharamani [11] obtained intuitionistic fuzzy critical path by using metric distance ranking method. Elizabeth and Sujatha [18] presented two different algorithms to obtain the critical path in a fuzzy network problem involving triangular intuitionistic fuzzy numbers and triangular fuzzy numbers.

In this paper, by using a new type of arithmetic operations and a new ranking method on triangular intuitionistic fuzzy numbers, we propose a new method for the solution of the fuzzy networking problems with triangular intuitionistic fuzzy numbers. The proposed method does not require the computation of floats of each activity to identify the critical path. A numerical example is provided to show the efficiency of the proposed method.

The rest of the paper is organised as follows: In section 2, basic definitions and results of intuitionistic fuzzy set theory have been reviewed. In sections 3, algorithm for the proposed method is presented for the computation of the critical path, when the activity durations are taken as triangular intuitionistic fuzzy numbers. Numerical example is provided to illustrate the efficiency of the proposed method in section 4.

2. PRELIMINARIES

Definition 2.1. Let X be a universe of discourse. An Intuitionistic Fuzzy Set (IFS) $\tilde{a}^I$ in X is given by $\tilde{a}^I = \left\{ (x, \mu_{\tilde{a}^I}(x), \gamma_{\tilde{a}^I}(x)) \, / \, x \in X \right\}$, where the functions $\mu_{\tilde{a}^I} : X \to [0,1]$ and $\gamma_{\tilde{a}^I} : X \to [0,1]$ determine the degree of membership and degree of non membership of the element $x \in X$, respectively, and for every $x \in X$, $0 \le \mu_{\tilde{a}^I}(x) + \gamma_{\tilde{a}^I}(x) \le 1$.

Note 2.1. Throughout this paper $\mu_{\tilde{a}^I}(x)$ represents membership values and $\gamma_{\tilde{a}^I}(x)$ represents non membership value of $x \in X$.

Definition 2.2. For every common fuzzy subset $\tilde{a}^I$ on X, Intuitionistic Fuzzy Index of X in $\tilde{a}^I$ is defined as $\pi_{\tilde{a}^I}(x) = 1 - \mu_{\tilde{a}^I}(x) - \gamma_{\tilde{a}^I}(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element X in $\tilde{a}^I$. Obviously, for every $x \in X$, $0 \le \pi_{\tilde{a}^I}(x) \le 1$.

Definition 2.3. An Intuitionistic Fuzzy Number (IFN) $\tilde{a}^I$ is

(i). an Intuitionistic fuzzy subset of the real line,

(ii). normal, that is there is any $x_0 \in R$, such that $\mu_{\tilde{a}^I}(x_0) = 1$, $\gamma_{\tilde{a}^I}(x_0) = 0$.

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(iii). Convex for the membership function $\mu_{\tilde{a}^I}(x)$, i.e. $\mu_{\tilde{a}^I}(\lambda x_1+(1-\lambda)x_2)\geq \min(\mu_{\tilde{a}^I}(x_1),\mu_{\tilde{a}^I}(x_2))$ for every $x_1,x_2\in R$, $\lambda\in[0,1]$.

(iv). concave for the non-membership function $\gamma_{\tilde{a}^I}(x)$, i.e. $\gamma_{\tilde{a}^I}(\lambda x_1+(1-\lambda)x_2)\leq \max(\gamma_{\tilde{a}^I}(x_1),\gamma_{\tilde{a}^I}(x_2))$ for every $x_1,x_2\in R$, $\lambda\in[0,1]$.

Definition 2.4. An Intuitionistic fuzzy number $\tilde{a}^I$ is a Triangular Intuitionistic Fuzzy Number (TIFN) with parameters $a'_1\leq a_1\leq a_2\leq a_3\leq a'_3$ and denoted by $\tilde{a}^I=(\langle a_1,a_2,a_3\rangle;\langle a'_1,a_2,a'_3\rangle)$ having the membership function and non-membership function as follows:

$$\mu_{\tilde{a}^I}(x)=\begin{cases}0 & \text{for } x<a_1\\ \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1\leq x\leq a_2\\ 1 & \text{for } x=a_2\\ \dfrac{a_3-x}{a_3-a_2} & \text{for } a_2\leq x\leq a_3\\ 0 & \text{for } x>a_3\end{cases}\quad \text{and}$$

$$\gamma_{\tilde{a}^I}(x)=\begin{cases}1 & \text{for } x<a'_1\\ \dfrac{a_2-x}{a_2-a'_1} & \text{for } a'_1\leq x\leq a_2\\ 0 & \text{for } x=a_2\\ \dfrac{x-a_2}{a'_3-a_2} & \text{for } a_2\leq x\leq a'_3\\ 1 & \text{for } x>a'_3\end{cases}$$

We use $F(R)$ to denote the set of all triangular intuitionistic fuzzy numbers. Also if $m=a_2$ represents the modal value (or) midpoint, $\alpha_1=(a_2-a_1)$ represents the left spread and $\beta_1=(a_3-a_2)$ right spread of membership function and $\alpha_2=(a_2-a'_1)$ represents the left spread and $\beta_2=(a'_3-a_2)$ right spread of non- membership function.

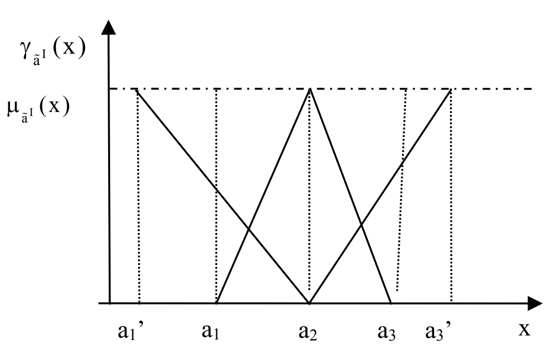

Figure-1. Membership and non-membership functions of a Triangular intuitionistic fuzzy number.

Note 2.2. Here $\mu_{\tilde{a}^I}(x)$ increases with constant rate for $x\in[a_1,a_2]$ and decreases with constant rate for $x\in[a_2,a_3]$, but $\gamma_{\tilde{a}^I}(x)$ decreases with constant rate for $x\in[a'_1,a_2]$ and increases with constant rate for $x\in[a_2,a'_3]$.

Particular Case: Let $\tilde{a}^I=(\langle a_1,a_2,a_3\rangle;\langle a'_1,a_2,a'_3\rangle)$ be a triangular intuitionistic fuzzy number then the following cases arises

Case1: If $a'_1=a_1$, $a'_3=a_3$, then $\tilde{a}^I$ represent Triangular Fuzzy Number (TFN).

Case2: If $a'_1=a_1=a'_3=a_3=m$, then $\tilde{a}^I$ represent a real number m.

Definition 2.5. A triangular intuitionistic fuzzy number $\tilde{a}^I\in F(R)$ can also be represented as a pair $\tilde{a}^I=(\underline{a},\overline{a};\underline{a}',\overline{a}')$ of functions $\underline{a}(r)$, $\overline{a}(r)$, $\underline{a}'(r)$ and $\overline{a}'(r)$ for $0\leq r\leq 1$ which satisfies the following requirements:

(i). $\underline{a}(r)$ is a bounded monotonic increasing left continuous function for membership function
(ii). $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function for membership function.
(iii). $\underline{a}(r)\leq\overline{a}(r)$, $0\leq r\leq 1$
(iv). $\underline{a}'(r)$ is a bounded monotonic decreasing left continuous function for non-membership function.
(v). $\overline{a}'(r)$ is a bounded monotonic increasing left continuous function for non-membership function.
(vi). $\underline{a}'(r)\leq\overline{a}'(r)$, $0\leq r\leq 1$.

Definition 2.6. For an arbitrary triangular intuitionistic fuzzy number $\tilde{a}^I=(\underline{a},\overline{a};\underline{a}',\overline{a}')$, the number $a_0=\left(\dfrac{\underline{a}(1)+\overline{a}(1)}{2}\right)$ or $a_0=\left(\dfrac{\underline{a}'(1)+\overline{a}'(1)}{2}\right)$ is said to be a location index number of membership and non membership functions. The non-decreasing left continuous functions $a_*=(a_0-\underline{a})$, $a^*=(\overline{a}-a_0)$ are called the left fuzziness index function and the right fuzziness index function for membership function and the non-decreasing left continuous functions $a'_*=(a_0-\underline{a}')$, $a'^*=(\overline{a}'-a_0)$ are called the left fuzziness index function and the right fuzziness index function for non-membership function respectively. Hence every Triangular Intuitionistic Fuzzy Number $\tilde{a}^I=(\langle a_1,a_2,a_3\rangle;\langle a'_1,a_2,a'_3\rangle)$ can also be represented by $\tilde{a}^I=(\langle a_0,a_*,a^*\rangle;\langle a_0,a'_*,a'^*\rangle)$.

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2.1 Arithmetic operations on triangular intuitionistic fuzzy numbers

Prabakaran and Ganesan[4] have proposed a new fuzzy arithmetic on triangular intuitionistic fuzzy numbers based upon both location index and fuzziness index functions for membership and non-membership functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b∈L we define $a \vee b = \max\{a,b\}$ and $a \wedge b = \min\{a,b\}$.

In particular for any two triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(\langle a_0, a_*, a^* \rangle; \langle a_0, a'_*, a'^* \rangle\right)$ and $\tilde{b}^I = \left(\langle b_0, b_*, b^* \rangle; \langle b_0, b'_*, b'^* \rangle\right)$, we define

(i) Addition

$$\begin{aligned}\tilde{a}^I + \tilde{b}^I &= \left(\langle a_0, a_*, a^* \rangle; \langle a_0, a'_*, a'^* \rangle\right) \\ &\quad + \left(\langle b_0, b_*, b^* \rangle; \langle b_0, b'_*, b'^* \rangle\right) \\ &= (\langle a_0 + b_0,\ \max\{a_*, b_*\},\ \max\{a^*, b^*\}\rangle; \\ &\quad \langle a_0 + b_0, \max\{a'_*, b'_*\},\ \max\{a'^*, b'^*\}\rangle)\end{aligned}$$

(ii) Subtraction

$$\begin{aligned}\tilde{a}^I - \tilde{b}^I &= \left(\langle a_0, a_*, a^* \rangle; \langle a_0, a'_*, a'^* \rangle\right) \\ &\quad - \left(\langle b_0, b_*, b^* \rangle; \langle b_0, b'_*, b'^* \rangle\right) \\ &= (\langle a_0 - b_0,\ \max\{a_*, b_*\}, \max\{a^*, b^*\}\rangle; \\ &\quad \langle a_0 - b_0,\ \max\{a'_*, b'_*\}, \max\{a'^*, b'^*\}\end{aligned}$$

2.2 Ranking of triangular intuitionistic fuzzy numbers

Many authors such as Abbasbandy and Hajjari [17], Deng Feng Li *et al.* [2], Grzegorzewski [10], Jayagowri and Geetharamani [11] and Mitchell [3] etc, Have proposed different ranking methods for intuitionistic fuzzy numbers. Prabakaran and Ganesan **Error! Reference source not found.** proposed a new ranking method for triangular intuitionistic fuzzy numbers based on the left and the right spreads at some α -levels of fuzzy numbers.

For an arbitrary triangular intuitionistic fuzzy number $\tilde{a}^I = \left(\langle a_0, a_*, a^* \rangle; \langle a_0, a'_*, a'^* \rangle\right)$ with parametric form $\tilde{a}^I = \left(\underline{a}, \overline{a}; \underline{a}', \overline{a}'\right)$, we define the magnitude of the triangular intuitionistic fuzzy number $\tilde{a}^I$ by

$$\begin{aligned}\mathrm{Mag}(\tilde{a}^I) &= \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + 2a_0 + \underline{a}' + \overline{a}')\ f(r)\ dr\right) \\ &= \frac{1}{2}\left(\int_0^1 (a^* + a'^* + 6a_0 - a_* - a'_*)\ f(r)\ dr\right),\end{aligned}$$

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0,\ f(1)=1$ and $\int_0^1 f(r)\ dr = \frac{1}{3}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r^2$. Hence

$$\begin{aligned}\mathrm{Mag}(\tilde{a}^I) &= \left(\frac{a^* + a'^* + 6a_0 - a_* - a'_*}{6}\right) \\ &= \left(\frac{\underline{a} + \overline{a} + 2a_0 + \underline{a}' + \overline{a}'}{6}\right).\end{aligned}$$

The magnitude of a triangular intuitionistic fuzzy number $\tilde{a}^I$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $\mathrm{Mag}(\tilde{a}^I)$ is used to rank intuitionistic fuzzy numbers. The larger $\mathrm{Mag}(\tilde{a}^I)$ is larger intuitionistic fuzzy number.

For any two triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(\langle a_0, a_*, a^* \rangle; \langle a_0, a'_*, a'^* \rangle\right)$ and $\tilde{b}^I = \left(\langle b_0, b_*, b^* \rangle; \langle b_0, b'_*, b'^* \rangle\right)$ in F(R), we define the ranking of $\tilde{a}^I$ and $\tilde{b}^I$ by comparing $\mathrm{Mag}(\tilde{a}^I)$ and $\mathrm{Mag}(\tilde{b}^I)$ on R as follows:

(i). $\tilde{a}^I \succeq \tilde{b}^I$ if and only if $\mathrm{Mag}(\tilde{a}^I) \geq \mathrm{Mag}(\tilde{b}^I)$

(ii). $\tilde{a}^I \preceq \tilde{b}^I$ if and only if $\mathrm{Mag}(\tilde{a}^I) \leq \mathrm{Mag}(\tilde{b}^I)$

(iii). $\tilde{a}^I \approx \tilde{b}^I$ if and only if $\mathrm{Mag}(\tilde{a}^I) = \mathrm{Mag}(\tilde{b}^I)$

3. INTUITIONISTIC FUZZY CRITICAL PATH ANALYSIS

A triangular intuitionistic fuzzy project network is an acyclic digraph, where the vertices represent events, and the directed edges represent the activities, to be performed in a project. We use $\tilde{N} = \left(\tilde{V}, \tilde{A}, \tilde{T}\right)$ to denote this triangular intuitionistic fuzzy project network. Let $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, ..., \tilde{v}_n\}$ be the set of triangular intuitionistic fuzzy events (vertices), where $\tilde{v}_1$ and $\tilde{v}_n$ are the initial and terminal events of the project respectively, and each $\tilde{v}_i$ belongs to some path from $\tilde{v}_1$ to $\tilde{v}_n$. Let $\tilde{A} = \{\tilde{a}_{ij} = (\tilde{v}_i, \tilde{v}_j) / \text{for } \tilde{v}_i, \tilde{v}_j \in \tilde{V}\}$ be the set of directed edges that represents the activities to be performed in the project. For each activity $\tilde{a}_{ij}$, we use $\tilde{t}_{ij} \in \tilde{T}$ to denote the intuitionistic fuzzy time required for the completion of $\tilde{a}_{ij}$. A critical path is a longest path from the initial event $\tilde{v}_1$ to

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the terminal event $\tilde{v}_n$ of the project network and an activity $\tilde{a}_{ij}$ on a critical path is called a critical activity.

3.1. Notations and meanings

V (j): The set of all predecessor nodes of node j.

$\tilde{t}_{ij}$: The triangular intuitionistic fuzzy activity duration of activity $\tilde{a}_{ij}$.

$\tilde{d}_j$: The distance between the source node and the node j.

P_i : The i – th path of the triangular intuitionistic fuzzy project network.

P : The set of all paths in a triangular intuitionistic fuzzy project network.

Property 3.1. If $\tilde{a}_{ij} = (\tilde{v}_i, \tilde{v}_j)$, $\tilde{a}_{mn} = (\tilde{v}_m, \tilde{v}_n)$ are two triangular intuitionistic fuzzy activities, activity $\tilde{a}_{ij}$ is predecessor of activity $\tilde{a}_{mn}$ if and only if there is a chain from event j to event m in project network.

Property 3.2. If $\tilde{a}_{ij} = (\tilde{v}_i, \tilde{v}_j)$, $\tilde{a}_{mn} = (\tilde{v}_m, \tilde{v}_n)$ are two triangular intuitionistic fuzzy activities, activity $\tilde{a}_{ij}$ is an immediate predecessor of activity $\tilde{a}_{mn}$ if and only if either $j = m$, or there exists a chain from event j to event m in the project network consisting of dummy activities only.
From the above properties we see that a node "i" is said to be predecessor node of node "j" if

(i). Node "i" is directly connected to node "j".
(ii). The direction of path, connecting node "i" and "j", is from "i" to "j".

Theorem 3.1. Assume that the triangular intuitionistic fuzzy activity duration of all activities in a project network are triangular intuitionistic fuzzy numbers. Then there exists Triangular Intuitionistic Fuzzy Critical Path in the project network.

3.2. Proposed algorithm

A new algorithm is proposed for finding the fuzzy critical path under intuitionistic fuzzy environment.

Step 1: Construct the triangular intuitionistic fuzzy project network.
Step 2: Determine the precedence relationships and numbering all the events.
Step 3: Identify each activity along with triangular intuitionistic fuzzy duration in a project.
Step 4: Let $\tilde{d}_1 = (\langle 0,0,0\rangle;\langle 0,0,0\rangle)$ be the source node of the project network.
Step 5: Find $\tilde{d}_j = \max\{\tilde{d}_i + \tilde{t}_{ij}\}$ such that $i \in V(j), j \neq 1, j = 2,3,...,n$. Then find $P_i = i \rightarrow j$ where i and j are the values of the maximum value which is in $\tilde{d}_j$.
Step 6: Repeat step 5 until $\tilde{P}_i$ is calculated for all the nodes starting from source node to destination node.
Step 7: Combine all the path obtained by step 5 and 6 and discard the nodes which don't have either direct or indirect link with both the source node and destination node.
Step 8: The intuitionistic fuzzy optimized path can be obtained by combining the remaining paths starting from source node to destination node.

4. NUMERICAL EXAMPLE

In this section triangular intuitionistic fuzzy project network problem is presented to demonstrate the computational process of intuitionistic fuzzy critical path analysis proposed above.

Example4.1. Consider an example discussed by Elizabeth and Sujatha [18].

Table-1. Triangular intuitionistic fuzzy arc length of each activity.

Activity	Activity duration
1-2	$(\langle 25,35,55\rangle;\langle 20,35,60\rangle)$
1-3	$(\langle 21,30,50\rangle;\langle 15,30,55\rangle)$
2-4	$(\langle 28,44,58\rangle;\langle 22,44,65\rangle)$
2-6	$(\langle 25,43,55\rangle;\langle 20,43,60\rangle)$
3-4	$(\langle 31,45,52\rangle;\langle 28,45,58\rangle)$
3-5	$(\langle 24,37,47\rangle;\langle 20,37,55\rangle)$
4-7	$(\langle 30,47,50\rangle;\langle 29,47,60\rangle)$
5-7	$(\langle 27,37,50\rangle;\langle 25,37,55\rangle)$
6-7	$(\langle 35,52,65\rangle;\langle 30,52,70\rangle)$

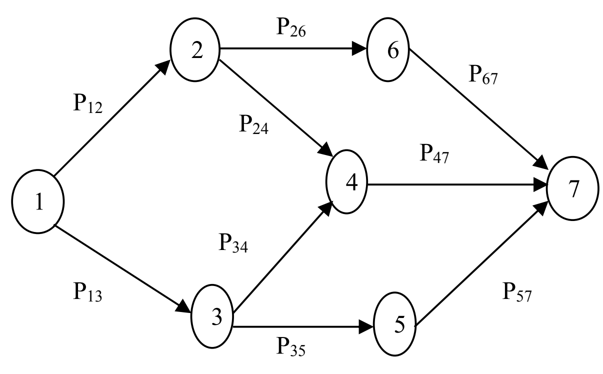

Figure-2. Intuitionistic fuzzy project network.

Given triangular intuitionistic fuzzy numbers $\tilde{a}^1$ can also be expressed as

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$$\tilde{a}^I = (\langle a_1, a_2, a_3\rangle; \langle a'_1, a_2, a'_3\rangle) = (\underline{a}, \bar{a}; \underline{a}', \bar{a}')$$
$$= \left(\langle a_0, a_*, a^*\rangle; \langle a_0, a'_*, a'^*\rangle\right).$$

Table-2. Triangular intuitionistic fuzzy arc length of each activity is in the form $\left(\langle \mathbf{a_0}, \mathbf{a_*}, \mathbf{a^*}\rangle; \langle \mathbf{a_0}, \mathbf{a'_*}, \mathbf{a'^*}\rangle\right)$.

Activity	Activity duration
1-2	(⟨35,10,20⟩;⟨35,15,25⟩)
1-3	(⟨30,9,20⟩;⟨30,15,25⟩)
2-4	(⟨44,16,14⟩;⟨44,22,21⟩)
2-6	(⟨43,18,12⟩;⟨43,23,17⟩)
3-4	(⟨45,14,7⟩;⟨45,17,13⟩)
3-5	(⟨37,13,10⟩;⟨37,17,18⟩)
4-7	(⟨47,17,3⟩;⟨47,18,13⟩)
5-7	(⟨37,10,13⟩;⟨37,12,18⟩)
6-7	(⟨52,17,13⟩;⟨52,22,18⟩)

Since node 7 is the destination node, so n=7.

Assume the source node as $\tilde{d}_1 = \left(\langle 0,0,0\rangle; \langle 0,0,0\rangle\right)$, the values of $\tilde{d}_j$; j=2, 3, 4, 5, 6, 7 can be obtained as follows:

Iteration 1: Since only node1 is the predecessor node of node 2, so putting i=1 and j=2 in step 5 of the proposed algorithm, the value of $\tilde{d}_2$ is

$$\tilde{d}_2 = \max\left\{\tilde{d}_1 + \tilde{t}_{12}\right\}$$
$$= \max\left\{\left(\langle 0,0,0\rangle; \langle 0,0,0\rangle\right) + \left(\langle 35,10,20\rangle; \langle 35,15,25\rangle\right)\right\}$$
$$= \left\{\left(\langle 35,10,20\rangle; \langle 35,15,25\rangle\right)\right\}$$

Therefore $P_1 = 1 \to 2$

Iteration 2: The predecessor node of the node 3 is 1, so putting i=1 and j=3 in step 5 of the proposed algorithm, the value of $\tilde{d}_3$ is

$$\tilde{d}_3 = \max\left\{\tilde{d}_1 + \tilde{t}_{13}\right\} = \max\left\{\begin{array}{l}\left(\langle 0,0,0\rangle; \langle 0,0,0\rangle\right) + \\ \left(\langle 30,9,20\rangle; \langle 30,15,25\rangle\right)\end{array}\right\}$$
$$= \left\{\left(\langle 30,9,20\rangle; \langle 30,15,25\rangle\right)\right\}$$

Therefore $P_1 = 1 \to 3$

Iteration 3: The predecessor nodes of the node 4 are node 2 and 3, so putting i=2, 3 and j=4 in step 5 of the proposed algorithm, the value of is $\tilde{d}_4$ is

$$\tilde{d}_4 = \max\left\{\tilde{d}_2 + \tilde{t}_{24}, \tilde{d}_3 + \tilde{t}_{34}\right\}$$
$$= \max\left\{\begin{array}{l}\left(\langle 35,10,20\rangle; \langle 35,15,25\rangle\right) + \\ \left(\langle 44,16,14\rangle; \langle 44,22,21\rangle\right), \\ \left(\langle 30,9,20\rangle; \langle 30,15,25\rangle\right) + \\ \left(\langle 45,14,7\rangle; \langle 45,17,13\rangle\right)\end{array}\right\}$$
$$= \left\{\left(\langle 79,16,20\rangle; \langle 79,22,25\rangle\right)\right\}$$

Therefore $P_2 = 2 \to 4$

Iteration 4: The predecessor node of the node 5 is 3, so putting i=3 and j=5 in step 5 of the proposed algorithm, the value of $\tilde{d}_5$ is

$$\tilde{d}_5 = \max\left\{\tilde{d}_3 + \tilde{t}_{35}\right\} = \max\left\{\begin{array}{l}\left(\langle 30,9,20\rangle; \langle 30,15,25\rangle\right) + \\ \left(\langle 37,13,10\rangle; \langle 37,17,18\rangle\right)\end{array}\right\}$$
$$= \left\{\left(\langle 67,13,20\rangle; \langle 67,17,25\rangle\right)\right\}$$

Therefore $P_3 = 3 \to 5$

Iteration 5: The predecessor node of the node 6 is 2, so putting i=2 and j=6 in step 5 of the proposed algorithm, the value of $\tilde{d}_6$ is

$$\tilde{d}_6 = \max\left\{\tilde{d}_2 + \tilde{t}_{26}\right\} = \max\left\{\begin{array}{l}\left(\langle 35,10,20\rangle; \langle 35,15,25\rangle\right) + \\ \left(\langle 43,18,12\rangle; \langle 43,23,17\rangle\right)\end{array}\right\}$$
$$= \left\{\left(\langle 78,18,20\rangle; \langle 78,23,25\rangle\right)\right\}$$

Therefore $P_2 = 2 \to 6$

Iteration 6: The predecessor nodes of the node 7 are node 4, 5 and 6, so putting i=4, 5, 6 and j=7 in step 5 of the proposed algorithm, the value of is $\tilde{d}_7$ is

$$\tilde{d}_7 = \max\left\{\tilde{d}_4 + \tilde{t}_{47}, \tilde{d}_5 + \tilde{t}_{57}, \tilde{d}_6 + \tilde{t}_{67}\right\}$$
$$= \max\left\{\begin{array}{l}\left(\langle 126,17,20\rangle; \langle 126,22,25\rangle\right), \\ \left(\langle 104,13,20\rangle; \langle 104,17,25\rangle\right), \\ \left(\langle 130,18,20\rangle; \langle 130,23,25\rangle\right)\end{array}\right\}$$
$$= \left\{\left(\langle 130,18,20\rangle; \langle 130,23,25\rangle\right)\right\}$$

Therefore $P_6 = 6 \to 7$.

Combine all the paths obtained and discard the nodes which don't have either direct or indirect link with both the source node and destination node, we have $1 \to 2$, $2 \to 6$, $6 \to 7$ and the critical path is $1 \to 2 \to 6 \to 7$. Hence the optimal duration of the project network is

$$\left(\langle 130,18,20\rangle; \langle 130,23,25\rangle\right)$$
$$= \left(\langle 112,130,150\rangle; \langle 107,130,155\rangle\right).$$

But the Optimal triangular intuitionistic fuzzy project duration obtained by Elizabeth and Sujatha [18] is $(\langle 85,130,175\rangle;\langle 70,130,190\rangle)$.

5. CONCLUSIONS

In this paper, we have proposed a modified algorithm to solve triangular intuitionistic fuzzy networking problems without following the traditional methods (i.e. starting time, finishing time, floats, etc for each activity). The project activity durations are expressed in the form of location index, left fuzziness and right fuzziness index functions for both the membership and non membership values. By applying the proposed algorithm, we have solved a network problem discussed by Elizabeth and Sujatha [18]. The result obtained by using the proposed algorithm is compared with their solution. We see that the proposed algorithm is easy for solving intuitionistic fuzzy networking problems and provides better solution to the problem.

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