Critical Path Method (CPM) is a project schedule modelling technique. Mr. Morgan R. Walker and James E. Kelly developed this technique in the late 1950s.

Critical Path

A network diagram has many paths originating from one point and ending at another point. Every path has a duration and the one with the longest duration is the critical path.

We can define a critical path as:

- The longest path in the network diagram, or
- The shortest duration to complete the project.

Notice that the first statement is talking about the **longest path** and the other is talking about the **shortest duration.**

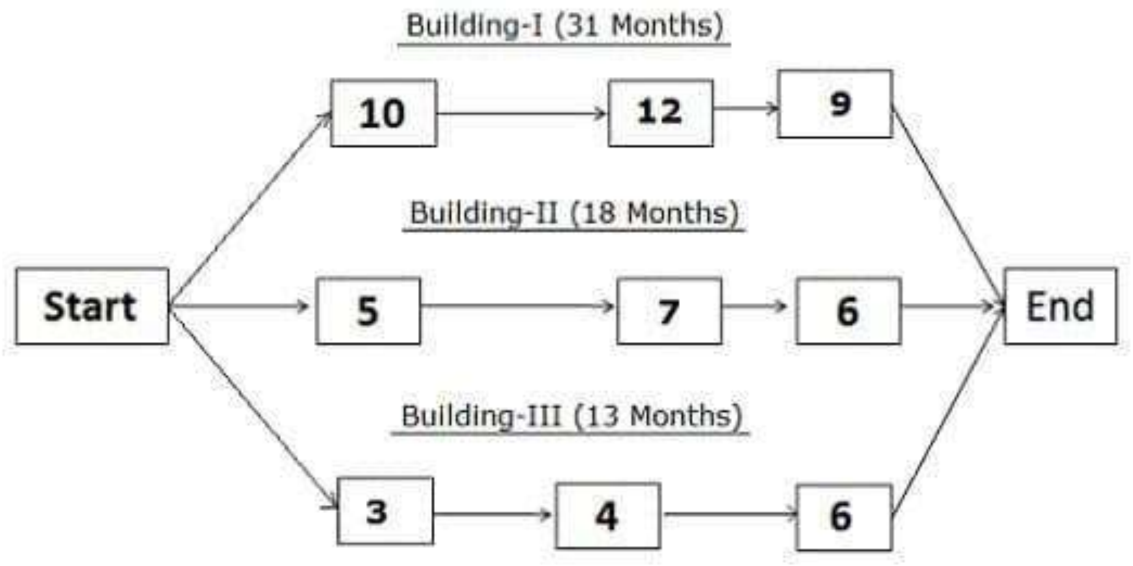
They may appear to be opposites, but they are conveying the same message.

For example, let’s say we have a project to construct three buildings. The first is the largest, the second is medium-sized, and the third is the smallest.

We may develop the network diagram which comprises three paths; each path represents each building.

Let us calculate the duration for each path. For the first building, the duration is 31 months, the second will take 18 months, and the third will require 13 months.

We can see that the first path is for the largest building, the second path is for the medium-sized one, and the third path is for the building.



Now, let us review the diagram.

We can notice that the first path is the longest?

It is 13 months longer than the second, and 18 months longer than the third.

This means that we can wait 13 months then we can start working on the second building because we can complete the second building in 18 months.

Likewise, we could wait 18 months and then start working on the third building because it will take only 13 months to complete. This means that, even if we start work on the third building after 18 months from the project start date, we can finish it on time.

This waiting period is known as the float or slack.

So, which is the critical path in this network diagram?

It is the **longest path on the network diagram** because we cannot complete our project before finishing the first building. Although we can complete the other two buildings quickly, our project is not considered complete until the first building is.

This proves the first statement that says “the critical path is the longest path on the network diagram.”

Now, what is the shortest duration to complete the project?

It is 31 months because we cannot complete our project in less time, and this is the duration of the critical path.

This proves the second statement that says, “The critical path is the shortest duration in which we can complete the project.”

So, both definitions are the same.

We can define the critical path as the sequence of activities from start to end, and it has the longest duration among all paths in a network diagram.

In ideal conditions, a network diagram should have one critical path. If it has more than one critical path, we will be in a difficult situation because we will have to manage more than one critical path.

The critical path has the longest duration, and it is the project’s duration. Activities on the critical path have no float; therefore, we must ensure that critical activities complete on time. Any delay in a critical activity will delay the project.

What if the Project is Delayed?

Schedule slippage is common in project management. However, there are some tools that can help webring things back on schedule. These are called schedule compression techniques. **Fast-tracking and crashing are two examples.** If our project is behind schedule, we can use these tools to get it back on time.

Procedure for Finding the Critical Path in a Network Diagram

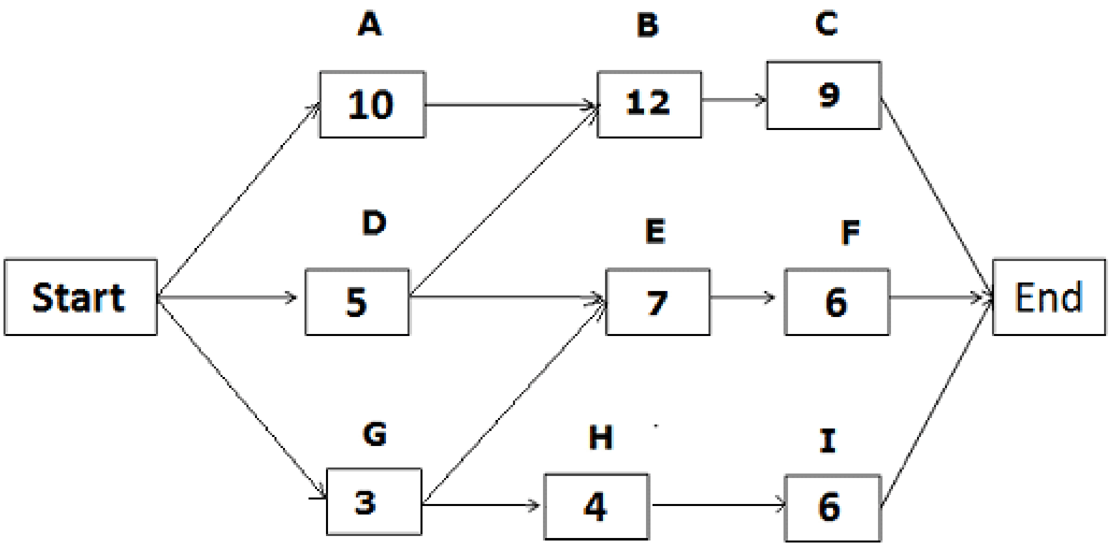
We can use the following steps to find a critical path in a network diagram:

- Draw the network diagram.
- Identify all paths in the network diagram.

- Find the duration of each path.
- The path with the largest duration is the critical path.

Example

Based on the network diagram below, identify the total number of paths, critical path, and float for each path.



The above network diagram has five paths. The paths and their durations are as follows:

1. Start -> A -> B -> C-> End {duration: 31 days.}
2. Start ->D -> E ->F -> End {duration: 18 days.}
3. Start -> D -> B -> C -> End {duration: 26 days.}
4. Start -> G ->H ->I -> End {duration: 13 days.}
5. Start -> G -> E ->F -> End {duration: 16 days.}

Since the duration of the first path is the longest, it is the critical path. The float on the critical path is zero.

The float for the second path "Start ->D -> E ->F -> End" = duration of the critical path – duration of the path "Start ->D -> E ->F -> End"

= 31 – 18 = 13

Hence, the float for the second path is 13 days.

Using the same process, we can calculate the float for other paths as well.

Float for the third path = 31 – 26 = 5 days.

Float for the fourth path = 31 – 13 = 18 days.

Float for the fifth path = 31 – 16 = 15 days.

Calculate Early Start, Early Finish, Late Start, and Late Finish

We have identified the critical path and the duration of the other paths. Now it's time to move on to more advanced calculations: Early Start, Early Finish, Late Start and Late Finish.

Calculating Early Start (ES) and Early Finish (EF)

To calculate the Early Start and Early Finish dates, we use the **forward pass**; we will start from the beginning and proceed to the end.

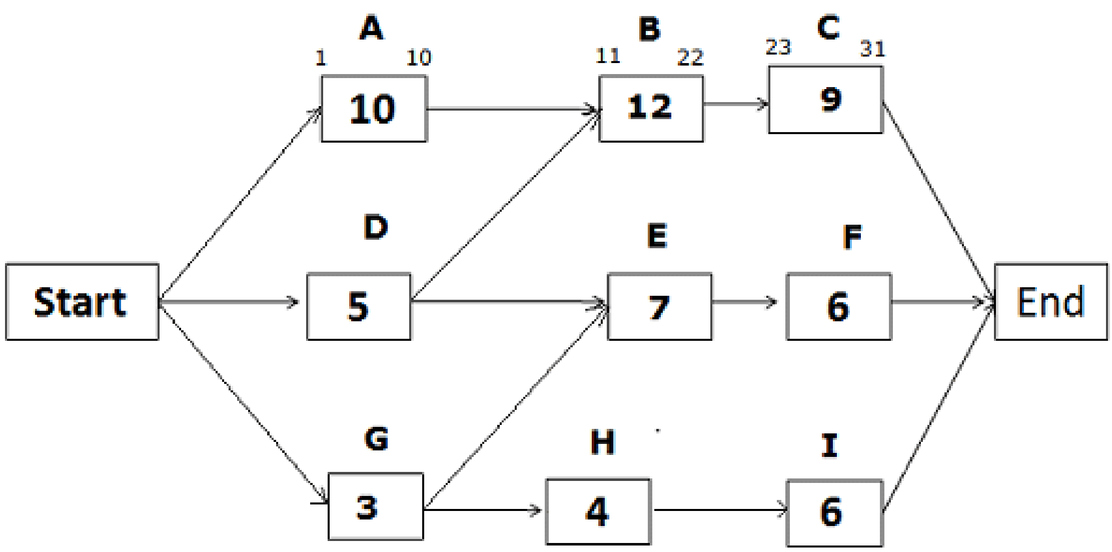
The **Early Start (ES) for the first activity on any path** will be 1 because we cannot start an activity before the first day of our project.

The starting point for any activity is the **endpoint of the predecessor activity** on the same path (plus one).

The formula used for calculating Early Start and Early Finish dates:

- Early Start of the activity = Early Finish of predecessor activity + 1
- Early Finish of the activity = Activity duration + Early Start of activity – 1

Early Start and Early Finish Dates for the path Start -> A -> B -> C -> End



Early Start of activity A = 1 (Since this is the first activity of the path)

Early Finish of activity A = ES of activity A + activity duration – 1

= 1 + 10 – 1 = 10

Early Start of activity B = EF of predecessor activity + 1

= 10 +1 = 11

Early Finish of activity B = ES of activity B + activity duration – 1

= 11 + 12 – 1 = 22

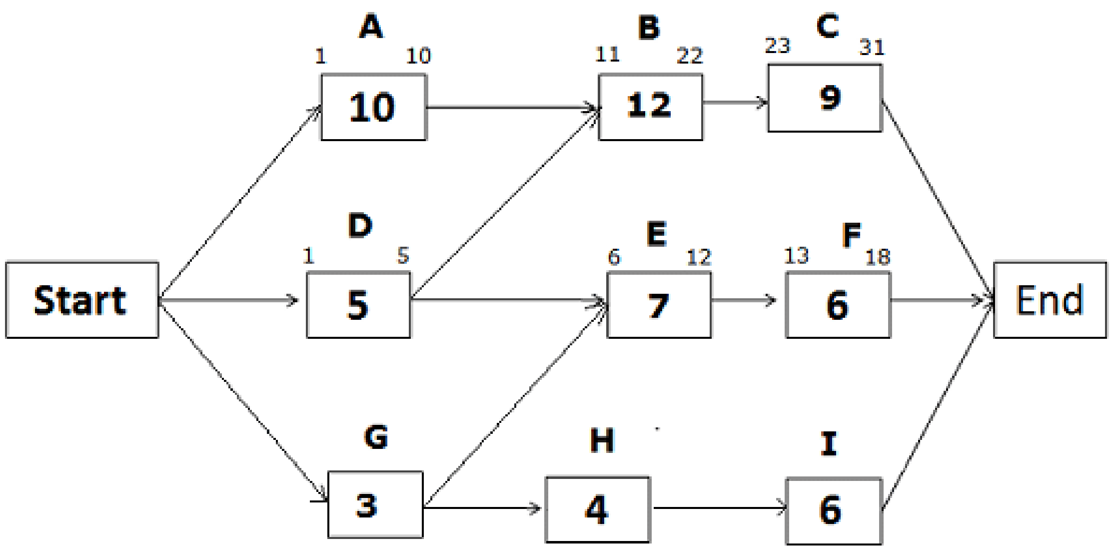
Early Start of activity C = EF of predecessor activity + 1

= 22 +1 = 23

Early Finish of activity C = ES of activity C + activity duration – 1

= 23 + 9 – 1 = 31

Early Start and Early Finish Dates for the path Start -> D -> E -> F -> End



Early Start of activity D = 1 (Since this is the first activity of the path)

Early Finish of activity D = 1 + 5 – 1 = 5

Early Start of activity E = EF of predecessor activity + 1

Since activity E has two predecessor activities, which one will we select? The answer is the activity with the greater Early Finish date. The Early Finish of activity D is 5, and the Early Finish of activity G is 3 (we will calculate it later).

Therefore, we will select the Early Finish of activity D to find the Early Start of activity E.

Early Start of activity E = EF of predecessor activity + 1

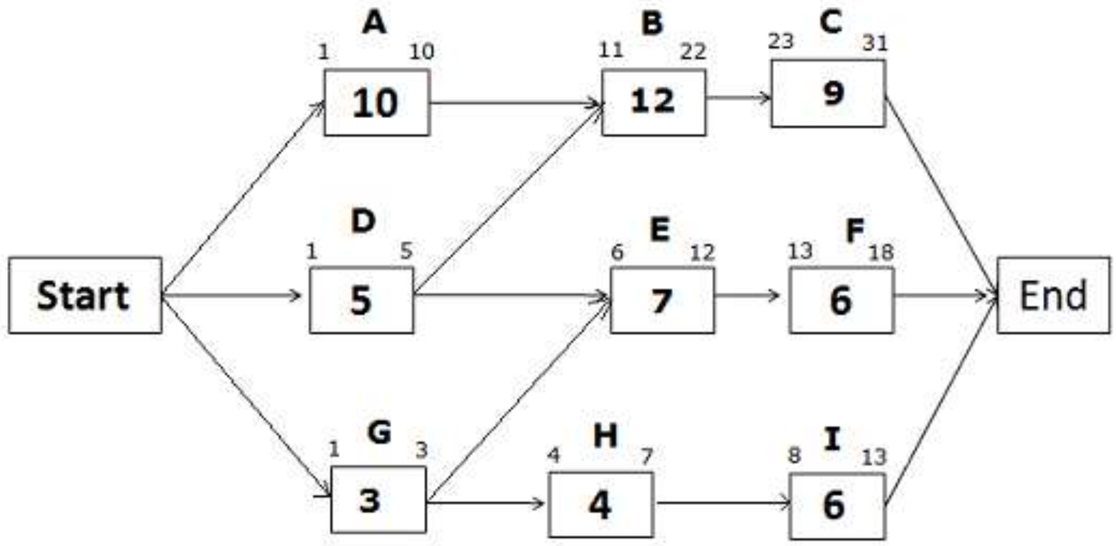
= 5 + 1 = 6

Early Finish of activity E = 6 + 7 – 1 = 12

Early Start of activity F = 12 + 1 = 13

Early Finish of activity F = 13 + 6 -1 = 18

Early Start and Early Finish Dates for the path Start -> G -> H -> I -> End



Early Start of activity G = 1 (Since this is the first activity of the path)

Early Finish of activity G = 1 + 3 – 1 = 3

Early Start of activity H = 3 + 1 = 4

Early Finish of activity H = 4 + 4 – 1 = 7

Early Start of activity I = 7 +1 = 8

Early Finish of activity I = 8 + 6 – 1 = 13

Calculating Late Start (LS) and Late Finish (LF)

We have calculated the Early Start and Early Finish dates of all activities. Now it is time to calculate the Late Start and Late Finish dates.

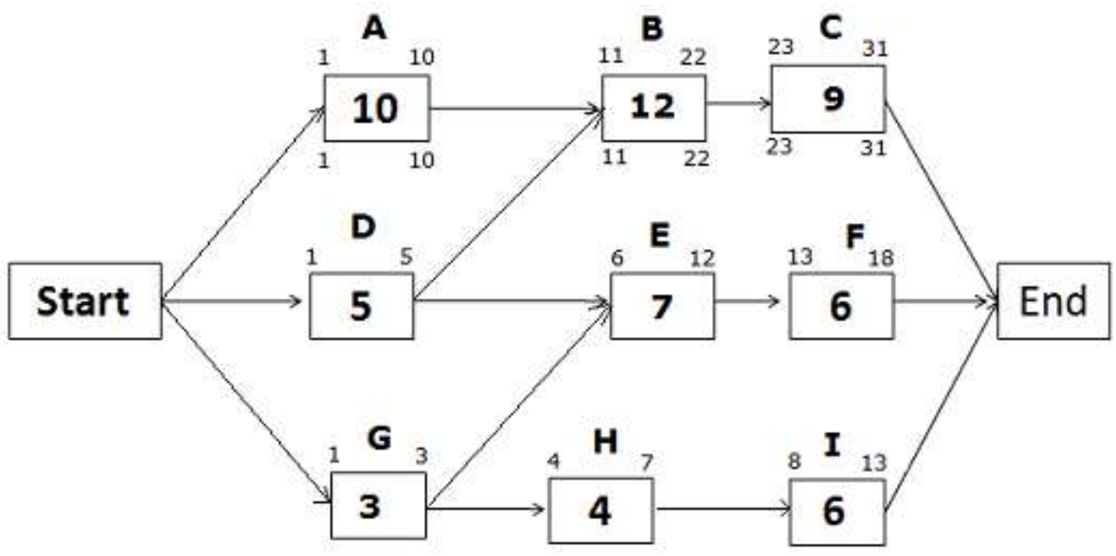
The Late Finish date of the last activity on all paths will be the same because no activities can continue once the project is completed.

The formula used for Late Start and Late Finish dates:

- Late Start of Activity = Late Finish of activity – activity duration + 1
- Late Finish of Activity = Late Start of successor activity – 1

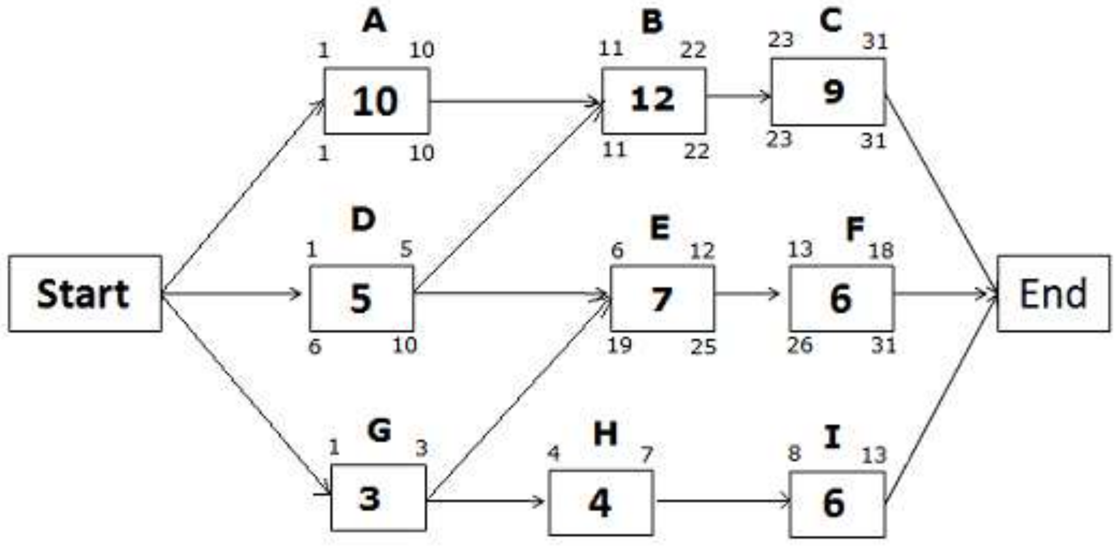
To calculate the Late Start and Late Finish, we use the backward pass; i.e. we will start from the last activity and move back towards the first activity.

Late Start and Late Finish Dates for the path Start -> A -> B -> C -> End



On a critical path, the Late Start, and Late Finish dates will be the same as the Early Start and Early Finish dates

Late Start and Late Finish Dates for the path Start -> D -> E -> F -> End



Late Finish of activity F = 31 (because wecannot allow any activity to pass the project completion date)

Late Start of activity F = LF of activity F – activity duration + 1

= 31 – 6 +1 = 26

Late Finish of Activity E = LS of successor activity – 1

= LS of Activity F – 1

= 26 – 1 = 25

Late Start of Activity E = LF of activity E – activity duration + 1

= 25 – 7 + 1 = 19

Late Finish of activity D = LS of successor activity – 1

If we look at the network diagram, we will notice that activity D has two successor activities, B and E. So, which activity would we select?

We will select the activity with the earlier (least) Late Start date. Here, the Late Start of activity B is 11, and the Late Start of activity E is 19.

Therefore, we will select activity B, which has the earlier Late Start date.

Hence,

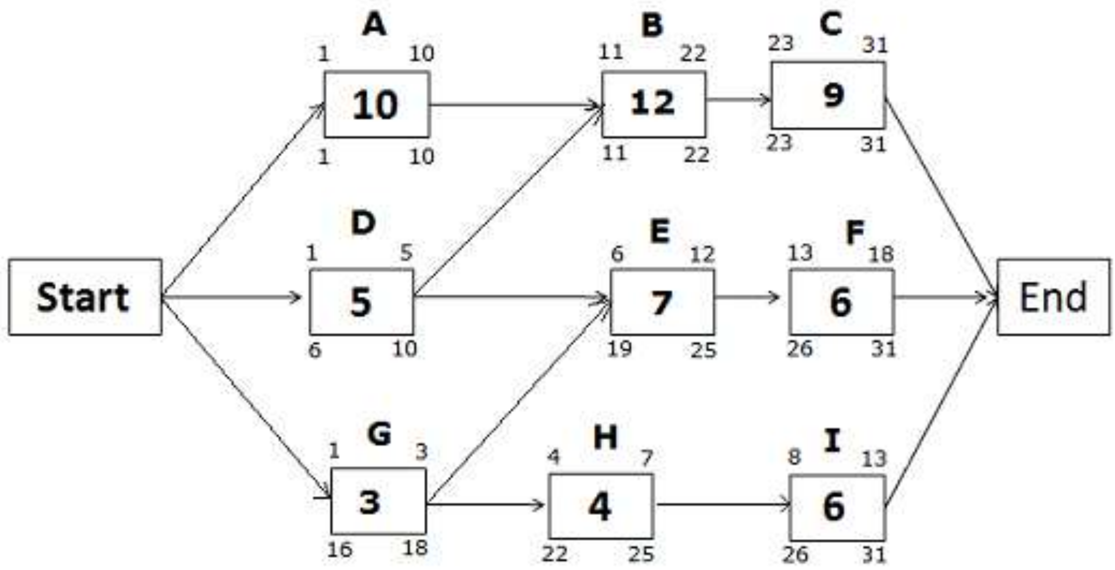
Late Finish of activity D = LS of activity B – 1

= 11 – 1 = 10

Late Start of Activity D = LF of activity D – activity duration + 1

= 10 – 5 + 1 = 6

Late Start and Late Finish Dates for the path Start -> G -> H -> I -> End



Late Finish of activity I = 31 (because wecannot allow any activity to pass the project completion date)

Late Start of activity I = 31 – 6 + 1 = 26

Late Finish of activity H = 26 – 1 = 25

Late Start of activity H = 25 – 4 + 1 = 22

Late Finish of Activity G = 19 – 1= 18 (we will choose the late start of activity E, not activity H because the Late Start of activity E is earlier than the Late Start of activity H).

Late Start of activity G = 18 – 3 + 1

= 16

Calculate the Free Float

The formula for the Free Float is:

- Free Float = ES of next activity – EF of current activity – 1

Benefits of the Critical Path Method

The following are a few benefits of the Critical Path Method:

- It shows a graphical view of the project.
- We can discover and visualize dependencies.
- It aids in project planning, scheduling, and controlling.
- It helps in contingency planning.
- We can see the critical path and identifies critical activities.
- It helps we assign the float to activities and flexibility to float activities.
- It shows we where we can take action to bring projects back on track.

Drawbacks of the Critical Path Method

Although the critical path is a very useful tool in project planning, it has some drawbacks, such as:

- The Critical Path Method is an optimal planning tool and assumes that all resources are available for the project at all times.
- It does not consider resource dependencies.
- There is a chance of misusing float or slack.
- Less attention paid to non-critical activities, though sometimes they may become critical activities.
- Projects based on the critical path often do not finish on time.

DEFINITION OF FLOAT AND TYPES OF FLOAT

Total floats and free floats have an important role in the development of a network diagram. A better understanding of these terms will help us draw, understand analyse Critical Path.

Total Float

Total float is how long an activity can be delayed, **without delaying the project completion date.**

Most important thing to be aware of is on a critical path, the total float is zero.

Total float is also known as 'slack'.

We can calculate the total float by subtracting the Early Start date of an activity from its Late Start date.

Total Float = Late Start date – Early Start date

Or

We can get it by subtracting the activity's Early Finish date from its Late Finish date.

Total Float = Late Finish date – Early Finish date

Free Float

Free float is how long an activity can be delayed, **without delaying the Early Start of its successor activity.**

We can calculate the free float by subtracting the Early Finish date of the activity from the Early Start date of the next.

Free Float = ES of next Activity – EF of current Activity

Please note that if two activities are converging into a single activity, only one of these two activities may have a free float.

A note on the convention used in the example:

We can refer to the first day of our project in two ways. Some experts consider it to be "one" and others think of it as "zero."

Both conventions are correct, and we are free to choose whichever we prefer.

I decided to refer to my first day of the project as "one."

Here are my reasons:

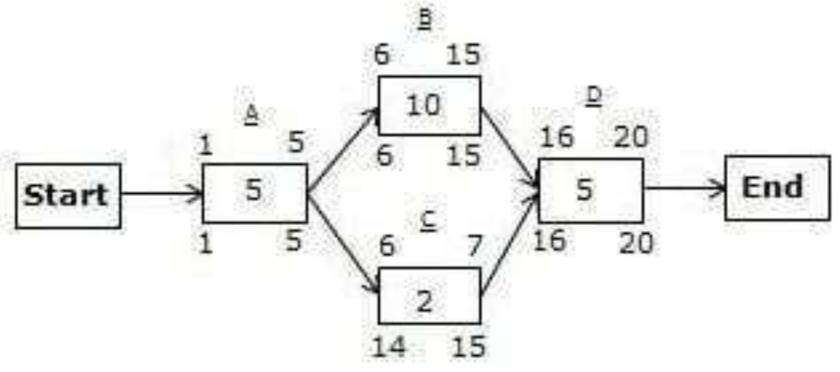
- The PMBOK Guide follows this convention.
- It seems more logical to me to say, "Hey, today is my first day of the project!" instead of saying, "Hey, today is my zero-day of the project."

Anyway, we are free to select our choice.

The formula used to calculate free float is different for these two situations, however, the result is the same.

Now, we will look at some examples. We will start with a simple one and then move on to a more complicated situation.

Example: 1



In the above network diagram, we can see two paths:

1. The first path is A->B->D with a 20-day duration.
2. The second path is A->C->D with a 12-day duration.

The path A->B->D is the critical path because it has the longest duration.

Calculating the Total Float

The path A->B->D is a critical path; therefore, it will not have a total float.

The path A->C->D is a non-critical path, so it can have a total float.

There are two methods to calculate the total float. In the first, we subtract the duration of the non-critical path from the critical path.

In the second method, we find the total float for any activity by subtracting the Early Start date from the Late Start date (LS – ES) or subtracting the Early Finish date from the Late Finish date (LF – EF) on any activity.

The first method of finding the total float

Total float = duration of the critical path – duration of the non-critical path

= (duration of the path A->B->D) – (duration of the path A->C->D)

= 20 – 12

= 8

Hence, the total float is eight days.

The second method of finding the total float

On the path A->C->D, Activity A and D lie on the critical path; therefore, they will not have a total float. Only Activity C can have a total float.

We can calculate the total float by using either the finish dates or start dates.

First, we will go with the Late Finish and Early Finish dates:

Total float for Activity C = (LF of Activity C – EF of Activity C)

= 15 – 7

= 8

Now, the second formula:

Total float for Activity C = (LS of Activity C – ES of Activity C)

= 14 – 6

= 8

The durations are the same. This means that both formulas will give us the same result.

Calculating the Free Float

From the figure, we can see that only Activity C can have a free float because all other activities are on the critical path.

Let’s find it.

Free float of Activity C = ES of next activity – EF of Activity C – 1

= 16 – 7 – 1

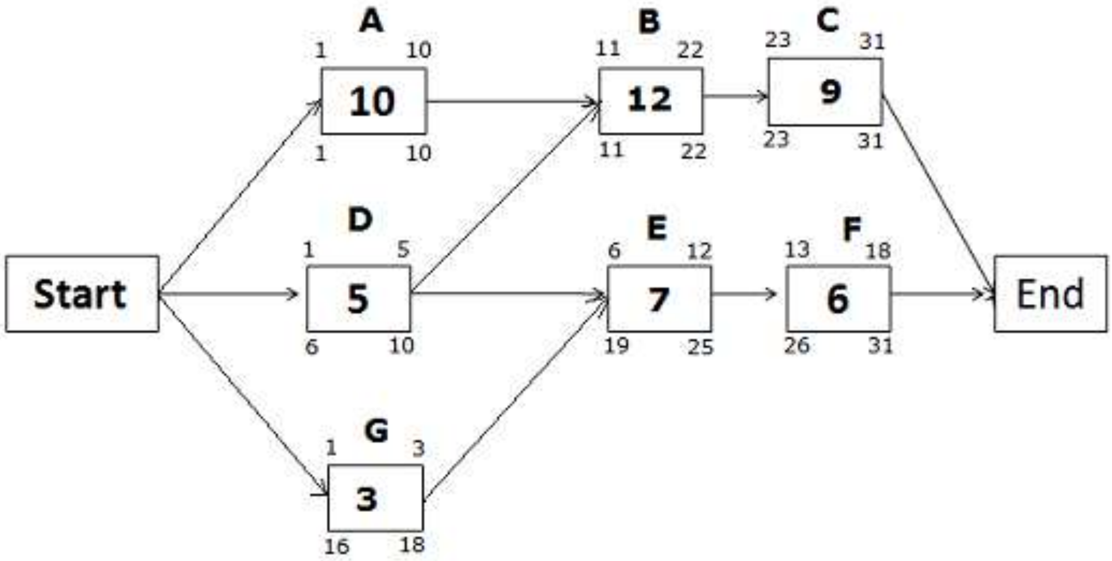
= 8

Hence, the free float for Activity C is eight days.

Now we will discuss a more complex example.

Example: 2

For the below-given network diagram, find which activities can have a free float and calculate the free and total float, considering the duration in days.



We know that:

Free float = ES of next activity – EF of current activity – 1

In the above diagram, Activity G can have the free float because Activity D and G converge on one common activity.

Activity D will not have a free float because its successor, Activity E, is starting the day after the completion of Activity D.

Free Float for Activity G

We know the formula for free float:

Free float of Activity G = Early Start of Activity E – Early Finish of Activity G – 1

= 6 – 3 – 1

= 2

Total Float for Activity G

Total float for Activity G = Late Finish of Activity G – Early Finish of Activity G

= 18 – 3

= 15

We can see here that the free float for Activity G is two days, and the total float is fifteen days. Both are different.

Summary

Total float and free float are important concepts in schedule management. Total float is commonly referred to as float. Activities on a non-critical path will have a total float. When two activities converge, one of the converging activities will have a free float.

FAST TRACKING AND CRASHING: SCHEDULE COMPRESSION TECHNIQUES

There are many reasons we may want to compress the schedule.

A project could have been delayed and we may have to bring it back on schedule.

Management may also ask us to compress the schedule.

Project delays can happen for many reasons. For example:

- An unrealistic schedule
- Unavailability of promised resources
- The occurrence of unidentified risks
- Force majeure

Other reasons for compressing the schedule are:

- The client wants to complete the project early.
- Client wants to launch a product or open a facility early.

We can use one of two schedule compression techniques, fast-tracking and crashing, to decrease the project's duration with no change in scope.

Fast-Tracking

As per the PMBOK Guide, 6th edition, fast-tracking is a schedule compression technique in which activities or phases normally performed in a sequence, are done in parallel for at least a portion of their duration.

In fast-tracking we must review the critical path and list all vital activities. Then we analyse which ones can be performed partially or fully parallel with other activities.

We will not review the activities on the non-critical paths, they have float. **Reducing the duration of those activities will not affect the schedule; it will only give more float to them.**

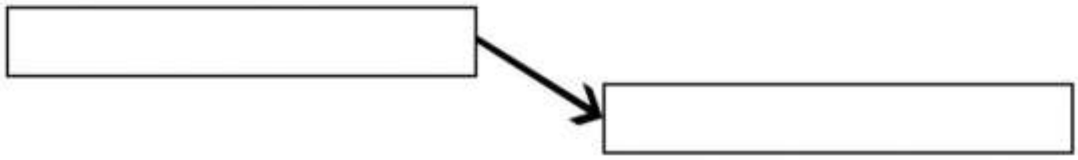
We should monitor other paths whose durations are close to the critical path. If any other path becomes critical, we will reduce the duration of the new critical path. In this case, our current path will no longer be critical.

Afterward, we will rearrange the fast-tracked activities and reanalyse the schedule.

To compress the schedule, project managers start with fast-tracking because it does not cost more. However, it increases risk as activities are overlapping.

As a rule of thumb, we can fast track sequential activities by 33%. This means we can start the next activity when the previous is 66% complete. Both activities partially overlap. It will increase the risk but within acceptable limits.

Fast-tracking helps we compress the schedule up to certain limits. Continuing beyond the limit will increase the risk, which may lead to rework and further delays.



Before Fast Tracking

After Fast Tracking

Example

Let's say that we are building a school and construction work is about to finish.

Later, we start carpentry and electrical works.

When we review our progress, we see that we are behind schedule; we have to move faster to complete the project on time.

We will review the carpentry and electrical work activities and see if we can perform them in parallel. Then, we can apply fast-tracking.

In this case, we can start the electrical and carpentry work at the same time.

Lead VS Fast-Tracking

On a compressed network diagram, activities with lead and fast-track activities look the same. Hence, many often think the lead is the same as fast-tracking.

Lead is a type of dependency that we use while creating the network. It is already factored into the schedule. On the other hand, fast-tracking is a forced overlap. We do it to shorten the schedule. Fast-tracking increases risks and possible rework, while lead is a dependency type on the network diagram and it does not affect the risk.

Crashing

As per the PMBOK Guide 6th edition, **crashing is a technique used to shorten the schedule duration for the least incremental cost by adding resources.**

In crashing, we review the critical path activities and find ones that can be completed early with extra resources, that can provide the highest compression with the least cost.

Afterward, we will apply crashing to those activities.

While crashing, we will monitor other paths as well. It is possible that the duration of other paths could become equal or greater than critical path.

Initially, we will get a greater reduction in duration with less input cost. However, as we continue further, the cost will increase and the reduction will dwindle.

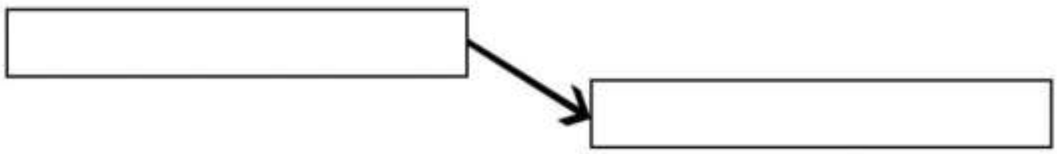
It is necessary to do a cost benefit analysis to arrive at a decision.

A few examples of crashing techniques are:

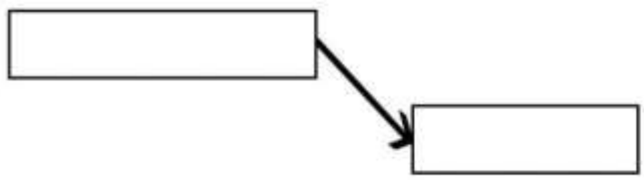
- Overtime
- More resources
- Monetary rewards

Crashing cannot be applied to all activities.

For example, we have to wait until the concrete dries before we can start next activity.



Before crashing



After crashing

Example

We are constructing a room. According to the duration estimate, two masons will take four days to complete it.

We have to reduce the duration of this activity by crashing. We add two more masons to complete the task in two days.(Provided four masons can be accommodated in that space, rule of 3 cannot be applied always.)

Sometimes, crashing may not produce the desired result. Getting skilled resources is not easy. and they take time to settle. We cannot bring in a new group of people and expect them to perform immediately.

Therefore, it is possible that the cost will increase without any significant gain.

Perform due diligence before using crashing.

Difference Between Fast Tracking and Crashing

The following are a few differences between fast-tracking and crashing:

- In fast-tracking, activities are rescheduled to be performed partially or fully in parallel, while in crashing, we add extra resources to the activities to finish them early.
- Fast-tracking does not cost we extra money, crashing does.
- Fast-tracking increases risks. Crashing does not, significantly.
- We use fast-tracking when activities can be overlapped to decrease their duration, while we use crashing on those where adding extra resources can decrease their duration.

When Should We Use Fast Tracking or Crashing?

This depends on the situation and requirements.

For example, if the client wants to complete the project early and is willing to pay, we use crashing.

Generally, we will start with fast-tracking to shorten the schedule. Once we are done, we can go for crashing if necessary.

Sometimes we may use both techniques. For example, the client is threatening to fine us for the delay. To avoid this, we will compare the cost of crashing with the fine. If the crashing cost outweighs the fine, we will use it with fast-tracking for maximum schedule compression.

We may also use crashing if the project delay can affect the company’s image or credibility.

Why are Schedule Compression Techniques Applied to the Critical Path?

As the name suggests, these are schedule compression techniques. The schedule is based on the critical path, the longest path of the network diagram and its duration is that of the project.

Reducing other paths won’t reduce the duration of the project. We will just give those paths more float.

If we want to reduce the duration of schedule, we have to shorten the duration of the critical path.

Summary

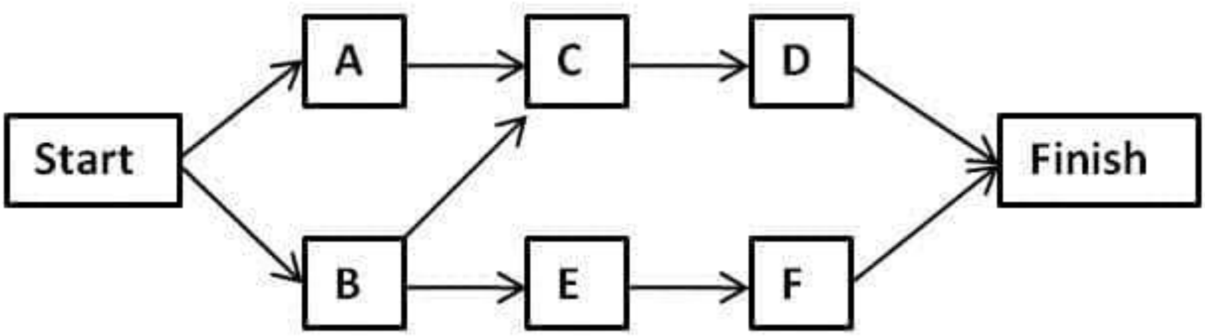
Projects often get delayed and we have to compress the schedule, fast-tracking and crashing are two ways to do that. These techniques help us decrease the duration of the project. Fast-tracking does not involve cost, but it increases risks. Crashing does not significantly increase risk, but it is a costly process. Use these techniques carefully because we are dealing with critical activities. Any wrong step can affect the project negatively.

Reference :PM Study Circle.
Blog by Fahad Usmani.
https://pmstudycircle.com/2012/09/fast-tracking-crashing-schedule-compression-techniques-in-time-management/

Precedence Diagram (Activity on Node) method of scheduling

A **Precedence Diagramming Method** (PDM) is a graphical representation technique. It shows the inter-dependencies of activities and is used in development of a project schedule . The other name for this technique is Activity on Node (AON). We use this method in drawing the project schedule network diagrams.

The Precedence Diagramming Method shows activity relationships. Hence, it is an important communication tool for stakeholders.



Precedence Diagram Method (PDM)

The Precedence Diagramming Method is made of boxes (rectangles or any shape), known as nodes. These boxes show the project activities. An arrow connects two boxes and shows the relationship. Therefore, these diagrams are also known as Activity on Node (AON) diagrams.

Type of Dependencies in PDM

The PDM uses four dependencies:

1. Mandatory Dependency
2. Discretionary Dependency
3. External Dependency
4. Internal Dependency

Mandatory Dependency

This dependency is also known as hard logic. We cannot avoid it. Starting the next activity depends on it.

For example, we cannot construct the roof until we build all the walls.

Discretionary Dependency

This dependency is also known as preferential or soft logic, it plays a role in optimizing resources.

For example, we can construct the four walls in any sequence. However, if constructing them in a certain sequence is beneficial we build them in that order. Here, we can change the sequence of activities as per your preferred logic.

External Dependency

The project management team has no control over an external dependency.

For example, we may need **government approval** before starting the next activity.

Internal Dependency

These are dependencies are within the control of your project or organization.

For example, we cannot get a resource until it is free from another project OR we cannot get cement unless we arrange for funds.

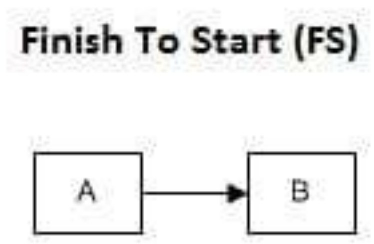
Dependency Relation

The Precedence Diagramming Method uses four relationships:

1. Finish to Start (FS)
2. Finish to Finish (FF)
3. Start to Start (SS)
4. Start to Finish (SF)

Finish to Start (FS)

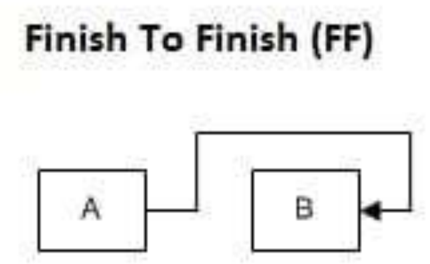
Here, the next activity cannot start until the first is complete. This is the most common relationship in PDM.



For example, to paint a wall we first we have to construct the wall. In this case, the first activity is constructing the wall and the second activity is painting. We cannot start painting the wall until the wall is ready.

Finish to Finish (FF)

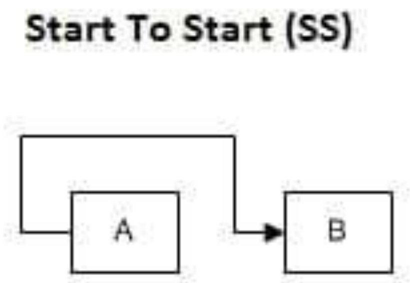
Here, we cannot complete the next activity until the first is finished. Put simply, both activities should be finished simultaneously.



For example, let us say that we are constructing a villa for a client. The client gives us the colour scheme for external paint. We cannot reach the milestone of completing the villa until we get the client’s complete requirements. Here, both activities should be finished simultaneously.

Start to Start (SS)

Here, the next activity cannot be started until the first starts. Both activities should start simultaneously.

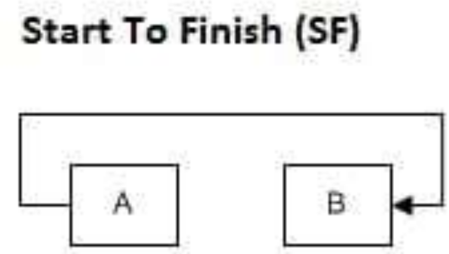


Suppose we have to apply a coating on a wall, but the wall must be cleaned in order to apply it.

Therefore, one team will clean the wall and second team will coat it. Both activities can start simultaneously.

Start to Finish (SF)

Here, we cannot finish the next activity until the first starts.



For example, let us say we are moving into a new home and your old home has to be demolished. In this case, we cannot move into your new home until it is ready. Hence, the second activity (construction of the new home) must be finished before the first activity starts (moving into a new home).

Put simply, we are moving into your new home. We cannot start vacating your old home until the new house is ready.

Although this relationship is rare, we must understand all the dependencies. It will help we draw the network diagram and develop the project schedule.

This concludes the precedence diagram method.

We may hear the term Activity on Arrow (AOA). This is a less commonly used technique in diagramming methods. The AOA method is a special case of the Precedence Diagramming Method.

AOA diagram only uses the Finish to Start relationship. It shows the duration over the arrows, that is why many experts call this diagramming method the Activity on Arrow diagram. PERT is an example of this technique.

There is a difference between the AON and AOA diagram. The AOA diagram emphasizes milestones (events), and the AON diagram emphasizes tasks.

How to Draw a Precedence Diagram

To draw a PDM, break Work Breakdown Structure down to the activity level.

Then we will create a table, list all activities, and sequence the activities.

The next step is to add relationships to each activity. We will add what activity comes next.

Finally, we will draw the diagram.

Benefits of Precedence Diagramming Method

This method offers many benefits to project management:

1. It helps we find relationships and dependencies among activities. This helps us in planning and avoiding risks. If any task is missing, we can easily identify it.

2. We can find critical activities and focus on them. Any delay in critical activities will delay your schedule.
3. A project schedule network diagram is a good communication tool. Stakeholders can visualize activities and understand the schedule.
4. Without the Precedence Diagram, we cannot develop your project schedule.

Summary

The Precedence Diagram Technique has an important role in project management. Your project schedule depends on it and it is a good communication tool. It is commonly referred to as AON, where nodes represent activities. The other PDM is AOA, where nodes represent milestones and duration is shown on the arrow.