

**CRITICAL PATH ANALYSIS USING SIMULATION TECHNIQUES AND  
SELECTION OF LEAN TOOLS TO MULTIPLE CRITICAL PATHS BASED ON COST  
FACTOR**

A Thesis by

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FACTOR**

The following faculty members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Industrial Engineering.

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## **ABSTRACT**

A production system converts raw materials into finished goods by various processes. The processes could occur parallel in order to reduce the time taken to complete the processes. Critical path in production systems is the maximum time taken to complete processes occurring in parallel. Identifying critical paths are important because, in order to improve the cycle time emphasis needs to be given to the critical path. Critical path is usually a single path in deterministic processing times. Due to variability more than one path might tend to be critical within the same system. The initial part of this research focuses on identifying changes in critical paths in variable processing times and prioritization of paths. Several metrics such as critical path severity index, all path severity indexes, probability of critical path beyond standard time are used to identify the criticality of paths. Next, suitable tools are implemented within these paths in order to improve the probability of completion and reduce the costs due to delay. An economic analysis for using lean tools within the paths is done. The allocation of improvement tools is based on the variability of each process and the processing time

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## CHAPTER 1

### INTRODUCTION

#### 1.1 Problem background

A facility is an integral component in a business enterprise. An efficient facility provides an edge over competitors such as faster delivery of parts without delay and with lower fluctuations in the processing time between workstations. Production systems convert raw materials into finished goods that provide value to the customer. Production systems consist of workstations either in parallel, series or a combination of series and parallel. In order to improve an existing complex production system, which involves many operations occurring in series and parallel, the dominating path needs to be analyzed. The path that takes the maximum time to complete the process is defined as the critical path for that process. But in production systems with high variability and low volume such as ship building industries and aircraft manufacturing industries, the processing times have a high variability as most operations are labor intensive. Various other factors such as part outages and tool unavailability also contribute to variability. In order to decrease processing time and variability, improvement needs to be done in the critical path. Else any improvements made on other non-critical paths would not help in decreasing the cycle time for the workstation. When the processing times are deterministic, it is easy to see that only one of the lines could be a critical path. But, in industries such as aircraft manufacturing, there is a trend of high variability in the processing times due to manual labor and process variation. Due to the high variability, more than one critical path could dominate the system. So, if emphasis is given only to one critical path, there might not be any significant difference in the time taken to complete a part. In such a scenario, more than one path needs to be analyzed simultaneously in order to reduce the time taken to complete a part. Thus, a study has been done

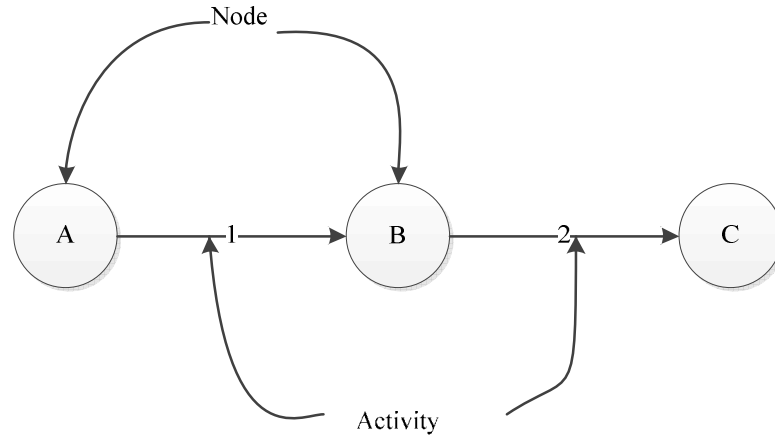
to identify the possibilities of more than one path being critical and the significance of each of the contributing factors.

## **1.2 Production System**

A production system involves a set of processes that convert raw materials into finished goods which is valued in the eyes of the customer. The set of processes involved can either be in series, parallel, or a combination of both. One of the problems faced in production systems is the completion of processes within a fixed standard time. The delays could be due to variability, shortage of resources, machine breakdowns, high travelling time between workstations, bottleneck machine and tool failure. In order to satisfy the customer, parts have to be supplied at the right cost, in the right quantity and at the right time. Delays in the production line could result in penalties in the form of lost orders and high work in process inventory cost.

## **1.3 Critical Path**

Critical path is the maximum time taken to complete the process when there are no external delays that are not accounted for. It is used to calculate the time taken to complete a set of operations when the processing times are certain. Non-critical path(s) completes the operations within the time taken at the critical path. To calculate time taken at each path, the critical path is represented as an activity and each process is represented by a node as shown in Figure 1.1.



**Figure 1.1 Representation used for critical path method**

#### **1.4 Critical path in workstations**

Critical path in a production system is the path that takes the maximum time to complete all the processes. Time taken at the critical path within a workstation of parallel and serial processes determines the cycle time for that workstation. Thus, in order to reduce the time taken at the workstation, critical path needs to be analyzed at first. In production systems, data is stochastic and variability leads to more than one part being critical at the same time.

#### **1.5 Variable processing times**

Many techniques have been used to calculate the time of completion within a machine. Research has been done to calculate the time of completion when the processing time is deterministic and the activities are independent by using Critical Path Method (CPM). Program Evaluation and Review Technique (PERT) analysis is used to calculate the time of completion when the processing time is uncertain assuming that the critical path calculated by the critical path method is the same. However, previous research does not consider the shifts in critical path when there are stochastic processing times. Variable processing times occur mostly in manufacturing systems which have manual labor. Suer (1996) classified manufacturing systems

as either labor intensive or machine intensive. Variability is dominant in labor intensive manufacturing system because of manual labor such as tool change, loading and unloading material from the machine transferring parts.

## **1.6 Lean Manufacturing**

Lean manufacturing is a philosophy implemented by Toyota Production System (TPS) in Japan. Lean manufacturing aims to identify all the activities and resources within a firm or industry as value added activities, non- value added activities and necessary non-value added activities. This philosophy identifies value added activity as any process or resource, that adds value to the product and non-value added activity as the process that do not add any value to the product and needs to be eliminated. Necessary non-value added activities are the activities that do not add value to the product, but are necessary and cannot be eliminated due to external factors like variability though they can be reduced. Various tools are used to improve the efficiency of the system, tools can be used to either reduce the processing time or reduce the variability in the system.

## **1.7 Research purpose**

In workstations which have several thousands of tasks mostly distributed in series and parallel, the cycle time of a workstation is defined by the time taken at the critical path. Due to variability, there are instances when more than one path tends to be critical within the same workstation, which could cause the time taken at the workstation to exceed the standard cycle time. In these cases, in order to reduce the cycle time, emphasis needs to be given to more than one workstation. This research studies how much influence does each path in the system has in the probability of completion and ensure that the parts are produced within the standard cycle



time. A study on the shift in critical path is also done along with the probability of each part being critical.

## **1.8 Research objectives**

The objectives of this research:

1. Identification of critical path(s) in stochastic processing times
2. Calculation of probability of a part being critical:
  - a. Without consideration to standard time
  - b. Considering the standard time
3. Calculation of the average time a path exceeds from the standard move rate
4. Improving the probability of completion
5. Implementing lean techniques within the system so as to reduce the costs due to delay

The research work has been organized into five chapters. Chapter 2 provides the existing literature and methodologies used to identify critical path(s) and lean tools used to reduce costs. Most of the literature is for deterministic values and chapter 3 provides a detailed methodology on the identification of critical paths and prioritizing them for improvement and chapter 4 includes application of lean tools to the critical paths and improving the probability of completion, thus, improving the existing system with the minimal cost and available resources. Chapter 5 is the conclusion of calculation of critical paths and application of lean techniques in order to improve the performance of the system. It also includes future research in the field of critical path in variable processing times.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

The purpose of this research is to analyze a given production system, which involves several operations in series and parallel. The research done in production systems is based on low volume and high variability, such as, aircraft manufacturing industries and ship building. Section 2.2 discusses production systems followed by section 2.3 which involves a study on the effects of variability in a production system. Section 2.4 focuses on network programming models used to calculate the time taken to complete a set of processes within a network based on complexity of the model used. Section 2.5 discusses the methods used to identify the critical path and calculate the time taken at the critical path which in turn is used to calculate the time taken to complete the set of operations. The next section is about using fuzzy numbers used to calculate critical paths. Section 2.7 deals with calculating the time taken to complete a set of operations using Project Evaluation Review Technique (PERT) and the drawbacks of using PERT analysis. Section 2.8 includes implementing lean techniques in critical path analysis to reduce the processing time based on economic analysis which is followed by the research motivation in section 2.9.

#### **2.2 Production Systems**

Production line is a system that converts the given raw material to the finished goods based on the customer's requirement by using the minimal tools in a highly efficient manner. Certain processes could take higher processing times than other processes. This is due to difference in time taken to complete the process, tool unavailability, waiting time and machine downtime. According to Suer (1996), production systems are classified into machine intensive

and labor intensive production system. In order to do accurate operations such as gripping, manual labor is required (Bi and Zhang, 2001).

Production systems such as aircraft manufacturing industries involve low volume and highly customized goods that are nearly all the aircrafts built are based on the customer's requirements. In such systems, there exists high variability in many processes within the line.

### **2.3 Variability in production systems**

Production system converts raw material into finished goods which are valued in the eyes of the customer through a series of operations or processes. Production systems can be classified into machine intensive production system and labor intensive production system (Suer, 1996). In a machine intensive production system, the factors that decide the productivity of the system is primarily based on the number of machines available in the production system and does not depend on the labor present in the system. In labor intensive production systems, the factors that influence the productivity are the labor present in the system. The workers are allotted with inexpensive equipment to perform the operations (Pachaimuthu, 2010). In labor intensive manufacturing systems, due to the high involvement of labor, the variability tends to be more due to worker absenteeism, skill level required to perform the task, change in worker productivity within the day and breaks.

Production lines were initially developed based on high volume and for standardized products. So emphasis was given to highest utilization and high labor specialization (Shtub and Dar-El, 1989 & Scholl, 1999). Due to product standardization, the lines could be balanced perfectly. But due to an increase in demand and global competition, the demand for customized goods and services were expected. Due to an increase in product volume within the line, the variations also increased.

## **2.4 Network Programming**

Various network programming techniques are used to evaluate the time taken to complete products and meet the deadlines of various lines. The various network programming techniques are Critical Path Method (CPM), Project Evaluation and Review Technique (PERT), Bar/Gantt chart, Line of Balance (LOB), Vertical Production Method (VPM), and Linear Scheduling Model (LSM).

Gantt chart is a tool used to schedule based on the start and finish time. The advantages of using are that Gantt charts are simple to use, they take less time to create. The disadvantage of using Gantt charts is that it becomes tough to implement in complex production systems having variable processing time. They also fail to show the dependencies of various processes. Gantt chart works better in simple production systems. When the complexity of a production system increases, or if the production systems has variable processing time, Gantt charts cannot be used to schedule start and finish time.

Line of Balance (LOB) is used in areas where repetition of jobs occurs. Linear Scheduling Method (LSM) is used to represent the location and time operations when they occur. The above networking models are not feasible for complex production systems.

Based on the type of industry and the work required, various models have proven to be efficient (Yamin & Harmelink, 2001). Table 2.1 shows which scheduling method is optimal for which type of industry based on complexity of production system and the number of activities required completing.

**Table 2.1 Recommended scheduling tool for different industries (Yamin & Harmelink, 2001)**

<b>Type of industry</b>	<b>Application</b>	<b>Scheduling Method</b>	<b>Main Characteristic</b>
Linear and continuous projects	Pipelines, railroads, tunnels and highways	LSM	<ul style="list-style-type: none"> <li>• Few activities</li> <li>• Executed along a linear path/ space</li> <li>• Hard sequence logic</li> <li>• Work continuity crucial for effective performance</li> </ul>
Multi-unit repetitive projects	housing complex, buildings	LOB	<ul style="list-style-type: none"> <li>• Final product a group of similar units</li> <li>• Same activities during all projects</li> <li>• Balance between different activities achieved to reach objective production</li> </ul>
High-rise buildings		LOB, VPM	<ul style="list-style-type: none"> <li>• Repetitive activities</li> <li>• Hard logic for some activities, soft for others</li> <li>• Large number of activities</li> <li>• Every floor considered a production unit</li> </ul>
Very complex processes	Refineries, aircraft manufacturing	PERT/CPM	<ul style="list-style-type: none"> <li>• Extremely large number of activities</li> <li>• Complex design</li> <li>• Activities discrete in nature</li> <li>• Crucial to keep project in critical path</li> </ul>
Simple projects	any kind	Bar/Gantt chart	<ul style="list-style-type: none"> <li>• Indicates only time dimension (when to start and end activities)</li> <li>• Relatively few activities</li> </ul>

From the table, it can be inferred that for an aircraft industry which involves a lot of complex operations, the most optimal networking techniques include are Project Evaluation Review Technique (PERT) and Critical Path Method (CPM). CPM method uses deterministic values to calculate the time for completion for various operations PERT method overcomes the deterministic concept of processing time by using a beta distribution. PERT method calculates the completion time by considering the highest time to complete the task, least time to complete

the task and the most common time to calculate the critical path. PERT cannot be used if the data follows a distribution other than beta distribution (Chanas & Zielinski, 2001).

## **2.5 Critical Path**

Critical Path Method (CPM) technique was developed by E.I. DuPont de Nemours and company in conjunction with the Univac Applications Research Center of Remington Rand (Wickwire and Smith, 1975). It has gained popularity ever since and is used to calculate the time taken to complete a project.

Critical path is the maximum time taken to complete a set of processes when parallel operations occur. Generally, the path that takes the maximum time to complete the given set of operations determines the rate at which a process comes out. This path is called the critical path. In deterministic data, the critical path can be calculated by simply calculating the sum of the times taken by individual processes within the workstation. In stochastic data, the critical path is calculated the same way by adding the individual processing times. But, due to variability more than one path could tend to dominate the system.

According to Winston (1994) various rules need to be considered when coining analysis using critical path method. The various rules used for critical path analysis are shown below.

- Rule 1. Activities for parallel processing must start at a node.
- Rule 2. The activities must end at the completing node.
- Rule 3. The number of nodes need to be numbered such that the completion node has a larger number than the starting node.
- Rule 4. An activity should not be represented with more than one node.
- Rule 5. If two nodes need to be connected with more than one arc then a dummy arc is used to trigger the activity.

Previous researches by Nasution (1994) analyzed critical path with the earliest time a part comes out and the latest time a path comes out based on interactive fuzzy logic techniques.

Mount Clemens General Hospital (MCGH) used critical path method to improve the total quality management in clinical areas (Hofmann P.A., 1991). The warm up time considered was around 44 patients and critical path analysis was considered after the warm up period. A complication rate of around 5% was in the critical path when compared to a 16% rate for no guided care. After the identification of the critical path, a steering committee was formed comprising of members from hospital administration, nursing, quality assurance and risk management for further investigation. It was found that critical path method was a very effective tool for an environment of communication and commitment.

## **2.6 Fuzzy critical path method**

According to Chanas and Zielinski (2001), critical paths assume the task time to be deterministic, but in real scenarios those are usually not true. They analyzed the criticality of each path for various cases using fuzzy logic. In the late 1900s, researches (J.J.Buckley, 1989; S.Chanas, 1982; S.Chanas and J. Kamburowski, 1981; S.Chanas and E. Radosinski, 1976; I.S. Chang, Y. Tsujimura, M. Gen, T. Tozawa, 1995, H. Prade, 1979; H. Rommelfanger, 1994; A.I. Slyeptsov, T.A. Tyshchuk, 1997; A.I. Slyeptsov, T.A. Tyshchuk, 1999, J.S. Yao, F.T. Lin, 2000) on critical path analysis was done using fuzzy logic in critical path method wherein fuzzy numbers were used to model activity times. Common deterministic numbers were replaced by fuzzy numbers which in turn had modified dependencies. The project characteristics were defined based on these properties.

## 2.7 Project Evaluation and Review Technique (PERT)

PERT overcomes the deterministic task time by assuming random variables following beta distributions to model the activity times (Malcolm, Roseboom, Clark & Fazar, 1959). Many papers have been published regarding PERT analysis ever since, but all the papers focused on the path task times following a beta distribution. PERT analysis is based on the determining activity times based on optimistic 'a', most likely 'm' and pessimistic 'b' activity times (Ginzburg, 1988).

Ginzburg (1988) concludes that the 'most likely activity time' is useless. PERT analysis is highly complex for large networks (Schoderbek, 1965). A survey was conducted based on 81 respondents which is shown in table 2.2

**Table 2.2 Disadvantages of PERT (Schoderbek, 1965)**

<b>Disadvantages</b>	<b>Number</b>	<b>% Respondents</b>
Excess of work	16	19.7
Training of personnel in PERT	9	11.1
Inflexibility of the system	54	66.6
Cost of application	19	23.5
Too much reliance on PERT	21	25.9
None	4	4.9
Other	20	24.7
Don't know	1	1.2
Not Ascertained	4	4.9



Research has not been done to calculate the change in a critical path and the probability of other paths being critical using simulation as a tool. This research aims to focus on a high variable system and identify the probability of each path being critical.

## **2.8 Lean Implementation**

According to Achanga, Shebah, Roy and Nelder (2006), productivity improvement techniques such as lean manufacturing can be used to improve the existing process. Successful implementation of lean can be achieved by enhanced critical decision processes. Their research suggests that proper funds need to be allocated to improvement processes or else no significant improvement can be seen within the company. Workforce training is also crucial to lean implementation.

Research was done in an aircraft manufacturing industry to improve the probability of completion within various workstations, so that parts complete the operations within the standard move rate. Standard move rate is the time within which the operations need to be completed. If the operations do not complete within the standard move rate, parts move to the next workstation and previous operations also take place at the next workstation. Various lean tools were used to improve the existing process in an aircraft manufacturing system. The lean metric used for the analysis was probability of completion ( $P_C$ ). Probability of completion is the probability that the processes complete within the standard time. Based on their research, the probability of completion was low, so lean tools were used to improve the probability of completion. Various strategies were used to improve the probability of completion ( $P_C$ ) to 75%, 80% and 85% all the time (Durgaparameswaran, 2010). The costs for implementation significantly increased when the probability of completion increased. Lean tools were used to improve the probability of completion of a set of tasks in an aircraft industry. Various combinations of options were

suggested based on the cost of implementing the tools and on the improvement of probability of completion. Along with an increase in the probability of completion, there was a decrease in the idle time for various tasks.

## **2.9 Research Motivation**

Critical path method is used to calculate the time taken to complete a set of processes. Since most of the research has assumed the processing time to be deterministic, this assumption is usually not true. Research has been done by using fuzzy numbers as processing times. No significant approach has been done to calculate the criticality of each path being critical using simulation techniques.

Thus, the initial objective of this research is to identify a procedure for calculating a methodology of identifying the criticality of each path. The procedure is based on the time taken to complete the processes within each path in a complex production system. This includes several thousands of operations occurring in combinations of series and parallel using simulation techniques.

The next step is to allocate a heuristic procedure to assign lean improvement tools within the processes. The tools are assigned by doing an economic study to calculate the tradeoff between the costs estimated in implementing the lean technique along with the reduced penalty cost caused due to delays in the production line. Penalty cost is any cost caused due to delays including lost order, rework cost and high work in process. Lean techniques could either be implemented in one or more than one processes based on the costs incurred by implementing the technique and the return on investment.

## CHAPTER 3

### CRITICAL PATH IDENTIFICATION METHODOLOGY

#### 3.1 Introduction

In high-variability low-volume production systems such as ship building and aircraft manufacturing, the number of stations is relatively low. However, the number of tasks assigned to each station is very high. In the aircraft industry, the number of stations may be five or six typically. But each station may have hundreds or more activities assigned to it. Some of these activities have no precedence requirements and may be performed in parallel, while others may have precedence constraints and restricts the job to a serial line. There may be several serial paths in a single station as well. Thus, there may be multiple parallel serial paths that are followed for production completion. Determining the critical path for such productionline involves the analysis of the serial lines. In general, analysis of critical path in a line with parallel processing involves calculating the time taken by each path to complete the production operations and then determining the maximum time taken. In deterministic situations, the process of identifying the critical path is relatively easy.

Based on previous research (Durgaparameswaran and Krishnan, 2010), in facilities such as aircraft industries which involve low volume production, most of the orders tend to exceed the standard move rate of each station which leads to a delay within the line. Move rate is the standard time within which the work needs to be transferred to the next workstation, irrespective of whether the job has been completed or not. When the production time exceeds the move rate, it results in losses in the form of reduced production rate, cost of additional workers, high inventory costs and lost orders. In order to reduce the completion time for a workstation, attention is usually given to the critical path. Aircraft manufacturing industry typically identifies

a single critical path based on the standard time allotted to each task. This method of identifying the critical path would be correct, if the processing times are deterministic. However, the aircraft manufacturing industry is characterized by high variability.

In high variability production systems with multiple parallel paths of production activity, the critical path may shift from one path to the next depending on the actual time of processing for each aircraft. Thus instead of a single critical path, there may be multiple critical paths with each path having a probability of being the critical path during a production run. This chapter focuses on identifying the critical paths for low volume and high variability production systems. The various notations that will be used in this chapter are in section 3.2, which is followed by the assumptions that are used in section 3.3. Critical path identification in both deterministic and stochastic cases is discussed in section 3.4. This section focuses on identifying the probability that a path is critical when it exceeds the standard time, the time exceeded by the critical path beyond the standard time, and the time exceeded by all paths beyond the standard time. The method is then used to test a case study which is discussed in section 3.5 which is finally followed by conclusions drawn from the procedure on the steps followed to calculate the criticality of paths in section 3.6.

## **3.2 Notations**

The notations used in the research are

i	Workstation number $\{i= 1,2,3,\dots,n\}$
j	Path number $\{j=1,2,3,\dots,\rho\}$
k	Process order within a path $\{k=1,2,3,\dots,n_k\}$
$P_c$	Probability of completion
$m_{ij}$	Number of times actual processing time exceeds standard time for workstation i for path j

$n_r$	Number of replications
$T$	Standard cycle time for each workstation
$t_{ij}$	Time path $j$ for workstation $i$ exceeded the standard move rate
$\alpha$	Severity ratio
$R_n$	Replication number
$P_j$	Probability of path $j$ being a critical path
$PT_{ijk}$	Processing time for path $j$ within process $k$ and workstation $i$
$P_R$	Standard processing time
$R_{ijk}$	Actual processing time for process $k$ at path $j$ in workstation $i$
$PC_{ij}$	Probability that line $j$ in workstation $i$ is the critical path
$PS_{ij}$	Probability that line $j$ in workstation $i$ is the critical path and it exceeds the standard time

### 3.3 Assumptions

The following assumptions were considered for the critical path problem:

1. The number of parts created for each run is one.
2. The distribution followed by the processing time is triangular.
3. The simulation run time was around 5000 minutes which is the maximum time taken to complete a part.
4. Warm up time is 0 because only one part is created in every run and that each run is independent of the previous one.
5. Parts at a particular workstation are completed within that workstation irrespective of the time taken.
6. Serial layout is considered.

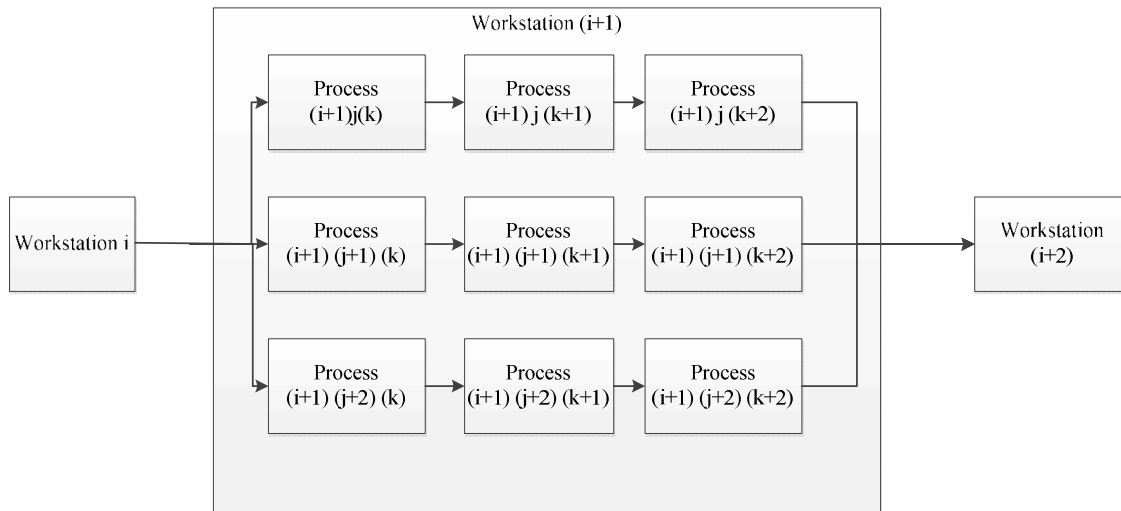
7. Tasks cannot be reallocated within the line due to large number of precedence constraints.

### 3.4 Critical path identification

This section focuses on the steps for identification of critical path in a production system for deterministic and stochastic processing times. The critical path decides the rate at which a part is processed. In order to reduce the time taken at a workstation within a production system, emphasis needs to be given on the critical path.

#### 3.4.1 Deterministic processing time

Consider a workstation with 9 operations (3 parallel paths) occurring in parallel as shown in Figure 3.1.



**Figure 3.1 Process configurations within a workstation.**

If all the 9 operations had deterministic processing times, then the critical path is the maximum of the processing times for the 3 parallel paths. The time taken at the critical path is shown in equation 3.1

$$\text{Time at Critical Path} = \max \begin{bmatrix} P_{jk} + P_{j(k+1)} + P_{j(k+2)} \\ P_{(j+1)k} + P_{(j+1)(k+1)} + P_{(j+1)(k+2)} \\ P_{(j+2)k} + P_{(j+2)(k+1)} + P_{(j+2)(k+2)} \end{bmatrix} \quad 3.1$$

But in real life cases, due to manual labor, most of the processing times follow stochastic processing times which are discussed in the next section.

### 3.4.2 Stochastic processing time

Most processing times are not deterministic and follow stochastic values. This is due to involvement of labor intensive production systems. Suer (1996) classified production systems based on the workers and the machines present in the industry. According to Bi and Zhang (2001) manual labor is accurate when the operations are highly variable. Also, due to cost factors and flexibility within the workstation manual labor is preferred in job shop production, although, manual labor leads to variability in the system (Mucklow et al., 1980). Variability exists in the system due to several factors such as delays due to operators (man), machine setups, raw material outages, maintenance and other factors. Variability can cause a huge difference in the task times for a particular process. The coefficient of variability is the ratio of the standard deviation and the mean (Hopp & Spearman, 2001). Coefficient of variation ( $C_v$ ) is used to measure variability. Based on coefficient of variation, variability is classified into three types as shown in Table 3.1. When there is variability in the system, then more than one path tends to be critical during multiple runs.

**Table 3.1 Types of variability (Hopp, & Spearman, 2001)**

Variability Type	Coefficient of variation ( $C_v$ )
Low variability	$C_v < 0.75$
Medium Variability	$0.75 < C_v < 1.33$
High Variability	$C_v > 1.33$

Assuming a triangular distribution for task times, the time taken at the critical path is the summation of individual triangular distributions. Since summation of many triangular distributions is complex, simulation is done.

A procedure that focuses on identifying critical paths and prioritizing them in order to reduce costs due to delay from the standard time is given below.

### 3.4.3 Identification of critical path(s)

A path that takes the most time for completing the given set of operations within a workstation is the critical path. This section aims to identify the probability of each path being critical. Critical path is calculated by summation of individual task times within each path and is the path that takes the maximum time. The mathematical formulation for calculation of critical path is shown in the next section.

#### 3.4.3.1 Mathematical formulation for identifying critical paths in Stochastic Systems

The expected time for processing each path within a workstation for all the workstations can be calculated in equation 3.2.

$$\text{Expected time for line } j \text{ within workstation } i, y_{ij} = \sum_{k=1}^{n_k} T_{ijk} X_{ijk} \quad \forall \begin{matrix} j = 1, 2, \dots, \rho \\ i = 1, 2, \dots, n \end{matrix} \quad 3.2$$

where,

$$X_{ijk} = \begin{cases} 1 & \text{when part is processed in process } k \text{ with line } j \text{ for workstation } i \\ 0 & \text{otherwise} \end{cases}$$

$X_{ijk} = 1$  when the part is in workstation 'i', within the 'j' line and in the 'k' process.

$T_{ijk}$  is processing time for process k within line j for workstation k.

Based on the expected time for each process, the time taken at the critical path within each workstation can be calculated using equation 3.3

$$S_i = \max (y_{i1}, y_{i2}, \dots, y_{ij}, \dots, y_{in\rho}) \quad \forall i \quad 3.3$$



Time at the critical path  $T_i$  for all the workstations is

$$T_i = \sum_{i=1}^n S_i \quad 3.4$$

When the processing time is stochastic, there is no guarantee that the critical path that is identified will remain as the critical path at all times. The variability in task times will result in other paths being critical.

Analytical methods can be used to predict the probability of each path being critical. However, identifying the critical path using analytical; process is tedious. In the case of triangular distribution, analytical models cannot be used. Hence, simulation is the best choice for predicting probabilities of critical path.

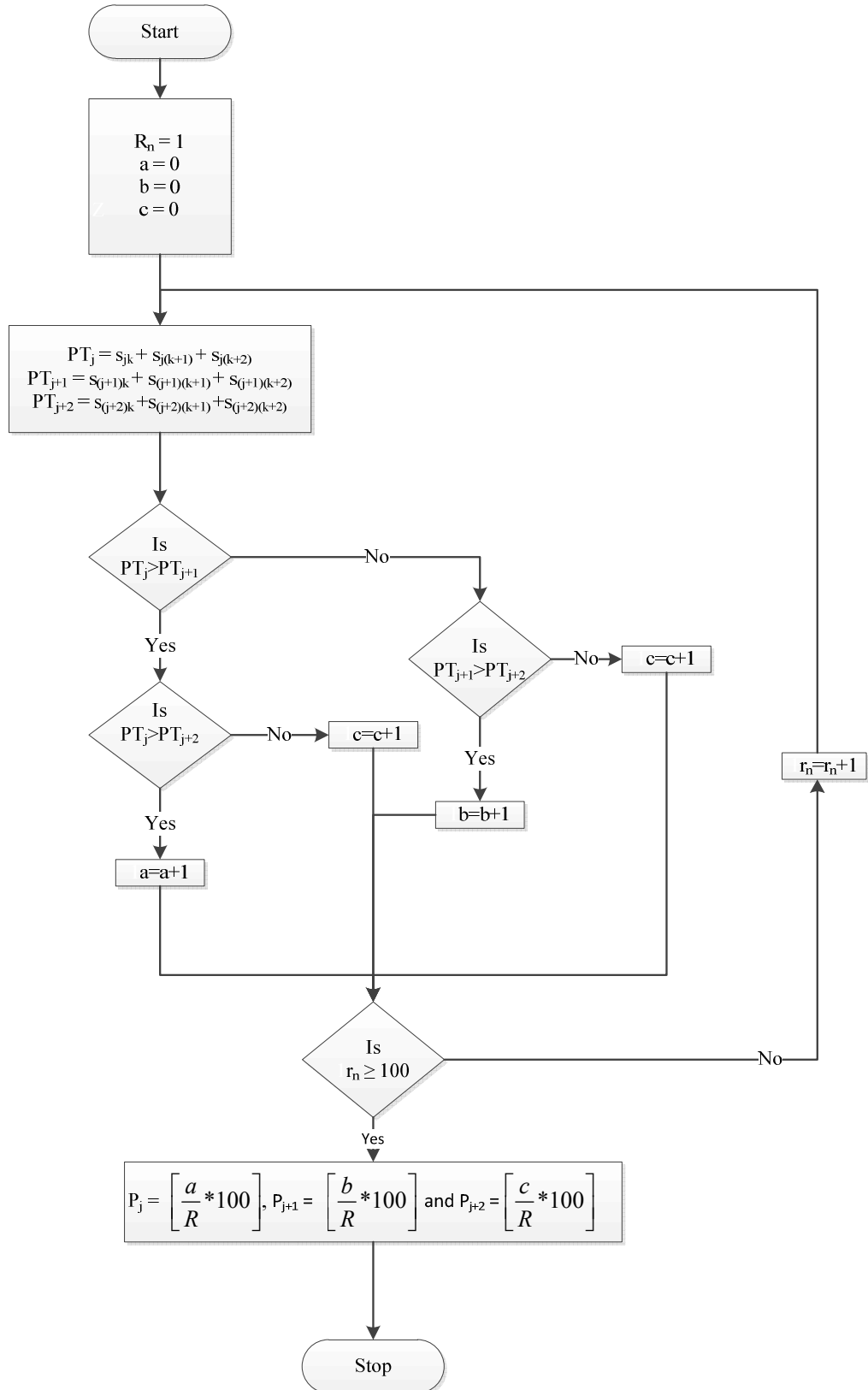
Since the processing time for each process within the workstations follow a triangular distribution, the total time taken for completion of each line within the workstation would involve convolution of the triangular distributions. Due to the complexity in the modeling of convolutions, especially when using triangular distribution, analytical models may not be easy to implement. As a result, simulation is used to determine the probability of each critical path.

#### **3.4.3.2 Algorithm**

- Step 1:        Start
- Step 2:        Calculate the processing time for all the three paths by adding the individual processing times.
- Step 3:        If the processing time along  $j^{\text{th}}$  path is greater than the processing time of  $(j+1)^{\text{th}}$  path then go to Step 5.
- Step 4:        If the processing time along  $(j+1)^{\text{th}}$  path is greater than the processing time of the  $(j+2)^{\text{th}}$  path then go to Step 7.

- Step 5: If the processing time along  $j^{\text{th}}$  path is greater than the processing time of  $(j+2)^{\text{th}}$  path then go to Step 6 else go to Step 8.
- Step 6: Increment  $j$  path.
- Step 7: Increment  $j+1$  path.
- Step 8: Increment  $j+2$  path.
- Step 9: Increment the replication number.
- Step 10: Goto step 2 if the simulation run is less than 100.
- Step 11: Calculate the probability of paths  $j$ ,  $j+1$  and  $j+2$  being critical
- Step 12: Stop

### 3.4.3.3 Flowchart



### 3.4.4 Move rate

Standard processing time is the standard time taken to complete the processes within a workstation. As mentioned in the previous sections, due to variability, a process might exceed the standard time to a great extent. Also, a critical path might not exceed the standard time. This move rate metric aims to capture a critical path only when a processing time exceeds the standard time alone. This metric neglects the probability of a path being critical when the total time taken to complete the process is within the standard time for that workstation. The mathematical model used to calculate the move metric is shown in the below.

#### 3.4.4.1 Mathematical formulation

The calculation of critical path and estimation of processing time for each workstation can be calculated using equations 3.2 to 3.4.

However, calculating the probability of each path being critical based on the standard move rate is shown in equation 3.6.

The probability that line 1 in workstation 1 is the critical path and it exceeds the standard time is represented as  $PS_{11}$ . The standard move rate for each workstation is  $t_s$  minutes.

$$\begin{aligned} PS_{11} &= P(y_{11} > y_{12}, y_{11} > y_{13} \text{ and } y_{11} > t_s) \\ &= P(y_{11} > y_{12}) * P(y_{11} > y_{13}) * P(y_{11} > t_s) \end{aligned} \quad 3.6$$

The probability that the remaining paths of workstations 1, 2 and 3 exceed the standard time is calculated.

$$PS_{12} = P(y_{12} > y_{11}) * (y_{12} > y_{13}) * P(y_{12} > t_s)$$

$$PS_{13} = P(y_{13} > y_{11}) * (y_{13} > y_{12}) * P(y_{13} > t_s)$$

$$PS_{21} = P(y_{21} > y_{22}) * (y_{21} > y_{23}) * P(y_{21} > t_s)$$

$$PS_{22} = P(y_{22} > y_{21}) * (y_{22} > y_{23}) * P(y_{22} > t_s)$$

$$PS_{23} = P(y_{23} > y_{21}) * (y_{23} > y_{22}) * P(y_{23} > t_s)$$

$$PS_{31} = P(y_{31} > y_{32}) * (y_{31} > y_{33}) * P(y_{31} > t_s)$$

$$PS_{32} = P(y_{32} > y_{31}) * (y_{32} > y_{33}) * P(y_{32} > t_s)$$

$$PS_{33} = P(y_{33} > y_{31}) * (y_{33} > y_{32}) * P(y_{33} > t_s)$$

Due to the complexity of combining more than two triangular distributions, simulation is done.

### 3.4.4.2 Algorithm

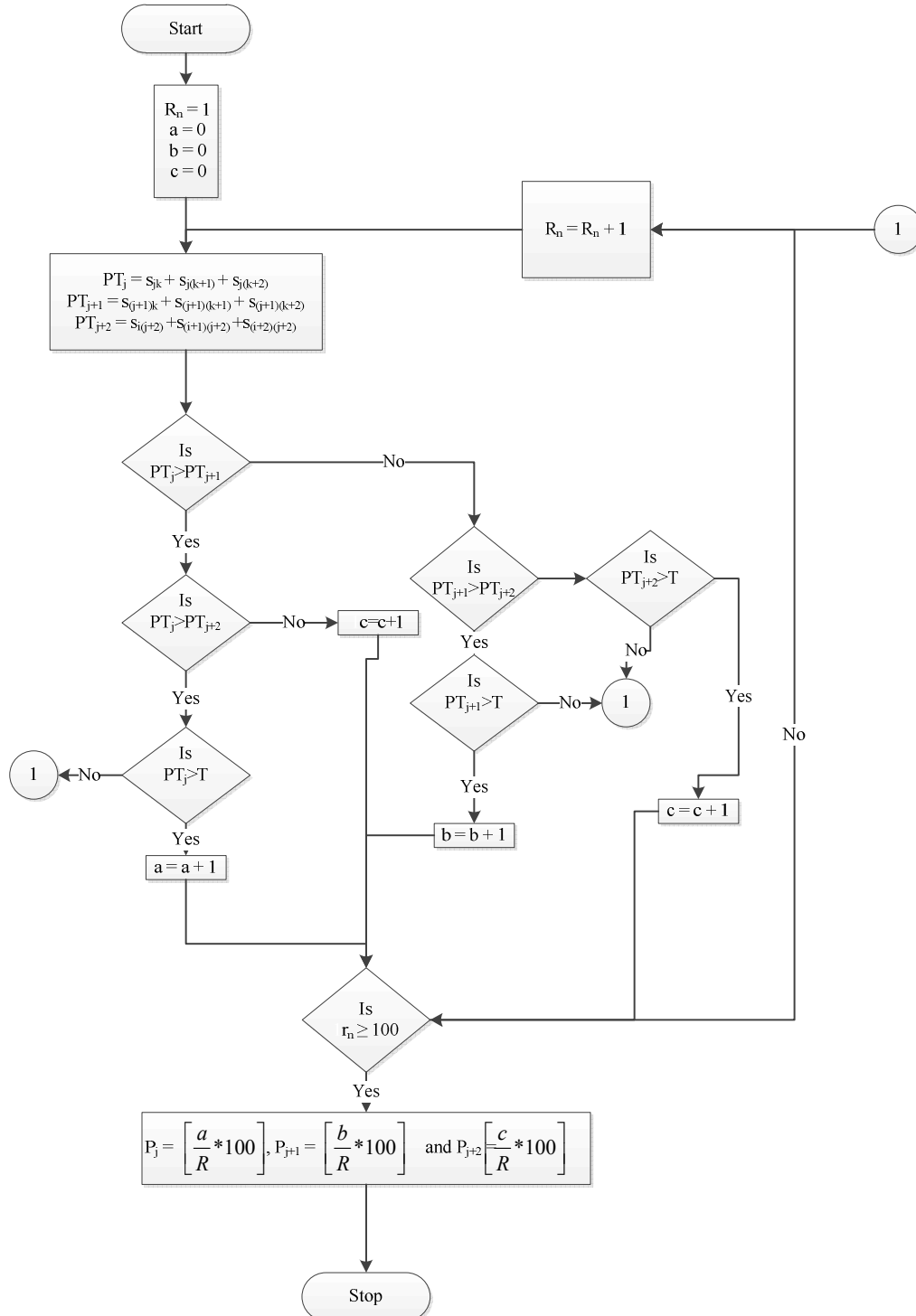
- Step 1: Start
- Step 2: Calculate the processing time for all the three paths by adding the individual processing times.
- Step 3: If the processing time along  $j^{\text{th}}$  path is greater than the processing time of  $(j+1)^{\text{th}}$  path .then go to Step 5.
- Step 4: If the processing time along  $(j+1)^{\text{th}}$  path is greater than the processing time of the  $(j+2)^{\text{th}}$  path then go to Step 7.
- Step 5: If the processing time along  $j^{\text{th}}$  path is greater than the processing time of  $(j+2)^{\text{th}}$  path then go to Step 6 else go to Step 8.
- Step 6: If  $j^{\text{th}}$  path is greater than the cycle time goto step 9.
- Step 7: If  $(j+1)^{\text{th}}$  path is greater than the cycle time goto step 10.
- Step 8: If  $(j+2)^{\text{th}}$  path is greater than the cycle time goto step 11.
- Step 9: Increment a. Calculate the increase in processing time. Goto step 12.
- Step 10: Increment b. Calculate the increase in processing time. Goto step 12.
- Step 11: Increment c. Calculate the increase in processing time. Goto step 12.

Step 12: Increment the replication number.

Step 13: Goto step 2 if the simulation run is less than 100.

Step 14: Stop

### 3.4.4.3 Flowchart



### 3.4.5 Critical Path Severity Index (CSI)

The previous metrics focused on calculating the probability of a path being critical and the probability of a path being critical beyond the standard time. No emphasis was given to the time with which the paths exceeded the critical path. The critical path severity metric focuses on calculating the average time a critical path exceeds from the standard time by using a critical path severity index. Critical Path Severity Index (CSI) is a scale for classifying the priority of critical paths exceeding the standard time in manufacturing industries. It is calculated by calculating the average of the times a process exceeded the standard time as shown in equation 3.9.

$$\text{Critical path severity index for path } j (\alpha_j) = \frac{\sum t_{ij}}{m_{ij}} \text{ (in minutes)} \quad 3.9$$

where,

$t_{ij}$  = time exceeded (in minutes) by critical path  $j$  for workstation  $i$ ,  $\forall i = 1, 2, 3, \dots, n$

$m_{ij}$  = number of occurrences in which the processing time exceeds the standard time of the workstation

**Table 3.2 Variability based on CSI**

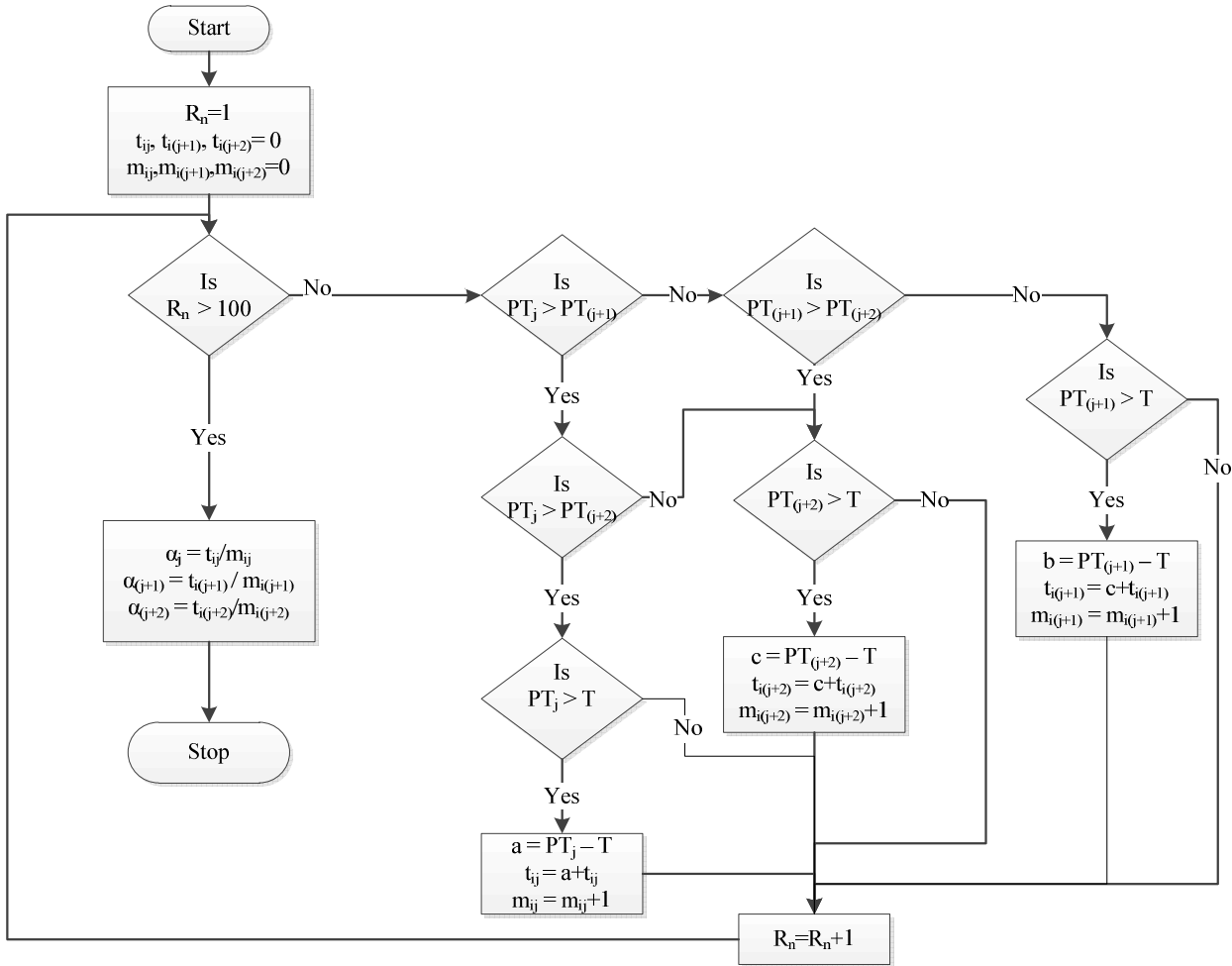
CSI (mins)	Priority
$CSI < t_l$	Low priority
$t_l \leq CSI < t_h$	Medium priority
$CSI \geq t_h$	High priority



### **3.4.5.1 Algorithm**

- Step 1: Start
- Step 2: Check which path is critical
- Step 3: Check whether the critical path exceeds the standard cycle time of the machine
- Step 4: Calculate the time the critical path exceeds the standard cycle time
- Step 5: Take the average time a path exceeds from the standard cycle time
- Step 6: Increase the replication number and if the number of replications is less than 100  
then Goto step 2
- Step 7: Calculate the severity index for each path
- Step 8: Stop

### 3.4.5.2 Flowchart



### 3.4.6 Severity Index for paths ( $\beta$ )

The previous index focused on the average time a critical path exceeded from the standard cycle time. There is a probability that more than one path might exceed the standard time. This metric aims to take the average of the time all the paths exceed the standard time. The paths may or may not be critical, it only depends on the time a path exceeded from the standard cycle time.

$$\text{Severity Index for path } j (\beta_j) = \frac{\sum \tau_{ij}}{m_{ij}} \text{ (in minutes)} \quad 3.10$$

where,

$\tau_{ij}$  = time exceeded from the standard time(in minutes) by path j for workstation i,  $\forall i =$

1,2,3,...,n

$m_{ij}$  = number of occurrence in which the processing time exceeds the standard time of the workstation

The procedure followed to calculate the severity index is shown in the algorithm and flowchart below.

#### **3.4.6.1 Algorithm**

Step 1: Start

Step 2: Identify the critical path for each replication

Step 3: If the critical path exceeds the standard cycle time, go to step 4 else go to step 6

Step 4: Calculate the time the critical path exceeded from the standard cycle time

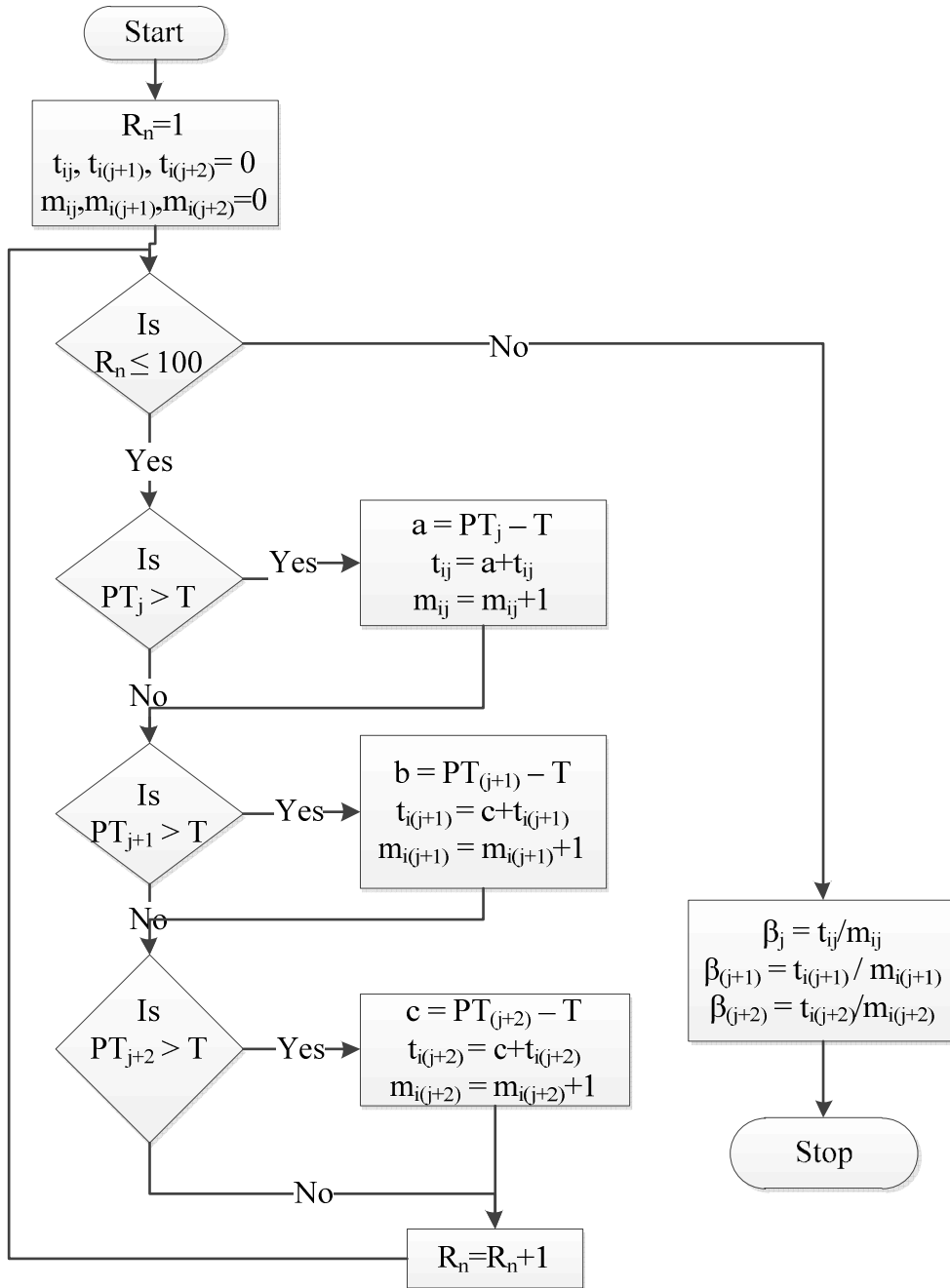
Step 5: If the other paths exceed the standard cycle time, calculate the average time exceeded by the path

Step 6: If the replication number is lesser than the total number of replications, goto step 1

Step 7: Calculate the average time exceeded by all the paths beyond the standard time

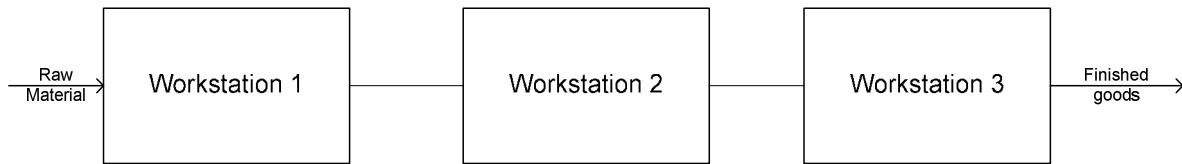
Step 8: Stop

### 3.4.6.2 Flowchart



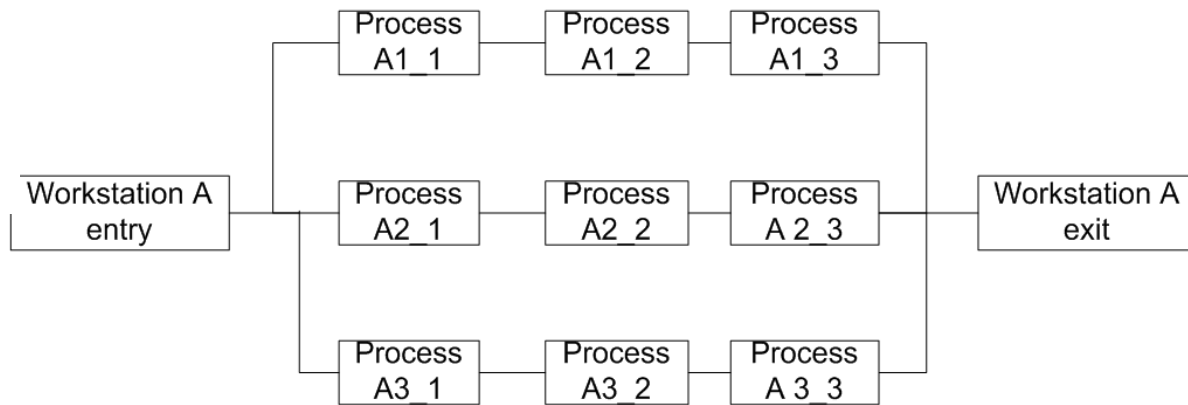
### 3.5 Case Study – 3 Workstation – 27 Processes

The methodology is verified using a case study which comprises of 3 workstations arranged in line and each comprising of 9 processes. The processes involve machining and assembly operations. The workstations are arranged in series layout as show in Figure 3.2.



**Figure 3.2 Line diagram for the production system**

The line diagram for a workstation is shown in the Figure 3.3.



**Figure 3.3 Line diagram of workstation 1**

The line diagrams for the second and third workstations are similar. Each workstation has 3 parallel paths and every path has 3 processes occurring in series. For a part to exit a workstation, it needs to complete all the processes occurring within the workstation.

The processing time for each process follows triangular distribution for workstations 1, 2 and 3 which are shown in Tables 3.3(a), (b) and (c) respectively.

**Table 3.3 (a) Processing times for workstation 1**

	Workstation 1 (i= 1)		
	Process 1	Process 2	Process 3
Path 1 (j=1)	1400,1550, 1650	1400,1448,1652	1402,1552,1653
Path 2 (j=2)	1370,1470,1700	1370,1471,1700	1470,1470,1700
Path 3 (j=3)	1350,1400,1750	1350,1400,1750	1350,1400,1750

**Table 3.3 (b) Processing times for workstation 2**

	Workstation2 (i =2)		
	Process 1	Process2	Process3
Path 1 (j=1)	1400,1440,1650	1400,1550,1650	1400,1550,1650
Path 2 (j=2)	1370,1470,1700	1372,1468,1703	1372,1468,1698
Path 3 (j=3)	1350,1401,1749	1351,1399,1749	1348,1405,1748

**Table 3.3 (c) Processing times for workstation 3**

	Workstation 3 (i= 3)		
	Process 1	Process 2	Process 3
Path 1 (j=1)	1401,1553,1657	1398,1551,1653	1401,1552,1648
Path 2 (j=2)	1371,1403,1752	1368,1401,1751	1368,1398,1753
Path 3 (j=3)	1351,1402,1753	1348,1398,1751	1351,1397,1748

All processing times are in minutes.

### 3.5.1 Deterministic case

If the data is assumed to be deterministic, path 1 is the critical path for all the workstations. In order to produce the parts within the standard time of 4600 minutes, focus needs to be given to path 1 for workstations 1, 2 and 3. Other paths need not be considered to improve the line.

But in real cases, processing time is mostly stochastic and tends to follow some distribution. The data obtained followed triangular distribution.

### 3.5.2 Stochastic case

Since the data followed a triangular distribution, the processing time for a particular line is the sum of all the individual processes within the line. To calculate the critical path for each time involved summing up for each line within a workstation leading to three distributions within each of the workstation. Calculation of the summed distribution for the entire line leads to 27 different distributions. Thus, simulation was done.

The number of replications done is calculated using half width calculation as shown in equation 3.11.

$$N_r = t_{\frac{\alpha}{2}, n-1}^2 \frac{\sigma^2}{h^2} \quad 3.11$$

Based on the half width analysis, and an H of  $\pm 17.5$  minutes, the number of replications required is 100. The simulation is done using discrete event simulation QUEST (QUEuing Event Simulation Tool) software. Simulation is run for 6000 minutes. It is assumed that the task needs to be completed within that workstation before the part is moved to the next workstation irrespective of the move rate.

The number of parts produced per run is one. This case study is done for parts that have high variability and low volume. Based on this, the average processing time per run was found to be about 4619 minutes.

### 3.5.2.1 Critical path due to variability

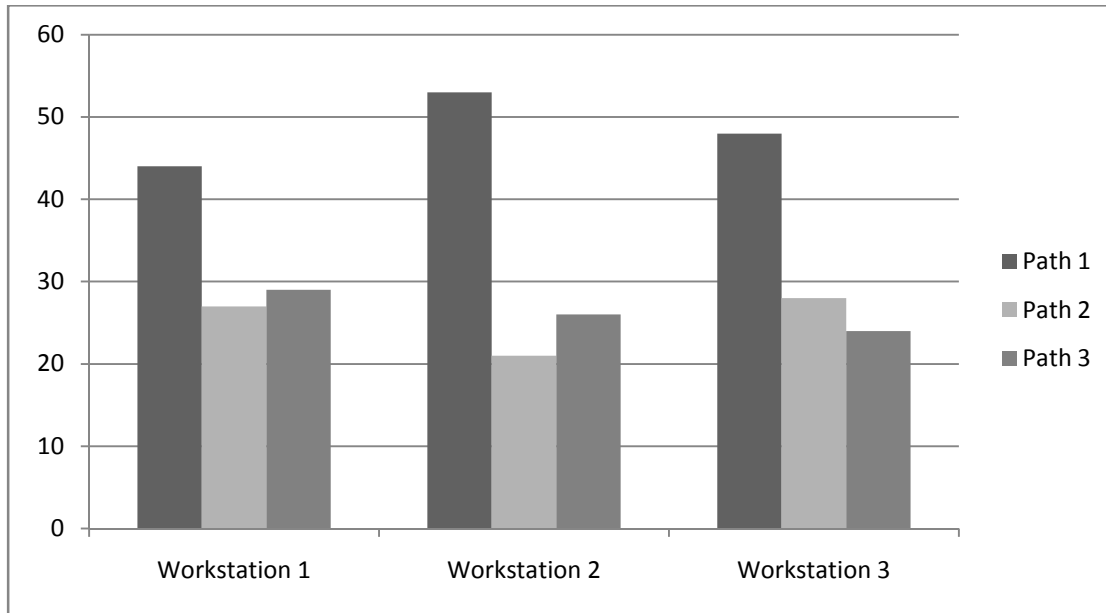
Under deterministic conditions, path 1 is the critical path. However, due to variability all the paths could tend to be a critical path for multiple runs. For the first workstation, the probability that a path was a critical path is shown in Table 3.4. As seen in the table, path 1 has a 44% probability of being the critical path. However, path 2 has a 27% probability and path 3 has a 29% probability.

**Table 3.4 Critical path for workstation 1**

Path	Probability of Critical Path
1	44 %
2	27 %
3	29 %

So, emphasis needs to be given to path 1 for reducing the probability of path 1 being critical. But, paths 2 and 3 can also be critical paths. If focus is given only to path 1 to reduce the completion time, then paths 2 and 3 would become the critical path. The probability for a path being critical for all the workstations is shown in Figure 3.4.





**Figure 3.4 Probability of path being critical for all the workstations**

One of the objectives is to successfully implement techniques to paths which may or may not be critical paths all the time, so as to reduce the average processing run of 4619 minutes closer to the expected move rate of the workstation with the available resources, such as man, machine and money.

### **3.5.2.2 Critical path shift based on standard time**

As discussed in the previous section, path 1 tends to be the critical path most of the time. But, the standard time for each workstation is 4600 minutes. The previous method did not consider whether the critical path exceeded the standard time. One of the objectives is to try to reduce the total cycle time to the standard time of the workstation. So, emphasis is given only when the total cycle time exceeds the standard time of the line. Based on the standard method, paths that completed the process before 4600 minutes were not considered. Only the parts that delayed beyond the standard time and the total time exceeded per path per run are considered.

### 3.5.2.3 Frequency beyond standard time

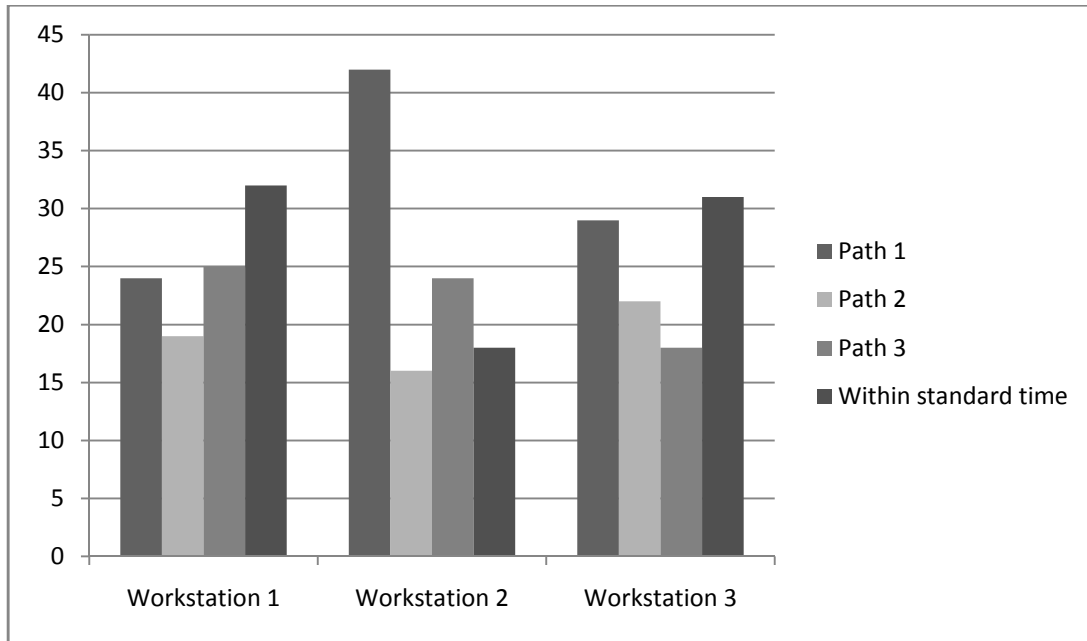
Critical paths that exceeded the standard cycle time are considered irrespective of the time it exceeded the standard cycle time. The frequency of the standard cycle time exceeding the standard cycle time is shown in Table 3.5.

**Table 3.5 Frequency of a critical path exceeding the standard cycle time**

Path number	No of times exceeded	Percentage of Path being critical	Probability that path is critical and exceeding the standard time
1	24	35%	24%
2	19	28%	19%
3	25	36.8%	25%

$\Sigma$  68%

The total number of times a critical path's processing time exceeded the standard cycle time is 68 times from the 100 replications. From the table, it can be inferred that the path one again tends to be more critical. The probability of a critical path exceeding the standard time for all the workstations is shown in Figure 3.5.



**Figure 3.5 Probability of a critical path exceeding the standard time**

Emphasis needs to be given on how much delay is caused by the critical path beyond the standard time. This was calculated by summing up the difference in the actual processing time for critical path and the standard cycle time. Based on that the total number of minutes the path exceeded the standard move rate is calculated.

For example in replication 1 workstation 1,

$$(\text{Critical Path})_{\text{replication 1}} = \max \begin{bmatrix} 4695.2 \\ 4535.4 \\ 4467.6 \end{bmatrix}$$

Time taken by critical path = 4695.2

As the time taken at the critical path is greater than the standard time, that path is considered.

$$P_A \text{ is the critical path} = \frac{\text{no of times path 1 exceeded the standard time}}{\text{no of occurrences}}$$

$$= \left( \frac{24}{68} \right) * 100$$

$$= 35.3\%$$

### 3.5.2.4 Critical path severity index (CSI)

The next step is to calculate the severity ratio, which is the ratio of the total number of minutes a path exceeded the standard time and the number of times it exceeded for workstation i and path j. The critical path severity index for path 1 for the first workstation is shown.

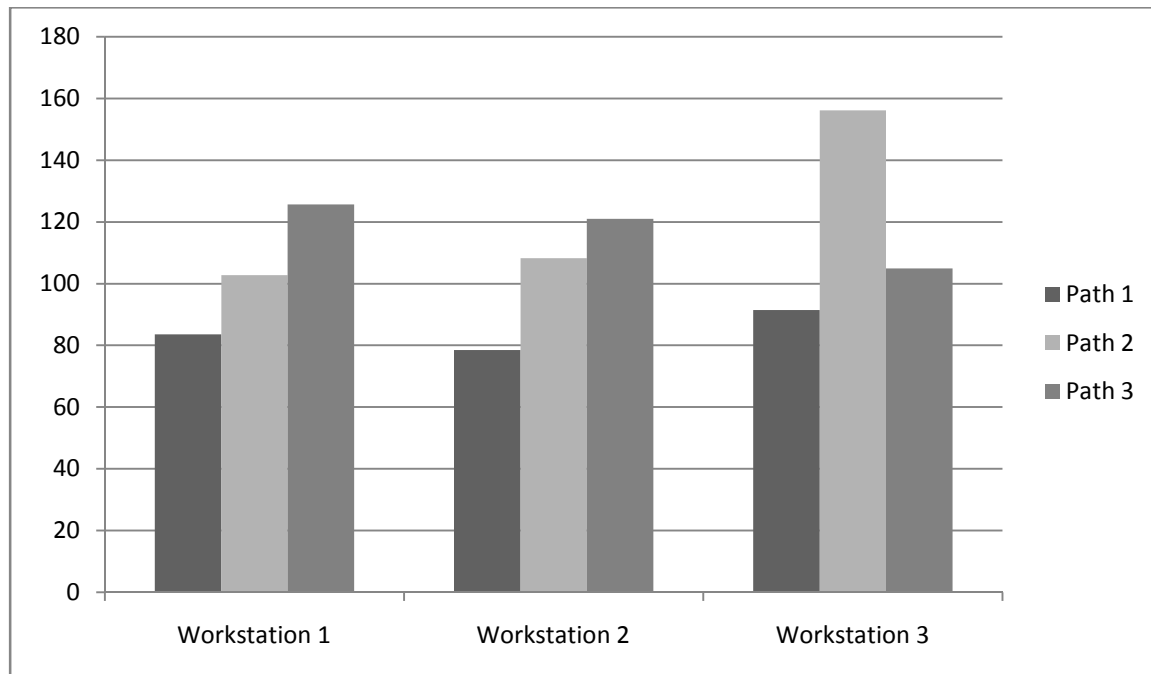
$$\begin{aligned}\text{Critical path severity index for path 1 } (\alpha_1) &= \frac{1742.238}{24} \\ &= 83.5\end{aligned}$$

The severity ratios for paths 1 to 3 are shown in Table 3.6

**Table 3.6 Critical path severity index for workstation 1**

Path	Total Exceeded Time (minutes)	No of times exceeded	Critical path severity index (minutes)	Percentage contribution (%)
1	2005.2	24	83.6	26.8
2	1953.2	19	102.8	32.9
3	3143.6	25	125.7	40.3

From the table, it can be inferred that the total number of times a critical path exceeds the standard time is 24 (it is the summation of the number of times exceeded). The probability that a critical path exceeds the standard time is 24%. The critical path severity index for all the workstations are shown in Figure3.6.



**Figure 3.6 Critical path severity indexes for all the workstations**

From the graph, it is clear that the severity index for path 3 for all the workstations are the maximum.

#### 3.5.2.5 Severity index for all paths

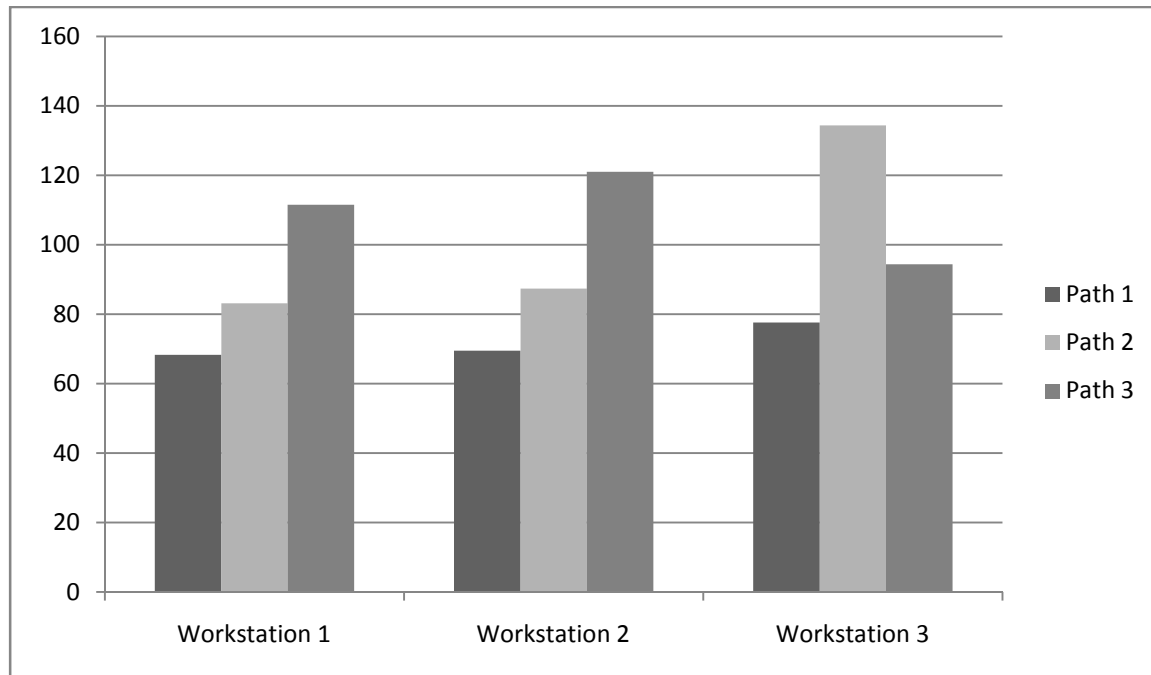
The last section is focused on the difference in the time exceeded by all the paths from the standard time. At times, paths other than the critical path could also exceed the standard cycle time. This metric focuses not only on the critical paths that exceed the standard cycle time, but also the non-critical paths that exceed the standard cycle time. The severity index for all paths for workstation 1 is shown in Table 3.7.

$$\begin{aligned}
 \text{Severity index for path 1 } (\beta_1) &= \frac{2663}{39} \\
 &= 68.3 \text{ minutes}
 \end{aligned}$$

**Table 3.7 All path severity index for workstation 1**

Path	Total Exceeded Time (minutes)	No of times exceeded	Severity index (minutes)
1	2663	39	68.3
2	2493.6	30	83.1
3	3343.7	30	111.5

Based on the severity index, it can be found that path 3 tends to be more critical than paths 1 and 2 as the average time exceeded by path 3 is higher than the other two. So, if improvement needs to be given based on time taken to complete the job, initial emphasis needs to be given to path 3, because even though the probability of path 3 being a critical path is not large, the average time exceeded by path 3 when is critical is greater than an hour.



**Figure 3.7 All path severity indexes for all the workstations**

**Table 3.8 Comparison of the Metrics**

<b>Metric</b>	<b>Order of dominance</b>
Probability of Critical Path	1-3-2
Based on move rate	1-3-2
Critical Path Severity Index	3-2-1
All path severity index	3-2-1

### **3.6 Conclusion**

In this chapter, new critical path identification methods are proposed to help improve the probability of completion of processes and measures such as critical path severity index, all path severity index and critical path due to standard time are proposed. Various methods are used to identify the critical path, such as what is the probability of a path being critical and the average time a path exceeded the standard time. From the metrics, it can be inferred that path 1 for all the workstations tends to be the critical path most of the time, although, when the average time a path exceeds the standard cycle time is considered, path 3 tends to have a larger value with respect to all the other paths for the workstations. Based on these methods, suitable suggestions can be given to various lines for improvement. Methods to improve the line and allotment of the improvement methods and the steps taken are suggested in the later chapters.

## **CHAPTER 4**

### **LEAN IMPLEMENTATION**

#### **4.1 Introduction**

The previous chapter explains about situations used to improve a production system which has low volume and high variability scenarios such as an aircraft manufacturing industry. Such industries have thousands of high variable operations occurring in series and in parallel. Some tasks could either take a few minutes to complete or can also take several hours or days. The variability is due to in availability of resources such as parts, labor and machines. In a production system which has hundreds of operations, resources are needed in most of the operations to reduce variability. But, due to limitations in cost and space, simply increasing resources for all the variable operations is not feasible. The previous chapter talks about cases where critical paths change over a period of runs due to variability and considers various metrics that can be used to capture the variability of paths. This chapter focuses on using these metrics as a tool to identify which improvement on which process will give the best results. Using this technique, an industry will follow a systematic procedure for bringing an improvement and will have control over the benefits of implementing an improvement technique. This will also help an industry in tackling problematic processes based on a decreasing order of criticality.

#### **4.2 Assumptions**

The assumptions for lean implementation are

1. All lean experiments are independent of each other (the results of a lean tool on one process would not change the results in another process).
2. Each process can undergo only one lean experiment.



3. Only one lean experiment can be implemented in one line.
4. The improvement can either be addition of extra labor, material or machinery.

### 4.3 Model input requirements

The proposed model requires the following input data for testing the lean implementation approach:

1. Penalty cost for delay in parts
2. Precedence constraints in parts
3. Cost of lean experiments for all the workstations

### 4.4 Mathematical Model

Implementing the lean experiments would involve identifying the workstation with the minimum probability of completion. Calculation of minimum probability of completion can be done using equation 4.1.

$$P_{c_i} = \frac{\# \text{ of simulation runs} - \sum_j \# \text{ of times path } j \text{ is critical and beyond standard time}}{\# \text{ of simulation runs}} \quad 4.1$$

After identifying the workstation with the least probability of completion, the next step is to capture the worst path within that workstation. The formulation used to calculate the critical path within a particular workstation is shown in equation 4.2.

$$\text{critical path } j \text{ for } i = \max \begin{matrix} (\# \text{ of times } j \text{ is critical and exceeds standard time} \\ * \text{ average time exceeded}) \end{matrix} \quad 4.2$$

Identifying the worst process would involve simulating the improvement on each process and calculating the magnitude of improvement based on penalty cost. The process that shows the

maximum improvement is chosen for improvement. Equation 4.3 is used for calculating the improvement on each process.

$$\text{Improvement for path } j = \frac{\Delta \text{Improvement after lean for path } j * \text{penalty cost}}{\text{number of replications}} \quad 4.3$$

The improvement for all the processes within the path is calculated using equation 4.3 and the maximum of the given values is chosen. The next step is to calculate the payback period based on the cost of lean implementation which is shown in equation 4.4.

$$\text{Pay back period} = \frac{\text{cost of lean implementation}}{\text{improvement for path } j} \quad 4.4$$

After calculating the improvement in the workstation, repeat the simulation and identify individual probability of completion and repeat the processes.

#### **4.5 Case Study: High variability 3 workstations – 9 paths and 27 processes**

The case study has been done on the 3 workstations, 9 paths and 27 process systems. Each workstation has 3 paths that are in parallel. The paths are independent of each other. Each path has 3 processes occurring in series and they are independent of each other. However, for the part to move to the next workstation all the processes within the paths need to be completed. Calculation of probability of critical path for each path is explained in the previous chapter. The lean experiments for each path for all the processes are shown in table 4.1.

Each path has a unique improvement technique which has different costs to implement and have different benefits. The improvement is reduction in processing time or reduction in the variability.

**Table 4.1 Cost of Lean Experiment and improvement**

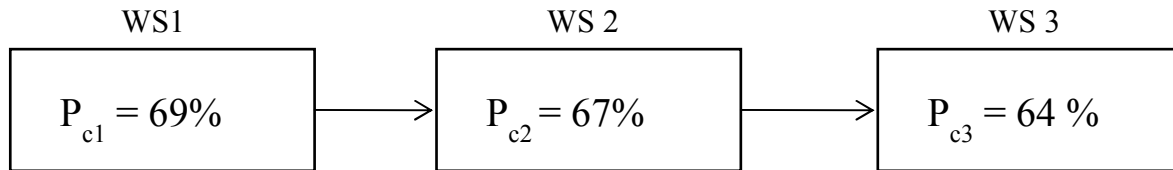
Lean experiment	Reduction in time	Cost to implement
1	(a-230,b-55,c-134)	\$ 160,000
2	(a-270,b-40,c-160)	\$ 240,000
3	(a-210,b-50,c-170)	\$ 150,000
4	(a-250,b-45,c-170)	\$ 200,000
5	(a-215,b-40,c-140)	\$ 155,000
6	(a-200,b-45,c-120)	\$ 115,000
7	(a-230,b-30,c-140)	\$ 175,000
8	(a-220,b-30,c-150)	\$ 100,000
9	(a-300,b-60,c-200)	\$ 300,000

The column reduction in time says that if lean experiment 1 is implemented, there would be a reduction in the minimum value by 230 minutes, maximum value by 55 minutes and in the mode by 134 minutes. Similarly, improvements from implementing the other lean experiments can be obtained using the above table. Each of this experiment can be any improvement such as adding men, equipment or improving the availability of raw material.

The first step is to identify the workstation with the least probability of competition. Probability of completion is the probability that the workstation would complete the operation within the standard time required to complete the given set of operations within that workstation. The formulation for calculating the probability of completion is calculated from equation 4.1. Probability of completion for workstation 1 is shown below.

$$PC_1 = \frac{100 - 31}{100} = 69\%$$

Similarly, probability of completion for each workstation is calculated and is shown in figure 4.1. The probability of completion for the entire production system is the product of probability of completion of individual workstations.



**Figure 4.1 Probability of completion for all workstations**

Attention is given to workstation 3 because of the minimum probability of completion of 64%. The next step is to identify the worst path in workstation 3. This is calculated as the maximum of the product of the average time each critical path exceeds the standard move rate (Metric 2) and the number of times the critical path exceeds the standard move rate (Metric 3). This is calculated using table 4.2.

**Table 4.2 Identification of the worst process**

	<b>Metric 2</b>	<b>Metric 3 (in minutes)</b>	<b>M2*M3</b>
Path 3 a	11%	45.1	496.54
Path 3 b	16%	92.6	1481.9
Path 3 c	10%	58.5	584.5

Based on the results shown in table 4.2, path 3 b is the worst path as it has the maximum amount of time exceeded from the standard move rate. The final step is to calculate the process that would be benefited the most with lean implementation, which is calculated by identifying the maximum of the product of the reduction probability of critical path and the reduction in the average time exceeded. The improvement for process is shown using equation 4.5.

$$\text{Improvement} = \frac{\Delta \text{Metric 2} * \Delta \text{Metric 3} * \text{Penalty cost}}{\text{Number of replications}} \quad 4.5$$

$$\text{Improvement} = \frac{10 * 15.7 * 10,000}{100} = \$15,700$$

The results for the reduction in the probability of each path being critical and the average time exceeded from the standard move rate are obtained from the simulation. The penalty cost per minute due to delay within any workstation is assumed to be \$10,000. The payback period is then calculated from the cost of implementing the improvement tool in that process and the minimum payback period is taken. The values of the payback period and the reduction of probability of critical path and the average time exceeded are shown in table 4.3.

**Table 4.3 Improvement for path b in workstation 3**

<b>Process</b>	<b>Δ Metric 2</b>	<b>Δ Metric 3 (minutes)</b>	<b>Payback period (months)</b>
1	10	15.7	6
2	8	12.3	11
3	1	46.98	21

Once the improvement has been done, apply the simulation again and calculate the least probability of completion and the above steps are repeated to identify the worst process within a particular line.

The results are then studied based on improvement on the most critical process which is shown in table 4.4.

**Table 4.4 Improvement based on criticality**

Lean Experiment	Probability of completion
8	32.8 (69%*67%*71%)
4	39.7
3	43.1
9	47.4
2	50.5
7	57
6	62.7
1	68.9

The probability of completion which is shown in table 4.4 can be obtained by taking the product of the probability of completion of each workstation since all the workstations are in a serial layout. The results obtained from table 4.4 are based on the criticality of parts. In other words, this method solely shows improvement based on how critical the line is. However, most of the time, improving a line might have more costs than the losses due to criticality of the line, or in a few cases, it could be more beneficial to improve other lines first. Due to other factors required to implement an improvement such as cost of improvement, the next table considers lean experiments based on the payback period. Payback period considers the cost of a particular improvement technique and it compares the cost of implementing the technique with the benefits in terms of reduction of penalty costs and calculates the period in which the investment can be regained. The results based on the rate of return are calculated and is shown in table 4.5.

The payback period is calculated using equation 4.6.

$$\text{Pay back period} = \frac{\text{Cost of implementing the improvement}}{\text{Reduction in penalty cost per month}}$$

4.6

For calculating the payback period for the path a at workstation 3

$$\text{Pay back period for workstation 3 path b} = \frac{100,000}{15,700} = 7 \text{ months}$$

**Table 4.5 Lean implementation based on payback period**

Lean Experiment	Payback period(months)
4	3
7	5
6	5
9	7
8	6
3	7
2	10
1	21

Table 4.5 explains about selection of lean experiment based on the payback period for each improvement technique. This chapter proposes a methodology of applying improvements either by just considering the criticality of each path. It also helps identify a method to apply improvements by considering the costs to implement an improvement by considering the payback period.

## **CHAPTER 5**

### **STUDY OF CRITICAL PATH BASED ON VARIABILITY**

The previous chapters showed methods to identify critical paths and implement improvements based on cost of implementation and rate of return on investment. The study was done for high variability conditions. This chapter studies the effect of the proposed methodology for improving the line based on changes in variability. In the previous chapter, the analysis was done on high variability scenarios. This chapter aims to study the effect of improving the line, with lesser levels of variability.

#### **5.1 Variability Levels**

The various levels of variability that were studied are follows:

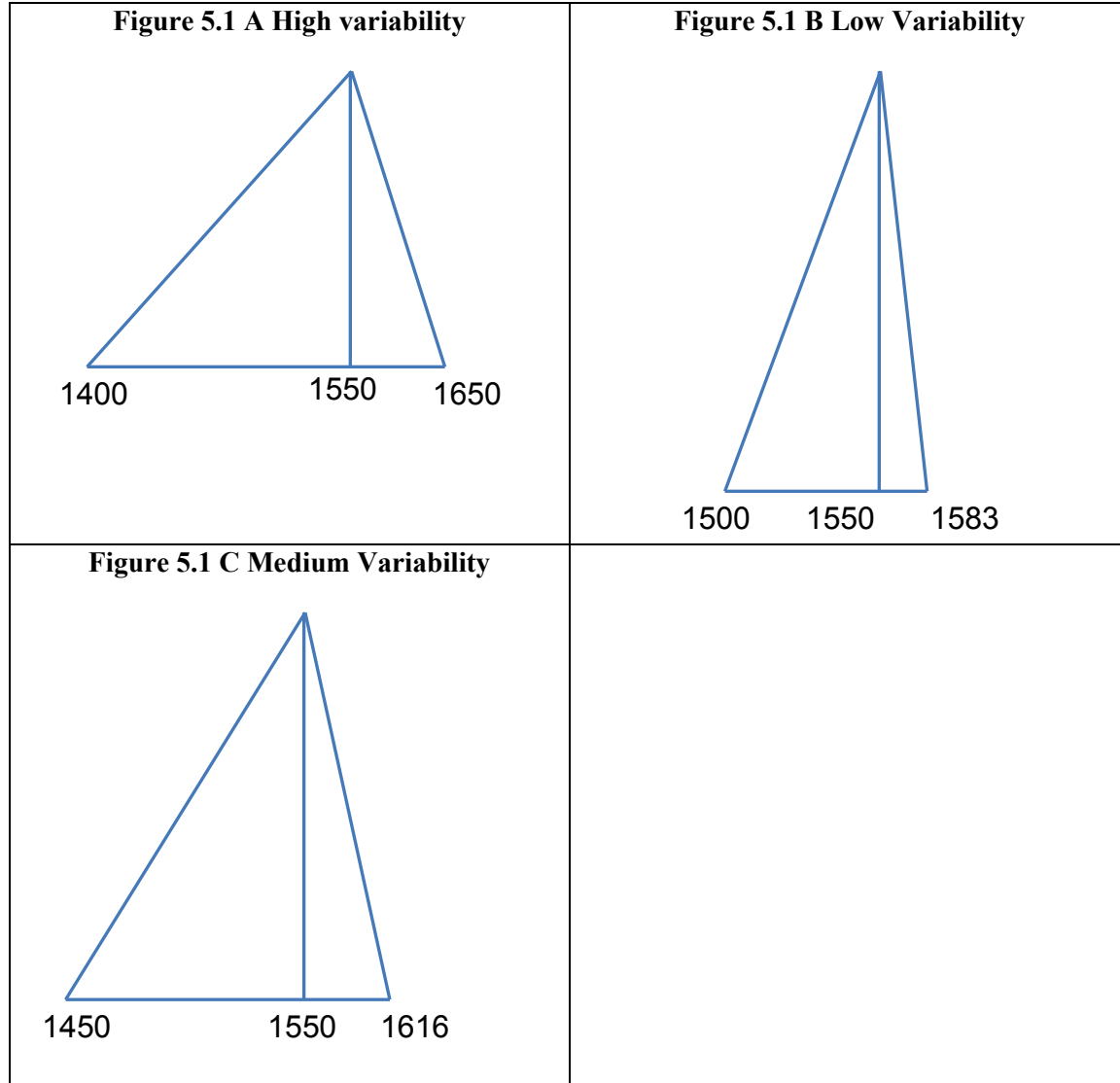
Case 1: no variability in the system

Case 2: low variability in the system

Case 3: medium variability in the system

The cases are set based on the initial case study. For example, consider the distribution for process 1 for workstation 1 at path 1 which is shown in figure 5.1 A. For the first case, the mean of the distribution is considered as the processing time. In the second case, the minimum is one third of the difference in the minimum and the mode. Similarly, the maximum is one third the difference in the maximum and the mode which is shown in figure 5.1 B. The third case is similar to the second case except that two third the difference is considered which is shown in figure 5.1C.





**Figure 5.1 Distributions for process 1 in line 1 for workstation 1**

## 5.2 No variability

In the no variability case, the processing time was taken as the mean processing time and based on this, the critical path was calculated. Critical path was identified to be only one line per workstation and there was no shift in the critical path within workstations. Similarly, there was no difference between the probability of a path being critical and the probability a critical path

exceeding the standard move rate. Metrics which are used to study the criticality of each path that is explained in the third chapter are shown in table 5.1.

**Table 5.1 Critical path analysis for deterministic case**

	<b>WS 1</b>	<b>WS 2</b>	<b>WS 3</b>
<b>Metric 1</b>			
Path 1	100%	100%	100%
Path 2	0%	0%	0%
Path 3	0%	0%	0%
<b>Metric 2</b>			
Path 1	100%	100%	100%
Path 2	0%	0%	0%
Path 3	0%	0%	0%
<b>Metric 3</b>			
Path 1	150	250	256
Path 2	0	0	0
Path 3	0	0	0
<b>Metric 4</b>			
Path 1	150	250	256
Path 2	11	6	12
Path 3	0	0	0

There is no change in the critical path in the deterministic case. Hence, lean experiments were implemented without any further simulation which is shown in table 5.2.

**Table 5.2 Order of Lean Experiments in Deterministic case**

Lean Experiment	Probability of Completion
6	0
2	0
8	73.6%
1	83.7%
5	94%
9	100%

The probability of completion was found to be zero, since all three workstation had a mean processing time greater than standard move rate. Hence, lean experiments were implemented on all three workstation in order to observe any improvement in the probability of completion. However, only six lean experiments were required to increase the probability of completion and the processes were done within the standard move rate. This case is ideal, wherein there is no variability in the system. Though this is impractical, two cases with different levels of variability were studied.

### **5.3 Low variability case**

After studying the deterministic case, the next step was to increase the variability. The variability was around one third of the high variability case. The metrics required to identify the criticality of each path were calculated. Lean experiments were implemented to the paths based on criticality based on the methodology which is shown in chapter 3.

**Table 5.3 Critical path analysis for low variability case**

	WS 1	WS 2	WS 3
<b>Metric 1</b>			
Path 1	92%	100%	100%
Path 2	8%	0%	0%
Path 3	0%	0%	0%
<b>Metric 2</b>			
Path 1	10%	85%	94%
Path 2	1%	0%	0%
Path 3	0%	0%	0%
<b>Metric 3</b>			
Path 1	8.7	46.1	48.0
Path 2	14.9	0	0
Path 3	0	0	0
<b>Metric 4</b>			
Path 1	8.7	46.1	48.0
Path 2	14.9	15.1	12.2
Path 3	0	0	0

After identifying the metrics, the next step was to calculate the order in which the lean experiments need to be done in order to improve the probability of completion which is shown in table 5.4.

**Table 5.4 Order of Lean Experiments in less variability scenario**

Lean Experiment	Probability of Completion
6	11.5%
2	60.5%
8	64.5%
1	69.8%
5	73.5%
9	81.9%

Like the deterministic case, only six lean experiments were required to increase the probability of completion and the processes were done within the standard move rate.

#### 5.4 Medium variability case

The study was also done in the medium variability scenario. The steps involved are similar to the previous cases. The criticality of each path for all the workstations is shown in table 5.5.

The results that were obtained from table 5.5 are similar to the high variability case. There is a shift in the criticality of paths, wherein the more variable path has a lesser probability of being critical. However, when it is critical, the average time exceeded is large.

The order of lean experiments and the probability of completion are shown in table 5.6.

**Table 5.5 Critical path analysis for medium variability case**

	WS 1	WS 2	WS 3
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<b>Metric 1</b>			
Path 1	54%	56%	60%
Path 2	33%	21%	22%
Path 3	13%	23%	18%
<b>Metric 2</b>			
Path 1	22%	31%	38%
Path 2	27%	16%	15%
Path 3	11%	19%	17%
<b>Metric 3</b>			
Path 1	29.8	24.6	25.8
Path 2	45.8	39.2	34.7
Path 3	54.3	59.7	63.7
<b>Metric 4</b>			
Path 1	15.6	17.4	17.6
Path 2	37.4	32.6	29.9
Path 3	45.1	52.6	51.2

**Table 5.6 Order of Lean Experiments in medium variability scenario**

<b>Lean Experiment</b>	<b>Probability of Completion</b>
8	25.7%
4	33.5%
3	40.8%
9	52.5%
2	56.3%
7	60.1%
6	65.8%
1	82.3%

Table 5.7 shows the lean experiments for all the cases based on probability of completion. These results do not take the cost of applying the lean experiment into consideration.

**Table 5.7 Order of lean experiments based on probability of completion**

<b>Deterministic</b>	<b>Less variability</b>	<b>Medium variability</b>	<b>High variability</b>
6	6	8	8
2	2	4	4
8	8	3	3
1	1	9	9
5	5	2	2
9	9	7	7
		6	6
		1	1

The results show that there is no significant difference in the medium and the high variability case.

Table 5.8 shows the lean experiments for all the cases based on return on investment. This table takes the cost of applying each experiment and the improvement from each experiment into account.

**Table 5.8 Order of lean experiments based on payback period**

<b>Deterministic</b>	<b>Less variability</b>	<b>Medium variability</b>	<b>High variability</b>
9	9	4	4
5	5	6	7
8	4	7	6
4	8	8	8
1	1	9	9
2	2	2	3
		1	2
		3	1

There is no significant difference in the order of experiments for the medium variability and the high variability case. The results shown in table 5.8 is similar to the results in table 5.7 stating that there is not much variation in the medium and high variability scenarios. However, there is a significant difference in the order of lean experiments for the order of experiment based

on the probability of completion and the payback period. By comparing the results, it can be inferred that the most critical path may not be the best path to improve due to the cost of implementing. It also states that the cheapest improvement may not have ideal results due to significance of the criticality of the path. Based on the study on different levels of variability, it can be inferred, that the shift in the critical path is more in the medium and the high variability cases. In such cases alone, simulation is a key tool that can be used to study the effects on critical path. In the less variability case, simulation need not be done



## **CHAPTER 6**

### **CONCLUSION AND FUTURE RESEARCH**

#### **6.1 Conclusion**

In this section, the conclusion and future research for this study is discussed. For products that are manufactured which have high variability in processing times and manufactured in low volume, improvement cannot be just given to the line that takes the maximum processing time. Due to different levels of variability within each process, different processes might be critical at different runs. Various metrics are used to study the paths and analyze the criticality of multiple paths. The metrics used to capture the criticality of paths for high variable production systems such as aircraft manufacturing industries are shown in chapter 3. After identifying the criticality of paths, a systematic procedure for improvement is discussed about in chapter 4. Different improvement techniques have different costs. The improvement techniques could be addition of labor, increasing machinery or improving the availability of material. Improvements are based on either to improvement in the probability of completion or based on return on investment. Chapter 5 discusses about implementing the proposed methodology for different levels of variability. It shows the results of implementing critical path using simulation for different levels of variability.

In order to improve a production system which has high variability and low volume, improvement on the path which has the highest processing time alone will not yield significant results. Improvement needs to be given to the process that has high variability and a significant processing time. In order to identify that process various metrics were used. The tool used to identify these metrics was simulation. The metrics used were based on the processing time and

the variability. By combining the various metrics, a method was shown to identify the paths that caused delays in production. Once the various critical paths were identified, the next step was to identify a procedure for improvement. Various improvement techniques were allotted for each process. Each improvement technique could either be allocating extra personnel, machine or increasing buffer before the process. The improvements were used to either reduce variability or processing time or a combination of both. Each improvement technique has a unique cost for implementation. Implementing improvement techniques were based on the costs to implement and the criticality of the path.

## **6.2 Future Research**

The research brings into account different levels of variability and considers different cost factors for different experiments. However, only one improvement could be used for each process. At times, more than one improvement could be necessary for a process. Besides, having more than one improvement technique for a process could increase flexibility in applying improvement techniques. One of the factors for implementing an improvement technique was considering the improvement to be independent of each other. Though most of the time, an improvement technique could either have a positive or a negative effect on other lines. When the problem becomes larger, simulating a model gets tedious and time consuming. Genetic algorithm or other heuristic methods can be used to solve problems which require a lot more operations to save time and resources.

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