Lecture 27

Critical Path in Network Analysis

27.1 Critical Path in Network Analysis

27.1.1 Basic Scheduling Computations

The notations used are
(i, j) = Activity with tail event i and head event j
E_i = Earliest occurrence time of event i
L_j = Latest allowable occurrence time of event j
D_{ij} = Estimated completion time of activity (i, j)
$(Es)_{ij}$ = Earliest starting time of activity (i, j)
$(Ef)_{ij}$ = Earliest finishing time of activity (i, j)
$(Ls)_{ij}$ = Latest starting time of activity (i, j)
$(Lf)_{ij}$ = Latest finishing time of activity (i, j)

The procedure is as follows

1. **Determination of Earliest time (E_j): Forward Pass computation**

- **Step 1**
  The computation begins from the start node and move towards the end node. For easiness, the forward pass computation starts by assuming the earliest occurrence time of zero for the initial project event.

- **Step 2**
  i. Earliest starting time of activity (i, j) is the earliest event time of the tail end event i.e. $(Es)_{ij} = E_i$
  ii. Earliest finish time of activity (i, j) is the earliest starting time + the activity time i.e. $(Ef)_{ij} = (Es)_{ij} + D_{ij}$ or $(Ef)_{ij} = E_i + D_{ij}$
  iii. Earliest event time for event j is the maximum of the earliest finish times of all activities ending in to that event i.e. E_j = max [$(Ef)_{ij}$ for all immediate predecessor of (i, j)] or E_j =max [$E_i + D_{ij}$]

2. **Backward Pass computation (for latest allowable time)**

- **Step 1**
  For ending event assume E = L. Remember that all E's have been computed by forward pass computations.

- **Step 2**
  Latest finish time for activity (i, j) is equal to the latest event time of event j i.e. $(Lf)_{ij} = L_j$

- **Step 3**
  Latest starting time of activity (i, j) = the latest completion time of (i, j) – the activity time or $(Ls)_{ij} = (Lf)_{ij} - D_{ij}$ or $(Ls)_{ij} = L_j - D_{ij}$

- **Step 4**
  Latest event time for event 'i' is the minimum of the latest start time of all activities originating from that event i.e. L_i = min $[(Ls)_{ij}$ for all immediate successor of (i, j)] = min $[(Lf)_{ij} - D_{ij}]$ = min $[L_j - D_{ij}]$

3. Determination of floats and slack times

There are three kinds of floats

- **Total float** – The amount of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time.
  Mathematically
  $(Tf)_{ij}$ = (Latest start – Earliest start) for activity (i – j)
  $(Tf)_{ij} = (Ls)_{ij} - (Es)_{ij}$ or $(Tf)_{ij} = (L_j - D_{ij}) - E_i$

- **Free float –** The time by which the completion of an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity.
  Mathematically
  $(Ff)_{ij}$ = (Earliest time for event j – Earliest time for event i) – Activity time for (i, j)
  $(Ff)_{ij} = (E_j - E_i) - D_{ij}$

- **Independent float** – The amount of time by which the start of an activity can be delayed without effecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.
  Mathematically
  $(If)_{ij} = (E_j - L_i) - D_{ij}$
  The negative independent float is always taken as zero.

- **Event slack** - It is defined as the difference between the latest event and earliest event times.
  Mathematically
  Head event slack = $L_j – E_j$, Tail event slack = $L_i - E_i$

4. Determination of critical path

- **Critical event** – The events with zero slack times are called critical events. In other words the event i is said to be critical if $E_i = L_i$

- **Critical activity** – The activities with zero total float are known as critical activities. In other words an activity is said to be critical if a delay in its start will cause a further delay in the completion date of the entire project.

- **Critical path** – The sequence of critical activities in a network is called critical path. The critical path is the longest path in the network from the starting event to ending event and defines the minimum time required to complete the project.

27.2 Worked Examples

Example 1
Determine the early start and late start in respect of all node points and identify critical path for the following network.

Solution
Calculation of E and L for each node is shown in the network

Activity(i, j)	Normal Time (D_{ij})	Earliest Time		Latest Time		Float Time $(L_i - D_{ij}) - E_i$
		Start (E_i)	Finish ($E_i + D_{ij}$)	Start ($L_i - D_{ij}$)	Finish (L_i)	
(1, 2)	10	0	10	0	10	0
(1, 3)	8	0	8	1	9	1
(1, 4)	9	0	9	1	10	1
(2, 5)	8	10	18	10	18	0
(4, 6)	7	9	16	10	17	1
(3, 7)	16	8	24	9	25	1
(5, 7)	7	18	25	18	25	0
(6, 7)	7	16	23	18	25	2
(5, 8)	6	18	24	18	24	0
(6, 9)	5	16	21	17	22	1
(7, 10)	12	25	37	25	37	0
(8, 10)	13	24	37	24	37	0
(9, 10)	15	21	36	22	37	1

Network Analysis Table

From the table, the critical nodes are (1, 2), (2, 5), (5, 7), (5, 8), (7, 10) and (8, 10)

From the table, there are two possible critical paths

i. $1 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10$
ii. $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 10$

Example 2

Find the critical path and calculate the slack time for the following network

Solution

The earliest time and the latest time are obtained below

Activity(i, j)	Normal Time (D_{ij})	Earliest Time		Latest Time		Float Time (L_i - D_{ij}) - E_i
		Start (E_i)	Finish (E_i + D_{ij})	Start (L_i - D_{ij})	Finish (L_i)	
(1, 2)	2	0	2	5	7	5
(1, 3)	2	0	2	0	2	0
(1, 4)	1	0	1	6	7	6
(2, 6)	4	2	6	7	11	5
(3, 7)	5	2	7	3	8	1
(3, 5)	8	2	10	2	10	0
(4, 5)	3	1	4	7	10	6
(5, 9)	5	10	15	10	15	0
(6, 8)	1	6	7	11	12	5
(7, 8)	4	7	11	8	12	1
(8, 9)	3	11	14	12	15	1

From the above table, the critical nodes are the activities (1, 3), (3, 5) and (5, 9)

The critical path is 1 → 3 → 5 → 9