Sandia National Laboratories

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Robust Project Scheduling

Motivation

Projects are often scheduled under the assumptions of complete information and a static execution environment. In reality, projects are subject to uncertainty/ disruptions which are gradually resolved during execution. Examples include: resources become unavailable, duration variability, deliveries delayed, due dates re-negotiated, and weather conditions. Such disruptions incur costs due to missed deadlines, resource idleness, higher work-in-progress inventory, and system nervousness. Thus, we seek to develop a (robust) pre-schedule that ensures stability in activity start times by incorporating uncertainty.

Assumptions

- Activities cannot be started before their foreseen starting time in the pre-schedule.
- Proper allocation of resources has been performed, thus reflected in the precedence constraints, or that allocation will occur in a later phase based on the pre-schedule.
- The effect of one disturbance will not interact with the effect of another.

Definitions

- A project can be represented by a directed acyclic graph G = (N, A).
  N: Activities   A: Finish-start precedence relations
- Path P(i, j) denotes any path from i to j in G(N, A).
- Set TA denotes the transitive closure of A, such that (i, j) ∈ TA if and only if ∃ some P(i, j).
- P*(i, j) is the path with the largest sum of activity durations between i and j, i.e. the longest path.

Parameters	
d_i	Duration of activity i
w	Final project deadline
c_j	Cost per unit-time overrun on start of activity j
$g_i(\cdot)$	Probability mass function for disturbance scenarios of activity i
l_{ik}	Disturbance duration due to scenario k of activity i
λ_{ij}	Length of P*(i, j) not including durations of i and j
D_i	Collection of disturbance scenarios for activity i
Decision Variables	
s_i, s_j	Start time of activity i or j, respectively
Δ_{ijk}	Delay in start of activity j due to disturbance scenario k of activity i

Formulation

$$\min \sum_{(i,j)\in TA} \sum_{k\in D_i} c_j g_i(l_{ik}) \Delta_{ijk}$$

s.t. (1) $s_j - s_i \geq d_i \quad \forall (i,j) \in TA$

(2) $s_n - s_0 \leq w$

(3) $\Delta_{ijk} + s_j - s_i \geq l_{ik} + d_i + \lambda_{ij} \quad \forall (i,j) \in TA \quad \forall k \in D_i$

(4) $\Delta_{ijk} \geq 0 \quad \forall (i,j) \in TA$

(5) $s_0 = 0;\ s_i\ unrestricted \quad \forall i \neq 0$

Intuition: Objective is to minimize the expected weighted deviation of actual versus planned activity start-times.

Current & Future Work

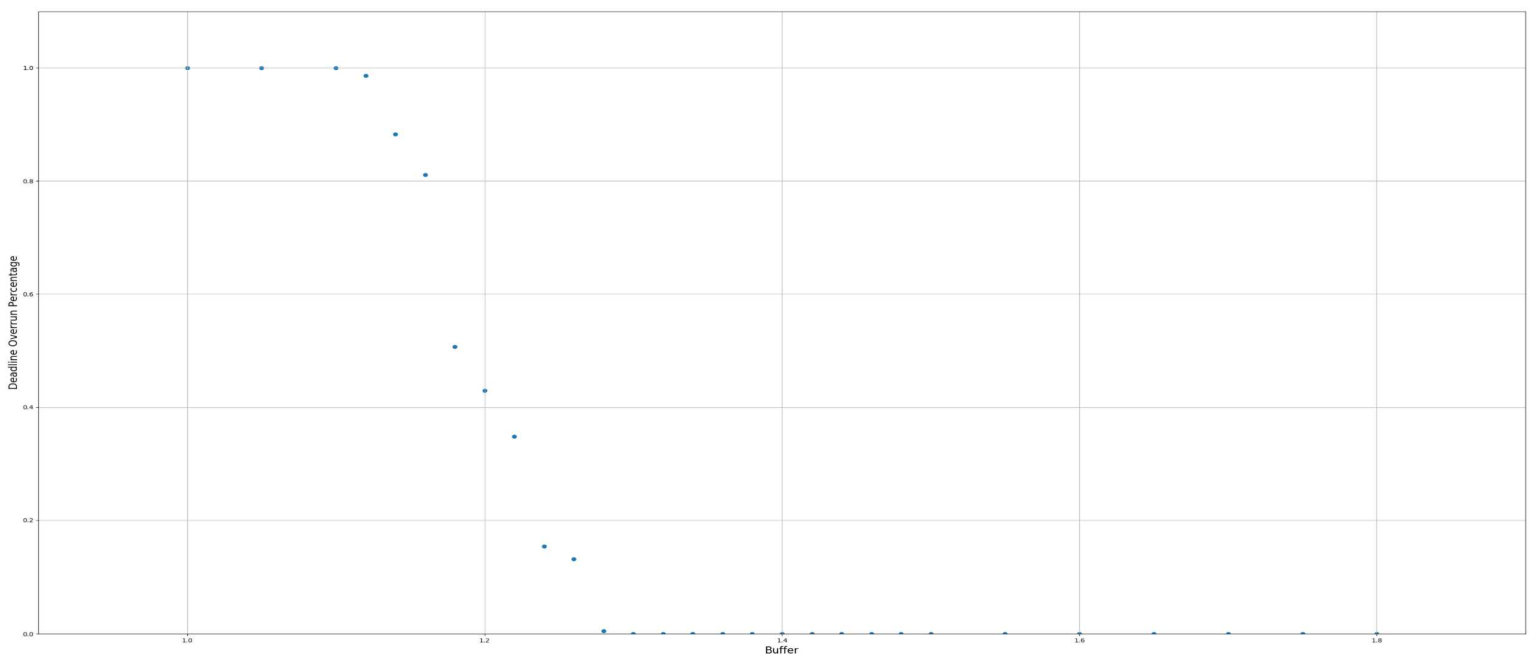
- Transforming formulation into resource-constrained integer programming format.
- Heuristic or exact procedure to restrict set TA to only include activity pairs that have close proximity within the precedence network.

Citation

Herroelen, Willy, and Roel Leus. “The Construction of Stable Project Baseline Schedules.” *European Journal of Operational Research*, vol. 156, no. 3, 2004, pp. 550–565., doi:10.1016/s0377-2217(03)00130-9.

Evaluation

Illustration 1: For a 30-node network with parameters $c_i = U[1,2,3,4]$ and $l_{ik} = Triangular(0, 0.15 \times d_i, 0.30 \times d_i)$ the overrun percentage of deadline *w* and the formulation’s objective value are observed to rapidly fall as buffer increases. Buffer is a numeric multiplier applied to network longest path *P*(0,n)*. Similar patterns are evident in 60-, 90-, and 120-node networks.



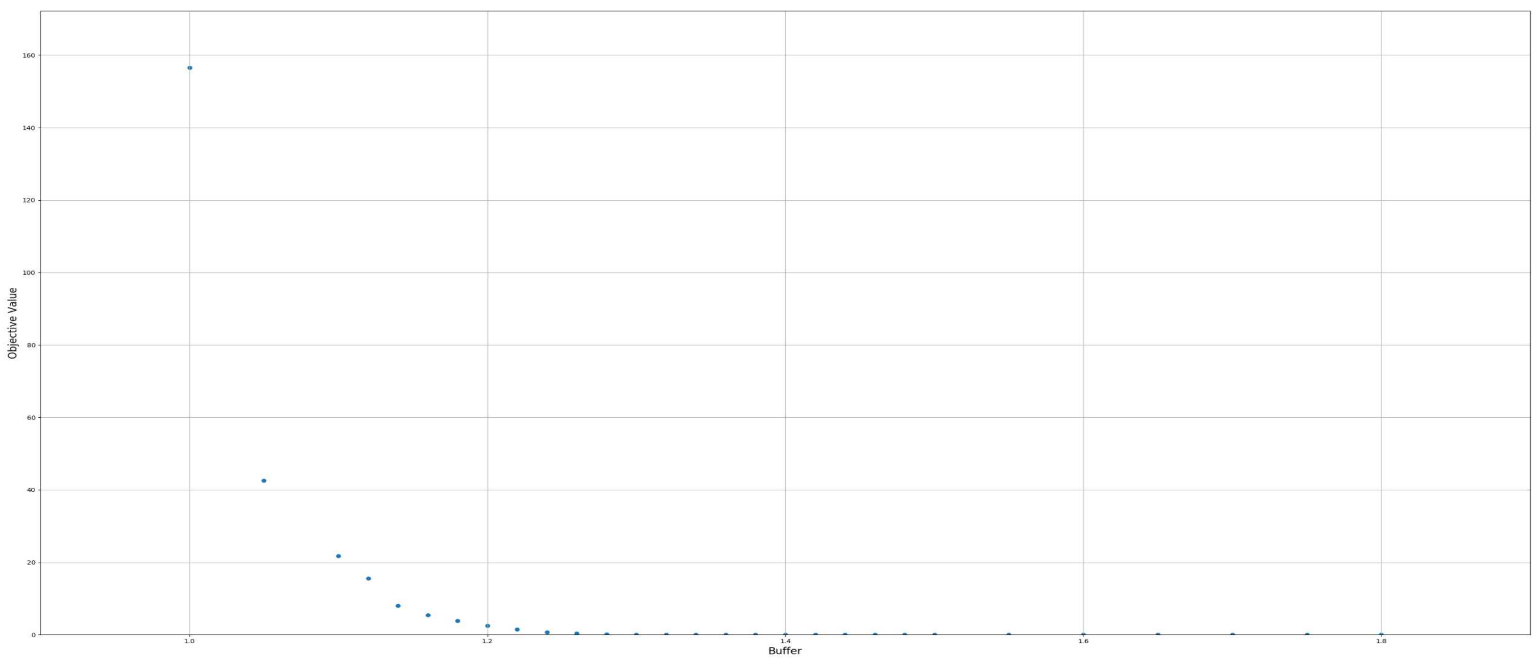


Illustration 2: For the same 30-node network, specified parameters, and buffer set to 1.25, a project Gantt chart can be produced. Spacing is interspersed between network longest path sequence 1-2-5-6-8-18-26-31-32. Such spacing distributes risk and ensures stable start times amidst delays. This is supported by simulation – where the simulated delays exhibit a linear relationship with the Δ_{ijk} decision variables over thousands of runs.

Illustration 3: For the same parameters, including buffer held constant at 1.25, fifteen distinct networks are sampled each from 30-, 60-, 90-, and 120-node network sizes to display the relationship between solution time and formulation complexity. A roughly linear relationship can be observed from lower-left to top-right in solve-time as both |N(G)| and |TA| increase in value.