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COORDINATING THE SUPPLIER-RETAILER SUPPLY CHAIN UNDER NOISE EFFECT WITH BUNDLING AND INVENTORY STRATEGIES

Ata Allah Taleizadeh

School of Industrial Engineering, College of Engineering, University of Tehran
Tehran, Iran

Leopoldo Eduardo Cárdenas-Barrón*

School of Engineering and Sciences, Tecnológico de Monterrey
E. Garza Sada 2501 Sur, C.P. 64849, Monterrey, Nuevo León, México

Roya Sohani

Department of Industrial Engineering, Islamic Azad University
South Tehran Branch, Tehran, Iran

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Abstract. In current competitive market, the products and their demand's uncertainty are high. In order to reduce these uncertainties the coordination of supply chain is necessary. Supply chain can be managed under two viewpoints typically: 1) centralized supply chain and 2) decentralized supply chain, and the coordination can be done in both types of chains. In the centralized supply chain there exists a global decision maker who takes all the best decisions in order to maximize the profit of the whole supply chain. Here, the useful information required to make the best decisions is open to all members of the chain. On the other hand, in the decentralized supply chain all members decide in a separate and sequential way, how to maximize their profits. In order to coordinate efficiently the supply chain, both supplier and retailer are involved in a coordination contract that makes it possible for the decentralized decisions to maximize the profit of the entire supply chain. In this context, the situation that the supplier-retailer chain faces is a two-stage decision model. In the first stage the supplier, based on former knowledge about the market, decides the production capacity to reserve for the retailer. In the second stage, after that demand information is updated, the retailer determines the bundle price and the quantity of bundles to order. This paper considers a supply chain comprised of one supplier and one retailer in which two complementary fashion products are manufactured and sold as a bundle. The bundle has a short selling season and a stochastic price dependent on demand with a high level of uncertainty. Therefore, this research considers that the demand rates are uncertain and are dependent on selling prices and on a random noise effect on the market. Profit maximization models are developed for centralized and decentralized

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* Corresponding author: Tel. +52 81 83284235, Fax +52 81 83284153. E-mail address: lecarden@itesm.mx (L.E. Cárdenas-Barrón).

supply chains to determine decisions on production capacity reservation, order quantity of bundled products and the bundle-selling price. The applicability of the developed models and solution method are illustrated with a numerical example.

1. **Introduction and literature review.**

1.1. **Motivation.** In the last decade, two complementary products coming from a duopoly market like a supply chain, where relationship was either found to be cooperative or non-cooperative among channel members (manufacturer, distributor, supplier, retailer, buyer, etc.). Duopoly market does not allow the channel members of the supply chain to have the same power when compared to the retailers or manufacturers. In the case of non-cooperative market, the party with more power plays a leader and the other as a follower. Also in integrated businesses' where complementary products like multimedia PC-sound system, conditioner and shampoo, operating system and computer are produced by manufacturers influencing consumer's demands in the market such a way that the purchase of one product affects and increases the possibility of purchasing the other product. Thereby, small changes in demand resulting in large changes in the decision. Therefore, when the manager investigates factors influencing demand, purchasing behaviour of the customers' may be affected by factors like selling price, seasonality, and inventory level. Two separate firms for pricing of complementary goods under information asymmetry were considered by Yue et al. (2006) and Mukhopadhyay et al. (2011).

Recently, the companies are practising commercial strategies like bundling, commonly known as tying. Selling of different products (or services) together is what bundling consists of. Practice of bundling comprises of a principal product and a secondary product, complementing each other, although, same number of products can also form a bundle. Some examples of bundling two or more services or products, such as shampoo with conditioner, toothbrush and toothpaste, computer and printer, flight with hotel, just to name a few illustrations. Basically, when a bundling strategy is used by a company for the selling of its products, the selling of a highly complementary product at the same time is an advantage that it gets. Many of the reasons behind companies implementing bundling strategies are: increasing of market share, reduction in packing costs, increase of sales, improving of customer service, thereby extending market power of one product to another. Bundling does not have a consistent, universally accepted definition but the integration and sale of two or more separate products at same price is what product bundling is. Categorized into pure bundling and mixed bundling, pure bundling a strategy where items are only sold as bundles and not separately whereas mixed bundling a strategy where products can be sold as both bundles and separately. Bundling price and its process influences consumer by offering two complementary products as a bundle price that generally increases the chain profit. A descriptive study on bundling of complementary products by Estelami (1999) clearly pointed out that minimize consumer costs from 18 to 57% through bundling all depending on amount of bundled items, the value of same items, and level of variations. Nalebuff (2004) provided an excellent discussion regarding motivations employing bundling strategy. Two types of bundling strategies exist: pure and mixed. Pure bundling strategy, products are only sold as a bundle. In contrast, to the mixed bundling where products can be sold as a bundle or individually.

Generally, the products and their demand's uncertainty are high if the market for the same is unpredictable. Therefore, reduction of uncertainty is important through the coordination of the supply chain. Consequently, coordination is a mandatory task in any supply chain. Supply chain can be managed under two viewpoints typically: 1) centralized supply chain and 2) decentralized supply chain, and the coordination can be done in both types of chains. In the centralized supply chain there is a global decision maker who makes all the best decisions with the aim to maximize the profit of the whole supply chain. Here, the useful information necessary to make the best decisions is open to all participants of the chain. In the decentralized supply chain all participants decide in an isolated and sequential way, how to maximize their profits. In order to coordinate efficiently the supply chain, both supplier and retailer are involved in a coordination contract that makes it possible for the decentralized decisions to maximize the profit of the entire supply chain. The situation that the supplier-retailer chain faces is a two-stage decision model. In the first stage the supplier, based on former knowledge about the market, decides the production capacity to reserve for the retailer. In the second stage, after that demand information is updated, the retailer determines the bundle price and the quantity of bundles to order. This paper considers a supply chain comprised of one supplier and one retailer in which two complementary fashion products are manufactured and sold as a bundle. The bundle has a short selling season and a stochastic price dependent on demand, with a high level of uncertainty. Furthermore, as the two products are selling together, there is random noise in the market due to the psychological effects of the bundling strategy that affects customers' purchases. The research works more closely related to our paper are Chen et al. (2010) and Yan and Bandyopadhyay (2011). Chen et al. (2010) study and analyze a coordination contract for a retailer-supplier chain in which only one type of product is processed and sold in a short vending season. The problem is also modeled as a two-stage news-vendor model in which an initial decision on ordering is made in the first stage and additional ordering and pricing decisions are done in the second stage. Conversely, Yan and Bandyopadhyay (2011) develop a profit maximization model to study the benefits of a bundling strategy. This paper combines the main ideas related to coordination and bundling from Chen et al. (2010), and Yan and Bandyopadhyay (2011) into a new profit maximization model to determine the bundle price of two products that are sold together and the bundle order quantity.

1.2. **Literature review.** Bundling permits monopolist extracting additional surpluses by diminishing the variance of average valuations by using the law of large numbers that it was showed by Armstrong and Vickers (2010). Li et al. (2013) defined a measure of consumer heterogeneity, which increases with costs, and as they presented that, an increase in the measure of consumer heterogeneity affects pure bundling and its performance is poorly relative to individual sales. Stochastic modelling of a retail firm selling two types of perishable products in a single period both as bundle and independent items is considered by Gürler et al. (2009) where they showed that when more bundles are formed by retailer, or higher prices are charged for the bundle or both as the products convert less substitutable and more complimentary. Bhargava (2012) found that the reason behind the less attractiveness of bundling is due to the conflicts in supply chain that produce an overpricing of component products by manufacturers.

There exist several studies by researchers regarding products' bundling allowing discount. For example see the works by Matutes and Regibeau (1992) and Gans, and King (2006). Price consumer evaluations of a discounted product coming from a bundle is considered by Sheng et al. (2007) where they showed the effects of price discounts interplay with complementarity of bundle components. Yan and Badyopadhyay (2011) investigate bundling of complementary products. They exposed how conditionally firms could be benefited from complementary bundling. On further investigation by Yan et al. (2014) regarding the advertising on the success of product bundling and strategic influence of product complementarity, showed that when bundled products are sold by a firm, both advertising and product complementarity significantly impacts its performance. Brito and Vasconcelos (2015) examined competitive effects of bundled discounts offered by pairs of independent firms in a setting with vertically differentiated goods.

The literature regarding bundling is also rich and vast. In this line, the following research works are relevant: Guiltinan (1987) proposes a normative framework to choose suitable kinds of services under different mixed-bundling forms. Later, Rosenthal et al. (1995) develop a mixed integer linear program to examine the relationship among various bundling strategies. Their model determines the optimal purchasing strategy for the buyer that minimizes the total purchase cost. Simultaneously, Simonin and Ruth, (1995) examine the impact of bundling policies on the reserve prices to the customers of the bundle and its components involving a new product and a tie-in product. Afterwards, Estelami (1999) investigate the savings to customers under complementary bundling products. Later, Bennett and Robson (2001) examine how associations balance their provision of distinct services, the potential for associations to offer new services, and the importance of bundling. Venkatesh and Kamakura (2003) discuss an analytical model of contingent valuations to maximize the profit of system. They survey different bundling strategies such as pure bundling and mixed bundling. Later, Oppewal and Holyoake (2004) study the impacts of retail accumulation and bundling on shopping behavior, especially on in-store purchase incidence and the sequencing of shopping activities. Subsequently, Vaubourg (2006) presents a theoretical argument for the market outcome when some enterprises use pure bundling strategies whereas others apply mixed bundling. One year later, Bitran and Ferrer (2007) analyze the problem of how to define the composition and price of a bundle such that it maximizes the total expected profit. At the same time, Hubbard et al. (2007) study the economic effects of pure bundling under the settings of duopoly and monopoly. They conclude that the bundled price is less than the summation of unbundled prices. McCardle et al. (2007) examine the properties of bundling products on retail trades. They determine the optimal bundle prices, order sizes, and profits under bundling strategy. Later, Arora (2008) study the efficacy of price bundling and message framing on intentions, beliefs and attitudes associated to characteristics of teeth whitening products. Afterwards, Bulut et al. (2009) develop a single period pricing model to determine the optimal bundling pricing policy of two perishable products that could be sold either as a bundle or individually, under a stochastic demand. Simultaneously, Gürler et al. (2009) present a stochastic modelling for a retail company, which vends two kinds of perishable items. Eckalbar, (2010) study the case of a monopoly selling two different products to a group of m traders determined by their reserve prices. In addition, they propose closed-form solutions to determine the optimal quantities, prices, customers' surplus and profit under the situations of pure-, mixed-bundling and individual sales. In the same year, Ferrer et al. (2010) develop a pricing model of bundles comprised of a service and a related product using a two-part tariff arrangement. Applying dynamic programming, they derive the optimal pricing policy that maximizes the firm's profit. Wappling et al. (2010)

examine some product bundling strategies offered to customers by selected companies in Swedish automobile, travel and banking organizations. Later, Yan and Bandyopadhyay (2011) build a profit-maximization model to find the optimum pricing and bundling policies for complementary items. Recently, Chakravarty et al. (2013) investigate bundling decisions in a two-level supply chain comprised of one retailer and several suppliers. Concurrently, Girju et al. (2013) study how channel interactions affect the product bundling decisions of channel members. Sheikhzadeh and Elahi (2013) explore two facets of bundling. They analyze the effects of risk and product heterogeneity on bundling decisions. Finally, Yan et al. (2014) develop a profit maximization model to analyze the strategic effect of advertising and product complementarity on the success of bundling products. Other related research works are Taleizadeh et al. (2010), Taleizadeh et al. (2013), Taleizadeh and Pentico (2013), Taleizadeh et al. (2015), Taleizadeh and Noori-daryan (2016) and Taleizadeh et al. (2016).

There are several papers on coordination contracts and bundling. Regarding the coordination contracts in supply chain, the following works are relevant: Donohue (2000) discusses the problem of developing supply contracts that permit the coordination of forecast information and production decisions of a manufacturer and a distributor for seasonal products. Afterwards, Barnes-Schuster et al. (2002) consider a two-period model and study the role of options in a buyer-supplier system. They also explain how options give flexibility to a buyer in response to market changes during the second period. In the same year, Taylor (2002) studies the coordination of supply chains with sales effort and channel rebate considerations. Cachon (2003) investigates the supply chain coordination under different transaction contracts, such as the wholesale-price contract, the buyback contract, the revenue-sharing contract, the quantity-flexibility contract, the sale-rebate contract and the quantity-discount contract. Later, Cachon and Lariviere (2005) apply revenue-sharing contracts in a supply chain with revenues calculated by each retailer's purchase quantity and price. Subsequently, Wang (2005) extend traditional quantity discount policies, which are merely based on order quantity of buyers, to discount policies based on both individual order quantity and the annual volume. Mathur and Shah (2008) consider a contract in a supply chain where a supplier needs to determine capacity under an offered-price regime before realizing the market demand. One year later, Yan and Pei (2009) focus on the strategic role played by the retail services in a dual-channel competitive market. Chen et al. (2010) study a coordination contract for a supply chain in which a fashionable product, with stochastic price-dependent demand, is produced and sold. Chen and Bell (2011) consider a decentralized supply chain, including a manufacturer and a retailer, in which the retailer determines the order quantity and the retail price at the same time that customer returns and price-dependent stochastic demand are occurring. Yan (2011) extends an analytical model to investigate impacts of profit sharing and differentiated branding strategies on the decisions of a multi-channel manufacturer-retailer supply chain's members.

Li et al. (2013) introduced a bundling sale strategy of the goods with a prevalent strategy in a manufacturing firm. They studied numerically the maximum profit of three bundling strategies (individual sale of the product, pure bundling of the product and mixed bundling of the product) and concluded that mixed bundling strategy is more profitable than other strategies. In manufacturing firms, the objective is to fabricate the product smoothly and maintain the service according to the customers' requirement. Due to this objective, manufacturing firms offer the good

Table 1. Some recent works related to bundling strategy

Literature	**Strategies**	**Selling price**	**Demand rate**	**Situation**
Chakravarti et al. (2013)	Bundling	Bundle price	Selling price	Decentralized supply chains
Li et al. (2013)	Mix bundling	Bundle price	Selling price	Bi-level programming
Yan et al. (2014)	Bundle pricing and advertising	Bundle price	Selling price	Product complementary and advertisement of bundle product
Wang et al. (2015)	Service bundling	—	Service and Price bundling	Duopoly competitive environment
Banciu and Ødegaard (2016)	Different bundling	—	—	Simulation technique
Giri et al. (2017)	Pricing	Bundling price	Linearly dependent on price	Duopoly market
Vamosiu (2018)	Imperfect Competition	Mixed bundling	—	Pure bundling
This paper	Bundling	Bundle selling price	Uncertain, selling price and random noise effect on market	Centralized and decentralized supply chains

price of the product as well as an excellent service facility in bundles form; this is named as price and service bundles (P&S bundles). Wang et al. (2015) introduced the firms' incentive and offer to customers' for buying the products in bundling form with maintenance or repair facility in a duopoly competitive marketing situations. Using this concept, Wang et al. (2015) developed three game models and analyzed the market situations for one firm, both firms and no firm when there exist the offer of P& S bundles in form. Banciu and Ødegaard (2016) introduced the problem of pricing a bundle of products when the underlying valuations of the bundle components are dependent. They used copula theory to model and solved the problem for different bundle strategies. Giri et al. (2017) studied a duopoly market situation where two manufacturers separately fabricate two complementary products and sell the fabricated products into together through a common retailer. In their paper, they considered that the demand depends on price. Giri et al. (2017) proposed two different situations: without and with bundling and described the problem mathematically to maximize the profit of each situation. They concluded that in the supply chain profit for the bundling product is better than the profit when products are sold separately. Vamosiu (2018) studied the profitability of bundling for a two-product seller of differentiated products facing competition from a one-product rival. Table 1 shows some recent works related to bundling strategy.

The contribution to existing literature of this research is to investigate a coordination contract for the supplier-retailer chain. Basically, this research focuses on a supply chain including one retailer and one supplier who manufactures two complementary products; their demand rates are price dependent and uncertain. Indeed, a random noise affects the market demand. Moreover, this research proposes a mathematical model to maximize the profit, and coordinates the supply chain considering bundling strategy, restriction on capacity and noise effect. In fact, this paper develops a two-stage optimization strategy in which the supplier decides the capacity amount in stage 1 and the retailer optimizes the order quantity and the selling price in stage 2. Furthermore, a risk and profit sharing contract under three parameters to coordinate the chain is developed. The proposed contract allows any agreed-upon division of the supply-chain profit, among the channel members, using controllable parameters. This is another contribution of this paper, and it should be noted that this structure has never been studied by existing research.

The remaining of this study is organized as follows. The section 2 describes briefly the problem, defines the notation and assumptions. Section 3 develops the profit maximization models for centralized and decentralized supply chains. Section 4 presents a numerical example. Section 5 provides the managerial insights. Finally, Section 6 gives some conclusions and future research directions.

2. Mathematical model and its description.

2.1. Problem description.

This paper deals with a supply chain comprised of one supplier and one retailer. The supplier manufactures two complementary products, which will be sold only as a bundle at; price $p_{B,}$ while previously both products were sold separately at prices p_1 and p_2. The demands of products are dependent on price, and there is a random noise effect on the market. In the chain, three main decisions must be made on different events that occur in two stages. These decisions are: production capacity reservation, order quantity and bundle price. In the first stage, the supplier forecasts the demand and then reserve a production capacity M for the retailer. After some time, the second stage occurs. Here, the retailer updates the forecast of the demand. Then, the retailer determines the bundling order quantity as well as the bundle price.

Yan and Bandyopadhyay (2011) proposed a linear demand function for the bundled products, and the random noise effect (x_B) is added to it. Thus, the demand function for the bundled products under noise effect is as follows:

$$D(p_1, p_2, p_B) = a_1 - a_2 p_B + \lambda(p_1 + p_2 - p_B) + x_B \tag{1}$$

Here, a_1 is the bundled product's acceptance by the market (potential demand),a_2 represents the response rate to the bundle price, λdenotes the bundling discount price sensitivity, p_1 is the price for product 1, p_2 is the price for product 2,$p_B$is the bundle price and$x_B$is the noise effect on the market demand. So when the bundle price increases, the demand quantity should be decreased ($-a_2 p_B$), and increase in the complementary degree should the demand rate of bundle products increases($+\lambda(p_1 + p_2 - p_B)$), if $p_1 + p_2 > p_B$which is the main assumption in bundle selling of items. Moreover, the demand quantity is affected by the noise effect, represented in the demand rate byx_B.

The demand function given by Equation (1) has two types of uncertainty which are the bundled products acceptance by market (a_1) and noise effect (x_B). It is assumed that a_1 has a probability density function $g(a_1)$ and the interval for$a_1$is $[l,h]$. On the other hand, the noise effect on demand is also a random variable distributed in the interval $[L,H]$ and it has a probability density function $f(x_B)$, cumulative distribution $F(x_B)$ and mean μ_B.

Only, at the end of the first stage, one can know the product's demand reaction to the bundle price. Then in the second stage, the value for a_1 is updated with$\tilde{a}_1$.

Decision variables:	
p_i :	The unit selling price of product i when there is no bundle selling(i=1 ,2)
Q_B^c :	The order quantity of products 1 and 2 in bundle pricing in centralized policy
M_B^c :	The reserved capacity of product 1 and product 2 in bundle pricing at stage 2 in centralized policy
Q_B^d :	The order quantity of products 1 and 2 in bundle pricing in decentralized policy
M_B^d :	The reserved capacity of product 1 and product 2 in bundle pricing at stage 2 in decentralized policy
p_{1B} :	The bundle price of products 1 and 2 when $Q_B < M_B$ such that $(p_{1B} < p_1 + p_2)$
p_{2B} :	The bundle price of products 1 and 2 when $Q_B = M_B$ such that $(p_{2B} < p_1 + p_2)$
M_i^c :	The reserved production capacity for product i when there is no bundling
Q_c^1 :	The order quantity of products 1 and 2 when there is no bundling in centralized policy
w_B:	The wholesale price that the supplier offers to its retailer
d_B:	The unit buyback price that the supplier pays to its retailer for each unit of the unsold bundle at the end of stage 2
α_B:	The risk subsidy
Parameters:	
c_{1i} :	Per unit cost of the reserved production capacity of product i at stage 1
c_{1B} :	Per unit cost of reserved production capacity in bundle policy at stage 1($c_{1B} = c_{11} + c_{12}$)
c_{2i} :	Per unit product cost of product i at stage 2
c_{2B} :	Per unit product cost in bundle policy at stage 2 $(c_{2B} = c_{21} + c_{22})$
v_{1i} :	Per unit salvage value of the unused production capacity of product i, the part that is established at stage 1 but not really used for production $(v_{1i} < c_{1i})$
v_{2i} :	Per unit salvage value of the leftover inventory of product i, at the end of the selling season
π_B:	Per unit penalty cost of stockout of bundled products
Π_B^r :	Retailer's profit in the decentralized model for $Q_B^d < M_B^d$ *and* $Q_B^d = M_B^d$
Π_B^m :	Supplier's profit in the decentralized model for $Q_B^d < M_B^d$ *and* $Q_B^d = M_B^d$
Π_B^c :	Supply chain's profit in the centralized model for$Q_B^c < M_B^c$ *and* $Q_B^c = M_B^c$
M^c :	The reserved production capacity in the centralized model
M^d :	The reserved production capacity in the decentralized model
θ :	The degree of complementarity between product 1 and product 2.$(0 \leq \theta \leq 1)$
λ :	The bundling discount price sensitivity $(0 < \lambda \leq a_2)$
s_i :	The safety stock of product i
s_B :	The safety stock in bundle policy, given by $s_B = Q_B - y(p_1, p_2, p_B)$
a_2 :	The response rate to the product price
a_1 :	Both product acceptances by the market (potential demand)
x_i :	Random noise of product i in the market
μ_i:	The expected value of random variable x_i
x_B :	Random noise in the market due to the bundle price
μ_B:	The expected value of random variable x_B
$F(s_B)$:	The safety stocking factor for bundling case
$f(x_i)$:	Probability distribution function of random variable x_i
$F(x_i)$:	Cumulative distribution function of random variable x_i
$y(p_1, p_2, p_B)$:	The demand function

Therefore, the demand function now becomes deterministic and known. It is given by:

$$y(p_1, p_2, p_B) = \tilde{a}_1 - a_2 p_B + \lambda(p_1 + p_2 - p_B) + x_B \tag{2}$$

Here, notice that Equation (2) does not consider the noise effect on demand market.

2.2. **Notation and assumptions.** The following notation is used throughout this paper. Some symbols and assumptions are similar to those given by Chen et al. (2010) and Yan and Bandyopadhyay (2011).

2.2.1. *Notation.* In the notation, the superscripts c and d refer to centralized and decentralized models, respectively.

With regard to per unit salvage value and per unit penalty cost of stock-out of bundled products, conditions $v_{2B} < v_{1B} + c_{2B}$and $\pi_B > v_{2B}$are established.

2.2.2. *Assumptions.* The following assumptions are made.

1. A single-supplier single-retailer supply chain is considered.
2. The supplier manufactures two complementary products, which are sold only as a bundle.
3. The demand depends on selling price and a random noise effect on the market is assumed.
4. During the two stages, three decisions related to production capacity reservation, order quantity and bundle price must be made.
5. During the first stage, the supplier predicts demand and a production capacity for the retailer is reserved.
6. During the second stage, the retailer updates the prediction of the demand and determines the bundling order quantity as well as the bundle price.
7. The following two types of uncertainty the bundled products acceptance by market and noise effect are considered.

3. Coordination supply chain model formulation and its solution.

3.1. Strategy without bundling.

The demand functions for product 1 and product 2 without bundle policy considering the complementarity (θ) and noise effect (x_i) are:

$$\begin{array}{l} D_1(p_1,p_2) = a_1 - a_2 p_1 - a_2\theta p_2 + x_1 \\ D_2(p_1,p_2) = a_1 - a_2 p_2 - a_2\theta p_1 + x_2 \end{array} \tag{3}$$

where $a_1 \in [l_i, h_i]$, $x_i \in [L_i, H_i]$. If the reserved production capacity is enough for the order quantity of both products ($Q_1^c < M_1^c,\ Q_2^c < M_2^c$) then the supply chain's profit at stage 2 is:

$$\Pi_{21}^c = \begin{cases} (p_1 - c_{21})(y_1(p_1,p_2) + x_1) - v_{11}(y_1(p_1,p_2) + s_1) \\ \quad -c_{21}s_1 + v_{21}(s_1 - x_1) + v_{11}M_1^c + (p_2 - c_{22})(y_2(p_1,p_2) + x_2) \\ \quad -v_{12}(y_2(p_1,p_2) + s_2) + v_{22}(s_2 - x_2) + v_{12}M_2^c - c_{22}s_2 \\ (x_1 \le s_1, x_2 \le s_2) \equiv (D_1 \le Q_1^c, D_2 \le Q_2^c) \\ \\ (p_1 - c_{21})(y_1(p_1,p_2) + x_1) - v_{11}(y_1(p_1,p_2) + s_1) \\ \quad -c_{21}s_1 + v_{21}(s_1 - x_1) + v_{11}M_1^c + (p_2 - c_{22})(y_2(p_1,p_2) + s_2) \\ \quad - v_{12}(y_2(p_1,p_2) + s_2) - \pi_2(x_2 - s_2) + v_{12}M_2^c - c_{22}s_2 \\ (x_1 \le s_1, x_2 > s_2) \equiv (D_1 \le Q_1^c, D_2 > Q_2^c) \\ \\ (p_1 - c_{21})(y_1(p_1,p_2) + s_1) - v_{11}(y_1(p_1,p_2) + s_1) \\ \quad -\pi_1(x_1 - s_1) + v_{11}M_1^c - c_{21}s_1 + (p_2 - c_{22})(y_2(p_1,p_2) + s_2) \\ \quad -v_{12}(y_2(p_1,p_2) + s_2) - \pi_2(x_2 - s_2) + v_{12}M_2^c - c_{22}s_2 \\ (x_1 > s_1, x_2 > s_2) \equiv (D_1 > Q_1^c, D_2 > Q_2^c) \\ \\ (p_1 - c_{21})(y_1(p_1,p_2) + s_1) - v_{11}(y_1(p_1,p_2) + s_1) \\ \quad -\pi_1(x_1 - s_1) + v_{11}M_1^c - c_{21}s_1 + (p_2 - c_{22})(y_2(p_1,p_2) + x_2) \\ \quad -v_{12}(y_2(p_1,p_2) + s_2) + v_{22}(s_2 - x_2) + v_{12}M_2^c - c_{22}s_2 \\ (x_1 > s_1, x_2 \le s_2) \equiv (D_1 > Q_1^c, D_2 \le Q_2^c) \end{cases} \tag{4}$$

Using algebra, the prices for products 1 and 2 when $Q < M$are calculated using Equation (4), when there is no bundle selling.

$$\begin{aligned} p_1(s) &= \left(\frac{\tilde{a}_1(1-\theta) + \mu_1 - \mu_2\theta}{2a_2(1-\theta^2)} + \left(\frac{c_{21} + v_{11}}{2} \right) - \frac{B_1(s) - \theta B_2(s)}{2a_2(1-\theta^2)} \right) \\ p_2(s) &= \left(\frac{\tilde{a}_1(1-\theta) + \mu_2 - \mu_1\theta}{2a_2(1-\theta^2)} + \left(\frac{c_{22} + v_{12}}{2} \right) - \frac{B_2(s) - \theta B_1(s)}{2a_2(1-\theta^2)} \right) \end{aligned} \tag{5}$$

Where$B_i\left(s\right) = \int_{s_i}^{H_i} \left(x_i - s_i\right) f\left(x_i\right) dx_i$and the detailed derivation of Equation (5) is given in Appendix A.

Moreover, the prices for product 1 and product 2 when $Q = M$ are determined using the following equations (See Appendix B).

$$\begin{aligned} &\int_{M_1^c - y_1(p_1,p_2)}^{H_1} [M_1^c - x_1 - y_1(p_1,p_2) + a_2(p_1 - c_{21} + \pi_1 - v_{21})] f(x_1) dx_1 \\ &\qquad + (y_1(p_1,p_2) + \mu_1 - a_2(p_1 - c_{21} - v_{21}) - a_2(p_2 - c_{22} - v_{22})) \\ &\qquad + \int_{M_2^c - y_2(p_1,p_2)}^{H_2} a_2\theta(p_2 - c_{22} + \pi_2 - v_{22}) f(x_2) dx_2 = 0 \\ &\int_{M_2^c - y_2(p_1,p_2)}^{H_2} [y_2(p_1,p_2) + x_2 - M_2^c + a_2(p_2 - c_{22} + \pi_2 - v_{22})] f(x_2) dx_2 \\ &\qquad + y_2(p_1,p_2) + \mu_2 - a_2(p_2 - c_{22} - v_{22}) - a_2\theta(p_1 - c_{21} - v_{21}) \\ &\qquad - \int_{M_1^c - y_1(p_1,p_2)}^{H_1} a_2\theta(p_1 - c_{21} + \pi_1 - v_{21}) f(x_1) dx_1 = 0 \end{aligned} \tag{6}$$

3.2. **Centralized policy model considering bundling.** First, the centralized model is considered for determining the joint decision on bundle price and order quantity of bundled products. In stage 2, the value of $a_1$is already known as $\tilde{a}_1$therefore the demand function for the product under the bundle policy is:

$$D(p_1, p_2, p_B) = \tilde{a}_1 - a_2 p_B + \lambda(p_1 + p_2 - p_B) + x_B \tag{7}$$

The safety stock of bundled products (s_B) is defined as follows$s_B = Q_B^c - y(p_1, p_2, p_B)$. Here, Q_B^c represents the ordering quantity of bundled products in the centralized model.

3.2.1. *When the reserved production capacity is enough.* If the reserved production capacity is enough for the order quantity of bundled products ($Q_B^c < M_B^c$) then the supply chain's profit at stage 2 is:

$$\Pi_B^c = \begin{cases} (p_{1B} - c_{2B}) \left[y(p_1, p_2, p_{1B}) + x_B \right] - (v_{11} + v_{12}) \left[y(p_1, p_2, p_{1B}) + s_B \right] \\ \quad + (s_B - x_B)(v_{21} + v_{22}) + M_B^c(v_{11} + v_{12}) - (c_{21} + c_{22}) s_B & x_B \le s_B \\ \\ (p_{1B} - c_{2B}) \left[y(p_1, p_2, p_{1B}) + s_B \right] - (v_{11} + v_{12}) [y(p_1, p_2, p_{1B}) + s_B] \\ \quad - \pi_B(x_B - s_B) + M_B^c(v_{11} + v_{12}) - (c_{21} + c_{22}) s_B & x_B > s_B \end{cases} \tag{8}$$

Calculating the expected value with respect to random noise variable (x_B) one obtains the expected value for the centralized supply chain profit under the bundle policy, given by:

$$\begin{aligned} E[\Pi_B^c] &= \textstyle\int_L^{s_B}\{(p_{1B}-c_{2B})[y(p_1,p_2,p_{1B})+x_B]+(v_{21}+v_{22})(s_B-x_B)\}f(x_B)dx_B \\ &+M_B^c(v_{11}+v_{12})+\textstyle\int_{s_B}^{H}\{(p_{1B}-c_{2B})[y(p_1,p_2,p_{1B})+s_B]-\pi_B(x_B-s_B)\}f(x_B)dx_B \\ &-(c_{21}+c_{22})s_B-(v_{11}+v_{12}))[y(p_1,p_2,p_{1B})+s_B] \end{aligned} \tag{9}$$

where $c_{2B}=c_{21}+c_{22}$.

Now let$U(s_B)=\int_L^{s_B}(s_B-x_B)f(x_B)dx_B$, $B(s_B)=\int_{s_B}^{H}(x_B-s_B)f(x_B)dx_B$, substituting the above relations into Equation (8) one obtains,

$$\begin{aligned} E[\Pi_B^c] &= (p_{1B}-c_{2B}-v_{11}-v_{12})\,(y(p_1,p_2,p_{1B})+\mu_B) \\ &-(c_{2B}+v_{11}+v_{12}-v_{21}-v_{22})U(s_B) \\ &-(p_{1B}+\pi_B-c_{2B}-v_{11}-v_{12})B(s_B)+(v_{11}+v_{12})M_B^c \end{aligned} \tag{10}$$

If there is no capacity constraint for$s_B$then we have (See Appendix C):

$$\begin{aligned} \frac{\partial E[\Pi_B^c]}{\partial p_{1B}} =& a_1-a_2p_{1B}+\lambda(p_1+p_2)-\lambda p_{1B}+\mu_B \\ &+(a_2+\lambda)(c_{22}+c_{21}+v_{11}+v_{12}-p_{1B})-B(s_B) \end{aligned} \tag{11}$$

Note that $E[\Pi_B^c]$ is concave with respect to bundle price (p_{1B}). Then the bundle price under bundling strategy is:

$$\begin{aligned} p_{1B} =& \frac{1}{2(a_2+\lambda)}\,(a_1+\lambda(p_1+p_2)+\mu_B+(a_2+\lambda)(c_{22}+c_{21}) \\ &+(a_2+\lambda)(v_{11}+v_{12})-B(s_B)) \end{aligned} \tag{12}$$

Assuming that the demand function at stage 2 is given by $D(p_1,p_2,p_{1B}) = \tilde{a}_1-a_2p_{1B}+\lambda(p_1+p_2-p_{1B})+x_B$ and if there is no production capacity constraint, then the optimal joint decision is to vend the bundle at $p_B^c$and the order quantity of bundled products is $Q_B^c=y(p_1,p_2,p_{1B}^c)+s_B^c$. The s_B^c corresponds to the largest solution that satisfies Equation (13) where $v_{2B}=v_{21}+v_{22}$.

$$-(c_{2B}+v_{1B}-v_{2B})+(p_{1B}^c+\pi_B-v_{2B})[1-F(s_B^c)]=0 \tag{13}$$

The expected profit of the supply chain in the centralized model ($E[\Pi_B^c]$) is unimodal ins_B^c orQ_B^c. Thus, there exists only one solution to Equation (13). This is because the following condition$p_{1B}^c-c_{2B}\geq v_{1B}-\pi_B$ holds. Otherwise, no bundled products should be vended at stage 2.

3.2.2. *When the reserved production capacity is not enough.* If the reserved production capacity is not enough for the order quantity of bundled products then $Q_B^c = M_B^c$ and the supply chain profit at stage 2 is:

$$\Pi_B^c=\begin{cases} (p_{2B}-c_{2B})[y(p_1,p_2,p_{2B})+x_B] \\ \qquad +(v_{21}+v_{22})[M_B^c-y(p_1,p_2,p_{2B})-x_B]-c_{2B}M_B^c & D\leq M_B^c \\ \\ (p_{2B}-c_{2B})M_B^c-\pi_B[y(p_1,p_2,p_{2B})+x_B-M_B^c]-c_{2B}M_B^c & D>M_B^c \end{cases} \tag{14}$$

Where $D=y(p_1,p_2,p_{2B})+x_B$. In this case, the expected profit can be calculated as follows:

$$\begin{aligned} E[\Pi_B^c] &= \textstyle\int_L^{M_B^c-y(p_1,p_2,p_{2B})}\,[(p_{2B}-c_{2B})[y(p_1,p_2,p_{2B})+x_B] \\ &+(v_{21}+v_{22})[M_B^c-y(p_1,p_2,p_{2B})-x_B]]\,f(x_B)dx_B \\ &+\textstyle\int_{M_B^c-y(p_1,p_2,p_{2B})}^{H}\,[(p_{2B}-c_{2B})M_B^c-\pi_B[y(p_1,p_2,p_{2B})+x_B-M_B^c]]\,f(x_B)dx_b \\ &-c_{2B}M_B^c \end{aligned}$$

$$E[\Pi_B^c] = (p_{2B} - c_{2B} - v_{21} - v_{22})[y(p_1, p_2, p_{2B}) + \mu_B] + (v_{21} + v_{22} - c_{2B})M_B^c \\ -(p_{2B} - c_{2B} - v_{21} - v_{22} + \pi_B) \int_{M_B^c - y(p_1,p_2,p_{2B})}^{H} [[y(p_1, p_2, p_{2B}) + x_B - M_B^c]]\times \\ \times f(x_B)dx_B \tag{15}$$

It is easy to show that $E[\Pi_B^c]$ is a concave function.

$$\frac{\partial E[\Pi_B^c]}{\partial p_{2B}} = y(p_1, p_2, p_{2B}) + \mu_B - (a_2 + \lambda)(p_{2B} - c_{2B} - v_{21} - v_{22}) \\ + \int_{M_B^c - y(p_1,p_2,p_{2B})}^{H} (M_B^c - y(p_1, p_2, p_{2B}) - x_B \\ +(a_2 + \lambda)(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22}))\, f(x_B)dx_B \tag{16}$$

$$\frac{\partial^2 E[\Pi_B^c]}{\partial p_{2B}^2} = -2(a_2 + \lambda) \int_{L}^{M_B^c - y(p_1,p_2,p_{2B})} f(x_B)dx_B \\ -(a_2 + \lambda)^2(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22})f\left(M_B^c - y(p_1, p_2, p_{2B})\right) < 0 \tag{17}$$

For the detailed derivation of Equation (17) see Appendix D.

If the reserved production capacity is less than the optimal level specified in the order, then the optimal selling price can be determined solving Equation (17),

$$y(p_1, p_2, p_{2B}) + \mu_B - (a_2 + \lambda)(p_{2B} - c_{2B} - v_{21} - v_{22}) \\ + \int_{M_B^c - y(p_1,p_2,p_{2B})}^{H} \left[\begin{array}{l} M_B^c - y(p_1, p_2, p_{2B}) - x_B \\ \quad +(a_2 + \lambda)(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22}) \end{array} \right] f(x_B)dx_B = 0 \tag{18}$$

3.3. **Production capacity reservation decision at stage 1.** In the centralized supply chain, the global decision maker needs to reserve production capacity at the beginning of stage 1. When production capacity constraint is not binding, the optimal order quantity of bundled products at stage 2 is defined as follows.

$$h^c(\tilde{a}_1) = \tilde{a}_1 - a_2 p_{1B} + \lambda(p_1 + p_2 - p_{1B}) + s_B^c = y(p_1, p_2, p_{1B}) + s_B^c \tag{19}$$

Therefore, from Equation (12) we have;

$$1 - F(s_B^c) - (p_{1B}^c + \pi_B - (v_{21} + v_{22}))f(s_B^c)\frac{\partial s_B^c}{\partial p_{1B}^c} = 0 \tag{20}$$

Equation (20) implies that $\frac{\partial s_B^c}{\partial p_{1B}^c} > 0$, hence, $\frac{\partial s_B^c}{\partial \tilde{a}_1} = \frac{\partial s_B^c}{\partial p_{1B}^c}.\frac{\partial p_{1B}^c}{\partial \tilde{a}_1}$. The first derivative of $h^c(\tilde{a}_1)$ with respect to $\tilde{a}_1$ is given by;

$$\frac{\partial h^c(\tilde{a}_1)}{\partial \tilde{a}_1} = 1 - a_2\frac{\partial p_{1B}^c}{\partial \tilde{a}_1} - \lambda\frac{\partial p_{1B}^c}{\partial \tilde{a}_1} + \frac{\partial s_B^c}{\partial \tilde{a}_1} = \frac{1}{2}\left\{1 + [1 + F(s_B^c)]\frac{\partial s_B^c}{\partial \tilde{a}_1}\right\} > 0 \tag{21}$$

According to equation (21) it is easy to see that $h^c(\tilde{a}_1)$ is increasing in$\tilde{a}_1$. The values for $p_{1B}^c, s_B^c$are jointly calculated by Equation (12). Now, we know that $M_B^c >$ 0because $p_{1B} > c_{1B} + c_{2B}$. Therefore, the demand function is positive. The full benefit under the bundle policy is:

$$E[\Pi_B^c] = -c_{1B}M_B^c + \int_{l}^{a(M_B^c)} E[\Pi_B^{c*}]g(a_1)da_1 + \int_{a(M_B^c)}^{h} E[\Pi_B^{c*}]g(a_1)da_1. \tag{22}$$

It is supposed that the reserved production capacity is in the interval $h^c(l) \leq M_B^c \leq h^c(h)$ and$a(M_B^c) = 2M_B^c + \mu + a_2(c_{2B} + v_{1B}) - B(s_B^c) - 2s_B^c$. At these values, the reserved production capacity matches the optimal order quantity of bundled products when there is no capacity limitation at stage 2. Therefore, the following theorem is proposed to determine the production capacity. This theorem affirms there is a unique solution for the optimal production capacity.

Theorem 3.1. *The optimal reserved production capacity reservation is the solution of Equation* (23).

$$\int_{l}^{a(M_B^c)}(v_{1B})g(a_1)da_1 + \int_{a(M_B^c)}^{h}[(v_{1B}-c_{1B})+(p_{2B}^c-c_{2B}+\pi_B-v_{1B})\int_{M_B^c-y(p_1,p_2,p_{2B})}^{H}f(x_B)dx_B]g(a_1)da_1 = c_{1B}=c_{11}+c_{12} \tag{23}$$

The derivation details of this theorem are shown in Appendix E.

3.4. **Decentralized policy under the bundle policy.** The decentralized supply chain is now considered in order to determine the best decisions. Here, the supplier decides the production capacity at stage 1 and the retailer decides the order quantity of the bundled products after the demand is updated at stage 2.

In the decentralized supply chain, both parties have sufficient information about market demand and costs. For the coordination of the supply chain, a three-parameter contract $(w_B,\alpha_B,d_B)$is used, as proposed by Chen et al. (2010). In contract(w_B,α_B,d_B), w_B is the wholesale price that the supplier offers to its retailer,$d_B$is the unit buyback price that the supplier pays to its retailer for each unit of the unsold bundle at the end of stage 2, and $\alpha_B$is the risk subsidy. When α_B is positive, there is a compensation that the retailer pays to its supplier to cover the loss due to the over reserved production capacity in stage 1. Conversely, when $\alpha_B$is negative, there is a bonus for the retailer from its supplier to avoid the over ordering of the retailer at stage 2. Following Chen et al. (2010)'s solution procedure, the following results are obtained.

$$\begin{aligned} w_B &= c_{1B}+c_{2B}+\rho\left[p_B+\pi_B-c_{1B}-c_{2B}-\tfrac{\pi_B(y(p_1,p_2,p_B)+\mu_B)}{M_B^d}\right]+\tfrac{\beta}{M_B^d} \\ \alpha_B &= (1-\rho)(c_{1B}-v_{1B})-\tfrac{\rho\pi_B(y(p_1,p_2,p_B)+\mu_B)}{M_B^d}+\tfrac{\beta}{M_B^d} \\ d_B &= v_{2B}+\rho(p_B+\pi_B-v_{2B}) \end{aligned} \tag{24}$$

Where$\rho$is on the interval [0, 1] andβ is any real number.

If the reserved production capacity is enough for the order quantity of product, $Q_B^d < M_B^d$, the benefit for the retailer under the contract is obtained as follows:

$$\Pi_B^r = \begin{cases} p_{1B}\left(y(p_1,p_2,p_{1B})+x_B\right)-(w_B-\alpha_B)\left(y(p_1,p_2,p_{1B})+s_B\right) \\ \quad +d_B(s_B-x_B)-\alpha_B M_B^d & x_B\le s_B \\ p_{1B}\left(y(p_1,p_2,p_{1B})+s_B\right)-(w_B-\alpha_B)\left(y(p_1,p_2,p_{1B})+s_B\right) \\ \quad -\pi_B(x_B-s_B)-\alpha_B M_B^d & x_B > s_B \end{cases} \tag{25}$$

Then the retailer's and supplier's expected profits are given by Equations (26) and (27), respectively.

$$\begin{aligned} E[\Pi_B^r] = &(p_{1B}-w_B+\alpha_B)\left(y(p_1,p_2,p_{1B})+\mu_B\right)-(w_B-d_B-\alpha_B)U(s_B) \\ &-(p_{1B}+\pi_B-w_B+\alpha_B)B(s_B)-\alpha_B M_B^d \end{aligned} \tag{26}$$

$$\begin{aligned} E[\Pi_B^m] = &(w_B-c_{2B}-v_{1B}-\alpha_B)\left(y(p_1,p_2,p_{1B})+\mu_B\right)+(v_{1B}+\alpha_B)M_B^d \\ &-(d_B-v_{2B})U(s_B) \end{aligned} \tag{27}$$

If the supplier's reserved production capacity fails to fulfil the retailer's optimal order quantity ($Q_B^d = M_B^d$) then the retailer's profit is determined as follows:

$$\Pi_B^r = \begin{cases} p_{2B}\left(y(p_1,p_2,p_{2B}) + x_B\right) - w_B M_B^d + d_B\left(M_B^d - y(p_1,p_2,p_{2B}) - x_B\right) \\ \qquad x_B \le M_B^d - y(p_1,p_2,p_{2B}) \\ \left(p_{2B} - w_B\right) M_B^d - \pi_B\left(y(p_1,p_2,p_{2B}) + x_B - M_B^d\right) \\ \qquad x_B > M_B^d - y(p_1,p_2,p_{2B}) \end{cases} \tag{28}$$

Now, the retailer's and supplier's expected profits functions are given by Equations (29) and (30), respectively.

$$\begin{aligned} E[\Pi_B^r] = {} & \left(p_{2B} - d_B\right)\left(y(p_1,p_2,p_{2B}) + \mu_B\right) - \left(w_B - d_B\right) M_B^d \\ & + \left(p_{2B} + \pi_B - d_B\right)\int_{M_B^d - y(p_1,p_2,p_{2B})}^{H}\left(M_B^d - y(p_1,p_2,p_{2B}) - x_B\right) f(x_B)dx_B \end{aligned} \tag{29}$$

$$\begin{aligned} E[\Pi_B^m] = {} & \left(d_B - v_{2B}\right)\left(y(p_1,p_2,p_{2B}) + \mu_B\right) + \left(w_B - c_{2B} - d_B + v_{2B}\right) M_B^d \\ & + \left(d_B - v_{2B}\right)\int_{M_B^d - y(p_1,p_2,p_{2B})}^{H}\left(y(p_1,p_2,p_{2B}) + x_B - M_B^d\right) f(x_B)dx_B. \end{aligned} \tag{30}$$

To verify that the (w_B, α_B, d_B)shown in Equation (24) can coordinate the chain, it is necessary to check first the decision of the retailer in stage 2. Substituting contract parameters into Equation (26) yields the profit of the retailer in the first case as below;

$$\begin{aligned} & E[\Pi_B^r] = (1-\rho)\times \\ & \times\left[\begin{array}{l} \left(p_{1B} - c_{2B} - v_{11} - v_{12}\right)\left(y(p_1,p_2,p_{1B}) + \mu_B\right) - \left(c_{1B} - v_{11} - v_{12}\right) M_B^d \\ -\left(c_{2B} + v_{11} + v_{12} - v_{21} - v_{22}\right) U(s_B) - \left(p_{1B} + \pi_B - c_{2B} - v_{11} - v_{12}\right) B(s_B) \end{array}\right] - \beta \end{aligned} \tag{31}$$

For the second case, by substituting (w_B, α_B, d_B)shown in Equation (24) into Equation (29), it is obtained:

$$\begin{aligned} & E[\Pi_B^r] \\ & = (1-\rho)\left[\begin{array}{l} \left(p_{2B} - c_{2B} - v_{21} - v_{22}\right)[y(p_1,p_2,p_{2B}) + \mu_B] + \left(v_{21} + v_{22} - c_{1B} - c_{2B}\right) M_B^d \\ -\left(p_{2B} - c_{2B} - v_{21} - v_{22} + \pi_B\right)\int_{M_B^c - y(p_1,p_2,p_{2B})}^{H} \times \\ \times\left[[y(p_1,p_2,p_{2B}) + x_B - M_B^d]\right] f(x_B)dx_B \end{array}\right] - \beta \end{aligned} \tag{32}$$

Additionally for the supplier, by substituting contract parameters in Equations (27) and (30), the profit functions for both cases are as below:

$$\begin{aligned} E[\Pi_B^m] = {} & \rho\left[\begin{array}{l} \left(p_{1B} - c_{2B} - v_{11} - v_{12}\right)\left(y(p_1,p_2,p_{1B}) + \mu_B\right) - \left(c_{1B} - v_{11} - v_{12}\right) M_B^d \\ -\left(c_{2B} + v_{11} + v_{12} - v_{21} - v_{22}\right) U(s_B) - \left(p_{1B} + \pi_B - c_{2B} - v_{11} - v_{12}\right) B(s_B) \end{array}\right] \\ & + \beta + c_{1B} M_B^d \end{aligned} \tag{33}$$

$$\begin{aligned} E[\Pi_B^m] = {} & \rho\left[\begin{array}{l} \left(p_{2B} - c_{2B} - v_{21} - v_{22}\right)[y(p_1,p_2,p_{2B}) + \mu_B] + \left(v_{21} + v_{22} - c_{1B} - c_{2B}\right) M_B^d \\ -\left(p_{2B} - c_{2B} - v_{21} - v_{22} + \pi_B\right)\int_{M_B^c - y(p_1,p_2,p_{2B})}^{H} \times \\ \times\left[[y(p_1,p_2,p_{2B}) + x_B - M_B^d]\right] f(x_B)dx_B \end{array}\right] \\ & + \beta + c_{1B} M_B^d \end{aligned} \tag{34}$$

So the retailer and the supplier decisions are the same as the results of the centralized case (See (8) and (14)), therefore the supply chain is coordinated.

In order to implement a contract, it is necessary that the supply chain profit can be allocated between the members under contract. Now, it is investigated whether the proposed contract can allocate any division of profit among the supply chain members:

TABLE 2. Effects of basic demand size a_1 to the contract for product 1 when $Q_1^c < M_1^c = 487$

a_1	p_1	w_1	d_1	α_1	Q_1^c	$F(s_1)$
500	237	151	133.50	2.64	316	0.887
550	259	161	144.50	1.85	343	0.896
600	282	172	156.00	1.08	369	0.903
650	304	183	167.00	0.29	397	0.910
700	326	192	178.00	-0.50	424	0.915
750	348	203	189.00	-1.29	450	0.920
800	371	213	200.50	-2.50	477	0.925
819.9	380	218	205.00	-2.36	487	0.927

Consider the case whenβ=0. It means the retailer takes up the entire profit of the supply chain. But if it is assumed that$\rho = 1$, the supplier takes the entire profit of the supply chain. When $0 < \rho < 1$, then ρ proportion of the supply chain's expected profit goes to the supplier and the remaining portion, $(1 - \rho)$, goes to the retailer. Thus, the proposed contract in this research is flexible enough to allocate an arbitrary profit between the members.

From Equations (31) to (34), it is concluded that β can be used as the transfer payment between the members. If$\beta = 0$, there is no transfer payment between retailer and supplier; but when$\beta > 0$, the retailer needs to pay the supplier. On the other hand, the supplier needs to pay the retailer when$\beta < 0$. Thus in addition toρ, β also reflects the relative powers between the supplier and the retailer.

For the proposed contract (w_B, α_B, d_B)satisfying Equation (24), w_B is increasing in ρ andβ, while$\alpha_B$is decreasing inρ, but increasing inβ. Moreover, d_B is increasing in$\rho$and independent ofβ. Thus, they have different effects on the contract (w_B, α_B, d_B)as well as the retailer's and the supplier's profit structures. Although $\rho$and β are in different scales, having different meanings, and affecting the profit structures differently, both can reflect the relative powers of the members.

4. **Computational and practical results.** In this part, a numerical example is constructed to illustrate the ideas developed in this research work. Here, we let the parameters as $H_i = 100$, $L_i = -100$, $H_B = 200, L_B = -200, h = 1000$, $\mu_i = 0\,, l = 500,\ c_{11} = 20,\ c_{12} = 30,\ c_{21} = 30, c_{22} = 40\,,\ a_2 = 1, \theta = 0.1, \lambda = 1,\ \rho = 0.5,\ \beta = 0, c_{1B} = 50,\ c_{2B} = 70,\ v_{1i} = v_{2i} = 0,\ \pi_1 = 30,\ \pi_2 = 40\,, \pi_B = 70, \mu_B = 0$, some of which are used based on Chen et al. (2010). The random variables s_B and a_1 follow a uniform distribution. Using these data, the optimal values for decision variables are obtained and expressed in Tables 2 to 7.

In Table 2 the effects of the basic demand size (a_1) on the decision variables are shown. Table 3 represents the effects of basic demand size (a_1) to the contract for product 1, when $Q_1^c = M_1^c = 487$. Table 4 reports comparison between coordination contract vs price-only contract for profit of product 1. Since the same results from the second product are obtained, then, in order to avoid repetition, we do not perform the same analysis. Table 5, shows the effects of the basic demand size of bundled products on the related decision variables when the reserved production capacity is enough for the order quantity of the bundled products. Table 6 shows the effects of basic demand size on the contract under bundling policy.

Table 3. Effects of basic demand size a_1 to the contract for product 1, $Q_1^c = M_1^c = 487$

a_1	p_1	w_1	d_1	α_1	$F(s_1)$
820	405	231	217.50	-1.50	0.975
850	427	242	228.50	-1.68	0.965
880	449	253	239.50	-1.86	0.950
910	472	264	251.00	-2.04	0.940
940	495	275	262.50	-2.16	0.905
970	518	287	274.00	-2.31	0.880
1000	541	298	285.50	-2.46	0.855

Table 4. Comparison between coordination contract vs price-only contract for profit of product 1, Coordination contract: M_1^c =487 and total profit=279270

w_1	Capacity	Supplier profit	Retailer profit	Total profit
163	449	142000	123210	265210
199	426	125720	105810	231530
250	401	10954	90740	101694
290	378	95801	75437	171238
320	356	81918	62549	144467

Table 5. Effects of basic demand size to the contract under bundling policy, $Q_B^c < M_B^c = 867$

a_1	p_{1B}	w_B	d_B	α_B	Q_B^c	$F(s_B)$
500	279	216	174.50	8.00	564	0.799
550	303	225	186.50	6.10	592	0.812
600	327	234	198.50	4.20	644	0.823
650	350	244	210.00	2.27	696	0.833
700	374	255	222.00	0.41	746	0.842
750	398	264	234.00	-1.48	796	0.850
800	421	273	245.50	-3.46	847	0.857
819.9	431	278	250.50	-4.18	867	0.860

Table 6. Effects of basic demand size on the contract under bundling policy when, $Q_B^c = M_B^c = 867$

a_1	p_{2B}	w_B	d_B	α_B	$F(s_B)$
820	437	279	253.50	-5.92	0.752
850	474	298	272.0	-5.90	0.750
880	510	315	290.00	-5.86	0.747
910	546	333	308.00	-5.60	0.742
940	585	353	327.00	-5.96	0.740
970	621	371	345.50	-6.25	0.739
1000	658	389	364.00	-6.35	0.736

TABLE 7. Profit with bundling policy, Proposed contract: M_B^c=867 and total profit=260621

w_B	Capacity	Supplier profit	Retailer profit	Total profit
234	838	142490	104540	247030
259	794	126730	87871	214601
299	745	112410	72037	184447
352	668	96453	60367	156820
386	630	85792	45607	131399

From Figures 1 and 2, it is easy to see that the impact of the basic demand a_1 on both pricing strategies with bundling and without bundling is positive; that is, the prices and wholesale prices always increase with an increase in the value of a_1.

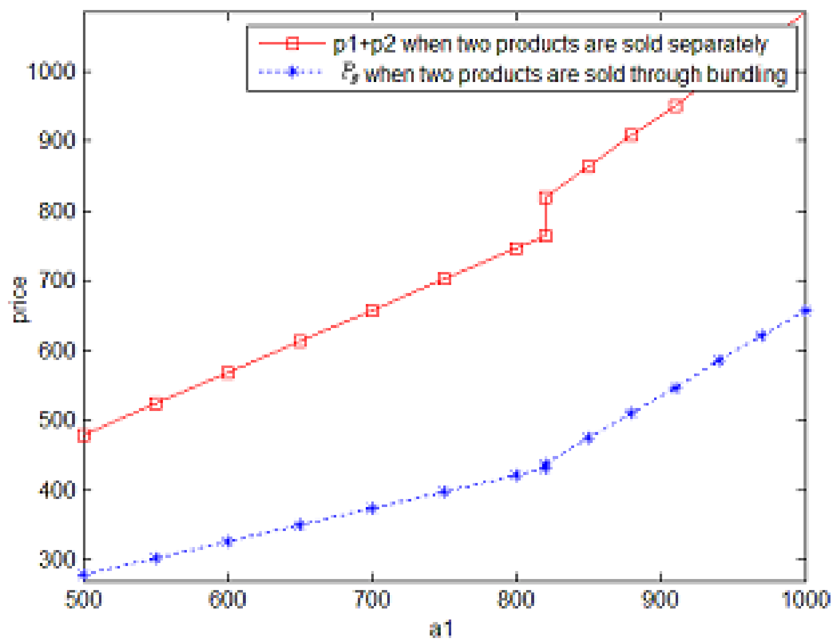

FIGURE 1. Impact a_1 between two products on the retailer's pricing strategy.

Now, a sensitivity analysis is conducted. Principally, we vary the value of one parameter and fix the others. Then we see its impact on the optimal values for of decision variables and the total profit for the supplier and retailer. The initial data for the sensitivity analysis are $a_1 = 750$, $H_i = 100$, $L_i = -100$, $\mu_i = 0$, $c_{11} = 20$, $c_{12} = 30$, $c_{21} = 30$, $c_{22} = 40$, $a_2 = 0.5$, $L_B = -200$, $H_B = 200$, $\mu_B = 0$, $l = 500$, $h = 1000$, $\lambda = 0.35$, $\rho = 0.5$, $\beta = 0$, $\pi_B = 70$, $v_{1i} = v_{2i} = 0$, $c_{1B} = 50$, $c_{2B} = 70$, $\pi_1 = 30$, $\pi_2 = 40$, $\theta = 0.25$, where the basic optimal values of decision variables based, on the mentioned initial data, are $p_1 = 240$, $p_2 = 242$, $p_B = 424$, $F(s_B) = 0.834$, $w_B = 278$, $\alpha_B = -3.60$, $Q_B^c = 624$, $d_B = 247$, retailer's profit= 81592 and supplier's profit=115320.

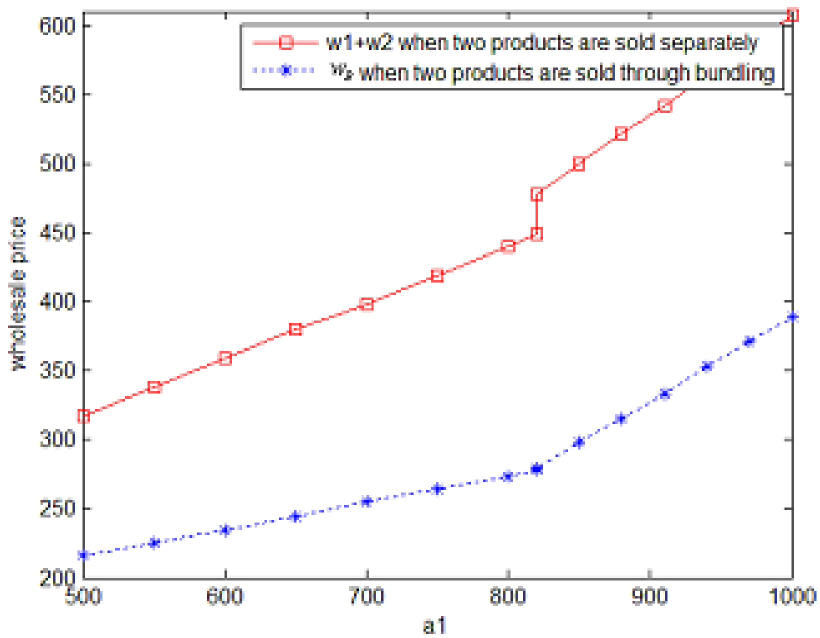

FIGURE 2. Impact a_1 between two products on the wholesale pricing strategy.

TABLE 8. The results in numerical analysis

	Percent change	p_1	p_2	p_B	$F(s_B)$	Q_B^c	w_B	α_B	d_B	Retailer profit	Supplier profit
$a_2 = 0.5$	+50	52.8	52.48	38.44	6.02	-29.97	33.09	-279.44	33	-7.07	-4.66
	+25	25.83	26.03	18.63	3.12	-12.82	15.83	-121.39	15.99	4.00	3.47
	+15	15.42	15.70	11.08	2.04	-7.05	9.35	-68.06	9.51	3.99	3.13
	-15	-15.42	-15.29	-10.61	-2.28	6.09	-8.99	51.94	-9.11	-7.31	-5.91
	-25	-25.42	-25.62	-17.22	-4.08	9.29	-14.03	90.83	-14.98	-14.60	-9.97
	-50	Infeasible									
$\theta = 0.25$	+50	10.42	10.33	1.65	0.36	2.08	1.08	17.78	1.42	4.89	3.53
	+25	5.00	4.96	0.94	0.24	0.96	0.72	8.33	0.81	2.29	1.86
	+15	2.92	3.31	0.47	0.12	0.64	0.36	6.39	0.40	1.34	1.55
	-15	-3.33	-3.31	-0.71	0.00	-0.32	-0.36	-3.06	-0.61	-1.91	-0.08
	-25	-5.42	-4.96	-0.94	-0.12	-0.96	-0.72	-6.39	-0.81	-2.32	-1.05
	-50	-10	-10.33	-1.65	-0.24	-1.06	-1.08	-14.72	-1.42	-4.72	-3.42
$\lambda = 0.35$	+50	Infeasible									
	+25	0.00	0.00	13.21	2.40	-7.05	11.15	-66.39	11.34	6.83	5.81
	+15	0.00	0.00	8.25	1.56	-4.01	6.83	-39.44	6.88	5.11	4.10
	-15	0.00	0.00	-8.25	-1.68	4.17	-6.47	36.67	-7.29	-7.63	-4.22
	-25	0.00	0.00	-13.44	-3.00	5.61	-11.15	55.83	-11.74	-12.06	-8.73
	-50	0.00	0.00	-25.71	-6.83	8.33	-20.86	91.67	-22.06	-27.45	-19.74

The results of the sensitivity analysis based only in changes on the parametersa_2, $\lambda$and θ are shown in Table 8.

5. **Managerial insights.** From Table 2 to Table 8 the following managerial insights are observed:

1. According to Table 2, increasing the basic demand size increases the selling price, wholesale price, order quantity and unit buyback price from the supplier to its retailer, while the risk subsidy decreases. Also by increasing the basic demand, the safety stock level increases. These results occur when the reserved production capacity is enough for the order quantity of the first product. When α_1 is positive, there is a compensation that the retailer pays to its supplier to cover the loss due to the over reserved production capacity in stage 1. Conversely, when α_1 is negative, there is a bonus for the retailer from its supplier to avoid the over ordering of the retailer at stage 2.
2. According to Table 3 results, when there is not enough capacity, the same behavior occurs for selling price, wholesale price, order quantity, unit buyback price and the risk subsidy. Conversely, increasing the basic demand decreases the safety stock level.
3. According to Table 4, one can observe that the total supply chain profits in the price-only supply chain case (transaction between the supplier and the retailer is solely based on the wholesale price and there is no risk sharing) are lower than under contract, with a range of loss between 5% and 48%. Also, notice that the supplier's profit is ever higher than the retailer's profit. It means retailer should try to set the contract parameters between himself and the supplier to gain more profit.
4. According to Table 5, increasing the basic demand size a_1 causes the bundle selling price to increase, as well as the wholesale price, order quantity and unit buyback price, but the risk subsidy decreases. Also by increasing the basic demand size of the bundled products, the safety stock level rises too.
5. According to Table 6, when there is not enough capacity, the same behavior occurs for the decision variables, but increasing the basic demand size decreases the safety stock level.
6. Table 7 shows the results for several wholesale prices. It can be noted that the total supply chain profits in the price-only supply chain are inferior to those of the centralized supply chain, with a range of loss between 5% and 50%.
7. According to Table 8, in the bundling policy, it should be noted that when a_2 and θ increase both p_1 and p_2 increase too. On the other hand, p_1 and p_2 are highly sensitive respect to the changes in the values of $a_2$while are sensitive respect to the changes in the values ofθ. Since p_1 and p_2 are not function ofλso they are not sensitive respect to the changes in the values ofλ. However, p_B is highly sensitive respect to the change in the values of a_2 and it increases when a_2 increases.
8. Also,$p_B$is sensitive respect to the increase in the values of λbut $p_B$is highly sensitive respect to the decrease in the values of λ. On the other hand,p_B,Q_B^c, w_B, and d_B are slightly sensitive respect to the change in the values of θ. When $a_2$decreases by 50%, there is no feasible solution.
9. On the other hand,$F(s_B)$is sensitive respect to the increase of a_2 while it is slightly sensitive respect to the increase in the values ofθandλ. Additionally, Q_B^c is highly sensitive respect to the increase in the values of a_2 while Q_B^c is sensitive to the decrease in the values ofa_2. ConverselyQ_B^c, w_B are sensitive respects to the change in the values ofλ. Furthermore, w_B is highly sensitive respect to the change in the values ofa_2. When λincreases by 50%, there is no

feasible solution. Moreover, α_B is highly sensitive respect to the changes in the values of $a_2$andλ. α_B is sensitive respect to the change in the values ofθ. It is noted that the supplier's profit and retailer's profit are sensitive respect to the change in the values of a_2 while the profits are slightly sensitive respect to the change in the values ofθ. Likewise, the supplier's profit is sensitive respect to the change in the values ofλ. In contrast, the retailer's profit is highly sensitive respect to the decrease in the values of λ and it is sensitive respect to the increase in the values ofλ.

The practical application of the proposed model in this paper is in the sense that in the real life-marketing situation, it is observed the bundle strategy for selling the products everywhere. Some manufacturing firms' sells their products in the way of buy one get one free. Due to this offers, the manufacturer tries to attract more customers in order that they buy the more products. In addition, someone tie up with other manufacturing firms and sells the products in bundle form with different products. This kind of strategy, it is seen everywhere in the market all over world. Main aim of the manufacturing firms is how to get more profit for sales of the products and attract more and more customers. Therefore, one objective of this paper is how to represent and solve mathematically this type problem.

6. **Conclusion.** In this research work, a supply chain optimization via a coordination contract, considering that demand has random noise, is studied. We used the profit and risk-sharing contract that can coordinate the chain in a decentralized situation. In the chain studied, two complementary fashion products are sold. This paper analyzes the following two pricing strategies: 1) pricing of complementary products without bundling policy and 2) pricing of complementary products under bundling policy. In the first strategy, the products are sold individually, while in the second strategy both products are sold together. In this study, it is shown that in both pricing strategies, the retail price and the wholesale price of products increase with an increment in the demand. Also, with increment in the market demand, the order of bundled products also increases. It was found that the retailer does not tend to order extra products and the retailer considers the selling price lower when the demand is high.

In the bundling policy, the wholesale and retail prices are lower than the prices without bundling. Moreover, the capacity has important effects on the decision variables, especially on the service level constraint. When there is not enough capacity, increasing the basic demand decreases the safety stock level, while when the capacity is high enough, increasing the basic demand raises the safety stock level. This behavior can be seen both, when the products are sold individually and when they are sold as a bundle. As mentioned above, the price of the bundled products is lower than the prices of the individual products. These results indicate that a firm that uses bundling strategy to sell its products should offer a larger discount.

Moreover, the contract is shown to be flexible and can split the total supply chain profit in any portion between the supplier and the retailer, in a decentralized situation, under the proposed contract.

Finally, we conclude that the contract with bundling is applicable to the supply chain coordination and practitioners and academicians can use it. It is especially useful for businesses that produce and/or sells complementary products, because the proposed model fits those supply chains very well. If the product combination is right, the decision to bundle can produce the following results: 1) increase unit sales volume; 2) increase margins; 3) offer new channel and cross-industry opportunities; and 4) offer exposure to new potential customers.

For future research directions, we would recommend to consider dynamic pricing or marketing efforts such as cooperative advertising. Also of interest, there are other types of demand functions with asymmetric or non-asymmetric information that can be explored. Moreover, consider different potential demand for both types of products could be of interest. These are undoubtedly research topics that can be investigated by researchers in the near future.

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Appendix A. Detailed derivation of Equation (5) From Equation (4), we have:

$$\begin{aligned}
E\left[\pi_{21}^c\right] = &\int_{L_1}^{s_1}\left(\left(p_1-c_{21}\right)\left(y_1\left(p_1,p_2\right)+x_1\right)+v_{21}\left(s_1-x_1\right)\right)f\left(x_1\right)dx_1 \\
&+\int_{L_2}^{s_2}\left(\left(p_2-c_{22}\right)\left(y_2\left(p_1,p_2\right)+x_2\right)+v_{22}\left(s_2-x_2\right)\right)f\left(x_2\right)dx_2 \\
&-c_{21}s_1-v_{11}\left(y_1\left(p_1,p_2\right)+s_1\right)+v_{12}M_2^c-c_{22}s_2-v_{12}\left(y_2\left(p_1,p_2\right)+s_2\right) \\
&+\mathrm{v}_{11}M_1^c+\int_{L_1}^{s_1}\left(\left(p_1-c_{21}\right)\left(y_1\left(p_1,p_2\right)+x_1\right)+v_{21}\left(s_1-x_1\right)\right)f\left(x_1\right)dx_1 \\
&+\int_{s_2}^{H_2}\left(\left(p_2-c_{22}\right)\left(y_2\left(p_1,p_2\right)+s_2\right)+\pi_2\left(x_2-s_2\right)\right)f\left(x_2\right)dx_2 \\
&-c_{21}s_1-v_{11}\left(y_1\left(p_1,p_2\right)+s_1\right)+v_{12}M_2^c-c_{22}s_2-v_{12}\left(y_2\left(p_1,p_2\right)+s_2\right) \\
&+\mathrm{v}_{11}M_1^c+\int_{s_1}^{H_1}\left(\left(p_1-c_{21}\right)\left(y_1\left(p_1,p_2\right)+x_1\right)+\pi_1\left(x_1-s_1\right)\right)f\left(x_1\right)dx_1 \\
&+\int_{L_2}^{s_2}\left(\left(p_2-c_{22}\right)\left(y_2\left(p_1,p_2\right)+x_2\right)+v_{22}\left(s_2-x_2\right)\right)f\left(x_2\right)dx_2 \\
&-c_{21}s_1-v_{11}\left(y_1\left(p_1,p_2\right)+s_1\right)+v_{12}M_2^c-c_{22}s_2-v_{12}\left(y_2\left(p_1,p_2\right)+s_2\right)+\mathrm{v}_{11}M_1^c
\end{aligned}$$

where

$$U(s_i)=\int_{L_i}^{s_i}(s_i-x_i)f(x_i)dx_i \quad ,B(s_i)=\int_{s_i}^{H_i}(x_i-s_i)f(x_i)dx_i$$

$$\begin{aligned}
\Rightarrow E[\Pi_{21}^c] = &((p_1-c_{21}-v_{11})(y_1(p_1,p_2)+\mu_1)-(c_{21}+v_{11}-v_{21})U(s_1) \\
&-(p_1-c_{21}+\pi_1-v_{11})B(s_1)) \\
&+((p_2-c_{22}-v_{12})(y_2(p_1,p_2)+\mu_2)-(c_{22}+v_{12}-v_{22})U(s_2) \\
&-(p_2-c_{22}+\pi_2-v_{12})B(s_2))+v_{11}M_1^c+v_{12}M_2^c \\
&+\left(\begin{array}{l}(p_1-c_{21}-v_{11})(y_1(p_1,p_2)+\mu_1)-(c_{21}+v_{11}-v_{12})U(s_1) \\ -(p_1-c_{21}+\pi_1-v_{11})B(s_1)\end{array}\right) \\
&+(p_2-c_{22}-v_{12})(y_2(p_1,p_2)+\mu_2)-(c_{22}+v_{12}-v_{22})U(s_2) \\
&-(p_2-c_{22}+\pi_2-v_{12})B(s_2),+v_{11}M_1^c+v_{12}M_2^c
\end{aligned}$$

If there is no capacity constraint for s_i then we have:

$$H(p_1,p_2)=\begin{bmatrix}\frac{\partial^2E[\Pi_{21}]}{\partial p_1^2} & \frac{\partial^2E[\Pi_{21}]}{\partial p_1\partial p_2} \\ \frac{\partial^2E[\Pi_{21}]}{\partial p_2\partial p_1} & \frac{\partial^2E[\Pi_{21}]}{\partial p_2^2}\end{bmatrix}=\begin{bmatrix}-4a_2 & -4a_2\theta \\ -4a_2\theta & -4a_2\end{bmatrix}=16a_2^2(1-\theta^2)\ >0$$

Since $\frac{\partial^2E[\Pi_{21}]}{\partial p_1^2}=-4a_2<0$ and $\|H(p_1,p_2)\|=16a_2^2(1-\theta^2)\ >0$, $H(p_1,p_2)$ is negative definite matrix and the profit function is concave. Therefore $E[\Pi_{21}^c]$ is concave. Thus,

$$\begin{aligned}
\frac{\partial E[\Pi_{21}]}{\partial p_1}&=2\left(-2a_2p_1+a_1+2a_2c_{21}+v_{11}a_2+\mu_1-B(s_1)+v_{12}\theta a_2+2c_{22}a_2\theta\right)-4a_2\theta p_2 \\
\frac{\partial E[\Pi_{21}]}{\partial p_2}&=2\left(2a_2\theta c_{21}+a_1+2a_2c_{22}+v_{11}a_2\theta+\mu_2-B(s_2)+v_{12}a_2-2a_2p_2\right)-4a_2\theta p_1
\end{aligned}$$

Now, setting two above equations equal to zero and solving them simultaneously, we obtain the optimal values for p_1, p_2 as given below:

$$p_1(s) = \left(\frac{a_1(1-\theta)+\mu_1-\mu_2\theta}{2a_2(1-\theta^2)} + \left(\frac{c_{21}+v_{11}}{2}\right) - \frac{B_1(s)-\theta B_2(s)}{2a_2(1-\theta^2)}\right)$$
$$p_2(s) = \left(\frac{a_1(1-\theta)+\mu_2-\mu_1\theta}{2a_2(1-\theta^2)} + \left(\frac{c_{22}+v_{12}}{2}\right) - \frac{B_2(s)-\theta B_1(s)}{2a_2(1-\theta^2)}\right)$$

Where $B_i(s) = \int_{s_i}^{H_i} (x_i - s_i) f(x_i)\, dx_i$.

Appendix B. Detailed derivation of Equation (6)

If the reserved production capacity is not enough for the order quantity of product 1 and product 2 then $Q_1^c = M_1^c$, $Q_2^c = M_2^c$. Thus, the supply chain profit at stage 2 is:

$$\Pi_{22}^c = \begin{cases} (p_1 - c_{21}) D_1(p_1,p_2) - v_{21}(M_1^c - D_1(p_1,p_2)) - c_{21}M_1^c + (p_2 - c_{22}) D_2(p_1,p_2) \\ +v_{22}(M_2^c - D_2(p_1,p_2)) - c_{22}M_2^c \qquad (D_1 \le M_1^c, D_2 \le M_2^c) \\ (p_1 - c_{21}) D_1(p_1,p_2) - v_{21}(M_1^c - D_1(p_1,p_2)) - c_{21}M_1^c + (p_2 - c_{22}) M_2^c \\ -\pi_2(D_2(p_1,p_2) - M_2^c) - c_{22}M_2^c \qquad (D_1 \le M_1^c, D_2 > M_2^c) \\ (p_1 - c_{21}) M_1^c - \pi_1(D_1(p_1,p_2) - M_1^c) - c_{21}M_1^c + (p_2 - c_{22}) M_2^c \\ -\pi_2(D_2(p_1,p_2) - M_2^c) - c_{22}M_2^c \qquad (D_1 > M_1^c, D_2 > M_2^c) \\ (p_1 - c_{21}) M_1^c - \pi_1(D_1(p_1,p_2) - M_1^c) - c_{21}M_1^c + (p_2 - c_{22}) D_2(p_1,p_2) \\ +v_{22}(M_2^c - D_2(p_1,p_2)) - c_{22}M_2^c \qquad (D_1 > M_1^c, D_2 \le M_2^c) \end{cases}$$

In this case, the expected profit is determined as follows:

$$\begin{aligned} &E[\pi_{22}^c] \\ &= \int_{L_1}^{M_1^c - y_1(p_1,p_2)} ((p_1 - c_{21}) D_1(p_1,p_2) - v_{21}(M_1^c - D_1(p_1,p_2))) f(x_1)\, dx_1 - c_{21}M_1^c \\ &+ \int_{M_1^c - y_1(p_1,p_2)}^{H_1} ((p_1 - c_{21}) M_1^c - \pi_1(D_1(p_1,p_2) - M_1^c)) f(x_1)\, dx_1 \\ &+ \int_{L_2}^{M_2^c - y_2(p_1,p_2)} ((p_2 - c_{22}) D_2(p_1,p_2) - v_{22}(M_2^c - D_2(p_1,p_2))) f(x_2)\, dx_2 - c_{22}M_2^c \\ &+ \int_{M_2^c - y_2(p_1,p_2)}^{H_2} ((p_2 - c_{22}) M_2^c - \pi_2(D_2(p_1,p_2) - M_2^c)) f(x_2)\, dx_2 \\ &+ \int_{L_1}^{M_1^c - y_1(p_1,p_2)} ((p_1 - c_{21}) D_1(p_1,p_2) + v_{21}(M_1^c - D_1(p_1,p_2))) f(x_1)\, dx_1 - c_{21}M_1^c \\ &+ \int_{M_1^c - y_1(p_1,p_2)}^{H_1} ((p_1 - c_{21}) M_1^c - \pi_1(D_1(p_1,p_2) - M_1^c)) f(x_1)\, dx_1 \\ &+ \int_{M_2^c - y_2(p_1,p_2)}^{H_2} ((p_2 - c_{22}) M_2^c - \pi_2(D_2(p_1,p_2) - M_2^c)) f(x_2)\, dx_2 \\ &+ \int_{L_2}^{M_2^c - y_2(p_1,p_2)} ((p_2 - c_{22}) D_2(p_1,p_2) - v_{22}(M_2^c - D_2(p_1,p_2))) f(x_2)\, dx_2 - c_{22}M_2^c \end{aligned}$$

$$\begin{aligned} &\Rightarrow E[\Pi_{22}^c] \\ &= 2\begin{pmatrix} (p_1 - c_{21} - v_{21})[y_1(p_1,p_2) + \mu_1] - (p_1 - c_{21} + \pi_1 - v_{21}) \\ \int_{M_1^c - y_1(p_1,p_2)}^{H_1} [y_1(p_1,p_2) + x_1 - M_1^c] f(x_1) dx_1 + (v_{21} - c_{21}) M_1^c \end{pmatrix} \\ &\qquad +2\begin{pmatrix} (p_2 - c_{22} - v_{22})[y_2(p_1,p_2) + \mu_2] - (p_2 - c_{22} + \pi_2 - v_{22}) \\ \int_{M_2^c - y_2(p_1,p_2)}^{H_2} [y_2(p_1,p_2) + x_2 - M_2^c] f(x_2) dx_2 + (v_{22} - c_{22}) M_2^c \end{pmatrix} \end{aligned}$$

We show that the order affects the optimal price of the product. First, we take the first and second derivatives with respect to p_1 and p_2 as follows:

$$\begin{aligned} \frac{\partial E[\Pi_{22}^c]}{\partial p_1} &= 2\begin{pmatrix} y_1(P_1,P_2) + \mu_1 - a_2(p_1 - c_{21} - v_{21}) \\ + \int_{M_1^c - y_1(p_1,p_2)}^{H_1} (M_1^c - x_1 - y_1(p_1,p_2) \\ +a_2(p_1 - c_{21} + \pi_1 - v_{21}))\, f(x_1) dx_1 \end{pmatrix} \\ &+2\left(-a_2(p_2 - c_{22} - v_{22}) + \int_{M_2^c - y_2(p_1,p_2)}^{H_2} a_2\theta(p_2 - c_{22} + \pi_2 - v_{22}) f(x_2) dx_2\right) \end{aligned}$$

$$\frac{\partial^2 E[\pi_{22}^c]}{\partial p_1^2} = \left\{ \begin{array}{l} 2\left(-2a_2 \int_{L_1}^{M_1^c - y_1(p_1,p_2)} f(x_1)\, dx_1 \right. \\ \left. -a_2^2(p_1 - c_{21} + \pi_1 - v_{21}) f(M_1^c - y_1(p_1,p_2))\right) \\ +2\left(-a_2^2\theta^2(p_2 - c_{22} + \pi_2 - v_{22}) f(M_2^c - y_2(p_1,p_2))\right) \end{array} \right\} < 0$$

$$\frac{\partial^2 E\left[\pi_{22}^c\right]}{\partial p_1 d p_2}=2\left\{-a_2\theta-a_2-a_2^2\theta\left(p_2-c_{22}+\pi_2-v_{22}\right)f\left(M_2^c-y_2\left(p_1,p_2\right)\right)\right\}$$

$$\begin{aligned}&\frac{\partial E[\pi_{22}^c]}{\partial p_2}\\&=2\left\{-a_2\theta\left(p_1-c_{21}-v_{21}\right)-\int_{M_1^c-y_1(p_1,p_2)}^{H_1}a_2\theta\left(p_1-c_{21}+\pi_1-v_{21}\right)f\left(x_1\right)dx_1\right\}\\&+2\left\{\int_{M_2^c-y_2(p_1,p_2)}^{H_2}\left(y_2\left(p_1,p_2\right)+x_2-M_2^c+a_2\left(p_2-c_{22}+\pi_2-v_{22}\right)\right)f\left(x_2\right)dx_2\right\}\\&+y_2\left(p_1,p_2\right)+\mu_2-a_2\left(p_2-c_{22}-v_{22}\right)\end{aligned}$$

$$\frac{\partial^2 E\left[\pi_{22}^c\right]}{\partial p_2^2}$$

$$=\left\{\begin{array}{l}2\left(-2a_2\int_{L_2}^{M_2^c-y_2(p_1,p_2)}f\left(x_2\right)dx_2-a_2^2\left(p_2-c_{22}+\pi_2-v_{22}\right)f\left(M_2^c-y_2\left(p_1,p_2\right)\right)\right)\\+2\left(-a_2^2\theta^2\left(p_1-c_{21}+\pi_1-v_{21}\right)f\left(M_1^c-y_1\left(p_1,p_2\right)\right)\right)\end{array}\right\}<0$$

$$\frac{\partial^2 E[\Pi_{22}^c]}{\partial p_1\partial p_2}=2\left(-a_2\theta-a_2-a_2^2\theta(p_2-c_{22}+\pi_2-v_{22})f(M_2^c-y_2(p_1,p_2))\right)$$

According to the above expressions, one obtains the Hessian (H):

$$H(p_1,p_2)=\left[\begin{array}{cc}\frac{d^2E[\Pi_{22}^c]}{dp_1^2}&\frac{d^2E[\Pi_{22}^c]}{dp_1dp_2}\\\frac{d^2E[\Pi_{22}^c]}{dp_2dp_1}&\frac{d^2E[\Pi_{22}^c]}{dp_2^2}\end{array}\right]$$

$$=\left[\begin{array}{c}\left(\begin{array}{l}-2a_2\int_{L_1}^{M_1^c-y_1(p_1,p_2)}f(x_1)dx_1\\-a_2^2(p_1-c_{21}+\pi_1-v_{21})f(M_1^c-y_1(p_1,p_2))\\-\left(a_2^2\theta^2(p_2-c_{22}+\pi_2-v_{22})f(M_2^c-y_2(p_1,p_2)\right)\end{array}\right)\\\left(\begin{array}{l}-a_2\theta-a_2\\-a_2^2\theta(p_2-c_{22}+\pi_2-v_{22})f(M_2^c-y_2(p_1,p_2))\end{array}\right)\\\left(-2a_2\theta-a_2^2\theta(p_1-c_{21}+\pi_1-v_{21})f(M_1^c-y_1(p_1,p_2))\right)\\\left(\begin{array}{l}-2a_2\int_{L_2}^{M_2^c-y_2(p_1,p_2)}f(x_2)dx_2\\-a_2^2(p_2-c_{22}+\pi_2-v_{22})f(M_2^c-y_2(p_1,p_2))\\+2a_2^2\theta^2(p_1-c_{21}+\pi_1-v_{21})f(M_1^c-y_1(p_1,p_2))\end{array}\right)\end{array}\right]$$

Since

$$\frac{\partial^2E[\Pi_{22}]}{\partial p_1^2}=-2a_2\int_{L_1}^{M_1^c-y_1(p_1,p_2)}f(x_1)dx_1-a_2^2(p_1-c_{21}+\pi_1-v_{21})f(M_1^c-y_1(p_1,p_2)),$$

$-\left(a_2^2\theta^2(p_2-c_{22}+\pi_2-v_{22})f(M_2^c-y_2(p_1,p_2)\right)<0$ and $\|H(p_1,p_2)\|>0$, $H(p_1,p_2)$ is negative definite matrix and the profit function is concave. According to the above equation we have$H(p_1,p_2)>0$ then the expected profit$E[\Pi_{22}^c]$ in new selling pricesp_1^c , p_2^c is concave. Hence, the optimal pricing strategy is determined as follows.

$$\left\{\begin{array}{l}2\left(\begin{array}{l}\int_{M_1^c-y_1(p_1,p_2)}^{H_1}[M_1^c-x_1-y_1(p_1,p_2)+a_2(p_1-c_{21}+\pi_1-v_{21})]f(x_1)dx_1\\+y_1(p_1,p_2)+\mu_1-a_2(p_1-c_{21}-v_{21})\end{array}\right)\\\quad+2\left(-a_2(p_2-c_{22}-v_{22})+\int_{M_2^c-y_2(p_1,p_2)}^{H_2}a_2\theta(p_2-c_{22}+\pi_2-v_{22})f(x_2)dx_2\right)=0\\\\2\left(\begin{array}{l}\int_{M_2^c-y_2(p_1,p_2)}^{H_2}[y_2(p_1,p_2)+x_2-M_2^c+a_2(p_2-c_{22}+\pi_2-v_{22})]f(x_2)dx_2\\+y_2(p_1,p_2)+\mu_2-a_2(p_2-c_{22}-v_{22})\end{array}\right)\\\quad+2\left(-a_2\theta(p_1-c_{21}-v_{21})-\int_{M_1^c-y_1(p_1,p_2)}^{H_1}a_2\theta(p_1-c_{21}+\pi_1-v_{21})f(x_1)dx_1\right)=0\end{array}\right.$$

Appendix C. Detailed derivation of Equation (11)

The derivation detail of Equation (11) is as follows;

$$E[\Pi_B^c] = (p_{1B} - c_{21} - c_{22} - v_{11} - v_{12})\,(y(p_1, p_2, p_{1B}) + \mu_B) - (c_{21} + c_{22} + v_{11} + v_{12} - v_{21} - v_{22})U(s_B) - (p_B + \pi_B - c_{21} - c_{22} - v_{11} - v_{12})B(s_B) + M_B^c(v_{11} + v_{12}) \tag{C1}$$

Substituting (2) into B1, one obtains

$$E[\Pi_B^c] = (p_{1B} - c_{21} - c_{22} - v_{11} - v_{12})\,(a_1 - a_2 p_{1B} + \lambda(p_1 + p_2 - p_{1B}) + \mu_B) - (c_{21} + c_{22} + v_{11} + v_{12} - v_{21} - v_{22})U(s_B) - (p_{1B} + \pi_B - c_{21} - c_{22} - v_{11} - v_{12})B(s_B) + M_B^c(v_{11} + v_{12})$$

Taking the first partial derivative with respect top_{1B}one has Equation (11).

$$\frac{\partial E[\Pi_B^c]}{\partial p_{1B}} = a_1 - a_2 p_{1B} + \lambda(p_1 + p_2) - \lambda p_{1B} + \mu_B - a_2(p_{1B} - c_{21} - c_{22} - v_{11} - v_{12}) - \lambda(p_{1B} - c_{21} - c_{22} - v_{11} - v_{12}) - B(s_B)$$

$$= a_1 - a_2 p_{1B} + \lambda(p_1 + p_2) - \lambda p_{1B} + \mu_B - a_2 p_{1B} + a_2(c_{21} + c_{22}) + a_2(v_{11} + v_{12}) - \lambda p_{1B} + \lambda(c_{21} + c_{22}) + \lambda(v_{11} + v_{12}) - B(s_B)$$

$$= a_1 - a_2 p_{1B} + \lambda(p_1 + p_2) - \lambda p_{1B} + \mu_B + (a_2 + \lambda)(c_{21} + c_{22}) + (a_2 + \lambda)(v_{11} + v_{12} - p_{1B}) - B(s_B)$$

Appendix D. Detailed derivation of Equation (17)

The derivation detail of Equation (17) is as follows. Taking the first derivative of $E[\Pi_B^c]$with respect to p_{2B} we have:

$$\frac{\partial E[\Pi_B^c]}{\partial p_{2B}} = y(p_1, p_2, p_{2B}) + \mu_B - (a_2 + \lambda)(p_{2B} - c_{2B} - v_{21} - v_{22}) + \int_{M_B^c - y(p_1, p_2, p_{2B})}^{H} (M_B^c - y(p_1, p_2, p_{2B}) - x_B + (a_2 + \lambda)(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22}))\, f(x_B)dx_B \tag{D1}$$

Substituting (2) into C1, one obtains

$$\frac{\partial E[\Pi_B^c]}{\partial p_{2B}} = a_1 - a_2 p_{2B} + \lambda(p_1 + p_2 - p_{2B}) + \mu_B - (a_2 + \lambda)(p_{2B} - c_{2B} - v_{21} - v_{22}) + \int_{M_B^c - y(p_1, p_2, p_{2B})}^{H} (M_B^c - y(p_1, p_2, p_{2B}) - x_B)\, f(x_B)dx_B + \int_{M_B^c - y(p_1, p_2, p_{2B})}^{H} (a_2 + \lambda)(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22}) f(x_B)dx_B$$

$$\frac{\partial E[\Pi_B^c]}{\partial p_{2B}^2} = -2(a_2 + \lambda) \int_L^{M_B^c - y(p_1, p_2, p_{2B})} f(x_B)dx_B - (a_2 + \lambda)\,(M_B^c - y(p_1, p_2, p_{2B}) - M_B^c - y(p_1, p_2, p_{2B}) f(M_B^c - y(p_1, p_2, p_{2B}))) + (a_2 + \lambda)^2(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22}) f\,(M_B^c - y(p_1, p_2, p_{2B}))$$

$$\frac{\partial E[\Pi_B^c]}{\partial p_{2B}^2} = -2(a_2 + \lambda) \int_L^{M_B^c - y(p_1, p_2, p_{2B})} f(x_B)dx_B - (a_2 + \lambda)^2(p_{2B} - c_{2B} + \pi_B - v_{21} - v_{22}) f\,(M_B^c - y(p_1, p_2, p_{2B})) < 0$$

Appendix E. Proof of Theorem

1. If $M_B^c \geq h^c(h)$then we have

$$Max_{M_B^c \geq h^c(a^H)} E[\Pi_B^c] = -c_{1B} M_B^c + \int_l^h E[\Pi_B^c] g(a_1) da_1$$

2. If $M_B^c \leq h^c(h)$ then we have

$$Max_{M_B^c \leq h^c(a^L)} E[\Pi_B^c] = -c_{1B} M_B^c + \int_l^h E[\Pi_B^c] g(a_1) da_1$$

3. If $h^c(l) \leq M_B^c \leq h^c(h)$ then we have:

$$Max_{h^c(a^L)\leq M_B^c \leq h^c(a^H)} E[\Pi_B^c]$$
$$= -c_{1B}M_B^c + \int_l^{a(M_B^c)} E[\Pi_B^c]g(a_1)da_1 + \int_{a(M_B^c)}^h E[\Pi_B^c]g(a_1)da_1.$$

When $h^c(l) \leq M_B^c \leq h^c(h)$ then we have;

$$\begin{aligned}\frac{\partial E[\Pi_B^c]}{\partial M_B^c} &= g(a(M_B^c)).\frac{\partial a(M_B^c)}{\partial M_B^c}.\left(E[\Pi_B^{c*}] - E[\Pi_B^{c*}]\right)|_{a_1=a(M_B^c)} \\ &\quad + \int_l^{a(M_B^c)} \frac{\partial E[\Pi_B^{c*}]}{\partial M_B^c} g(a_1)da_1 + \int_{a(M_B^c)}^h \frac{\partial E[\Pi_B^{c*}]}{\partial M_B^c} g(a_1)da_1 - c_{1B}\end{aligned}$$

$$\begin{aligned}\Rightarrow \frac{\partial E[\Pi_B^c]}{\partial M_B^c} &= \int_l^{a(M_B^c)} (v_{11}+v_{12})g(a_1)da_1 \\ &\quad + \int_{a(M_B^c)}^h \left[\begin{array}{l}(v_{21}+v_{22}-c_{21}-c_{22}) \\ +(p_B^c - c_{21} - c_{22} + \pi_B - v_{21} - v_{22})\times \\ \times \int_{M_B^c - y(p_1,p_2,p_{2B})}^H f(x_B)dx_B\end{array}\right] g(a_1)da_1 - c_{1B}\end{aligned}.$$

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E-mail address: taleizadeh@ut.ac.ir
E-mail address: lecarden@itesm.mx
E-mail address: roya_sohani@yahoo.com