FUNDAMENTAL RESEARCH IN BIDDING AND ESTIMATING

Paper Presented to

The International Council for Construction Research Studies and Documentation CIB W-55

HAIFA, ISRAEL

R.M. Skitmore MSc PhD FRICS MCIOB

Department of Civil Engineering
The University of Salford
Salford M5 4WT, UK

May 1988

FUNDAMENTAL RESEARCH IN BIDDING AND ESTIMATING

Martin Skitmore
Department of Civil Engineering
The University of Salford

Abstract

Since Friedman, fundamental research in construction contract bidding and estimating has been concerned with the full problem definition, formulation and calibration. The general problem definition is now virtually complete in that bidding involves sequential and simultaneous decisions to be made over time, under conditions of uncertainty, and with multiple, conflicting objectives. Problem formulation has been rather haphazard and many *ad hoc* models have been used, often with very little empirical support. A major reason for this appears to be the lack of any real rival to Friedman's implicit theoretical base. This paper examines the approaches and models used in research with a view to the ultimate development of some theory on the subject.

Firstly, the relationship between bidding and estimating is considered along with the role of cost, price and other estimates. Secondly, the criticisms of Friedman's model are examined and in particular the efficacy of statistical models representing the various parts of the problem. Finally, research n estimating accuracy is considered in terms of influencing factors and research methodology. *Key words:* Bidding, estimating, statistical models.

1. Introduction

My experience of estimating and bidding (tendering) to date has been in two roughly equal phases. Firstly, as a student and practising quantity surveyor, in which I learned basic techniques and attempted to apply these in providing good service to clients. Secondly, as a teacher and researcher, I have tried to develop the subject by critical analysis of the techniques and the way in which they seem to be employed. This movement from practise to research seems to be quite common, in the UK at least, as the subject lends itself well to the usual perceived requirements of 'scientific' study (ie it involves numerical data) whilst seemingly providing a logical progression of knowledge acquisition over the years. In my case, the combination of conventional wisdom required through practise and knowledge gained by systematic observation has usually created more problems than solutions.

The major difficulty I think is the basic motivation for the research. It is natural for practitioners to prefer applied research – the development of systems, techniques etc. which will improve performance or profits or both. While pragmatic work of this type is undeniably valid, it is essentially incremental by nature and always relies on, and often prolongs the existence of, some assumptions about the problem under study. If these assumptions are reasonable in some sense then I have no complaint. On the other hand, if the assumptions are ill-founded then we have the makings of a different type of research, a research involving the identification, examination and testing of assumptions. Practitioners, in my experience, seldom distinguish between fact and opinion and tend to regard such research as at best naive and inconsequential, and at its worst destructive or vindictive. Whilst not sharing this view myself, I nevertheless have suffered equally from assumptions of 'blindness'. History has shown however that real progress in any field ultimately depends on the establishment of an explicit base. Ideally this base consists of fundamental 'truths'. In reality, as far as we know, the best we can hope for is a stated set of laws, axioms, or propositions that have gained general acceptance. To the best of my knowledge no such laws, axioms, or propositions exist or have existed in estimating and bidding, nor do there appear to be any serious attempts to work towards this goal.

My good friend and advisor, the mathematician Dr Ernest Wilde, now Pro Vice Chancellor of Salford University, once suggested to me that the first 90% of the time spent on research projects was in having fun playing around with ideas and the last 10& of the time was the 'serious bit'. Whilst it will be a sad day when estimating and bidding research ceases to be fun, it is perhaps time that we started to think a little more about the serious bit yet to come.

1. What is Meant by Estimating an Bidding?

People in the construction industry seem to have a clear impression of both these terms. Estimating is the process of working out likely costs and bidding is the process of converting an estimate into a tender price. People outside the industry have different ideas. Statisticians, for example, understand estimating to be a collection of techniques for arriving at a figure (an estimator) to denote the value of some measure describing an (often hypothetical) very large population based on analysis of a sample of the population. The importance of the subject for statisticians is in the ability of the estimator to accurately represent the unknown measure with a minimum degree of bias. Bidding on the other hand is often associated with auctions in which beating the opposition is the primary objective. The essential feature of most auction bidding is that the true value of the desired acquisition is uncertain or unknown, the behaviour of the bidder often being strongly influenced by the actions of competitors. The card game of contract bridge is a typical example. Here bids are made based on the system used by the players and taking into account the bids made or not made by the 'opposition', and the possible rewards and penalties that my ensue when the hand is eventually played out. On the majority of occasions, bridge players make no direct attempt to count in advance the number of tricks that can be made, or work out a playing plan, during the bidding phase. They rely instead on well established rules combined with a knowledge of the risks involved.

Applying these observations to the construction arena is easy enough. If we substitute the construction contract bidder for the card player, then bidding is seen as a skill involving the consideration of many subtle and partially known factors, not least the actions of competitors, with a special regard to the risks involved. That the rules or ethics of the construction contract 'game' do not normally allow bartering or repeated bidding does not significantly alter or simplify this position. Bartering or repeated bidding actually provides *more* information to the bidder, making the risks potentially more calculable. Construction contract bidding does however involve a few more complexities than bridge bidding. Firstly, construction bidders are faced with multiple choices in that they may choose not to bid at all, or pretend to bid by taking 'cover prices', or offer non-price alternatives such as a reduced construction period. Secondly, construction companies have multiple and conflicting objectives, such as meeting target profits or turnover, entering new markets, courting new clients etc. Thirdly, contract bids often have to be made simultaneously rather than sequentially, the result of the bidding for contract A may not be known until after bids are entered for contracts B, C and D. Fourthly, there are limits to the number of contracts that can be managed at any one time, bidders seek to obtain the best set of contracts. This means the occurrence of future contract opportunities have to be taken into account.

Seen in this way, the construct contract bidding problem invokes a mixture of feelings. For the student, horror or resignation that he will never learn how to cope with the situation. Indeed, if he relies on our present way of teaching of the subject, his fears are well justified. The construction bidder's reaction can range from satisfaction that his difficulties have been recognised (occasionally) to annoyance of the high sounding words used to describe common sense situation (frequently). The main difficulty for the researchers in this field is that the problem is not reduced to a simple enough form for a single comprehensive treatment, it also means having to learn a lot more about the problem itself.

Despite trepidations, my own view as a researcher is that the problem is not insuperable. Many techniques are already in existence to handle specific aspects. Game theory for instance is well suited to the analysis of reactionary competitors, decision theory helps in situations of uncertain outcomes, operations research in optimising outcomes of simultaneous decisions, multicriteria decision making in resolving objectives, simulation in coping with intractable formulations, behavioural science in predicting human behaviour, and statistics and economics contain a host os valuable techniques for developing new approaches.

2. The Role of Estimating

In analysing the nature of the bidding problem, I am frequently reminded that estimating seems to get overlooked or regulated to a fairly trivial position in the overall picture. This is not a deliberate effort on my part. Indeed, I have a vested interest in promoting the importance of the estimating role in order to support our current research activities in this area. My view of estimating is becoming increasingly aligned to that of the statistician, who spends most of his time examining potential models of the problem. The business of estimation is left until the final stages of his work. In terms of the total problem, the detailed study of estimation is essentially as esoteric exercise although it can sometimes lead to some striking insights into the nature of the problem itself. An example of this is in the analysis by McCaffer (1976a) and several others of the impact of intensity of competition on estimation bias in bid forecasting. By treating the estimate as a base line, it is possible to provide empirical evidence of underlying market influences on price levels which, in turn, can then be incorporated into models of the bidding problem.

In the context of the construction contract bidding problem, estimation is clearly needed on several fronts. Firstly, some estimate of the market price has to be obtained. Secondly, estimates of the outcomes of decisions are needed to evaluate the changes in monetary, human, and other physical states both inside and outside the bidding organisation. Thirdly, estimates of future contract opportunities are needed to identify potential contract portfolios.

Estimates are also often independent. Those with poor predictive properties for instance can lead to even poorer estimates in other parts of the system. Risk analysis is essentially a means of informing these situations.

Estimates are also needed of the characteristics of the estimates themselves – in statistical terms, the properties of the estimates.

Estimating market price and changes in monetary states seems now to have acquired the general title of cost modelling a term which, although covering the activities of researchers who work in both fields at once, can lead to a mistaken belief that both kinds of estimation are essentially the same. Contractors have always had a clear view of the separate and distinct roles of cost estimation and price estimation. The standard texts on cost estimation are consistent in exhorting the estimator to act independently of any perceptions he may have of the market price. Market pricing is the function of the adjudicator. Although there is evidence, by Whittaker (1970) for instance, that cost estimators are sometimes influenced by market related events such as failure to obtain contracts, the phenomena does seem to be largely restricted to lesser experienced estimators. The non-repetitive nature of most construction contracts (houses and factories excepted) means that a genuine pre-cost estimate is needed as a base line for market price calculations. If the cost estimate already contains some implicit market allowances, it can only make adjudication that much more difficult.

Turning to designers', or engineers', estimates as they are sometimes called, their task is clearly that of market price estimation. All the current techniques used in performing this function utilise known and assumed relationships between characteristics of the design and the market price. The most well known of these characteristics is the total floor area of a building which, together with the building type, provides a very good correlation with the market price. Other important determinants of market price, such as the intensity of competition, seem to be used to a lesser degree. The reason for this is probably due to the main interest of the designer being in the *relative* prices associated with different designs, so that non-design factors are taken to be constant. This does seem to be a rather narrow view however as better 'value for money' may also be achievable be decisions taken outside the design domain. Inviting more bids, open tendering, off-season procurement, simpler contract arrangements, enhancing the contract status etc. may all lower the market price, but are seldom contemplated. The major problem is that current estimation techniques inadequately handle such issues. Instead, there is a body of conventional wisdom in the industry that believes that more bids will produce false economies to the industry as a whole, owners want construction 'now', simpler contracts will result in more claims, interesting contracts will generate higher rather than lower prices. Little of this 'wisdom' has been tested nor do any explicit models exist that even demonstrate the point. Such models that do exist seem in fact to show the opposite to the conventional view.

Newton (1987) has recognised the underlying fault with much of the work that has been done in cost modelling, and that is the failure to consider 'purpose' (the danger with treading the well worn path is that it might not actually be leading anywhere!), the lack of a suitable theoretical base, and the lack of criteria for assessment of different models. He suggests that purpose may be established by analysis of the purpose of current models, which he believes to be that of 'informing design'. Like Newton, I think the lack of a theoretical base is the main difficulty, and I agree that the absence of assessment criteria does not help, but y analysis of the situation in the context of bidding suggests 'informing design' to be inappropriate and overrestrictive. Cost modelling should in my opinion be ultimately concerned with the industry, and possibly beyond.

3. Research in Bidding

There is now a voluminous literature on bidding, most of which is applicable to construction contract bidding in some way, which has now become beyond the reading capacity of anyone recently entering the field. Fortunately, some effort is now taking place to provide a classified review of the progress made to date, notably by Engelbrecht-Wiggans (1980) and King and Mercer (1988). These classifications are based on the two main stream decision theoretic and game theortetic treatments together with various supplementary additions to cover additional features of what is seen as the 'real world' problem. The increasing size of the supplementary sections suggests that a major rethink is needed in the basic theory behind the techniques used.

An appropriate point to start such a task is with what King and Mercer term the 'basic probabilistic' approach first introduced by Friedman (1956) in a seminal paper based on his PhD thesis – the first thesis on operations research to be produced in the USA. Friedman proposed a method of calculating optimum bids for a series of contracts by utilising cost estimate/bid price ratios collected for the various competitors involved. From frequency distribution of these ratios, Friedman estimated the probability of entering the lowest bid for a range of potential markup factors, taking the optimal markup to be the one which generated the largest probability/markup product. Apart from Friedman's method of estimating the probability of entering the lowest bid, a hotly disputed topic over the years, the technnique can be criticised for the several assumptions necessary for its application.

Firstly, the model may not be a reasonable representation of reality. This raises two issues. (a) Are any factors of the problem missing? There may be other decision variables than markup involved (eg. not bid at all, enter cover price, offer non-price features) or objectives other than maximising long term profit (eg. short term gains or losses), non monetary objectives may apply (eg. human welfare), time related aspects may be important (eg. cash flow, future contract opportunities), monetary and other constraints may exist.
(b) How well do statistical models fit the factors? Statistics, as all of mathematics, exists as a body of knowledge and techniques independent of the 'real world'. That it happens to mirror real world events rather well on many occasion is our good fortune. There is no guarantee that cost estimates, actual costs, and competitors' bids will nicely follow the statistical properties assumed. The statistical requirements of complete randomness and , in Friedman's model, independence are unattainable in practice. The question is 'how close is the fit?' and then 'how close is close enough?'.

Secondly, the model may be difficult to calibrate. This brings us back to estimating again. Even if an acceptable model is developed, that is intuitively pleasing and accords with all the known factors, it may not be possible to estimate the values of the variables involved. This particular difficulty comes in three interdependent parts. (a) Data may be lacking. The data demands of Friedman's model is known to be heavy. Many competitors' bids and their associated cost estimates are needed to construct reliable frequency distributions. (b) Techniques for converting data into usable information may not yet exist. As has already been mentioned, there is some disagreement on the method of probability estimation. (c) The person making the bidding decision is not infallible. Again Friedman's method has frequently been misrepresented, even by experts. Even with good data and good techniques, mistakes can still be made.

Several attempts have been made to improve on Friedman's method in answer to the criticisms made. One of these tries to incorporate more subjective objectives by replacing monetary values with utility values (Willenbrock, 1972, for instance) so bringing the subject into line with what is now known as decision analysis. The major problem with using utilities is that stable preferences seldom seem to exist and, mainly because of this, decision analysis is now generally regarded as being a theoretical novelty rather than a robust practical technique.

Another approach, initiated by Hanssmann and Rivett (1956), tries to reduce the data load by considering only the bids of the lowest competitors. Unfortunately, in reducing the amount of data used, this approach also equally reduces the power of the method by omitting relevant data. Another data reducing approach involves further assumptions of the statistical model used by taking the bidding patterns of competitors to follow some convenient probability distribution such as the normal distribution. Whilst this is a device often used by applied statisticians when using robust techniques such as the t-test or analysis variance, when even major deviations from the normal assumption are known to have little effect on the analysis, it is not known yet what the effect of this assumption may be on bidding models.

More recent work has developed these latter ideas in considering the efficacy of current statistical models and their extension into other aspects of the bidding problem (Skitmore, 1986). This work considers the application of statistical models to the three primary features of construction bidding; contract opportunities, decision outcomes, and competitors' bidding patterns.

4. Efficacy of statistical models

Contract Opportunities

The frequency of the size of contract opportunities was found by Hossein (1977) to be closely modelled by the exponential distribution, although Skitmore's (1986) analysis found a lognormal distribution to be more appropriate. The number of competing bidders has been suggested by Friedman 91956) to follow poisson distribution (Skitmore found a three parameter lognormal fit to be better), with an average derived by a regression analysis on contract size, an approach investigated by Wade and Harris (1976) who found a logarithmic relationship between the number of bidders and contract value. Others have questioned this latter approach as providing very poor predictive results (Gates, 1967), sometimes finding no relationship at all (Sugrue, 1977), or subject to the influence of market conditions (Skitmore, 1981). Such lack of predictive models has lead researchers to conduct simulation studies based on a randomised number of bidders (Rickwood, 1972). Finally the identify of competitors has been treated probabilistically by a predictive technique termed the multidistribution model (MD) which "…represents the local structure of the construction industry, a structure which allows the contractor to predict with a high level of confidence who is competitors will be on a specific project" (Shaffer and Micheau, 1971, p.116). Details of this model are apparently given in Casey and Shaffer (1964), a publication to which this writer has been unable to gain access. A method has been used by Skitmore (1986) however by means of regression analysis to predict the probability that a specific competitor will enter a bid.

Decision outcomes

Whilst no probabilistic approaches to modelling human factors have yet been attempted, many researchers have tackled the monetary aspects of the problem. Monetary aspects are dependent on two major factors, the probability of acquiring the contract and the probability of the occurrence of certain monetary states conditional on the contract being acquired. This later aspect is considered here in terms of anticipated expenditure (cost) and anticipated income.

Several attempts have been made to formulate the problem in a quantitative manner, which allows treatment of the variation between expenditure and estimates. One approach adopts the concept of a 'true' cost (Whittaker, 1970), sometimes known as "God's cost" (McCaffer, 1976a) or, perversely, as the "Devil's" cost (Fine, 1974). This is essentially Friedman's (1956) approach in taking the view that the true cost can only be known after the event, and assumes that the distribution of the ratio's' of the true cost to the estimated cost can be ascertained from the contractor's records. A different perspective is provided by McCaffer (1976b) whose model is derived on the basis that different estimators will assess the effects of factors on casts differently and hence a number of estimators are liable to produce a range of estimated costs, suggesting a probability distribution of estimates around some mean. This mean has been termed the ‘likely cost’ (Cauwelaert and Heynig, 1979) and several simulation studies (Fine and Hackemar, 1970; Rickwood, 1972: Morrison and Stevens, 1980, for instance) have been conducted on this basis. The advantage of the likely cost approach is that each contract cost estimate can be considered to be a random value drawn from a distribution of possible cost estimates unique to each project, whilst Friedman’s approach implies one distribution to apply irrespective of any non-random differences that may occur between contracts. This, according to Benjamin (1969) is an important factor for, in his view; there is no single distribution of the ratio of true to estimated cost that applies to all contracts without regard to the characteristics of that contract. An alternative line is to consider actual costs to be distributed about the estimated costs, this distribution being regarded by Vergara (1977) as symmetrical with actual costs equal on average to estimated costs.

The difference between the ‘true’ versus ‘likely’ cost models is essentially that some authors consider estimated costs as a random variable and the true cost as fixed whilst others take the true cost as being random and estimated cost to be fixed. A more fruitful approach is to treat both the costs *and* the estimates as random variables (as do Fuerst, 1977, and Rothkopf, 1980). There is some justification in this for, as Fine and Hackemar (1970) have demonstrated, variability in estimates of production and costs exists both before and after the event as accounts are guesses at past costs just as much are estimates guesses at future costs!

Some authors have criticised the use of statistical models of estimated costs on the grounds that such estimates depend on company position (Curtis and Maines, 1973), or they are conditioned by the work content of the project (Ortega-Reichert, 1968), they are dependent on the combination of contracts obtained (Stark and Mayer, 1971). Nevertheless, construction costs have frequently been modelled in this way, usually by the ratio of actual to estimated costs. Table 1 summarises some of the work that has been reported.

Table 1 Distribution parameters for costs/estimates

Modeller	Shape	Spread	Location
Barnes (1971)[h]		cv 5.8%	
Barnes & Lau (1974)[a]		cv 4-15%	
Beckmann (1974)[a]	Normal		
Beeston (1974)[g]		cv 4%	
Capen et al (1971)[a]	Lognormal		
Case (1972)[c]			
Cauwelaert & Heynig (1979)[a]	Uniform	± A%	
Fine (1974)[b]	Uniform	± 10%	
Fine & Hackemar (1970)[b]	Uniform	± 8-10%	
Friedman (1956)[c]	Gamma		
Gates (1967)[d]	Normal	cv 7.5%	
Greismer et al (1967)[a]	Uniform		
Hackemer (1970)[b]		± 5-15%	
Harris & McCaffer (1983)[a]	Uniform	± A%	
Leech & Earthrowl (1972)[c]	Lognormal		
Liddle (1979)[a]		± 5%	
Mitchell (1979)[a]	Normal		
Morin & Clough (1969)[c]	Symmetrical	cv 2%	1.0 (median)
Morrison & Stevens (1980)[a]		± 5-7 ½ (mean)	
Moyles (1973)[f]		± 5%	
Naert & Weverberg (1978)[i]			close to 1
Oren & Rothkopf (1975)[a]	Weibull		
Park (1966)[a]		± 5%	
Rickwood (1972)[b]	Normal		
Rothkopf (1969)[a]	Weibull		1.0 (exp. Val)
Rothkopf (1980)[a]	Weibull		
Rubey & Milner (1966)[a]		less than 10%	
Smith & Case (1975)[c]	Lognormal		
Smith & Case (1975)[c]	Loglogistic		
Vickery (1961)[a]	Uniform		
Whittaker (1970)[a]	Uniform		
Willenbrock (1972)[j]			+ 3%

[a] assumed for theoretical purposes
[b] assumed for simulation purposes
[c] source of data unknown
[d] analysis of 110 USA road projects
[e] analysis of 153 UK construction project
[f] opinion survey of UK contractors
[g] analysis of extent of agreement between UK construction estimators
[h] analysis of 160 British construction projects
[I] discussion with Dutch construction companies
[j] analysis of 20 USA road projects

At a more detailed level, random variables have been said to include: labour costs (Benjamin, 1969; Fine, 1970; Gates, 1971; Bennett and Fine, 1980; Bennett and Ormerod, 1984; Wilson, 1982, among many); material costs (Benjamin, 1969; Armstrong, 1972); subcontractors' costs (Benjamin, 1969); quantity related costs (Fine and Hackemar, 1970; Gates, 1971; Grinyer and Whittaker, 1973); effects of weather and seasons (Benjamin, 1969; Armstrong, 1972; Hillebrandt, 1974); costs of estimating (Leech and Jenkins, 1978); additional costs (Gates, 1971); and other costs such as insurance, bonding, fringe benefits (Benjamin, 1979).

Income is normally assumed to be some function of the value of the bid, the majority of modellers assuming a one to one relationship. It is clear however that such an assumption is far from realistic in construction contract bidding as many factors influence changes between the bid value and the income ultimately received. Most of the factors are dealt with on a contractual basis, remuneration often being provided for unpredicted events such as inflation, additional work caused by fire and flood, and quantity errors and delays outside the company's control. Further income may accrue outside the contractual position in the form of *ex gratia* payments for perhaps exceptionally inclement weather, interest received on invested capital, receipts from the leasing of advertising space on hoardings etc. Of these variations in income only one factor, the incidence of design changes, has been modelled statistically.

Profit however is another matter. Estimates of profit are invariably taken as the difference between the bid and the cost estimate, usually in percentage terms, and the probability distribution of profit as the difference between the bid (a constant) and cost (a variable). It is clear though that the probability distribution of profit is the difference between the two variables, income and expenditure, and is often best left in this disaggregated form particularly when modelled over time.

Such models of income and expenditure over time, or cash flow models, tend to be deterministic however. Atkins (1975) for instance has used deterministic models to represent the "cash flow patterns" for several differing project sizes and types. The only probabilistic model known to date is that by Kangari and Boyer (1981) in the form of a beta distribution.

Competitors' bidding patterns

As with decision outcomes, the use of statistical models to describe the bidding patterns of competitors has met with some criticism from several quarters. One criticism is that the events taking place are not truly random in the classical statistical sense as the data are not generated from repeated measures of the same experiment – each bidding event is unique (Benjamin, 1972). Another criticism is that the assumption of randomness is invalidated by the presence of many subjective factors influencing bidding behaviour (Curtis and Maines, 1974). A further criticism is of the basic assumption that competitors will follow the same general bidding patterns in the future as they have in the past (Park, 1962). An individual competitor may for instance change his strategy, 'rendering past data about him misleading' (Beeston, 1983).

All of these criticisms are essentially concerned with the random assumption contained in statistical models and the existence of several techniques to test this assumption suggests that the validity or otherwise of the criticisms may be resolved empirically. Apart from McCaffer (1976a), who conducted a time series analysis of individual bidders to ascertain trends, surprisingly little work has been done in this respect. Unfortunately, McCaffer's results, although suggesting that some contractors behave in a manner not entirely consistent with the random model, were thought to be generally inconclusive and further work is urgently needed in this area.

Despite these criticisms, competitive behaviour has been modelled statistically by many researchers. These models classify neatly into collective and individual models of competitors. One approach to estimated costs (Larew, 1976), true costs (Rothkopf, 1980; Rickwood, 1972) or that estimated costs are similar (Broemser, 1968) or vary around some common mean (Oren and Roghkopf, 1975; Morrison and Stevens, 1980), and that the markup component of bids is a random variable. This allows the ration of 'our' cost estimate to competitors' bid to have some direct meaning. Empirical evidence however does not support this supposition, Grinyer and Whittaker (1973) finding markups to vary very little between firms (6.8%$\pm$ 0.35%) although a similar analysis of Shaffer and Micheau's (1971) data and Skitmore's (1986) data revealed a rather larger 5.40% (1.84 standard deviation) and 5.57% (2.25 standard deviation) respectively, none of which are sufficient to account for a general coefficient of variation of around 5 to 9% for whole bids. For most purposes, assumptions about the nature of bid components are unnecessary, and the distribution of bids alone is of interest. Table 2 summarises some of the models found in the literature.

A little work has been carried out on the changing nature of these bid distributions. McCaffer (1976a), Johnston (1978), and Skitmore (1987) seem to have detected a relationship between market conditions and distribution shape, whilst McCaffer (1976a), Skitmore (1981a), Morrison (1884) and Flanagan and Norman (1985) claim to have found a negative correlation between spread and contract value.
Again, further work is needed in this area.

The distribution of low bids has been modelled by the winning bid/cost estimate ratios (Ackoff and Sasieni, 1968; Sugrue, 1977, 1980) or the winning bid/estimated contract value ratios (Hanssmann and Rivett, 1959). These ratios are often assumed to follow a normal distribution (Ackoff and Sasieni, 1968; Sugrue, 1980), an assumption tested empirically by Beeston (1983) and Sugrue (1977) with confirmatory results. A slightly different version by Sasieni *et al* (1959) considers the ratio (B_K)/K, where B is the winning bid and K the cost estimate, to also follow a normal distribution. An interesting feature of lowbid/cost estimate ratios is that their mean tends to be very close to unity (Skitmore, 1986), a fact that lends

Table 2 Distribution parameters for bids

Modeller	Shape	Spread	Location
AICBOR (1967)[o]		cv 6.8%	
Alexander (1970)[d]	Normal		
Arps (1965)[d]	Lognormal		
Barnes (1971)[m]		cv 6.5%	
Beeston (1971)[I]	Pos. skewed	cv 5.2-6%	
Brown (1966) [d]	Lognormal		
Capen et al (1971)[d]	Lognormal		
Cauwelaert & Heynig (1979)[a]	Uniform		
Cauwelaert & Heynig (1979)[g]	Normal		
Crawford (1970)[d]	Lognormal		
Dougherty & Nozaki (1975)[d]	Gamma		
Emond (1971)[d]	Normal		
Fine & Hackemar (1970)[b]	Uniform	cv 5%	
Friedman (1956)[a]	Gamma		
Grinyer & Whittaker (1973)[c]	Uniform	cv 6.04%	
Hossein (1977)[k]	Gamma		
Klein (1976)[a]	Lognormal		
McCaffer (1976a)[f]	Normal	cv 6.5%	
McCaffer (1976a)[n]	Normal	cv 7.5%	
McCaffer (1976a)[j]	Normal	cv 8.4%	
McCaffer & Pettit (1976)[j]	Pos. skewed	cv 8.4%	
Mitchell (1977)[a]	Normal		
Morrison & Stevens (1980)[a]	Normal	19.1% av. range	
Oren & Rothkopf (1975)[a]	Weibull		
Park (1966)[h]	Pos. skewed		
Pelto (1970)[d]	Lognormal		
Shaffer & Micheau (1971)[p]		cv 7.65%	
Skitmore (1981a)[l]		cv 7.65%	
Skitmore (1986)[q]	Normal	cv 6.8%	
Skitmore (1986)[r]	3 param lognormal	cv 13.5%	
Skitmore (1986)[s]	3 param lognormal	cv 7.8%	
Weverberg (1982)[a]	Lognormal		
Whittaker (1970)[c]	Uniform		1.068

a Assumed for theoretical purposes
b Analysis of an ‘adequate’ sample of UK construction contracts
c Analysis of 153 UK Government construction contracts
d USA oil and mineral tracts – source of data unknown
e Assumed for simulation studies
f Analysis of 183 Belgian building contracts
g “Consistent with work of other researchers”
h USA construction projects – source of data unknown
I Large sample of PSA building contracts
j
k Analysis of 545 US civil engineering and 63 mechanical engineering contracts
l Analysis of 269 UK building contracts
m Analysis of 159 UK construction contracts
n Analysis of 16 Belgian bridges contracts
o Analysis of 213 UK motorway contracts
p Analysis of 50 USA construction contracts
q Analysis of 51 UK construction contracts
r Analysis of 218 UK local authority construction contracts
s Analysis of 373 UK construction contracts

Some substance to the argument that 'cost' estimates may be closely related to market price estimates.

Models of collective bidding behaviour have been criticised for neglecting the inherent competitive advantage of some companies in differential relative efficiencies of production or estimating differential relative efficiencies of production or estimating abilities. Several researchers have modelled individual bidders, starting with Friedman (1956) and including Taylor (1963) and Morin and Clough (1969). The approaches are similar to that of modelling competitors collectively in that competitors' bid/cost estimates ratios are obtained and probability density functions fitted to the ensuing frequency distributions (Friedman, 1956; Taylor, 1963; Benjamin, 1972, for instance). Beeston (1982) has suggested using D ratios, in a similar manner to Sasieni *et al*, where D + {(lowest bid – estimated cost)/estimated cost} expressed as a percentage. Morin and Clough (1969), on the other hand, have used the relative frequencies of competitors' bid/cost estimate to won bid/cost estimate ratios.

Whilst al of the distribution parameters postulated for collective models necessarily apply to individual bidders, some modellers have proposed probability distributions specifically for the individual case. Griesmer *et al* (1967) assume bidders draw from a uniform distribution unique to each, and Winkler and Brookes (1980) have proposed models in which differing amounts of information (ie different variances) exist between bidders. Capen *et al* (1971), Curtis and Maines (1973) and Fuerst (1977) have attempted to derive parameter estimates for each bidder by simulation techniques, and Weverbereg (1982) has used a multivariate technique to estimate parameters of coalition bidding for oil leases, assuming the winning bid to be a constant. This latter approach has been developed by Skitmore (1986) as a means of both parameter estimation and significance testing via analysis of variance, showing that individual bidders do have different relative means and variances when modelled in this way.

Further considerations involve the independence assumption; factors influencing individual bidding behaviour such as changing market conditions (Whittaker, 1970; de Neufville *et al*, 1977; Carr and Sandahl, 1978; Park, 1980, for instance), contract characteristics (Christenson, 1965; Broemser, 1968; Benjamin, 1972; de Neufville *et al*, 1977; Sugrue, 1971; Carr and Sandahl, 1981; Morin and Clough, 1969) such as the class of construction (Morin and Clough, 1969; Shaffer and Micheau, 1971; Coke, 1981), contract size (McCaffer, 1976a; Harvey, 1979; Lange, 1973), and geographical location (Pelto, 1971; Harvey, 1979), the number of competitors (Benjamin, 1970; McCaffer, 1976a; Pelto, 1971; Harvey, 1979), the "make-up" of competitors (Carr and Sandahl, 1978); responsive actions of competitors, ie game theory (Griesmer and Shubik, 1963a, 1963b, 1963c; Griesmer *et al* (1967; Wilson, 1969; Rothkopf, 1969, 1980a, 1980b; Oren and Rothkopf (1975); Smith and Case, 1975; Knode and Swanson, 1978; Palfrey, 1980); the degrading effects of time (Morin and Clough, 1969); collusion (Mitchell 1977; Sheldon, 1982); and non-serious and unrealistic bids (Whittaker, 1970; Moyles (1973); Pim, 1974; McCaffer, 1976a; Johnston, 1978; Weverberg, 1981; Beeston, 1983).

5. Research in estimating

Research in estimating construction costs and prices has proceeded largely independently of research in bidding, the major emphasis being on systems development. Raftery (1987) has classified these developments into what he terms first, second and third generation models. First generation models are said to be characterised by the procedural approach typified by (design) elemental 'cost' planning, relying on a data bank of 'cost' analyses obtained by the analysis of priced bills of quantities and used in a "mathematically naive" way as a guide to market price estimation. Second generation models involve the intensive use of multiple regression analysis in investigation the use of linear models for market price forecasting. Third generation models are said to be knowledge based and attempt to incorporate some measure of expert judgement in the forecasting process. Whilst Raftery's typology does reflect the general trend in the literature relating to market price forecasting, it is by no means agreed that each generation represents an improvement on the last, but rather a series of attempts to hit the pot of gold by the application of some likely looking techniques.

It has been argued, correctly, that estimating requires a good structure or theory before any real progress can be made. Unfortunately, research has yet to reveal the nature of such a structure. Current opinions seem to be polarised into the resource based approach and the design based approach, and a reconciliation of these two schools is well overdue. Such a reconciliation seems possible through the more broader considerations of bidding research.

Unlike estimating research, bidding research does have a history does have a history of innovation and controversy resulting in a wealth of empirical data relating to its central issues. Moreover, biding research, subsuming as it does, cost and price estimating, is likely to provide the framework from which theories of estimation may emerge. On the other hand however, it is possible that empirical data derived from estimating research may help in the development of bidding theories. The problem meanwhile of course is to know just what data to collect that may be relevant to the problem, and this si reflected in the empirical work to date.

Perhaps the easiest question to answer on estimating is what constitutes a good estimate? This must ultimately concern the accuracy of the estimate, the cost of achieving this accuracy, and the trade off between the two – an optimisation problem – within the context of the estimating situation. Several factors have emerged regarding one part of this relationship – accuracy, and its influencing factors. Although some of these factors, such as the amount of information used in preparing the estimate, will have cost consequences, we have not yet reached the stage of quantifying these costs. Many other factors however do not involve any substantial cost considerations.

7. *Factors affecting estimating accuracy*

Two methods have been applied to the analysis of factors affecting accuracy in estimating (a) retrospective analysis of existing data files, and (b) experimental work. The usual approach has been to calculate the ratios of estimate to lowest bid, in the manner first adopted in bidding research (eg. Hanssmann and Rivett, 1959), and to measure the strength of association of these ratios with any other variables of interest. With very few exceptions, the work has been aimed at designers' estimation of market prices. The factors associated with estimating accuracy fall into four categories (1) the market (2) the information available (3) the technique used, and (4) the estimator himself.

Market factors

The construction contract market has four major characteristics – the product (building type and size), the procurement system (client, tendering system, contractual arrangements, documentation system), the geographical location of the product, and the nature of the competition for the contract.

The type of project has been found to be associated with estimating accuracy by several researchers. McCaffer 91976a) found statistically significant differences between the mean ratios for a total of 300 Belgian road contract and building contract estimates. Harvey (1979) found similar differences between a total of 2401 Canadian building, non-building, special trades, and other contracts. Morrison and Stevens (1980) also found differences between the mean ratios for a total of 648 UK school, housing and other contracts. Skitmore's (1985) experimental work with 12 estimators in the UK again found significant differences between early stage estimates for a school, housing, factory, offices and health centre contracts.

Wilson *et al* (1987) found significant differences between mean ratios for a total of 408 Australian public works contracts when divided into value ranges of less that $A50000, $A50000-$A250000, and over $A250000. Harvey also found a trend with mean ratios tending toward unity (ie more accurate estimates) for larger contracts, the best fit being the square of the inverse of the contract value. Other workers, notably by McCaffer 91976a) and Morrison and Stevens 919800 have found less conclusive results.

No research has yet been done to test the relationship between estimating error and the type of client, or general procurement method except for Wilson et al who claim to have found significant differences between contracts with and without the provision of bills of quantities.

Harvey's (1979) analysis of variance showed significant differences in mean ratios across the six Canadian regions studied. Significant regional/project type interactions were also found.

The nature of the competition has been studied at two levels. Firstly, the number of competitors bidding for the project, has on many occasions (McCaffer, 1976a; De Neufville, Hani and Lesage, 1977; Harvey, 1979; Hanscomb and Associates, 1984; Runeson & Bennett, 1983; Skitmore and Tan, 1988) been found to have a significant negative correlation with the low bid/estimate ratios (ie estimates are too low with few bidders and too high with many bidders). Strangely, this trend has also been found using mean bid/estimate ratios (McCaffer, 1976a; Runeson & Bennett, 1983), and even with highest bid/estimate number of bidders relationship is so striking that reserchers have sometimes tried to fit a curved relationship (Harvey, 1980; Hanscomb, 1984), the inverse number of bidders seeming the most appropriate (Harvey, 1980). This latter relationship coincides well with the statistical models of bidding distributions which imply a curved trend to this, although the mean and high bid trends are not anticipated. Some theoretical developments are clearly needed to accommodate this.

Work at the second level of competitive states has been concentrated on analysis over time, usually on a yearly basis. Morrison and Stevens (1980) found the modulus ratios to be much larger between 1973-75, a period of much uncertainty in the UK construction industry. De Neufville, Hani and Lesage 91977) also found differences between what they term “good” and “bad” years in the USA (good and bad contractors). The results of this analysis show that underestimates and overestimates are generally associated with good and bad years respectively.

Informational factors

The level of information available to the estimator increases as the design progresses. The effect of increasing information can therefore be assessed by comparing the accuracy of estimates made in the early stages of design (conceptual estimates) with those made when the design is substantially complete (detailed estimates). A comprehensive review of the general accuracy expected for these two types of estimates has been made by Ashworth and Skitmore (1983) indicating a standard deviation of 15 to 20 percent for detailed estimates. Two experimental studies, by Jupp and McMillan 91981) for detailed estimates and Skitmore (1985) for conceptual estimates, have been conducted aimed at quantifying the incremental effect of information on estimating accuracy.

Jupp and McMillan observed the effect of increasing levels of historical price data on the estimating accuracy of three quantity surveyors. The results of this exercise indicated that estimating accuracy improved only slightly with the increasing information. Skitmore’s experiment involved the provision of increasing amounts of information about the contract to be estimated in addition to price information relating to similar past contracts. In this study the use of past contract data was found to have no significant effect on accuracy levels. The provision of current contract information produced an increase in average estimate levels form –5.63 percent error (18.28 standard deviation) with one piece of information to 11.13 percent (14.59 standard deviation) with all of the 16 pieces of information provided. A closer analysis of the data however indicated that the subjects claiming a greater expertise made far less use of the information provided in terms of improved accuracy. These subjects were also able to estimate far more accurately, as is noted later. This propensity for experts to use less overt information has also been found in other behavioural studies (eg Silva & Regan, 1988).

Another study by Bennett (1987) investigated the reliability of data sources by an experiment involving eight subjects pricing fifty items of building work four times, using a different standard price books on each occasion. The eight prices received for each item using each book were examined and awarded a score on a scale of 0 to 8, where 0 indicated that all the prices were identical, 1 a marginal variation, 2 one serious error, 4 two serious errors, 6 three serious errors, and 8 four serious errors. The resulting totals for each book show a marked difference between accuracy measured in this way, with one book considerably outperforming the others.

Technical factors

The standard estimating texts assert that more detailed estimating techniques such as those using approximate quantities are *ipso facto* necessarily more accurate than more coarser techniques such as the floor area method. Apart from the two limited informational studies mentioned above, very little research has been attempted in establishing the validity of this.

One study (James, 1954), has compared the accuracy of the cube, floor area, and storey enclosure methods of estimating in terms of the number of estimates falling inside a rand of ±10% or ±20%. The results turn out to be statistically significant (chi-square 5.99, 2df), with the storey enclosure mehtod being better than the cube method.

Measures of accuracy for the various multiple regression methods have also received little press. Most of the work is in the form of academic theses and a review is currently in progress. One work of particular interest is that of Ross (1983) in which three methods of approximate quantities estimating were examined in terms of accuracy Ross' first method uses the simple average of the value of sections of work from a set of bills of quantities or previous contracts. The second method uses a regression procedure to predict total value form sectional values, and the third method uses a regression on the unit value of items. Thus the methods are generally arranged in order of increasing usage of information. Ross' results indicate the first method to be the most accurate (cv 24.50 percent), followed by the second method (cv 30.49 percent) and the third method (cv 52.66 percent), which suggests that the more sophisticated methods utilising more of the data available produced less accurate results.

Human factors

One of the factors emerging form Jupp and McMillan's (1981) research on informational effects was claimed to be the marked difference between the accuracy levels of the subjects employed in the studies, though little supporting evidence is available. Morrison and Stevens (1980) also seemed to find some association between groups of quantity surveyor estimators and accuracy levels, although it was not clear how much of the observed effect was confounded by the type and size of contract.

Skitmore's (1985) experiments provided clear evidence of significant differences in estimating accuracy between the individual surveyors involved. Although handicapped by the limited amount of data collected in this study, it was possible to tentatively conclude that the most consistent estimators were associated with high recall abilities, self-professed expertise, low mental imaging of the physical characteristics of the building, and high general and specific contract estimating experience. Low estimates were also found to be associated with self-professed expertise and high estimates with high recall abilities, high mental imaging and specific project experience. In addition, subjects exhibiting the greatest expertise were generally thought to be (a) more relaxed and confident (b) more concerned with maintaining familiarity with the market and overall price levels than other who believed the routine collection and careful analysis of project information to be of major importance, and (c) possibly able to recall the overall price of the projects undertaken.

6. Conclusions

Research in bidding and estimating is progressing on three levels – systems, techniques, and models. The usefulness of any system or assumptions, contained therein. Ultimately, we would hope for a general model expressed in terms of well established laws, axioms, or propositions upon which future systems and techniques may be developed. The task of fundamental research is to produce such a model.

The most important contribution to the modelling of bidding decisions was the first operations research approach of Friedman in 1956 who used a statistical model to represent bid/estimate quantity lies in its representation of the relationship between estimates of resource costs, or market prices, or some mixture of the two, and the other values of interest such as individual or collective competitors' bids, or the lowest bid itself. The research that has been done to date indicates that the distribution of bid/cost estimate ratios (Table 10 and bid/price estimate ratios (Table 2) is not sufficiently stable to justify any sophisticated applications such as that suggested by Friedman. Current work aimed at refining our understanding of the nature of the ratios and influencing factors seems to offer a possible way through this difficulty. It also seems likely that additional benefits may result in the development of some simple debiasing techniques for estimators (Beeston, 1988).

It is instructive to not that the many criticisms of Friedman's general proposals have been instrumental in furthering work in the bidding problem. Questions concerning the comprehensiveness of his model have led to the consideration of many other relevant factors, and difficulties in calibration have led to the consideration of the nature of estimation generally. There are signs however that the impetus provided by Friedman is beginning to dissipate. His model is now looking very outdated and the absence of an adequate alternative is inhibiting empirical work in the field. What seems to be needed is some theoretical base from which empirically testable hypotheses can be proposed. It may be that sufficient knowledge already exists to enable a theory to emerge. If so, it really is time to 'get serious'.

References

ACKOFF, R.L, & SASIENI, M. (1968) *Competitive problems of operations research* ch.13. John Wiley & Sons, New York

AICBOR, ASSOCIATED INDUSTRIAL CONSULTANTS LIMITED AND BUSINESS OPERATIONS RESEARCH LIMITED (1967) *Report of the Joint Consulting Team for Serial Contracting for Road Construction Ministry of Transport*

ALEXANDER, A.B (1970) *What price estimating accuracy?* Paper no 534. Metal Fabricating institute Inc., Rockford, Illinois

ARMSTRONG, K. (1972) *The development and appraisal of a computerised estimating system* MSc thesis Loughborough University of Technology

ARPS, J.J. (1965) *Journal of Petroleum Technology* vol.17. p1033-1039. "A strategy for sealed bidding"

ASHWORTH, A., & SKITMORE, R.M (1983) *Accuracy in estimating* Occasional Paper no.27. The Chartered Institute of Building ISBN 0-906600-57-X

ATKINS, K.J. (1975) *Bidding, finance and cash flow in the construction industry* PhD thesis University of Bradford

BARNES, N.M.L. (1971) *The design and use of experimental bill of quantities for civil engineering contracts* PhD Thesis. P1263 Feb University of Manchester Institute of Science and Technology

BARNES, N.M.L., & LAU, K.T. (1974) *Operations Research* vol.22. p510-513. "A note on cost estimation and the optimal bidding strategy" in Third International Cost Engineering Symposium. Association of Cost Engineers

BECKMAN, M.J. (1974) *Operations Research* vol.22. p510-513. "A note on cost estimation and the optimal bidding strategy"

BEESTON, D.T. (1982) *Estimating market variance* in building cost techniques: new directions ed. P.S. Brandon. E & F.N. Spon ISBN 0-419-12940-5 p265-277

BEESTON, D.T. (1988) *Adjustments for bias in forecasting* Unpublished, Property Services Agency, Room 266, St Christopher House, Southwark St, London

BENJAMIN, N.B.H. (1969) *Competitive bidding for building construction contracts* PhD dissertation Stanford University

BENNETT, J (1987) *Report on a test of Griffiths, Laxtons, Spons and Wessex price books* Unpublished. Thomas Skinner Directories, East Grinstead, West Sussex

BENNETT, J., & FINE, B. (1980) *Measurement in complexity in construction projects* SRC Research Report GR/A/1342.4 (Final Report) by Department of Construction Management, University of Reading April

BENNETT, J., & ORMEROD, R.N. (1984) *Construction Management and Economics* vol.2.p.225-263. "Simulation applied to construction projects"

BROESMER, G.M. (1968) *Competitive bidding in the construction industry* PhD dissertation, Stanford University

BROWN, K.C. (1966) *A theoretical and statistical study of decision making under uncertainty – competitive bidding for leases of offshore petroleum tracts* PhD dissertation, Southern Methodist University, Dallas

CAPEN, E.C., CLAPP, R.V. & CAMPBELL, W.M. (1971) *Journal of Petroleum Technology* June p641-653 "Competitive bidding in high risk situations"

CARR, R.I., & SANDAHL, J.W. (1978) *Journal of the Construction Division* ASCE vol.104. no.C01. March. P15-26 "Bidding strategy using multiple regression"

CASE, K.E. (1972) *IEE Transactions on Engineering Management* vol.19. no.4. November "Consideration of variability in cost engineering"

CASEY, B.J., & SHAFFER, L.R. (1964) *An evaluation of some competitive bid strategy models for contractors* Report n.4. Department of Civil Engineering, University of Illinois

CAUWELAERT, F.V., & HEYNIG, E. (1978) *Journal of the Construction Division* ASCE vol.105. no.C01. March p13-23 "Correction of bidding errors: the belgian solution"

CHRISTENSON, C. (1965) *Strategic aspects of competitive bidding for corporate securities* Boston Division of Research, Graduate School of Business Administration, Harvard University, p72-89

COOKE, B. (1981) *Contract planning and contractual procedures* MacMillan ISBN 0-333-30720-8

CRAWFORD, P.B. (1970) *Journal of Petroleum Technology* vol.22. p283-289 "Texas offshore bidding patterns"

DOUGHERTY, E.L., & NOZAKI, M. (1975) *Journal of Petroleum Technology* March. P349-356 "Determining optimum bid fraction"

EDMOND, L.J. (1971) *Cost and Management* Sept, Oct. p6-11 “Analytical strategy for the competitive price setter”
ENGLEBRECHT-WIGGANS, R. (1980) *Management Science* vol 26 p119-142 “Auctions and bidding models: a survey”
FINE, B. (1970) *Construction Progress* 14 July. P3-4 “Simulation technique challenges management”
FINE, B. (1974) *Building* 25 October. P115-121 “Tendering Strategy”
FINE, B., & HACKEMAR, G. (1970) *Building Technology and Management* Sept. p8-9 “Estimating and bidding strategy”
FLANAGAN, R., & NORMAN, G. (1985) *Construction Management and Economics* vol.3. p.145-161 “Sealed bid auctions: an application to the building industry”
FRIEDMAN, L. (1956) *Operations Research* vol.4. p104-112 “A competitive bidding strategy”
FUERST, M. (1977) *Journal of the Construction Division* ABCE vol.103. no.C01. March. P139-152 “Theory for competitive bidding”
GATES, M. (1967) *Journal of the Construction Division* ASCE vol.93. no. C01. March. P75-107 “Bidding strategies and probabilities”
GRIESMER, J.H., & SHUBIK, M. (1963a) *Naval Research Logistics Quarterly* vol 10 p11-12 “Toward a study of bidding processes, part 1: some constant sum games:
GRIESMER, J.H., & SHUBIK, M. (1963b) *Naval Research Logistics Quarterly* vol 10 p151-173 “Toward a study of bidding processes, part 2: games with capacity limitations”
GRIESMER, J.H., & SHUBIK, M. (1963c) *Naval Research Logistics Quarterly* vol 10 p191-217 “Toward a study of bidding processes, part 3: some special models”
GRIESMER, J.H., LEVITAN, R.E., & SHUBIK, M. (1967) *Naval Research Logistics Quarterly* vol.14. p415-433 “Toward a study of bidding processes, part 4: games with unknown costs”
GRINYER, P.H., & WHITTAKER, J.D (1973) *Operational Research Quarterly* vol.24. no.2. p181-191 “Managerial judgement in a competitive bidding model”
HACKEMAR, G.C (1970) *Building Technology and Management* Dec. p6-7 “Profit and competition: estimating and bidding strategy”
HANSCOMB ASSOCIATES (1984) *Area Cost Factors* Report for the US Army Corps of Engineers, Hanscomb Associates Inc, 600 West Peachtree Street, NW, Suite 1400, Atlanta, Georgia 30308, USA
HANSSMANN, F. & RIVETT, B.H.P (1959) *Operational Research Quarterly* vol.10. no.1 p49-55 “Competitive Bidding”
HARRIS, F. & McCAFFER, R. (1983) *Modern Construction Management* 2nd ed. Gromach ISBN 0-246-11818-0
HARVEY, J.R. (1979) *Competitive bidding on candadian public construction contracts, stochastic analysis for optimization* PhD thesis, School of Business Administration, University of Western Ontario.
HELLEBRANDT, P.M. (1974) *Economic theory and the construction industry* MacMillan ISBN 0-333-14944-0
HOSSEIN, B.R. (1977) *Risk analysis of tendering policies for capital projects* PhD thesis, University of Bradford.
JAMES, W. (1954) *The Chartered Surveyor* May “A new approach to single price-rate approximate estimating”
JOHNSTON, R.H. (1978) *Optimisation of the selective competitive tendering system by the construction client* Transport and Road Laboratory