Domain: **Ratios and Proportional Relationships (RP)**
Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.
Standard: **7.RP.1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*
<u>Standards for Mathematical Practice (MP):</u> MP.2. Reason abstractly and quantitatively. MP.6. Attend to precision
<u>Connections</u>: This cluster is connected to the Grade 7 Critical Area of Focus #1, **Developing understanding of and applying proportional relationships** and Critical Area of Focus #2**, Developing understanding of operations with rational numbers and working with expressions and linear equations.** This cluster grows out of Ratio and Proportional Relationships (Grade 6) and the Number System (Grade 6), and relates to Expressions and Equations (Grade 7). Cross Curricular connections - economics, personal finance, reading strategies.
<u>Explanations and Examples</u> **7.RP.1** Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. For example, if 1/2 gallon of paint covers 1/6 of a wall, then the amount of paint needed for the entire wall can be computed by 1/2 gal divided by 1/6 wall. This calculation gives 3 gallons. This standard requires only the use of ratios as fractions. Fractions may be proper or improper. **<u>Instructional Strategies</u>** Building from the development of rate and unit concepts in Grade 6, applications now need to focus on solving unit-rate problems with more sophisticated numbers: fractions per fractions. Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a "rule" that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc. This is not the time for students to learn to cross multiply to solve problems. Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on "out of 100"; now percents above 100 are encountered. Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions. Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate mastery.

Domain: **Ratios of Proportional Relationships (RP)**

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard: **7.RP.2.** Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Standards for Mathematical Practice (MP):

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #1, **Developing understanding of and applying proportional relationships** and Critical Area of Focus #2**, Developing understanding of operations with rational numbers and working with expressions and linear equations.**

This cluster grows out of Ratio and Proportional Relationships (Grade 6) and the Number System (Grade 6), and relates to Expressions and Equations (Grade 7).

Cross Curricular connections - economics, personal finance, reading strategies.

Explanations and Examples:

7.RP.2 Students' understanding of the multiplicative reasoning used with proportions continues from 6th grade. Students determine if two quantities are in a proportional relationship from a table. For example, the table below gives the price for different number of books. Do the numbers in the table represent a proportional relationship?

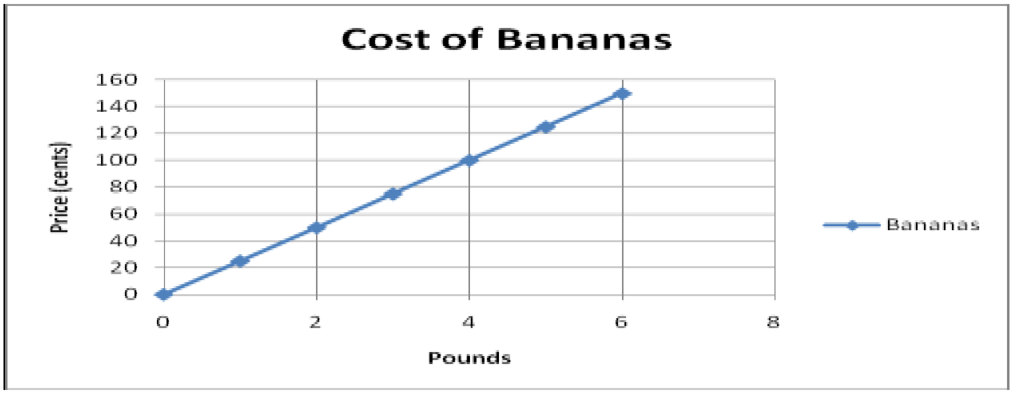
Students can examine the numbers to see that 1 book at 3 dollars is equivalent to 4 books for 12 dollars since both sides of the tables can be multiplied by 4. However, the 7 and 18 are not proportional since 1 book times 7 and 3 dollars times 7 will not give 7 books for 18 dollars. Seven books for $18 are not proportional to the other amounts in the table; therefore, there is not a constant of proportionality.

Number of Books	Price
1	3
3	9
4	12
7	18

Students graph relationships to determine if two quantities are in a proportional relationship and interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books cost 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when x = 1 will be the unit rate.
The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

The graph below represents the price of the bananas at one store. What is the constant of proportionality? From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph. Note: Any point on the graph will yield this constant of proportionality.



The cost of bananas at another store can be determined by the equation: P = $0.35n, where P is the price and n is the number of pounds. What is the constant of proportionality (unit rate)? Students write equations from context and identify the coefficient as the unit rate which is also the constant of proportionality.

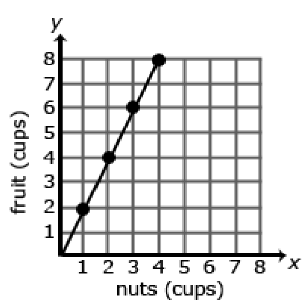
Note: This standard focuses on the representations of proportions. Solving proportions is addressed in 7.SP.3.

Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.

Examples:

- A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

Serving Size	**1**	**2**	**3**	**4**
Cups of Nuts (x)	1	2	3	4
Cups of Fruit (y)	2	4	6	8



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1).
The constant of proportionality is shown in the first column of the table and by the slope of the line on the graph.

- The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

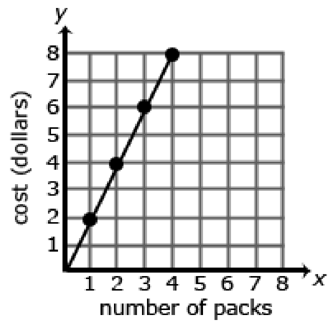


Table:

Number of Packs of Gum (g)	Cost in Dollars (d)
0	0
1	2
2	4
3	6
4	8

Equation: $d = 2g$, where d is the cost in dollars and g is the packs of gum

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as "the number of packs of gum times the cost for each pack is the total cost in dollars". They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost
($g \times 2 = d$).

Domain: **Ratios and Proportional Relationships**

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard: **7.RP.3.** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Standards for Mathematical Practice (MP):

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #1, **Developing understanding of and applying proportional relationships** and Critical Area of Focus #2**, Developing understanding of operations with rational numbers and working with expressions and linear equations.**

This cluster grows out of Ratio and Proportional Relationships (Grade 6) and the Number System (Grade 6), and relates to Expressions and Equations (Grade 7).

Cross Curricular connections - economics, personal finance, reading strategies.

Explanations and Examples:

7.RP.3 In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication.

Students understand the mathematical foundation for cross-multiplication.

For example, a recipe calls for 3/4 teaspoon of butter for every 2 cups of milk. If you increase the recipe to use 3 cups of milk, how many teaspoons of butter are needed?

Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

The use of proportional relationships is also extended to solve percent problems involving tax, markups and markdowns simple interest (I = prt, I = interest, p = principal, r = rate, and t = time), gratuities and commissions, fees, percent increase and decrease, and percent error.

For example, Games Unlimited buys video games for $10. The store increases the price 300%? What is the price of the video game?

Using proportional reasoning, if $10 is 100% then what amount would be 300%? Since 300% is 3 times 100%, $30 would be $10 times 3. Thirty dollars represents the amount of increase from $10 so the new price of the video game would be $40.

Continued next page

Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error.

$$\% \text{ error} = \frac{|\text{your result} - \text{accepted value}|}{\text{accepted value}} \times 100\ \%$$

For example, you need to purchase a countertop for your kitchen. You measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is the percent error?

$$\% \text{ error} = \frac{|5\text{ ft.} - 4.5\text{ ft.}|}{4.5} \times 100$$

$$\% \text{ error} = \frac{0.5\text{ ft.}}{4.5} \times 100$$

Several problem situations have been represented with this standard; however, every possible situation cannot be addressed here.

Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

Examples:

- Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs \$4.17. What is the projected cost of a gallon of gas for April 2015?

  A student might say: "The original cost of a gallon of gas is \$4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of \$4.17 is about \$1.04, the projected cost of a gallon of gas should be around \$9.40."

$$\$4.17 + 4.17 + (0.24 \bullet 4.17) = 2.24 \times 4.17$$

100%	100%	24%
\$4.17	\$4.17	?

Continued next page

- A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

37.50 Original Price of Sweater	
33% of 37.50 Discount	67% of 37.50 Sale price of sweater

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

- A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

Discount 40% of original price	Sale Price - $12 60% of original price
Original Price (p)	

$0.60p = 12$

- At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.

- A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal?

After eating at a restaurant, your bill before tax is $52.60 The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.
The amount paid = 0.20 x $52.50 + 0.08 x $52.50 = 0.28 x $52.50

Domain: **Geometry (G)**

Cluster: Draw, constructs, and describes geometrical figures and describes the relationships between them.

Standard: **7.G.1.** Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Standards for Mathematical Practice (MP):

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #3, **Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**.
Connections should be made between this cluster and the Grade 7 Geometry Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (7.G.4-6).
Grades 6 and 7: Ratios and Proportional Relationships
This cluster leads to the development of the triangle congruence criteria in Grade 8.

Instructional Strategies

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of time you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown.

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	1 unit		
Triangle	1 unit		
Rhombus	1 unit		

This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. In other words, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale?

Shape	Original Side Length	Created Side Length	Scale
Square	2.5 inches		
Parallelogram	3.25 cms		
Trapezoid	(Actual measurements)	Length 1 Length 2	

Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.
After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-number side lengths allows all students access to discover and understand the relationships. An interesting discovery is the relationship of the scale of the side lengths to the scale of the respective perimeters (same scale) and areas (scale squared). A sample recording sheet is shown.

Shape	Side Length	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3 in.	2	10 inches	20 inches	2	6 sq. in.	24 sq in.	4
Triangle	1.5 inches	2	4.5 inches	9 inches	2	2.25 sq. in.	9 sq in.	4

Students should move on to drawing scaled figures on grid paper with proper figure labels, scale and dimensions. Provide word problems that require finding missing side lengths, perimeters or areas. For example, if a 4 by 4.5 cm rectangle is enlarged by a scale of 3, what will be the new perimeter? What is the new area? or If the scale is 6, what will the new side length look like? or Suppose the area of one triangle is 16 sq units and the scale factor between this triangle and a new triangle is 2.5. What is the area of the new triangle?
Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation.
Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles.
Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.
This cluster is related to the following Grade 7 cluster "Solve real-life and mathematical problems involving angle measure, area, surface area, and volume." Further construction work can be replicated with quadrilaterals, determining the angle sum, noticing the variety of polygons that can be created with the same side lengths but different angle measures, and ultimately generalizing a method for finding the angle sums for regular polygons and the measures of individual angles. For example, subdividing a polygon into triangles using a vertex (N-2)180° or

subdividing a polygons into triangles using an interior point 180°N - 360° where N = the number of sides in the polygon. An extension would be to realize that the two equations are equal.
Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found. Challenges can also be given: "See how many different two-dimensional figures can be found by slicing a cube" or "What three-dimensional figure can produce a hexagon slice?" This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Instructional Resources/Tools

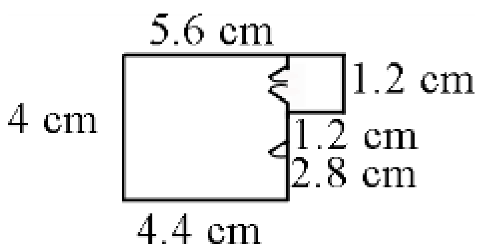
Straws, clay, angle rulers, protractors, rulers, grid paper
Road Maps - convert to actual miles Dynamic computer software - Geometer's SketchPad. This cluster lends itself to using dynamic software. Students sometimes can manipulate the software more quickly than do the work manually. However, being able to use a protractor and a straight edge are desirable skills.

Explanations and Examples:

7.G.1 Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

Example:

- Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.



Common Misconceptions:

Student's may have misconceptions about correctly setting up proportions, how to read a ruler, doubling side measures, and does not double perimeter.