

Proportional Relationships

Mathematics Grade 7

The focus of this unit is to develop an understanding of proportional relationships. Students continue their work with ratio tables from grade 6 to examine multiplicative relationships to determine proportionality. Students learn that a proportion is a relationship of equality between two ratios and the ratio of two quantities remains constant as the corresponding values of the quantities change. As students gain a deeper conceptual understanding of proportional relationships they begin to look for and make use of structure by understanding the roles of “for every,” “for each,” and “per.”

As students work with proportional relationships, they recognize that graphs that are not lines through the origin and tables in which there is not a constant ratio in the entries do not represent proportional relationships. Students then apply this understanding to equations in the form of $y = mx$ where m is the constant of proportionality (unit rate). Students connect their work with equations to their prior work with tables and diagrams.

*This unit addresses the grade 7 major cluster **7.RP.A.2** recognize and represent proportional relationships between quantities.*

- **CCSS.Math.Content.7.RP.A.2a**

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- **CCSS.Math.Content.7.RP.A.2b**

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- **CCSS.Math.Content.7.RP.A.2c**

Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

- **CCSS.Math.Content.7.RP.A.2d**

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Coherence:

Across-Grade Coherence: Content Knowledge from Earlier Grades

- **CCSS.Math.Content.6.RP.A.2**

Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."

- **CCSS.Math.Content.6.RP.A.3**

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- **CCSS.Math.Content.6.RP.A.3a**

Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

- **CCSS.Math.Content.6.RP.A.3b**

Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Within-Grade Coherence: Content from other Standards in the Same Grade that Provide Reinforcement

- **CCSS.Math.Content.7.RP.A.1**

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $1/2 / 1/4$ miles per hour, equivalently 2 miles per hour.

- **CCSS.Math.Content.7.EE.B.4**

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- **CCSS.Math.Content.7.EE.B.4a**

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Across-Grade Coherence: Looking Ahead for Content Connections

- **CCSS.Math.Content.8.FB.B.4**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- **CCSS.Math.Content.8.EE.B.5**

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

- **CCSS.Math.Content.8.EE.B.6**

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

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Connecting Equivalent Ratios to Proportional Relationships

Grade Level Standards Addressed in this Lesson:

- **CCSS.Math.Content.6.RP.A.3**
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- **CCSS.Math.Content.6.RP.A.3a**
Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Across-Grade Coherence: Looking Ahead for Content Connections

- **CCSS.Math.Content.7.RP.A.2**
Recognize and represent proportional relationships between quantities.
- **CCSS.Math.Content.7.RP.A.2a**
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Mathematical Goals:

This lesson is intended to help you assess whether students know and understand:

- How to create a set of equivalent ratios.
- How to justify a set of equivalent ratios by using multiple methods (ratios tables, graphing, etc.).
- How to justify that a proportional relationship exists using multiple methods (ratio tables, graphing, etc.).
- How to determine if two quantities are in a proportional relationship by testing for equivalent ratios.

Rigor:

This lesson requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

Application: Students continue to build their understanding of equivalent ratios through analyzing real world situations involving mixing different quantities of paint.

Conceptual understanding: Students develop their understanding of what it means when two quantities are in a proportional relationship, connecting this new understanding with their prior understanding of equivalent ratios.

Procedural Skill and Fluency: This lesson builds on the student's prior understanding of how to create sets of equivalent ratios through different strategies such as creating ratio tables, using the multiplicative comparison, graphing, etc. Students moving into 7th grade should have a strong foundation of these different strategies and have the opportunity in this lesson to determine which strategy will help them to create equivalent ratios in the most fluent way.

Overview of Lesson:

For students to be successful in the 7th grade domain of ratios and proportional reasoning, it is important that they have a strong conceptual understanding of the 6th grade standards identified in this lesson. The purpose of this lesson is to provide teachers with a resource that allows them to informally assess readiness by engaging in the activities. Student's tap into their prior knowledge of ratios by creating different sets of equivalent ratios. The students are able to use any method they learned in sixth grade. The teacher is able to quickly assess the students' understanding of creating a set of equivalent ratios along with the students preferred strategy. Students use their prior knowledge and understandings as stepping stones to identifying the connection between equivalent ratios and proportional relationships.

Estimate Time: 120 minutes

Essential Questions:

1. How does one test two quantities for equivalent ratios?
2. How does ratio reasoning differ from proportional reasoning?
3. How are equivalent ratios related to proportional reasoning?
4. What is a proportional relationship?

Common Misconceptions:

- When students find equivalent ratios, some tend to confuse additive versus multiplicative reasoning.
- When plotting the pairs of values on a coordinate plane, some students may believe that any two plotted values (points) that form a straight line are equivalent without taking into account whether the line does or does not go through the origin.

Universal Design for Learning:

- **[Engagement]** The lesson optimizes individual choice when students are allowed to determine which strategy they want to use when solving the paint problem.
- **[Engagement]** The lesson fosters collaboration and community through the use of the cross-sharing protocol.
- **[Action & Expression]** The lesson builds fluencies with graduated levels of support for practice and performance through the facilitation of the jigsaw activity.
- **[Representation]** The lesson activates and supplies background knowledge through the connection of equivalent ratios from grade 6.

Standards for Mathematical Practice:

- **[SMP #1]** Students are making sense of problems and persevering when provided a graph representing different quantities of paint mixtures and are asked to determine which mixtures of orange paint are the same as the original shade of paint.
- **[SMP #3]** Students are critiquing the reasoning of their classmates when they are asking clarifying questions about the chosen strategy used.
- **[SMP #7]** Students are looking for and making use of structure when they are creating and examining ratio tables, double number lines, tape diagrams, etc.

Resources:

- The Paint Organizer Student Handout (page 14)
- Teacher Reference RP Strategies (page 16)
- Exit Ticket (page 17)
- Colored Sticky Notes for Group Activity

Additional Resources in the Public Domain:

- Common Core Toolbox Web Link:
http://ccsstdtoolbox.agilemind.com/animations/standards_content_mathematics_proportionality.html

Opportunities for Assessment within the Lesson:

- Formative – Exit Ticket (page 13)
- Informal Assessment (Page 9 & 12)

Additional Assessment Tasks in the Public Domain:

- Illustrative Mathematics Task: Art Class, Assessment Variation
<https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1183>
- NRICH: Mixing Paints
<http://nrich.maths.org/4793>
- Inside Mathematics: Mixing Paint (enrichment)
<http://www.insidemathematics.org/assets/common-core-math-tasks/mixing%20paints.pdf>

Lesson Sequence:

Each student will need a copy of the **Paint Organizer** (handout page 14).

Teacher says to the class: “Last year you spent a significant amount of time on building your understanding of equivalent ratios. You used several different strategies when creating equivalent ratios and determining when ratios were equivalent. This understanding is the basis of our new unit on proportional relationships. So, to kick off this unit, tap into prior knowledge and get working on your ‘warming up’ problem!”

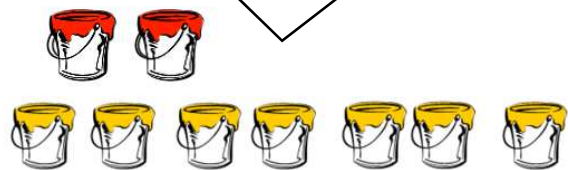
Warming up: (Independent)

Pose the following question on the board. Students should work independently on the problem and record their work on the section of the **Paint Organizer** labeled “Student Work.” Ask students to use what they know about ratios to answer the following question.

Kevin’s favorite color is orange. His mom is painting his bedroom. Suppose that Kevin’s mom has made a batch of orange paint by mixing 2 cans of red paint with 7 cans of yellow paint. What are some other combinations of numbers of cans of red paint and yellow paint that his mom can mix to make the same shade of orange?

UDL Representation:

It is helpful to present the problem with words, pictures, and animations (page 6 additional resources).



Once students have produced several combinations of paint that make the same shade of orange, have students identify the strategy they used to create new amounts of orange paint. Students should record their answer in the box labeled “Strategy I Used.”

Jigsaw Activity: (Productive Group Work)

Round 1: Same Strategy Group Share

- Group students according to the strategy they used to determine different batches of orange paint. (For example all students who used ratio tables should be placed into a group, however remember productive groups are between 3 and 4 students.)
- Identify different areas of your classroom that each group should report to for a mini discussion on their strategy.
- Explain to students that they are going to become experts in the strategy they used. To do this they are going to work for the next 5 to 10 minutes with their strategy group.

Teacher Note (Essential Question 1):

You may need to help students identify the name of the strategy used. Possible strategies used:

- Multiplicative Comparison
- Iterating and Partitioning
- Ratio Table
- Graph
- Double Number Line

Once students are in their strategy group pose the following questions:

- Why did you choose the strategy you used?
- How would you explain this strategy to someone that did not use this strategy?
- How could you use this strategy to solve a problem that you have not yet been given?
- What do you think are the Pros and Cons of using this strategy?

Students should record their answers on the section of the **Paint Organizer** labeled “Round 1 Same Strategy Group Share.”

Jigsaw round 2 directions for grouping students. You will need a 1 to 1 ratio of different colored sticky notes to the number of same strategy groups. Give each group member the same colored sticky note. Students will form a new group to share their strategy that they became an expert on during the first round of the jigsaw activity. Each new group formed should not have two students with the same colored sticky note.

Round 2: Different Strategy Group Share

- Have students form a new group of 3 to 4 students. No student in the new group should have the same colored sticky note.
- Cross Sharing Protocol (6 minutes per student, 24 minutes total)
 - Step 1: Presenting student shares their strategy to the group ensuring the group knows the responses to the posed questions from part 1. (3 minutes)
 - The presenting student discusses the strategy without interruption.
 - The remaining group listens, takes notes and jots down possible clarifying questions.
 - Step 2: Clarifying Questions (3 minutes)
 - The partner students ask clarifying questions. The presenting student responds succinctly.
 - Repeat steps 1 and 2 for each member of the group.

Standards for Mathematical Practice:

- Students are critiquing the reasoning of their classmates when they are asking clarifying questions about the chosen strategy used. [SMP 3]
- Students are making use of structure when they are sharing their explanations as to which strategy they used and how the chosen strategy can be used to help answer questions they've never seen before. [SMP 7]

Connecting Prior Knowledge to Proportional Relationships: (Whole Class)

In whole class discussion:

- Highlight the fact that students used multiple strategies as tools to determine equivalent ratios. In this unit we will be learning about proportional relationships. The definition of a proportional relationship is a collection of pairs of numbers that are in equivalent ratios (Progressions Document, 14). We can connect our understanding of equivalent ratios to proportional relationships by testing two ratios for equivalency in tables, graphs, and other strategies discussed in today's lesson (Essential Question 1). For a proportional relationship to exist all of the quantities must be in proportion to one another (Essential Question 4).

Teacher Note:

Create a poster that shows the different strategies for determining equivalent ratios.

- Multiplicative Comparison
- Iterating and Partitioning
- Ratio Table
- Graph
- Double Number Line

This poster should hang in the classroom during the entire unit as a resource for students.

In partners:

- Pose the following problem and have students respond.

Suppose that you have made a batch of pink paint by mixing 4 cups of red paint with 3 cups of white paint. If you have 21 cups of red paint and 28 cups of white paint, are the two given quantities of pink paint in proportion to one another (producing the same shade of pink paint)?

Solution:

No, the two given quantities are not in proportion to one another.

Teacher Note:

A common error in setting up proportions is placing numbers in incorrect locations. This is especially easy to do when the order in which quantities are stated in the problem is switched within the problem statement (Progressions Document, 9).

Student Work Analysis:

If a student answers the question with yes... A possible confusion is that they thought you could multiply by 7; however, they did not identify the correct number of cups of paint for the red and white paint. One suggestion to help students with this confusion is to have them write red paint and white paint in their ratio table. This tends to be an organizational issue that can easily be addressed.

After students discuss their responses with their partners and share with the class, extend students thinking by posing the following questions.

- How many cups of white paint will you need if you have 21 cups of red paint? Explain your reasoning?
 - $15\frac{3}{4}$ cups of white paint.
- How many cups of red paint will you need if you have 28 cups of white paint? Explain your reasoning?
 - $37\frac{1}{3}$ cups of red paint.

In this lesson we focused on only working with whole numbers. Once students have built fluency with using the different strategies for finding equivalent ratios, it is important to move students to using those strategies with problems that involve grade-level appropriate quantities. Teachers should use the following problems to determine if students understand the concept of equivalent ratios and are ready to use that understanding to work with more complex numbers.

Informal Assessment: (Independent)

Each table below represents different Paint Mixtures that create shades of pink paint. Examine the tables below and determine which tables represent proportional relationships between the number of red cups of paint and the number of white cups of paint.

Paint Mixture #1	
Red	White
1	0
2	3
3	6

Paint Mixture #2	
Red	White
1	1
2	3
3	5

Paint Mixture #3	
Red	White
1	2
2	4
3	6

Paint Mixture #4	
Red	White
1	3
2	5
3	7

Student Work Analysis:

- *The correct answer is Paint Mixture #3*
 - *Paint Mixture #3 – Students should notice that the number of cups of red paint increase by one each time and the number of cups of white paint increase by two each time. However, that does not define a proportional relationship. Students need to be able to show that a proportional relationship exists by identifying the multiplicative comparison ($\times 2$). Students may use strategies such as iterating and partitioning or by graphing the quantities on a coordinate plane.*
- *If a student answers the above question with response like those provided below, they may be using additive reasoning.*
 - *Paint Mixture #1 – Students might see that the number of cups of red paint increases and that the number of cups of white paint increases; however, they are not increasing proportionally. A common misconception is when students believe that a proportional relationship exists when the number of red paint is increasing by the same amount (in this case increasing by 1) and the number of white paint is increasing by the same amount (in this case increasing by 3).*
 - *Paint Mixture #2 and Paint Mixture #4 – Students might see that the number of cups of red paint increases and that the number of cups of white paint increases; however, they are not increasing proportionally. A common misconception is when students believe that a proportional relationship exists when the number of red paint is increasing by the same amount (in both of these cases increasing by 1) and the number of white paint is increasing by the same amount (in both of these cases increasing by 2).*
- *If you see students choosing paint mixtures 1, 2, or 4. Have them create a visual representation of the relationship by graphing the given quantities on a coordinate plane. This should help struggling learners easily identify which of the tables represent a proportional relationship by connecting the points on the graph and noticing that the line is straight and goes through the origin. This is a key concept learned in 6th grade [6.RP.A.3a] that students need to carry forward in their work throughout this unit.*

Moving from Ratio Reasoning to Proportional Reasoning: (Whole Class)

In whole class discussion:

- Early in this lesson we spent time focusing on building equivalent ratios by iterating and partitioning. We moved that understanding into what does it mean when two or more quantities are in a proportional relationship by examining the situation as a whole and understanding the overall relationship that exists. (Essential Question 3)
- Proportional reasoning involves thinking about relationships and making comparisons of quantities or values. The essential characteristic of proportional reasoning is that it must involve a relationships between two relationships. For example in our independent practice, paint mixture 3, three mixtures of pink paint were examined together as a whole creating a proportional relationship rather than simply a relationship between two concrete objects such as our warm-up problem of 2 cans of red paint to 7 cans of yellow paint. (Essential Question 2)

Teacher Note:

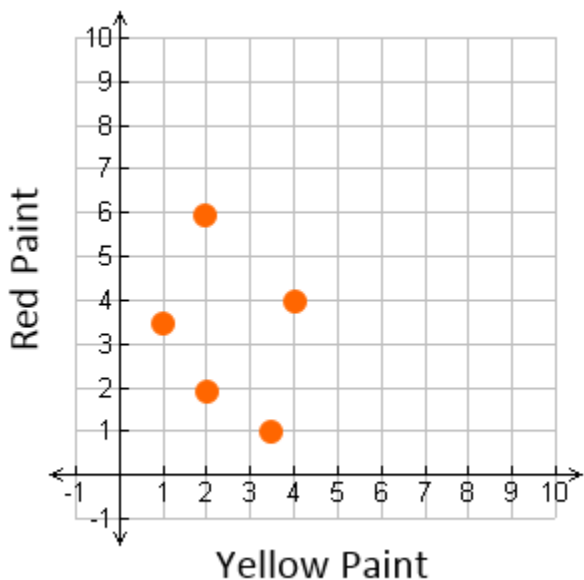
Proportional reasoning is the consideration of number in relative terms rather than absolute terms. “An important part of developing more sophisticated proportional reasoning is the ability to truncate the work of iterating a composed unit by using the arithmetic operation of multiplication. To accomplish this, students need to move from simply repeating a composed unit multiple times until they reach a particular goal to being able to anticipate the number of groups that they need” (Lobato, 38).

In Partners:

- Display the graph below to your students.
- Pose the following question:
 - Is each quantity of orange paint (created by mixing red and yellow paint) in proportion to the shade of orange paint created by mixing 7 cans of red paint and 2 cans of yellow paint? Be prepared to justify your answer using one of the strategies from today's lesson.
 - *Solution: When examining the values represented by the points on the graph, not all quantities of orange paint can be connected with a straight line that originates at the origin. Only one quantity of orange paint represented by the point (1, 3.5) is in proportion to the given quantity of 7 red to 2 yellow.*
 - If a student is struggling to get started on this problem, ask the student: "What information is missing from the graph?" This question should spark the student to plot the given ratio of 7 red: 2 yellow.

Teacher Note:

The ratio of A to B does not always imply (x,y) when plotted on a graph.



Standards for Mathematical Practice:

- Students are making sense of problems and persevering when provided a graph representing different quantities of paint mixtures and are asked to determine which mixtures of orange paint are the same as the original shade of paint. [SMP 1]
- Students are making use of structure when they are examining the graph to determine which of the points plotted form a straight line that goes through the origin. [SMP 7]

- Extend the learning by asking the following question below:
 - Are there any two quantities of orange paint (represented by the points on the graph) in proportion to one another?
 - *The ratio of 2 cans of red paint to 2 cans of yellow paint are in proportion to 4 cans of red paint to 4 cans of yellow paint.*
 - *Common misconception: if a student plots the value of the ratio of 7 cans of red paint to 2 cans of yellow paint incorrectly on the graph (7,2) then the student may conclude that the ratio of 3.5 cans of yellow to 1 can of red is in proportion to the quantity represented by the point they plotted.*

Problems that contain rational numbers: (Independent)

Teacher Directions: In order for 7th grade students to fully meet the rigor of the standard, they need to work with rational numbers. Below are set of problems that include rational numbers. All students need to be able to complete questions 1, 2 and 3. Question 1 is provided as an entry point for students because it resembles scenarios discussed in class. This question should be completed in a small group with the teacher for those students who struggled through the lesson prior to the students attempting questions 2 and 3 independently. Question number 4 includes the word “rate” which has not been officially defined in this lesson. If students easily complete questions 2 and 3 and are ready to extend their learning, provide them with question 4.

Questions:	Possible Solutions:																		
1. Suppose that you have made a batch of orange paint by mixing 2 cups of red paint and $\frac{3}{5}$ cups of yellow paint. What are some other combinations of numbers of cups of red paint and yellow paint that you can mix to make the same shade of orange?	<table><tr><td>Red</td><td>2</td><td>10</td><td>1</td><td>20</td><td>4</td></tr><tr><td>Yellow</td><td>$\frac{3}{5}$</td><td>3</td><td>$\frac{3}{10}$</td><td>6</td><td>$1\frac{1}{5}$</td></tr></table>	Red	2	10	1	20	4	Yellow	$\frac{3}{5}$	3	$\frac{3}{10}$	6	$1\frac{1}{5}$						
Red	2	10	1	20	4														
Yellow	$\frac{3}{5}$	3	$\frac{3}{10}$	6	$1\frac{1}{5}$														
2. It takes you $\frac{1}{7}$ of a week to complete 3 chores. What are some other combinations of time and chores that result in the same time per chore?	<table><tr><td>Weeks</td><td>$\frac{1}{7}$</td><td>1</td><td>2</td><td>3</td><td>4*</td></tr><tr><td># of chores</td><td>3</td><td>21</td><td>42</td><td>63</td><td>84</td></tr></table> <p>*It takes 4 weeks or 28 days to complete 84 chores.</p>	Weeks	$\frac{1}{7}$	1	2	3	4*	# of chores	3	21	42	63	84						
Weeks	$\frac{1}{7}$	1	2	3	4*														
# of chores	3	21	42	63	84														
3. You are making cookies to sell at your local fundraiser. The original recipe calls for $\frac{1}{2}$ of a cup of sugar for every $\frac{2}{3}$ of a cup of flour, resulting in approximately half a dozen cookies. You need to make multiple batches of cookies, what are some other combinations of sugar and flour can you mix that would result in the same cookie taste?	<table><tr><td>Sugar</td><td>$\frac{1}{2}$</td><td>1</td><td>3</td><td>$1\frac{1}{2}$</td><td>2</td></tr><tr><td>Flour</td><td>$\frac{2}{3}$</td><td>$\frac{4}{3}$</td><td>4</td><td>2</td><td>$2\frac{2}{3}$</td></tr><tr><td># of cookies</td><td>6</td><td>12</td><td>36</td><td>18</td><td>24</td></tr></table>	Sugar	$\frac{1}{2}$	1	3	$1\frac{1}{2}$	2	Flour	$\frac{2}{3}$	$\frac{4}{3}$	4	2	$2\frac{2}{3}$	# of cookies	6	12	36	18	24
Sugar	$\frac{1}{2}$	1	3	$1\frac{1}{2}$	2														
Flour	$\frac{2}{3}$	$\frac{4}{3}$	4	2	$2\frac{2}{3}$														
# of cookies	6	12	36	18	24														
4. Evan is catering lunch. He completes $\frac{1}{5}$ of his orders in $\frac{5}{8}$ of an hour. At this rate, what are some other combinations of numbers of completed orders?	<table><tr><td>Completed Orders</td><td>$\frac{1}{5}$</td><td>1</td><td>8</td><td>4</td><td>16</td></tr><tr><td>Hours</td><td>$\frac{5}{8}$</td><td>$\frac{25}{8}$</td><td>25</td><td>$\frac{25}{2}$</td><td>50</td></tr></table>	Completed Orders	$\frac{1}{5}$	1	8	4	16	Hours	$\frac{5}{8}$	$\frac{25}{8}$	25	$\frac{25}{2}$	50						
Completed Orders	$\frac{1}{5}$	1	8	4	16														
Hours	$\frac{5}{8}$	$\frac{25}{8}$	25	$\frac{25}{2}$	50														

Closure / Wrap-Up: (Whole Class)

In this lesson we worked on fine tuning our skills with generating equivalent ratios using a variety of strategies. We defined a proportional relationship as a collection of pairs of numbers that are in equivalent ratios. Through the exploration of different real world situations, we determined if our newly generated ratios were in proportion to the original given quantity. Remember for a proportional relationship to exist, all of the quantities must be in proportion to one another. In our next lesson we will continue to build on these strategies to determine when a situation is or is not proportional.

Exit Ticket: (Independent)

- Give each student a copy of the Lesson 1 Exit Ticket (handout page 17) to complete independently.
- If needed, students should also be given a copy of the Assessment Reference Sheet.

<http://avocet.pearson.com/PARCC/Home#9812>

Assessment Reference Sheet		
Grade 7		
1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallons
		1 liter = 1000 cubic centimeters

- Solution to question 1:
 - Store A, C and D costs \$0.48 per ounce
 - Store B costs \$0.56 per ounce
 - Store E costs \$0.74 per ounce
 - Store F costs \$0.30 per ounce
 - Cody should buy his fudge at Store F because they sell fudge at the cheapest price.
- Solution to question 2:
 - Store A, C and D all charge the same amount, \$0.48 per ounce of fudge.

Name: _____ Date: _____

Proportional Reasoning Unit

Lesson 1 Exit Ticket

Cody researched the cost of fudge at six different locations in his town.

Store	Quantity	Cost
Store A	20 ounces of fudge	\$9.60
Store B	\$1.40 for 2.5 ounces of fudge	
Store C	1 pound of fudge	\$7.68
Store D	12.5 ounces of fudge	\$6.00
Store E	10 ounces of fudge	\$7.40
Store F	\$2.40 for 0.5 pounds of fudge	

1. Which location should Cody go to buy his fudge? Explain your reasoning.

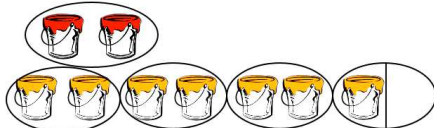
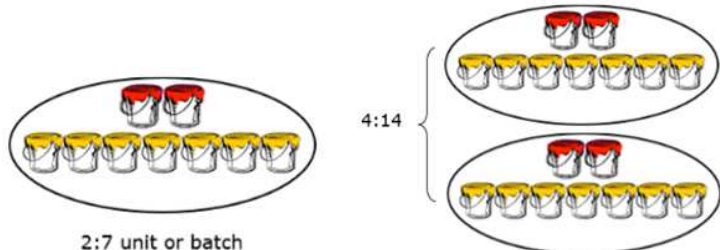
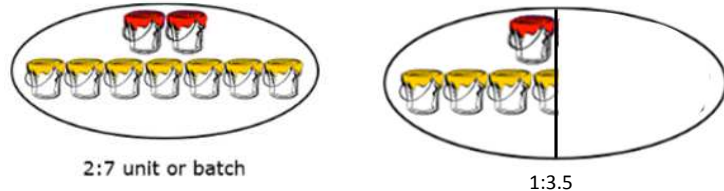
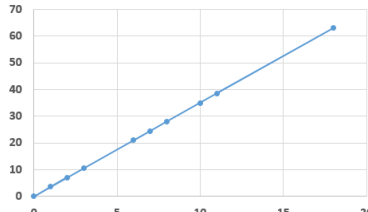
2. Do any of the locations charge the same amount for fudge? Justify your reasoning.

The Paint Organizer

Independent Work: Paint Problem	
Student Work	Strategy I Used
Round 1: Same Strategy Group Share	
Why did you choose the strategy you used?	How would you explain this strategy to someone that did not use this strategy?
How could you use this strategy to solve a problem that you have not yet been given?	What do you think are the Pros and Cons of using this strategy?

Round 2: Different Strategy Group Share	
Name of Strategy being shared:	How would you explain this strategy to someone that did not use this strategy?
How could you use this strategy to solve a problem that you have not yet been given?	What do you think are the Pros and Cons of using this strategy?
Round 2: Different Strategy Group Share	
Name of Strategy being shared:	How would you explain this strategy to someone that did not use this strategy?
How could you use this strategy to solve a problem that you have not yet been given?	What do you think are the Pros and Cons of using this strategy?
Round 2: Different Strategy Group Share	
Name of Strategy being shared:	How would you explain this strategy to someone that did not use this strategy?
How could you use this strategy to solve a problem that you have not yet been given?	What do you think are the Pros and Cons of using this strategy?

Teacher Reference: 6th Grade RP Strategies

Strategy:	Explanation:																								
Multiplicative Comparison	<div></div> <p>Form a group of 2 cans of red paint. Then you find the number of groups of 2 cans that you can make with 7 cans of yellow paint. Ratio of yellow paint to red paint is 3 ½.</p>																								
Iterating and Partitioning	<p>Iterating: Repeating</p> <div></div> <p>2:7 unit or batch</p> <p>4:14</p> <p>Partitioning: Breaking into equal-sized parts</p> <div></div> <p>2:7 unit or batch</p> <p>1:3.5</p>																								
Ratio Table	<table><tr><td>Red Paint:</td><td>2</td><td>4</td><td>1</td><td>3</td><td>7</td><td>6</td><td>8</td><td>10</td><td>18</td><td>11</td></tr><tr><td>Yellow Paint:</td><td>7</td><td>14</td><td>3.5</td><td>10.5</td><td>24.5</td><td>21</td><td>28</td><td>35</td><td>63</td><td>38.5</td></tr></table> <p>Ratio Tables do not need to be in sequential order just as long as they are equivalent ratios.</p>	Red Paint:	2	4	1	3	7	6	8	10	18	11	Yellow Paint:	7	14	3.5	10.5	24.5	21	28	35	63	38.5		
Red Paint:	2	4	1	3	7	6	8	10	18	11															
Yellow Paint:	7	14	3.5	10.5	24.5	21	28	35	63	38.5															
Graphing	<div></div> <p>Students should be able to explain that in order to be equivalent ratios, the graph needs to form a straight line that goes through the origin.</p>																								
Double Number Line	<div><table><tr><td></td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Red Paint</td><td> </td><td> </td><td> </td><td> </td><td> </td></tr><tr><td>Yellow Paint</td><td> </td><td> </td><td> </td><td> </td><td> </td></tr><tr><td></td><td>7</td><td>10.5</td><td>14</td><td>17.5</td><td>21</td></tr></table></div> <p>The order does matter in a double number line.</p>		2	3	4	5	6	Red Paint						Yellow Paint							7	10.5	14	17.5	21
	2	3	4	5	6																				
Red Paint																									
Yellow Paint																									
	7	10.5	14	17.5	21																				

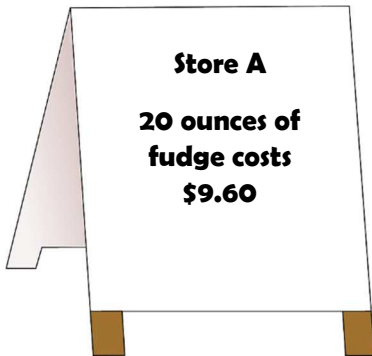
Name: _____

Date: _____

Proportional Reasoning Unit

Lesson 1 Exit Ticket

Cody researched the cost of fudge at six different locations in his town.



1. Which location should Cody go to buy his fudge? Explain your reasoning.
2. Do any of the locations charge the same amount for fudge? Justify your reasoning.

To Be or Not to Be Proportional

Standards Addressed in This Lesson:

- **CCSS.Math.Content.7.RP.A.2**

Recognize and represent proportional relationships between quantities.

- **CCSS.Math.Content.7.RP.A.2a**

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Across-Grade Coherence: Content Knowledge from Earlier Grades

- **CCSS.Math.Content.6.RP.A.3**

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- **CCSS.Math.Content.6.RP.A.3a**

Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Mathematical Goals:

This lesson is intended to help you assess whether students know and understand:

- A proportional relationship is determined when a collection of quantities are in equivalent ratios.
- How the variation in one ratio coincides with the variation in the other.
- When two quantities in a situation describe a proportional relationship.
- How to determine proportionality using multiple methods (ratio tables, graphing, etc.).
- How to determine the composed unit and/or the multiplicative comparison of a proportional relationship.

Rigor:

This lesson requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

Application: Students apply their understanding of equivalent ratios to determine whether a situation is or is not proportional in multiple real world situations.

Conceptual Understanding: Students develop their understanding of what it means to be proportional by examining a set of ratios and calculating a multiplicative comparison or the composed unit.

Procedural Skill and Fluency: This lesson builds on the students' prior knowledge of equivalent ratios through strategies such as ratio tables, double number lines, and/or graphing. Students moving into 7th grade should have a strong foundation of these different strategies and have the opportunity in this lesson to determine which strategy will help them to determine whether the presented situation is or is not proportional.

Overview of Lesson:

In this lesson students will be guided through a variety of activities to help them discover the properties of proportional relationships paying particular attention to composed units and multiplicative comparison. This lesson builds on the students' prior knowledge solidified in lesson one by asking students to use what they know about equivalent ratios to make an educated guess as to whether or not the situation presented represents a proportional relationship. Each of the situations in this lesson are represented by either words, graphs, or tables. The lesson then goes on to build students understanding of the multiplicative comparison and composed units. Several of the strategies reviewed in lesson one come up in the productive group work section when using ratio tables and double number line strategies.

Estimate Time: 180 minutes

Essential Questions:

1. How does comparing quantities describe the relationship between them?
2. How are equivalent ratios, values in a table, and ordered pairs connected?
3. What characteristics define the graphs of all proportional relationships?
4. How can one determine from a graph if the relationship is proportional?
5. What is a composed unit?

Common Misconceptions:

- Students often have misconceptions about proportional relationships because they do not have a sense of covariation. Meaning, students don't understand the relationships in which two quantities vary together and they have a difficult time seeing how the variation in one coincides with the variation in the other. For example, 3 sodas cost \$2.40 (two quantities in a multiplicative relationship); as the number of sodas vary (for example, to 6 sodas), so does the cost. And, as the cost changes, so does the number of sodas you will get. Once you know either a new price or a new number of sodas, you can determine the missing value.

Universal Design for Learning:

- **[Engagement]** The lesson fosters collaboration and community through the use of the Placemat Activity.
- **[Action & Expression]** The lesson facilitates managing information and resources by providing students with graphic organizers and templates for organizing information.
- **[Representation]** The lesson maximizes transfer and generalization by incorporating explicit opportunities for review and practice.
- **[Representation]** The lesson guides information processing, visualization and manipulation by progressively releasing information in the lesson.

Standards for Mathematical Practice:

- **[SMP #1]** Students make sense of problems and persevere in solving them by examining the cards and determining whether the given relationships is or is not proportional.
- **[SMP #2]** Students reason abstractly and quantitatively when they examine a ratio table to determine the composed unit or multiplicative comparison.
- **[SMP #3]** Students construct viable arguments when they justify how they sorted the cards.
- **[SMP #7]** Students look for and make use of structure when they analyze the given situation to make conjectures on whether the relationship shows properties of proportionality.

Resources:

- Cut out Proportional/Non-Proportional Cards (page 31)
- Proportional: Yes or No Student Handout (page 32)
- Cut out Placemat Problems (page 33)
- Number Cards #1 - 4
- Poster Paper & Markers
- Dry erase boards, pens and erasers

Opportunities for Assessment:

- Pre-assessment Proportional Activity (page 21)
- Check for Understanding (page 24)
- Formative – Exit Ticket (page 30)

Additional Assessment Tasks in the Public Domain:

- New York City Department of Education: Leaky Faucets
http://schools.nyc.gov/NR/ronlyres/FFF98A0A-16D1-4E10-BF2C-01A83F011B67/140497/NYCDOE_G7_Math_LeakyFaucets_FINAL.pdf

Lesson Sequence:

Teacher Notes: Prior to the lesson ensure you have cut out enough cards for your students (handout page 31). The following should take place prior to the implementation of this lesson.

Sorting Activity: (Independent)

- Have students sort the cards to determine whether they think the situation represented on the card is or is not proportional.
- Collect students responses on the graphic organizer provided (handout page 32)
- Give the Card Sorting Activity **Proportional: Yes or No**, in class a few days before the lesson. This will give you an opportunity to examine the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in this lesson.
- It is important that students complete this activity without assistance. Some students may find it difficult, but try to encourage students to make a reasonable attempt at the task. Reassure them the task will not be graded and they will have time to revisit and revise their solutions after the lesson.

Name: _____

Date: _____

Proportional Reasoning Unit

Proportional: Yes or No

Card	Proportional Yes or No	Justification/Reasoning

Standards for Mathematical Practice

- Students make sense of problems and persevere in solving them by examining the cards and determining whether the given relationships is or is not proportional. [SMP 1]
- Students construct viable arguments when they justify how they sorted the cards. [SMP 3]
- Students look for and make use of structure when they analyze the given situation to make conjectures on whether the relationship shows properties of proportionality. [SMP 7]

Card A:

Johnny weighs 312 pounds on the planet Jupiter and weighs 120 pounds on earth.

Marco weighs 520 pounds on the planet Jupiter and weighs 200 pounds on earth.

Sally weighs 208 pounds on planet Jupiter and weighs 80 pounds on earth.

Proportional: Yes or No

Card B:

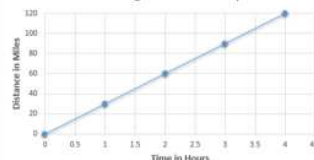
The table below shows the number of ounces of berries in a basket. Is the relationships between the number of ounces of blueberries and the number of ounces of strawberries proportional?

Number of Ounces of Blueberries	2	3	5	6	28
Number of Ounces of Strawberries	3	4.5	7.5	9	42

Proportional: Yes or No

Card C:

Car Traveling at a Constant Speed



Proportional: Yes or No

Card D:

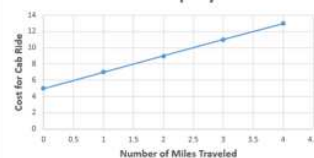
Dominique has a Cupcakery. The table below shows the number of cupcakes she has made and the amount of time it took her to make them. Does the table represent a proportional relationship?

Cupcakes	Time (In Minutes)
6	18
12	36
18	54

Proportional: Yes or No

Card E:

Cab Company



Proportional: Yes or No

Card F:

Gibber Jabber Phone Company charges \$0.70 for every 14 minutes on the phone.

Transcontinental Connectors Phone Company will allow you to talk for 21 minutes for the low price of \$2.10.

S-Mobil Phone Company advertises \$3.15 for every 63 minutes on the phone.

Proportional: Yes or No

During the Lesson:

Teacher says to the class, “Yesterday, we took a look at several different strategies to determine equivalent ratios. We spoke about the connection between equivalent ratios and proportionality by examining the situation as a whole and understanding the overall relationship. Today, we are going to deepen our understanding of what it means when a proportional relationship exists.”

Warming-Up: (Independent)

Display the following table.

Acres x	5	10	1	15	150	9
Maple Trees y	75	150	15	225	2250	135

- Have students jot down 2 – 3 things they notice about the table.
- Connect what students reveal that they noticed throughout the discussion in part 1.

Part I: Teacher engages students in a discussion. (Whole Class)

Introduction: *Covariation and its impact on Proportional Understandings*

Directions for Discussion:

Give each student a dry erase board, a pen, and an eraser.

Draw students’ attention to patterns they may notice in the table by having students answer the following questions:

- What are the two quantities (variables) in this situation?
 - *Acres, Maple Trees*
- What do you notice about the relationship between them?
 - *Encourage collaboration here. At this time have students turn to their elbow partner and share what they noticed prior to sharing out to the class. Our goal is to have all students actively participating in the lesson.*
- As the number of acres change, do the number of Maple trees change? How do they change?
 - *Yes. For every one acre of land there are 15 maple trees. If I decrease the number of acres then I will have a fewer number of maple trees growing. If I increase the number of acres then I will have a greater number of maple trees growing.*
 - *Students need to have a sense of covariation. In other words they need to understand situations where a change in one quantity causes a corresponding change in another when analyzing proportional relationships. For example, students need to see that as one quantity is doubled the other quantity also doubles.*
- Connecting back to your 6th grade experience with ratios; how does the statement, “for every one acre of land there are 15 maple trees” relate to your understanding of the word rate?
 - *As students are discussing the answer to this question, be sure to come to a class consensus for a definition of the word rate and unit rate. **Rate is defined** in the progressions document as a unit that is derived from the units of two quantities such as m/s, which is derived from meters and seconds (Progressions Document, 13). A rate is also defined as a set of infinitely many equivalent ratios (Labato, 13). **Unit Rate is defined** as the numerical part of the rate; the unit is used to highlight the 1 in “per 1” unit of the second quantity (Progressions Document, 13).*

Teacher Note (Essential Question 1):

In this activity, students need to pay close attention to the relationship that exists between the given quantities. “Students will need to identify situations in which one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship” (Lobato, 12).

Guided Classroom Discussion:

Ask students to see if they notice any patterns when they look at the relationship in each column.

- One relationship students may identify is the multiplicative comparison. The multiplicative comparison is the value of the ratio $A:B$ telling how A and B compare multiplicatively; specifically, it tells how many times as big A is as B (Progressions document, 13). In this example the multiplicative comparison is 15. The relationship between acres and maple trees, which is the multiplicative relationship of $5 \times 15 = 75$, showing that number of maple trees is 15 times greater than the number of acres.
- A second relationship, that students may identify, is all of the ratios of acres to maple trees when written in fractional form will always be simplified to $\frac{1}{15}$. The relationship is called the composed unit. A composed unit is defined as a way to form a ratio by composing (joining) two quantities to create a new unit. (Lobato, 19). The composed unit showing the relationships between acres and maple trees are $\frac{5}{75}, \frac{10}{150}, \frac{1}{15}, \frac{15}{225}$. These composed units refer to thinking of the ratio as one unit versus the relationship between acres and maple trees. The simplified composed unit is $\frac{1}{15}$. (Essential Question 5)

Acres x	5	10	1	15
Maple Trees y	75	150	15	225

Composed Unit $\frac{5}{75} = \frac{1}{15}$ Simplified Composed Unit

$x \cdot 15$

Engage students in a whole class discussion unpacking each vocabulary terms multiplicative comparison and composed unit based on the discussion / example above. The definitions below are provided as a resource for the teacher. However, in the discussion, the teacher needs to discuss these understandings in conjunction with the students' understandings. (Essential Question 5)

- Composed Unit:** Another way to form a ratio is by composing (joining) two quantities to create a new unit. (Lobato, 19).
- Multiplicative Comparison:** The value of the ratio $A:B$ tells how A and B compare multiplicatively; specifically, it tells how many times as big A is as B . (Progressions document, 13) Another way to name the multiplicative comparison is to use the term scale factor, which is simply a number that scales, or multiplies some quantity.

Teacher Note:

The term scale factor is used when referring to the multiplicative comparison between lengths, areas and other measurements (Progressions Document, 11).

Give students the following ratio table and ask them to identify if the given situation describes a proportional relationship and to explain their reasoning.

- Derrick is an artist. He paints portraits. The table below shows the number of portraits Derrick painted compared to the time (in hours) it took him to paint.

Number of Portraits x	Time (In Hours) y
3	9
4	15
5	21
6	27

Standards for Mathematical Practice

- Students reason abstractly and quantitatively when they examine a ratio table to determine the composed unit or multiplicative comparison. [SMP 2]

- Is the number of portraits proportional to the time it took Derrick to paint the portraits? How do you know?
 - Students may think that this is a proportional relationship based on the fact that as the number of portraits increase by one the number of hours increases by 6 (additive reasoning). However for covariation to occur the change in one variable must be caused by the change in the other variable. Since a multiplicative comparison is not present in the relationship, the situation does not represent a proportional relationship. (i.e. each additional portrait created does not take the same amount of time to create).*
 - To justify that the proportional relationship does not exist, the student may model the multiplicative comparison and show they are not constant for each provided ratio or the students may show that the composed units for each ratio are not equivalent.*

Number of Portraits x	Time (In Hours) y
3	9
4	15
5	21
6	27

$$\frac{3}{9} = \frac{1}{3} \quad \text{Composed Unit}$$

$$\frac{1}{3} \neq \frac{5}{21}$$

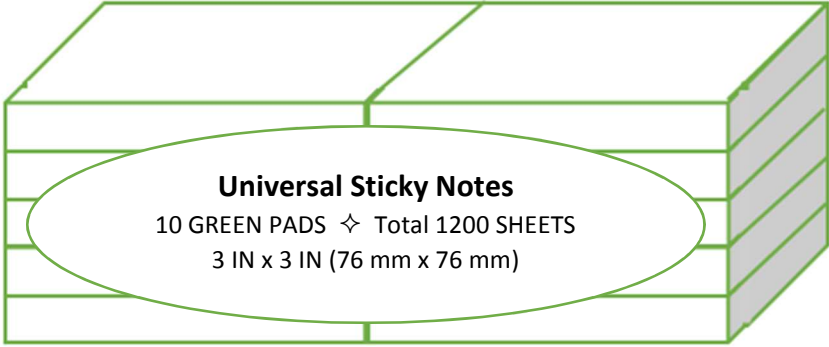
$$x \bullet 3$$

Multiplicative Comparison does not work with 5 \rightarrow 21

$$5 \times 3 \neq 21$$

Check for Understanding: (Independent)

Teacher Directions: In order for 7th grade students to fully meet the rigor of the standard, they need to work with rational numbers. Below are set of problems that include rational numbers. All students need to be able to complete questions 2 and 3 independently. Question 1 is provided as an entry point for students because it resembles scenarios discussed in class. This question should be completed in a small group with the teacher for those students who struggled through the lesson prior to the students attempting questions 2 and 3 independently.

Questions:	Solutions:										
<div>1. The table below shows the relationship between the cost (in dollars) to the number of movies rented. Determine if a proportional relationship exists? How do you know?</div> <table><tr><th># of Movies <i>x</i></th><th>Cost (dollars) <i>y</i></th></tr><tr><td>3</td><td>2.10</td></tr><tr><td>5</td><td>3.50</td></tr><tr><td>7</td><td>4.90</td></tr><tr><td>9</td><td>6.30</td></tr></table>	# of Movies <i>x</i>	Cost (dollars) <i>y</i>	3	2.10	5	3.50	7	4.90	9	6.30	<div>Yes, the composed unit is $\frac{10}{7}$ and the multiplicative comparison is 0.7.</div>
# of Movies <i>x</i>	Cost (dollars) <i>y</i>										
3	2.10										
5	3.50										
7	4.90										
9	6.30										
<div>2. The table below represents the relationship of the amount of total snowfall (in inches) to the amount of time (in hours) of a recent winter storm.</div> <table><tr><th>Time (h) <i>x</i></th><th>Snowfall (in.) <i>y</i></th></tr><tr><td>1</td><td>$1\frac{1}{3}$</td></tr><tr><td>2</td><td>$\frac{8}{3}$</td></tr><tr><td>3</td><td>4</td></tr><tr><td>4</td><td>$\frac{16}{3}$</td></tr></table> <div>Nicole describes the table using the following statements. Determine which of the statements are true or false and justify why.</div> <div>A. It takes $\frac{3}{4}$ of an hour to snow 1 inch.</div> <div>B. The multiplicative comparison is $\frac{3}{4}$</div> <div>C. In $5\frac{1}{2}$ hours it will snow $7\frac{1}{3}$ inches.</div>	Time (h) <i>x</i>	Snowfall (in.) <i>y</i>	1	$1\frac{1}{3}$	2	$\frac{8}{3}$	3	4	4	$\frac{16}{3}$	<div>A. True</div> <div>B. False the multiplicative comparison is $\frac{4}{3}$</div> <div>C. True</div>
Time (h) <i>x</i>	Snowfall (in.) <i>y</i>										
1	$1\frac{1}{3}$										
2	$\frac{8}{3}$										
3	4										
4	$\frac{16}{3}$										
<div>3. Joe buys the following pack of sticky notes. He needs 330 sheets for a class presentation. Joe measures the height of one pad of sticky notes and finds that 1 pad = 1 cm. How might Joe remove 330 sheets from the package without counting?</div> <div></div> <div>Figure Not Drawn to Scale</div>	<div>1 cm = 1 pad 1 cm = 120 sheets</div> <table><tr><td>1 cm</td><td>120 sheets</td></tr><tr><td>2</td><td>240</td></tr><tr><td>0.5</td><td>60</td></tr><tr><td>0.25</td><td>30</td></tr><tr><td>2.75</td><td>330</td></tr></table> <div>Joe can measure a height of 2.75 cm to remove 330 sheets without counting.</div>	1 cm	120 sheets	2	240	0.5	60	0.25	30	2.75	330
1 cm	120 sheets										
2	240										
0.5	60										
0.25	30										
2.75	330										

Part II: (Productive Group Work)

Arrange your students into groups of 4. Provide students with a whiteboard, marker and eraser. Display the following problem on the board.

BUYING COOKIES

Two dozen Chocolate Chip Cookies costs \$4.80

Nicole wants to calculate the cost for different amounts of candy for her upcoming party. If she buys ____ pounds of candy, it will cost her \$_____.

- Using what we have learned thus far in today's lesson, how might you suggest two reasonable quantities to put in the scenario provided to make this situation proportional?

While students are working in groups, hand out number cards 1 – 4 to each group member. Use the following protocol:

- Allow time for students to think through the problem independently.
- Allow time for group members to discuss the problem and collectively come up with a solution.
- Ask students to show their thinking and the strategy they chose to use to make sense of the problem.
- Use a spinner (1 – 4) and spin it to determine which group member gets to share out.
- Ask students to share their methods. These will vary. Some possible methods could be...

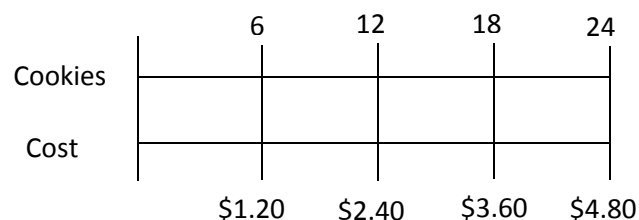
- Ratio Table Strategy:

$\div 2$

Cookies	24	12	6	18
Cost	\$4.80	\$2.40	\$1.20	\$3.60

Add

- Double Number Line Strategy:



Teacher Note (Essential Question 2):

On pages 25 – 26, it is important that students are able to clearly identify the connection between equivalent ratios, values in a table, and ordered pairs on a graph.

Now ask students for ideas on the properties of proportional relationships. Write the students' ideas on the board.

Then ask the following questions:

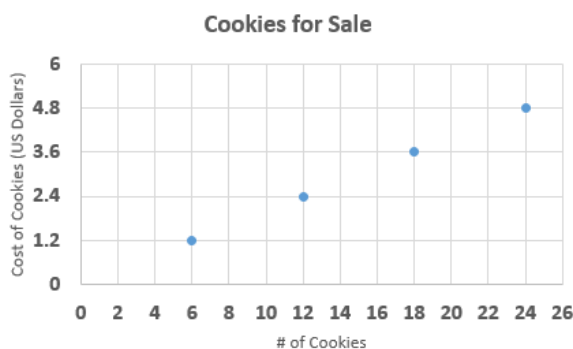
- How much would it cost to buy zero pounds of candy?
 - *Zero dollars*
- What happens to the total cost if you double the amount you buy?
 - *The cost would double*
- What happens to the total cost if you halve the amount you buy?
 - *The total cost would be cut in half as well*
- If you were to graph this relationship, what do you think the graph will look like?
 - *Don't provide the answer here. Let them explore this in their groups. (Answer: Graph goes through the origin is discussed at the bottom of page 26.)*

Now ask students to graph the relationship of "Buying Cookies" with their groups.

While students are working in groups, assign each group member a number between 1 and 4. To ensure accountability for all members of the group, spin a spinner to determine which member in the group shares out.

- Place a coordinate grid in a sheet protector. (Have students use their dry erase markers for graphing).
- Allow time for students to think through the problem independently.
- Allow time for group members to discuss the problem and collectively come up with an idea.
- Ask students to show their thinking by graphing the relationship as a group.
- Spin and have member x share out.

- Students' graphs should look similar to this:



Teacher Note:

For this problem you are working with discrete data points. This represents discrete data points because you typically do not buy part of a cookie. A continuous graph would be used when you can purchase parts of a unit. For students to determine whole number unknowns in a proportional relationship, they would need to connect the points in their graphs and notice that the points formed a straight line through the origin. They could then use this "connected line" to determine the cost of "x" cookies.

- Ask students to discuss in their groups if it would ever be ok to determine the cost of $\frac{1}{2}$ a cookies, or any other fractional piece of a cookie. Why or why not?

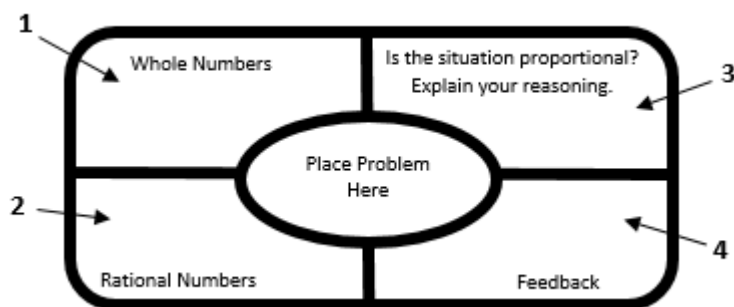
Now that students have graphed the relationships, ask the students what characteristics they notice with this proportional relationship if they were to connect their plotted ratios (Essential Question 3).

- Graphs a straight line
- Graph goes through the origin (0, 0)

Ask students, how does this information compare to when they determined in 6th grade whether or not two ratios were equivalent? It is important for students to move past determining whether two given ratios are equivalent to the greater understanding that every set of equivalent ratio pairs creates a proportional relationship modeled in the graph above (i.e. straight line going through the origin – Essential Question 4).

Part III: (Productive Group Work)

Round 1: Organize students in pairs or groups of three. Provide each group with poster paper for a “placemat activity.” Placemat problem cards (handout page 33) will need to be cut out and one card should be placed in the center of each placemat poster. The poster paper should be divided into sections like the model below:



Teacher Note:

The purpose of the activity is to have students place numbers in the blanks to productively struggle and look for and make use of structure in the given situations. Students should develop a conjecture as to whether they think the situation describes a proportional relationship based on the numbers they chose and their calculations.

Give each group one of the placemat cards and explain the following:

Each group has been given a different situation to examine. In the situation there are quantities represented with blanks instead of numbers. You are going to put numbers into these blanks and then answer the question.

Section 1: When first attempting the problem stick with numbers that are easy to work with, such as whole numbers.

Section 2: When attempting the problem the second time use rational numbers.

Section 3: Classify the situation on whether or not the two quantities vary proportionally. Write your answer and your reasoning.

Section 4: Feedback completed during rotations in round 2.

Standards for Mathematical Practice:

- Students make sense of problems and persevere in solving them by examining the cards and determining whether or not the situation describes a proportional relationship. [SMP 1]
- Students construct viable arguments when they provide feedback to their peers. [SMP 3]
- Students look for and make use of structure when they analyze the given situation to make conjectures on whether the relationship shows properties of proportionality. [SMP 7]

While students are working on round 1, determine the directions in which the students will rotate for round 2.

Once each group has had the opportunity to complete sections 1, 2 and 3, provide the following directions:

Round 2: Now we are going to provide each other feedback on our work completed in round 1. Each group is going to visit another groups' work and analyze the situation and the information provided. When analyzing each other's work:

- Carefully read each solution: Is there anything you don't understand? Do you notice any errors?
- Complete the problems in section 1 and 2 on your own.
- Compare your work to the groups work: Did you use the same strategy? Do you have the same answer?
- Do you agree that quantities represented in the situation are in proportion to one another?
- Write feedback in section 4. Your feedback should be based on the strategy used and the reasoning provided.

Teacher Note:

Depending on the time allotted for this activity, students can rotate to one group, all groups, etc. However, at minimum please build in time for students to provide feedback at least once and be able to visit all posters and read scenarios, answers and feedback.

Round 3: Return to your original poster. Read feedback provided and make edits and/or adjustments as needed.

Answer Key for Placemat Activity:

Placemat Problem:	Answer:
1	The quantities do not vary proportionally.
2	The quantities do vary proportionally.
3	The quantities do vary proportionally.
4	The quantities do vary proportionally.
5	The quantities do not vary proportionally.
6	The quantities do vary proportionally.

Part IV: Closure / Wrap-Up (Whole Class)

Have students identify properties of proportional relationships that have been identified throughout today's lesson. The teacher and students will work together to refine and create a master list with some guidelines to follow when determining if a proportional relationship exists. Once the master list has been finalized, the teacher should create a large poster of the list to display in the classroom for the duration of the unit. Be sure to include the following...

PROPERTIES OF PROPORTIONAL RELATIONSHIPS

- All composed units can be simplified down to a ratio that is equal to all of the other provided ratios.
- The scale factor is a constant multiplier of the independent term (x).
- If you double one quantity, the other quantity doubles.
- If the first quantity is zero, the second quantity has to be zero
- The graph of the relationship is a straight line through the origin

At this time it is important for students to be provided an opportunity to go back to the sorting activity from the beginning of this lesson to be able to apply what they have learned to the card sort to make necessary adjustments if needed.

- Hand back the graphic organizer from the "Before the Lesson" card sort.
- Hand out the cards for the activity.
- Have students re-examine their work and re-sort. Some students will need to edit their justifications as to whether the relationship is or is not proportional.

Answer Key for Card Sort:

Card:	Answer:
A	Yes
B	Yes
C	Yes
D	Yes
E	No
F	No

Part V: Exit Ticket (Independent)

Teacher Directions: All students need to be able to complete all three questions independently. Question 1 is provided as an entry point for students because it is similar to questions discussed in class. This question should be completed in a small group with the teacher for those students who struggled through the lesson prior to the students attempting questions 2 and 3 independently.

Questions:		Solutions:																
<p>1. The table above represents the relationship between the numbers of hours worked and the amount paid for three different jobs. Which job(s) pays their employees at a rate representing a proportional relationship? Justify your choice by explaining your reasoning.</p> <table><tr><th>Amount of Time Worked (hours)</th><th>Job #1 (dollars)</th><th>Job #2 (dollars)</th><th>Job #3 (dollars)</th></tr><tr><td>10.5</td><td>96.10</td><td>135.40</td><td>57.75</td></tr><tr><td>15.2</td><td>134.64</td><td>195.56</td><td>83.60</td></tr><tr><td>20</td><td>174.00</td><td>257.00</td><td>110.00</td></tr></table>		Amount of Time Worked (hours)	Job #1 (dollars)	Job #2 (dollars)	Job #3 (dollars)	10.5	96.10	135.40	57.75	15.2	134.64	195.56	83.60	20	174.00	257.00	110.00	Job #3.
Amount of Time Worked (hours)	Job #1 (dollars)	Job #2 (dollars)	Job #3 (dollars)															
10.5	96.10	135.40	57.75															
15.2	134.64	195.56	83.60															
20	174.00	257.00	110.00															
<p>2. A straight line passes through the points (0, 0) and (5, 1). It also passes through the point (2.5, 0.5). Determine if the values represented by the line represent a proportional relationship. Explain your reasoning.</p>		A proportional relationship does exist because you have a straight line that goes through the origin. Also if you double 2.5 and 0.5 you get 5 and 1.																
<p>3. My toaster has four slots for bread. It takes 2.5 minutes to make 4 slices of toast. Does a proportional relationship exist between the quantities of minutes and slices of toast? Explain your reasoning. Based on your reasoning, how long will it take to make 6 slices of toast?</p>		This situation does not represent a proportional relationship. Based on the information given it will take 5 minutes to make 5, 6, 7 or 8 slices of toast.																

Card A:

Johnny weighs 312 pounds on the planet Jupiter and weighs 120 pounds on earth.

Marco weighs 520 pounds on the planet Jupiter and weighs 200 pounds on earth.

Sally weighs 208 pounds on planet Jupiter and weighs 80 pounds on earth.

Proportional: Yes or No

Card B:

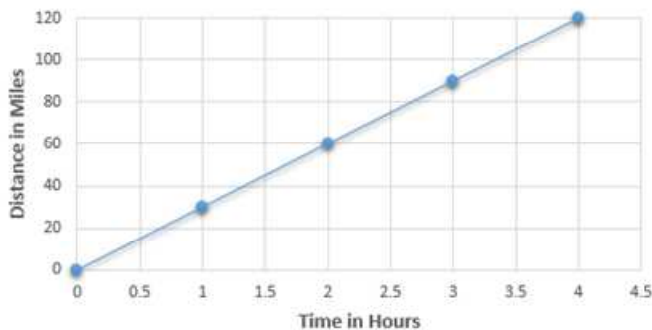
The table below shows the number of ounces of berries in a basket. Is the relationships between the number of ounces of blueberries and the number of ounces of strawberries proportional?

Number of Ounces of Blueberries	2	3	5	6	28
Number of Ounces of Strawberries	3	4.5	7.5	9	42

Proportional: Yes or No

Card C:

Car Traveling at a Constant Speed



Proportional: Yes or No

Card D:

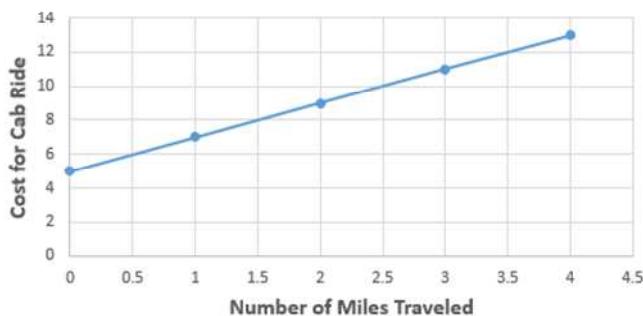
Dominique has a Cupcakery. The table below shows the number of cupcakes she has made and the amount of time it took her to make them. Does the table represent a proportional relationship?

Cupcakes	Time (In Minutes)
6	18
12	36
18	54

Proportional: Yes or No

Card E:

Cab Company



Proportional: Yes or No

Card F

Gibber Jabber Phone Company charges \$0.70 for every 14 minutes on the phone.

Transcontinental Connectors Phone Company will allow you to talk for 21 minutes for the low price of \$2.10.

S-Mobil Phone Company advertises \$3.15 for every 63 minutes on the phone.

Proportional: Yes or No

Name: _____

Date: _____

Proportional Reasoning Unit

Proportional: Yes or No

Card	Proportional Yes or No	Justification/Reasoning

Placemat Problem #1:

A cell phone provider provides _____ GB (gigabits) free each month. If I go over my allotted GB they charge \$_____ per GB. I used _____ GB last month.

How much was my bill for last month?

Placemat Problem #2:

A distance is _____ inches long on a map which is _____ miles in real life. A river is _____ inches on the map.

How long is the river in real life?

Placemat Problem #3:

To make _____ smoothie(s) you need _____ cups of orange juice, _____ banana(s) and _____ cups of strawberries. How many bananas, cups of strawberries and cups of orange juice do you need to make _____ smoothies?

Placemat Problem #4:

A straight line passes through the points (0, 0) and (____, ____). It also passes through the point (____, y).

Calculate the value of y.

Placemat Problem #5:

Bryce wants to go home in a taxi. He lives _____ miles away. The taxi company charges \$_____ plus \$_____ per mile.

How much will the fare be?

Placemat Problem #6

Juan buys _____ greeting card and pays \$_____ amount. Julia pays \$_____ to the store clerk.

How many cards did Julia purchase?

Exploring Proportional and Non-Proportional Relationships in the Real World

Grade Level Standards Addressed in this Lesson:

- **CCSS.Math.Content.7.RP.A.2**

Recognize and represent proportional relationships between quantities.

- **CCSS.Math.Content.7.RP.A.2a**

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- **CCSS.Math.Content.7.RP.A.2d**

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Across-Grade Coherence: Content Knowledge from Earlier Grades

- **CCSS.Math.Content.6.RP.A.3**

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- **CCSS.Math.Content.6.RP.A.3a**

Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

- **CCSS.Math.Content.6.RP.A.3b**

Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Across-Grade Coherence: Looking Ahead for Content Connections

- **CCSS.Math.Content.8.FB.B.4**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- **CCSS.Math.Content.8.EE.B.5**

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

- **CCSS.Math.Content.8.EE.B.6**

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Mathematical Goals:

This lesson is intended to help you assess whether students know and understand:

- How to apply equivalent ratio understandings to proportionality in the context of a real world problem.
- The pre-requisite skills necessary to begin the development in the understanding of when a linear equation represents a statement of proportionality $y = mx$ and when a linear equation represents a statement of proportionality combined with a vertical translation represented by the addition of b , $y = mx + b$.

Rigor:

This lesson requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

Application: Students continue to build their understanding of proportionality in the context of real world situations.

Conceptual understanding: Students develop their understanding of what it means to be proportional when graphing equivalent ratios from a given real world context.

Procedural Skill and Fluency: This lesson builds on the student's prior understanding of graphing sets of equivalent ratios from a table to determine whether a real world problem is or is not describing a proportional relationship.

Overview of Lesson:

In the prior lesson, students gained the necessary understandings to be able to determine whether or not two quantities were proportional. In this lesson students are provided an opportunity to solidify their understandings of proportionality by comparing two similar real life scenarios to be able to determine whether the provided situation describes a proportional relationship. Students use their prior knowledge and understandings of proportionality to examine linear situations. Through their examination they develop a deep conceptual understanding around the importance of a linear graph originating at the origin (0, 0).

Estimate Time: 90 minutes

Essential Questions:

1. How, in real world situations, does one determine if a given situation is or is not describing a proportional relationship?
2. What is a rate (speed) and how is it related to proportional reasoning?
3. What is the significance of the origin and proportionality?

Common Misconceptions:

- When students find equivalent distances and elapsed times, they might not take into account the different starting points.
- When trying to determine equivalent ratios, students might not understand the importance of the origin and miscalculate the speed for situation 2.

Universal Design for Learning:

- **[Engagement]** The lesson optimizes relevance, value, and authenticity by providing problems in the context of a real world situation.
- **[Action & Expression]** The lesson builds fluencies with graduated levels of support for practice and performance through the facilitation of questioning.
- **[Representation]** The lesson activates and supplies background knowledge through the connection of equivalent ratios from grade 6.
- **[Representation]** The lesson highlights patterns, critical features, big ideas, and relationships in the domain of proportional relationships.

Standards for Mathematical Practice:

- **[SMP #1]** Students are making sense of problems and persevering in solving them when provided a real world problem with horse speeds.
- **[SMP #3]** Students are constructing viable arguments when determining if a proportional relationship exists and justifying their response.
- **[SMP #4]** Students are modeling with mathematics as they translate the real world situation in a mathematical representation using both tables and graphs.
- **[SMP #6]** Students are attending to precision when they are communicating the importance of having a linear graph go through the origin.
- **[SMP #7]** Students are looking for making use of structure as they decontextualize the real world situations provided.

Resources:

- Racing Around Task (page 45)
- Graph paper
- Straight edge
- Poster paper

Opportunities for Assessment:

- Formative – Embedded Questions
- Formative – Racing Around Task (page 42)

Additional Assessment Tasks in the Public Domain:

- Illustrative Mathematics: Gym Membership Plans
<https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1983>
- Illustrative Mathematics: Robot Races, Assessment Variation
<https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1178>

Lesson Sequence:

Teacher says to the class, “Yesterday, we examined the properties of proportional relationships. Today, we will be given two different real world scenarios and asked to examine the key characteristics of the situations, to be able to graph them and determine whether the relationship that exists is proportional.”

Warning-Up: (Independent)

Today we are going to be examining real world situations that involve rate, ratio, speed, unit rate. As we warm our brains up, we are going to participate in a “quick write.” Take the next 3 – 4 minutes and write down and capture everything you already know about the four terms (rate, ratio, speed, and unit rate).

Guided Classroom Discussion:

Engage students in a whole class discussion unpacking each vocabulary term. The definitions below are provided as a resource for the teacher. However, in the discussion, the teacher needs to discuss these understandings in conjunction with the students’ understandings that they have written down during their quick write activity.

- **Ratio:** A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit (Lobato, 12).
- **Rate:** A rate is expressed in terms of a unit that is derived from the units of the two quantities such as m/s, which is derived from meters and seconds (Progressions Document, 13). A rate is also defined as a set of infinitely many equivalent ratios (Lobato, 13).
- **Unit Rate:** The term unit rate is the numerical part of the rate; the unit is used to highlight the 1 in “per 1” unit of the second quantity (Progressions Document, 13). Unit rates are also known as the constant of proportionality.
- **Speed:** The rate or a measure of the rate of motion: distance traveled divided by the time of travel.

Wrap up the discussion with the following question:

- How is speed related to proportional reasoning?
 - *If an infinite number of equivalent ratios will yield a rate. And we just defined speed as a unique rate representing quantities of distance and time. Then we can say that a proportional relationship exists if we have at least two equivalent rates (Essential Question 2).*

Exploration One: (Independent)

Pose the following situation on the board. Students should work individually for at least 3 to 4 minutes and have at least 4 other ratios to share before they begin working with a partner or small group.

You decide that you are going to go horseback riding. The horse you want to ride is in the barn. You go out and get on the horse and begin your ride. Traveling in a straight line from the barn, after 5 minutes, the horse is 440 yards. Generate several distances from the barn and elapsed time values for other parts of your ride so that the horse is traveling at the same speed throughout your journey.

Teacher Note:

Students may generate equivalent ratios based on one of the strategies from the previous lessons.

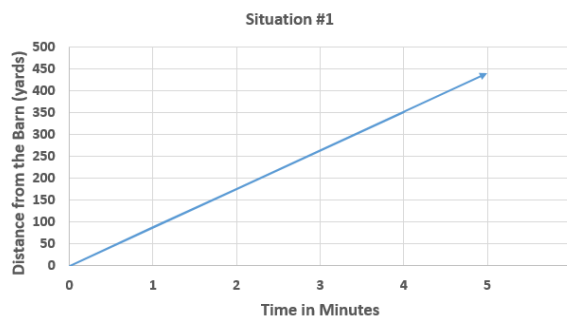
Continuation of Exploration One: (Productive Group Work)

With a partner or small group, students should share the equivalent ratios they have found and the strategy they used.

Teacher says, “Today we are going to focus on making tables and graphing.” If students created a table, have them plot values in the ratios they created. If students created a graph, have them create a table of ratios from their graph. If students used an alternative strategy, have them first create a table of equivalent ratios, and then plot those values.

Sample Table and Graph:

Time (minutes)	Distance from the Barn (Yards)
5	440
0	0
1	88
2	176
4	352



Standards for Mathematical Practice

- Students are making sense of problems and persevere in solving them when provided a real world problem with horse speeds. [SMP 1]
- Students Model with Mathematics as they translate the real world situation in a mathematical representation using both tables and graphs. [SMP 4]
- Students are looking for making use of structure as they decontextualize the real world situations provided. [SMP 7]

After students have constructed their equivalent ratios, created their tables and graphs; pose the following questions.

- By examining your generated ratios, can we determine if a proportional relationship exists? Justify your reasoning.
 - *Yes, a proportional relationship does exist. The multiplicative comparison is 88.*
- Should the ratio 0 to 0 be included in your table? Discuss your reasoning with your partner/group.
 - *At this time, do not provide students with the answer. The goal is for them to discover this understanding throughout this lesson. (The answer is provided to students on page 41).*
- What does the ratio of 0 to 0 represent in this situation? Discuss your reasoning with your partner/group.
 - *At this time, do not provide students with the answer. The goal is for them to discover this understanding throughout this lesson. (The answer is provided to students on page 41).*

Exploration Two: (Productive Group Work)

Pose the following situation on the board.

Situation 2:

You decide that you are going to go horseback riding. The horse you want to ride is currently 25 yards from the barn. You go out get on the horse and begin your ride. Traveling in a straight line from the barn, after 5 minutes, the horse is 465 yards. Generate several distances from the barn and elapsed time values for other parts of your ride so that the horse is traveling at the same speed throughout your journey.

Students should work individually for at least 3 to 4 minutes prior to working with a partner or small group. Small group or partner work should take about 10 minutes. Ask students the following questions:

- How is this situation similar or different then Situation 1?
 - *Situation 1 – the horse starts right at the barn. In Situation 2 – the horse starts 25 yards away from the barn.*
- How do you think this difference will impact the problem?
 - *It will impact the way in which I determine other distances and elapsed time traveling at the same speed.*
- How fast is the horse traveling per min in situation one compared to situation two?
 - *The horse is traveling at the same speed for each situation.*

Scaffolding questions to help guide students' thinking...

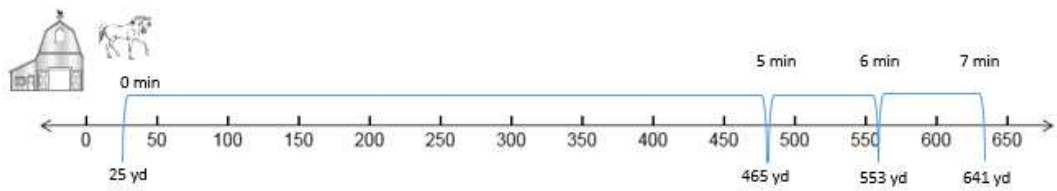
- In situation 2, how far from the barn is the horse after 5 min?
 - *465 yards – given in the problem.*
- In situation 2, how far did the horse travel after 5 minutes?
 - *440 yards. (465yds. – 25yds. = 440 yds.)*
- In situation 2, how fast is the horse traveling?
 - *88 yards per minute (440 yds. ÷ 5 min = 88 yds. per 1 min.)*

Teacher Note:

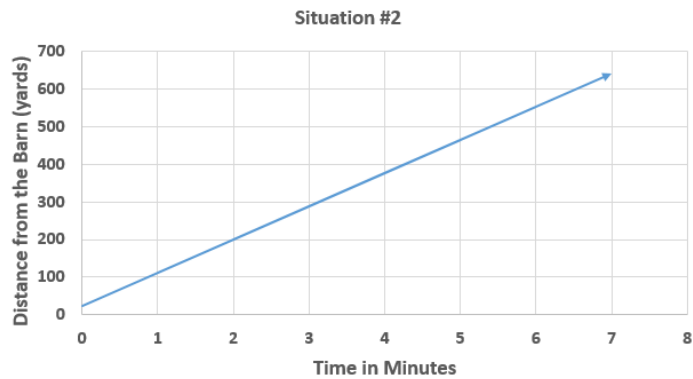
One way that students might approach this problem is to consider the distance the horse traveled in the first 5 minutes. This distance is the difference between the distance the horse was from the barn after 5 minutes and the horse’s initial distance. $465 \text{ yards} - 25 \text{ yards} = 440 \text{ yards}$. To determine the horse’s speed we can look at the composed unit of $440:5$. Students can find the distance traveled in 1 minute by partitioning $440:5$ into 5 equal parts. Splitting 440 into 5 equal parts give us 88 yards and splitting 5 into 5 equal parts gives us 1 minute. The horse traveled 88 yards in 1 minute. Students can use this ratio to determine how far the horse is from the barn over time. After 6 minutes, the horse is traveling an additional 88 yards from the barn. The horse’s total distance is $465+88=553 \text{ yards}$ from the barn.

See figure 1

Figure 1:



Have students create a table and a graph to represent situation 2.



Time (minutes)	Distance from the barn (yards)
0	25
5	465
1	113
6	553
7	641
2	201
4	377

Once students have completed their tables and graphs for situation 2, have them compare the graph from situation 1 to situation 2. Then pose the following questions.

- How does the graph of situation 2 compare with the graph of situation 1?
 - Both graphs form a straight line; however, situation 1 goes through the origin and situation 2 does not.
- What is the significance of having a linear graph go through the origin?
 - Allow time for students to think about this question and discuss in groups; however, do not provide them with the answer at this time. (The answer is provided to students on page 41).

In groups have students discuss the following:

Does the relationship presented in situation 1 represent a proportional relationship? Why or why not be ready to justify your answer.

Does the relationship presented in situation 2 represent a proportional relationship? Why or why not be ready to justify your answer.

Standards for Mathematical Practice

- Students are constructing viable arguments when determining if a proportional relationship exists and justifying their response. [SMP 3]
- Students need to attend to precision when they are communicating the importance of having a linear graph go through the origin. [SMP 6]

Solution: Graphically: Situation 1 does represent a proportional relationship. The graph of Situation 1 shows a line that goes through the origin. Situation 2 does not represent a proportional relationship because the line does not pass through the origin (Essential Question 3).

Teacher Note (Essential Question 3):

In lesson 2 you and your class created a poster with properties of proportional relationships. At this time, if students haven't made the connection, direct them back to the poster.

Whole Class Discussion:

We have just determined which situation was proportional through the strategy of graphing. In the previous lesson we learned about the multiplicative comparison and composed units. Discuss in your groups how these two situations can be proved to be proportional or not through the use of those strategies (Essential Question 1). As the teacher walks around, he/she should hear conversations consisting of the following...

Multiplicative Comparison:

- *Situation 1 does represent a proportional relationship because you can show that any two ordered pairs on the line are related by some factor a/b . For example the points (2, 176) and (4, 352) are related by a factor of 2.*
- *Situation 2 does not represent a proportional relationship because no two ordered pairs on the line are related by some factor a/b . For example (2, 201) and (4, 377), for the x-coordinate: if I multiply 2 by 2 I do get 4, however for the y-coordinate if I multiply 201 by 2 I get 402 which is not 377.*

Composed Unit:

- *Situation 1 does represent a proportional relationship because every point on the line represents the horses speed by simply forming a ratio of the y-value to its corresponding x-value. (5, 440) is the ratio 440:5 representing the composed unit of 88 yards per minute or $\frac{88}{1}$. (4, 352) is the ratio 352:4 representing the composed unit of 88 yards per minute or $\frac{88}{1}$. This composed unit holds true for all quantities represented in this situation.*
- *Situation 2 does **not** represent a proportional relationship. (5, 465) is the ratio of 465:5 which represents a composed unit of 93 yards per 1 minute or $\frac{93}{1}$. (4, 377) is the ratio of 377:4 which represents a composed unit of 94.25 yards per 1 minute or $\frac{94.25}{1}$. These two composed units are not the same. To determine the rate or speed of the horse in situation 2, you need two points because a proportional relationship does not exist.*

Give each student a copy of the task (handout page 45) and share the following:

- Name: _____ Date: _____
- Proportional Reasoning Unit
- Racing Around
- Nicole and Jackie are running in a race. The map (drawn to scale) below shows the route of the race.
- It takes Nicole 8 minutes to run a mile.
- It takes Jackie 10 minutes to run a mile.
-
1. Mark on the map where Nicole "N" and Jackie "J" will be one hour into the race. Explain how you know.
2. Will one runner pass the other runner at any time during the race?
- If so, how many times will this occur during the race and where? Label the map with an X where this occurs and explain your reasoning.
 - If not, explain how you know.

In this task there are two people running in a race – Nicole and Jackie. Read the questions carefully and answer them as fully as you can.

- Try to describe what the question is asking.
- How far is a lap in miles?
- How long does it take Nicole/Jackie to complete a lap?
- Suppose Nicole and Jackie raced just one block. Who would finish first?
- How many laps will Nicole/Jackie have run after 1 hour? 2 hours?
- How far apart are Nicole and Jackie when they have completed one lap? Two laps?
- You have calculated different distances and times. Do you notice anything about your work? How might this information help you explain the problem?
- How can you organize your work to make things easier to understand?
- Are there any patterns in your calculations?
- If you were organizing the race, describe how you could change the problem so that the runners would have a close finish?

While students are completing the task independently, note different student approaches to the task as scaffolded guidance may be necessary. By carefully examining student work, you will get a better idea of students' range of understanding and be in a better position to ask questions to help them progress. Notice strategies students use to solve the problem.

Partner Work:

Once students have had an opportunity to complete the racing around task, assign each student a partner identifying who is Partner A and who is Partner B.

Now you are going to work with a partner on the task to produce a joint solution that is better than your individual work. Before you make another attempt at the task you will each need to share your written work and verbally share your thinking. We are going to use the following protocol:

- Step 1: Partner A shares their written response and their thinking (4 minutes), while partner B listens without interruption.
- Step 2: Partner B asks questions to clarify their understanding (4 minutes), while partner A answers succinctly.
- Repeat the process switching roles.

Hand out to each pair a sheet of poster paper.

Now you are going to work collaboratively to produce a joint solution.

Emphasize to students that they must justify their decision if they improve one of their individual strategies, combine strategies, or use a completely different strategy in their joint solution. It is important that the joint solution builds on the thinking and knowledge they gained from working on the task individually and should not merely be a copy of their original work. The final product should be a joint effort and an improved version of the pair's individual solutions.

Note different approaches: Notice if the student's strategy is the same or different from their original response. If different, how did they decide? What are the student's reasoning for the approach they took? Are students aware of any assumptions they made? How do the students organize their work? Do students notice any patterns in their calculations? Are students concerned if their answers make sense? Questions to consider posing to students:

- How does your approach help you to explain your answer to question two?
- Can you justify your solution?
- How might you explain your solution without referring to the diagram?

Support student problem solving: If you notice that students are struggling to develop a joint solution, try not to make suggestions that move them towards a particular approach. Instead ask questions to help students clarify their thinking and encourage them to identify strengths and weaknesses of the individual solutions. Questions to consider posing to students:

- What have you done that you both agree on?
- What else do you need to find out?
- What do you know now that you did not know when working independently?
- Can you think of any other approaches to try?

Teacher Note:

While students are working on their joint solution you should note different student approaches and support student problem solving.

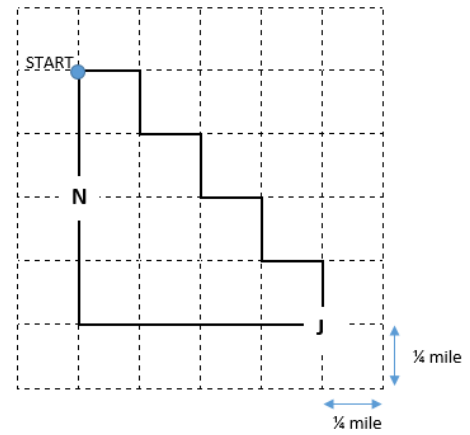
Teacher Note:

If a pair of students have completed the task, encourage them to extend the task to think about ways they could adapt the race to ensure a close finish for the two runners.

Solution:

- Question 1:

Nicole	1 mile	5 miles	2.5 miles	7.5 miles
	8 minutes	40 minutes	20 minutes	60 minutes
Jackie	1 mile	2 miles	3 miles	6 miles
	10 minutes	20 minutes	30 minutes	60 minutes

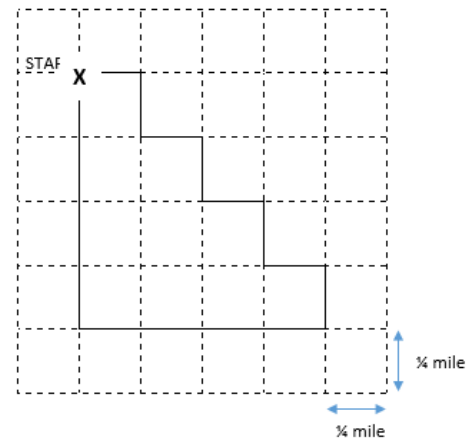


- Question 2:

- One possible solution is provided below; however, note that there are many solution pathways for students to correctly solve this task. Nicole will pass Jackie at the start of Jackie's 4th lap and Nicole's 5th lap. Since the race only consists of 6 laps there is no way that Nicole will pass Jackie a second time.

Table Method:

Time (minutes)	Distance (miles) Nicole	Distance (miles) Jackie	Difference in distance (miles)
0	0	0	0
10	1.25	1	0.25
20	2.5	2	0.5
40	5	4	1
160	20	16	4 (one full lap around the track)



Gallery Walk:

Have the students hang their joint solution in a designated location in the room. Give each student a set of sticky notes.

We are now going to complete a gallery walk to examine the different approaches that were taken in completing the task. As you walk around, write questions, comments and feedback on the sticky notes and stick them to your classmates' posters. Your final stop in the Gallery Walk will be your poster to review the sticky notes your classmates provided.

Closure:

Review the feedback and share understandings and learnings from the activity in a whole class discussion.

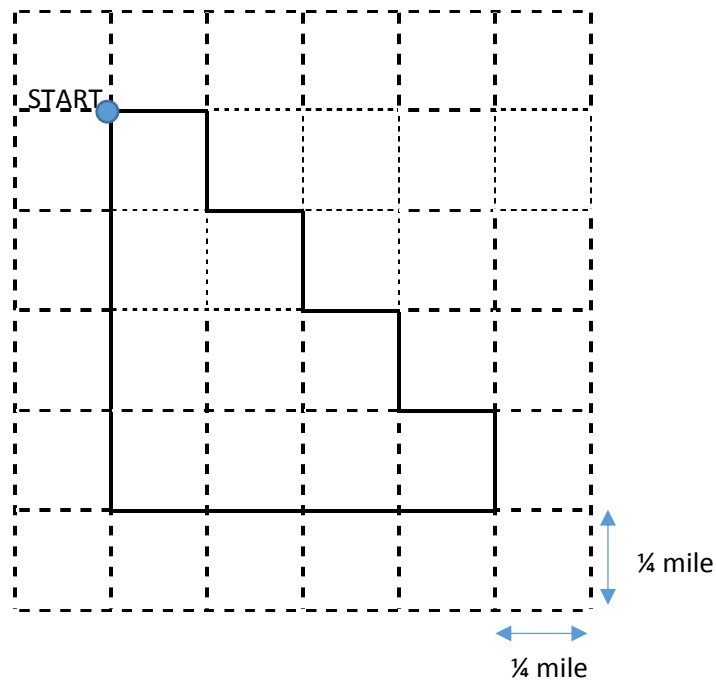
Proportional Reasoning Unit

Racing Around

Nicole and Jackie are running in a race. The map (drawn to scale) below shows the route of the race.

It takes Nicole 8 minutes to run a mile.

It takes Jackie 10 minutes to run a mile.



Nicole and Jackie run at a constant speed clockwise along the route. To finish the race, Nicole and Jackie must run six laps.

1. Mark on the map where Nicole "N" and Jackie "J" will be one hour into the race. Explain how you know.

2. Will one runner pass the other runner at any time during the race?
 - a. If so, how many times will this occur during the race and where? Label the map with an X where this occurs and explain your reasoning.
 - b. If not, explain how you know.

Equations and Proportional Reasoning

Grade Level Standards Addressed in this Lesson:

- **CCSS.Math.Content.7.RP.A.2c**

Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

- **CCSS.Math.Content.7.RP.A.2d**

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Within-Grade Coherence: Content from other Standards in the Same Grade that Provide Reinforcement

- **CCSS.Math.Content.7.EE.B.4**

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- **CCSS.Math.Content.7.EE.B.4a**

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Across-Grade Coherence: Looking Ahead for Content Connections

- **CCSS.Math.Content.8.FB.B.4**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- **CCSS.Math.Content.8.EE.B.5**

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

- **CCSS.Math.Content.8.EE.B.6**

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Mathematical Goals:

This lesson is intended to help you assess whether students know and understand:

- The pre-requisite skills necessary to begin the development in the understanding of when a linear equation represents a statement of proportionality $y = mx$ and when a linear equation represents a statement of proportionality combined with a vertical translation represented by the addition of b , $y = mx + b$.
- How to write an equation representing a proportional relationship.

Rigor:

This lesson requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

Application: Students are applying their prior knowledge of equations and proportional reasoning to identify the pattern that exists between the equation and the table of values.

Conceptual understanding: Students develop their understanding of the relationship between the point $(1, r)$ and the constant of proportionality when a table of values represents a proportional relationship.

Procedural Skill and Fluency: At this point, students should be fluent in plotting values of a set of equivalent ratios in a coordinate plane.

<p>Overview of Lesson: In this lesson students create a table of values based on a given equation and then examine the table to determine if a proportional relationship exists. Students identify that the constant of proportionality (unit rate) is represented in a table of values by the point $(1, r)$ and they use this information when writing an equation that describes the proportional relationship using the form $y = mx$. [In order for students to be successful with the material in this lesson, they must have had prior experience with the concepts covered in lessons one, two and three; such as graphing, creating ratio tables, constant of proportionality, multiplicative comparison, etc.]</p> <p>Estimate Time: 50 minutes</p>	
<p>Essential Questions:</p> <ol style="list-style-type: none"> 1. How is the equation $y = mx$ related to a set of equivalent ratios? 2. How can the constant of proportionality be represented? 3. How do you use the constant of proportionality to write an equation representing a proportional relationship? 	<p>Common Misconceptions:</p> <ul style="list-style-type: none"> • Students often have the misconception that the unit rate/ the point $(1, r)$ can be used to write the equation even if a proportional relationship does not exist.
<p>Universal Design for Learning:</p> <ul style="list-style-type: none"> • [Engagement] The lesson fosters collaboration and community when students are working with a small group to create their tables and determine patterns that exist between the table of values and the equation. • [Action & Expression] The lesson builds fluencies with graduated levels of support for practice and performance through building on student's prior knowledge and understanding of proportional relationships. • [Representation] The lesson clarifies vocabulary and symbols when discussing the equation $y = mx$ and solidifies the definition of constant of proportionality. 	<p>Standards for Mathematical Practice:</p> <ul style="list-style-type: none"> • [SMP #3] Students construct viable arguments when they justify how they sorted the cards. • [SMP #7] Students look for and make use of structure when they analyze the tables to make conjectures about the relationship between the equation and the table of values. • [SMP #8] Students look for and express regularity in repeated reasoning when they apply their understanding of proportional relationships to create an equation in the form $y = mx$.
<p>Resources:</p> <ul style="list-style-type: none"> • Activity One Cards (page 51) cut out and have ready prior to the lesson. • Investigating Tables Handout (page 52) • Poster 1 – Proportional Relationships (page 53) • Poster 2 – Not A Proportional Relationships (page 54) 	<p>Opportunities for Assessment:</p> <ul style="list-style-type: none"> • Formative – Exit Ticket (page 50) <p>Additional Assessment Tasks in the Public Domain:</p> <ul style="list-style-type: none"> • Illustrative Mathematics: Buying Coffee https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/104

Lesson Sequence:

Prior to the lesson, cut out “Activity One Cards” (handout page 51).

Teachers says to the class, “Today we are going to tap into your prior knowledge about creating a table of values given a linear equation. Once a table of values has been created, we are going to determine whether or not a proportional relationship exists using the skills and strategies we have been using throughout this unit.”

Students should be placed in groups of 3 to 4. Each group will need a card from Set A and a card from Set B.

Directions for Activity One: (Productive Group Work)

Hang up on opposite sides of the room Posters 1 and 2 (handout pages 53 – 54). Each Student should be given a copy of **Investigating Tables** Handout (handout page 52).

Have each group work as a team to do the following:

- Using what you know about equations, create a table of values for each given equation.
- Using what you know about ratios and proportional relationships, describe the table of values.

As the teacher walks around listening to the students work through the two problems, the teacher should be listening for the following:

- *The equation from Set A creates a proportional relationship. When we plot the values in the table they form a straight line that goes through the origin.*
- *The equation from Set B does not create a proportional relationship. When we plot the values in the table they form a straight line, but the line does not go through the origin.*

Have students cut their graphic organizer in half on the bold line. Then have students place the entire table next to the poster that says either Proportional Relationship or Not a Proportional Relationship.

Once all tables from the handout have been cut and posted, divide your class into two groups. One group stands next to the poster **Proportional Relationship** and the other group stands next to the poster **Not a Proportional Relationship**. Ask students to discuss in their groups what they notice about the equations around each poster. Have students switch groups and continue the discussion.

Teacher Note:

Students should be able to note that a proportional relationship exists if the equation is in the form $y = mx$. A proportional relationship does not exist when you have an equation in the form $y = mx + b$. However, they may not use precise language of $y = mx$ or $y = mx + b$ but students should be able to notice that set B equations all have a constant added.

Directions for Activity Two: (Productive Group Work)

Have students examine the equations that were placed under **Proportional Relationship**. Ask the students to identify a pattern that exists between the table of values and the equation. Once students have come up with a prediction, ask them to test their conjecture with a different table of values.

Have each group share their predications.

Pose the following questions...

- Take one of the tables that are in the category **Proportional Relationship**. Calculate the multiplicative comparison. What do you notice about the multiplicative comparison and the number in your equation?
- Calculate the value of y when $x = 1$. What do you notice about your answer and the equation?

Scaffolding:

If the students are struggling, first suggest that the students expand their table to determine additional values for x and y . Second suggest that the students find the value of y when x is equal to one. Students should be able to tell you that when $x = 1$ the y value is the same as the m value in $y = mx$. (Essential Question 2)

Teacher Note:

To sum up the learning from this activity. Be sure to highlight to the students that when $x = 1$, the y value is your multiplicative comparison which is your unit rate, which is also your constant of proportionality. (Essential Question 3)

Directions for Activity Three: (Whole Class)

Have students now examine the tables under the poster **Not a Proportional Relationship** to see if the same relationship exist for the tables under **Proportional Relationship**. Students should be able to tell you that this pattern does not exist when a proportional relationship does not exist.

Pose the following question:

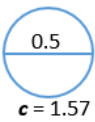
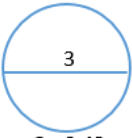
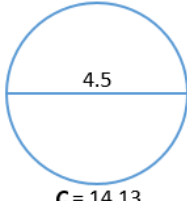
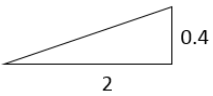
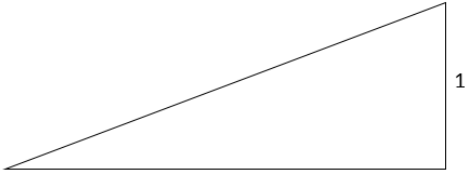
- Using what we just discovered, how can we determine the equation that represents the relationship that exists for a set of equivalent ratios?
 - *Students should be able to tell you that if you can find the value of y when $x = 1$ or the multiplicative comparison or the unit rate, you can write the equation in the form $y = mx$ because m is all of these.* (Essential Question 1)
- At this time students should be introduced to the term constant of proportionality. The constant of proportionality is the common factor multiplied by x to produce y which we refer to as the multiplicative comparison. The constant of proportionality refers to m in the equation $y = mx$. As a class, revisit the poster from lesson two in this unit and add the term “constant of proportionality” to the second bullet, connecting the term to scale factor and multiplicative comparison.

Standards for Mathematical Practice

- Students construct viable arguments when they justify how they sorted the cards. [SMP 3]
- Students look for and make use of structure when they analyze the tables to make conjectures about the relationship between the equation and the table of values. [SMP 7]
- Students look for and express regularity in repeated reasoning when they apply their understanding of proportional relationships to create an equation in the form $y = mx$. [SMP 8]

Practice: (Independent)

Teacher Directions: All students need to be able to complete questions 1 and 2. Question 1 is provided as an entry point for students because it resembles scenarios discussed in class. This question should be completed in a small group with the teacher for those students who struggled through the lesson prior to the students attempting questions 2 independently. Questions 2 and 3 applies students' understandings from the lesson to properties of geometric shapes. We have not formally worked with similar figures using the term scale factor, however highly able students may be able to productively struggle with question 3 applying understandings from this unit.

Questions:	Solutions:										
<p>1. Use the table below to determine if a proportional relationship exists. Write the equation that represents the relationship.</p> <table border="1"> <thead> <tr> <th>X</th><th>Y</th></tr> </thead> <tbody> <tr> <td>2</td><td>0.8</td></tr> <tr> <td>3</td><td>1.2</td></tr> <tr> <td>5</td><td>2</td></tr> <tr> <td>10</td><td>4</td></tr> </tbody> </table>	X	Y	2	0.8	3	1.2	5	2	10	4	<p>A proportional relationship does exist.</p> <p>If the table is expanded: $(1, 0.4)$ or $(1, \frac{2}{5})$.</p> <p>The equation is $y = \frac{2}{5}x$</p>
X	Y										
2	0.8										
3	1.2										
5	2										
10	4										
<p>2. Describe the relationship that exists between the variables c and d. Where c represents the circumference of the circle and d represents the diameter.</p> <p>Figures Not Drawn to Scale</p> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p>$c = 1.57$</p> </div> <div style="text-align: center;">  <p>$c = 9.42$</p> </div> <div style="text-align: center;">  <p>$c = 14.13$</p> </div> </div>	<p>The constant of proportionality between the diameter and circumference is 3.14 units.</p> <p>Because π is an irrational number, we used 3.14 as an approximation.</p>										
<p>3. Determine the scale factor between a side length of the small triangle to a side length of the larger triangle and write the equation that describes the relationship.</p> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p>2 0.4</p> </div> <div style="text-align: center;">  <p>5 1</p> </div> </div> <p>Figures Not Drawn to Scale</p>	<p>Scale factor is 2.5 $y = 2.5x$</p>										

Closure: (Whole Class Discussion)

From this lesson we have now learned how the connections between the graphs, equations, tables, and vocabulary associated with proportional reasoning are tools to help us make sense of real world situations.

Exit Ticket:

Which equations show a proportional relationship between **m** and **n**? Select **all** equations that show a proportional relationship and justify your work.

(a) $\frac{m}{9} = 8n$

(c) $5m = n + 4$

(e) $\frac{1}{4}m = 20 - n$

(b) $m = 12$

(d) $m = 3n$

Solution:

A, D

Set A	Set B
$y = 2x$	$y = 2x + 3$
$y = \frac{1}{2}x$	$y = \frac{1}{2}x + 1$
$y = x$	$y = x + 3$
$y = \frac{2}{3}x$	$y = \frac{2}{3}x + 1$
$y = 5x$	$y = 5x + 2$
$y = \frac{1}{3}x$	$y = \frac{1}{3}x + 3$
$y = \frac{3}{2}x$	$y = \frac{3}{2}x + 4$
$y = \frac{5}{2}x$	$y = \frac{5}{2}x + 2$

Table Investigation Handout:

Equation:		
x	y	Describe the table using what you know about ratios and proportional relationships.

Equation:		
x	y	Describe the table using what you know about ratios and proportional relationships.

Proportional Relationship

Not a Proportional Relationship

Connecting Scale Factor, Unit Rates, & Constant of Proportionality

Grade Level Standards Addressed in this Lesson:

- **CCSS.Math.content.7.RPA.2a**
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- **CCSS.Math.Content.7.RP.A.2b**
Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **CCSS.Math.Content.7.RP.A.2c**
Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.
- **CCSS.Math.Content.7.RP.A.2d**
Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Across-Grade Coherence: Content Knowledge from Earlier Grades

- **CCSS.Math.Content.6.RP.A.2**
Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."

Across-Grade Coherence: Looking Ahead for Content Connections

- **CCSS.Math.Content.8.FB.B.4**
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Goals:

This lesson is intended to help you assess whether students know and understand:

- The relationship between scale factor, constant of proportionality and unit rates.
- What the point on a graph of a proportional relationship represents.
- How to determine the constant of proportionality through multiple means, ie. graphs, tables, etc.
- The importance of the points $(0, 0)$ and $(1, r)$ in relation to a proportional relationship.

Rigor:

This lesson requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

Application: Students are decontextualizing real world situations that represent proportional relationships.

Conceptual understanding: Students develop a deeper understanding of unit rate and what it means in the context of the given problem.

Procedural skill and Fluency: Students become fluent in the calculations of unit rates, constants of proportionality, multiplicative comparisons, scale factors and composed units.

<p>Overview of Lesson:</p> <p>Building on the information presented in the prior lessons, this lessons helps students to construct that deep conceptual understanding of what each point (values represented by a ratio) means in the context of the problem. Students identify the relationship of the key vocabulary of proportionality; unit rate, scale factor, constant of proportionality, multiplicative comparison, and scale factors. Students also connect their prior understandings to the importance of the point $(0, 0)$ and $(1, r)$ by applying those points to the given situations.</p> <p>Estimate Time: 60 minutes</p>	
<p>Essential Questions:</p> <ol style="list-style-type: none"> 1. What characteristics define the graphs of all proportional relationships? 2. What is the relationship between the scale factor, unit rate and constant of proportionality? 3. What is the significance of the point $(1, r)$? 	<p>Common Misconceptions:</p> <ul style="list-style-type: none"> • Students often have misconceptions that there is only one unit rate. • Students often have the misconception that the unit rate is the constant of proportionality when $y = 1$.
<p>Universal Design for Learning:</p> <ul style="list-style-type: none"> • [Engagement] The lesson fosters collaboration and community through the use of productive groups. • [Action & Expression] The lesson uses multiple tools for expression and composition through the Notice and Wonder organizer. • [Representation] The lesson maximizes transfer and generalization by providing explicit supported opportunities to generalize learning to new situations. 	<p>Standards for Mathematical Practice:</p> <ul style="list-style-type: none"> • [SMP #1] Students make sense of problems and persevere in solving them by interpreting the meaning of a point on the line of a proportional relationship. • [SMP #4] Students model with mathematics when graphing real world situations. • [SMP #7] Students look for and make use of structure when they connect the relationship between the unit rate, constant of proportionality, scale factor and multiplicative comparison.
<p>Resources:</p> <ul style="list-style-type: none"> • Notice and Wonder Handout (page 62) • The Student Handout for Warm-up Activity (page 63) • Mable's Famous Orange Drink Handout (page 64) • Activity 4 Productive Group Work Handout (page 65) • Graph paper and straight edge if needed 	<p>Opportunities for Assessment:</p> <ul style="list-style-type: none"> • Informal Assessment – The student responses on Mable's Famous Orange Drink Handout (page 59) • Check for Understanding (page 61) • Informal Assessment – The products produced by students for the closure activity (page 61)

Lesson Sequence

Teachers says to class, “Building on the connection between the graph, the table, and the equation of a proportional relationship, we are going to closely examine the importance of the classifying characteristics of proportional relationships with an emphasis on the points (0, 0) and (1, r) in the context of a real world problem.”

Warming-Up: (Independent)

Ask your students to complete the **Notice and Wonder** graphic organizer (handout page 62) while examining the examples provided on the student handout for the warm-up activity (handout page 63).

- *In the Notice Column – have students jot down everything that they “see” in the presented material.*
- *In the Wonder Column – have students jot down anything that they “wonder” or have questions about from the presented material. This will help prepare them for the lesson.*

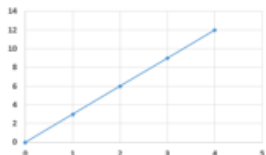
Activity 1: (Productive Group Work)

Once students have had time to complete their **Notice and Wonder** handout, place students into groups of 3 or 4. Each group will need their **Notice and Wonder** handout along with the **Student Handout** (handout pages 62 – 63) for this warm-up activity.

Have each group work as a team to complete the following:

- Using what you know about the multiplicative comparison, write a common definition for scale factor.
 - *This was a vocabulary word that was informally introduced in To Be or Not to Be Proportional from this unit. Scale factor is defined as a number that scales, or multiplies some quantity. The scale factor is also referred to as the multiplicative comparison.*
- Define constant of proportionality in your own words.
 - *We have defined this is previous lessons. However, some students may still struggle with the definition. One suggestion would be to ask the student to define the word constant separately from proportionality before trying to define constant of proportionality.*
 - Refer to properties of proportionality poster created in previous lesson.

Student Hand Out for Warm-Up Activity

<p>Example A:</p> $y = 3x$	<p>Example B:</p> 																
<p>Example C:</p> <table border="1" data-bbox="1029 1316 1131 1352"><tr><td>x</td><td>1</td><td>3</td><td>5</td></tr><tr><td>y</td><td>3</td><td>9</td><td>15</td></tr></table> <p>Multiplicative Comparison</p> $\times 3$	x	1	3	5	y	3	9	15	<p>Example D:</p> <table border="1" data-bbox="1315 1316 1417 1352"><tr><td>x</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y</td><td>15</td><td>18</td><td>21</td></tr></table> <p>Scale Factor</p> 3	x	5	6	7	y	15	18	21
x	1	3	5														
y	3	9	15														
x	5	6	7														
y	15	18	21														
<p>Example E:</p> <p><i>It cost \$3.00 for every 1 pound of bananas.</i></p>	<p>Example F:</p> (x, y) $(1, 3)$																

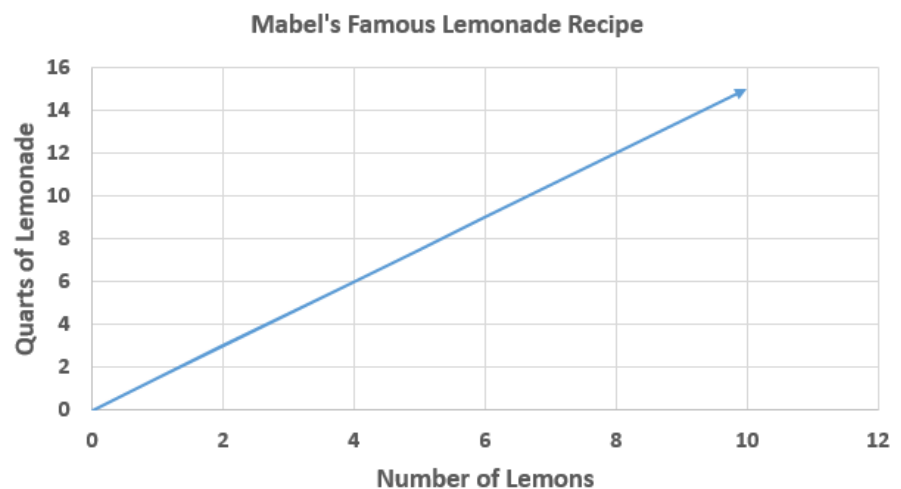
Activity Two: (Whole Class)

Post on the board the following.

Mabel is making her famous lemonade recipe which makes 3 quarts of lemonade for every 2 lemons. She wonders what other batches of lemonade she can make that will have the same lemony taste as this recipe.

Mabel started by creating a table comparing the number of lemons to the number of quarts of lemonade she makes. She then used this information to create the following graph.

Number of Lemons	Quarts of Lemonade
2	3
4	6
1	1.5
0.5	0.75
3.5	5.25
5	7.5
6	9
10	15



Pose the following questions:

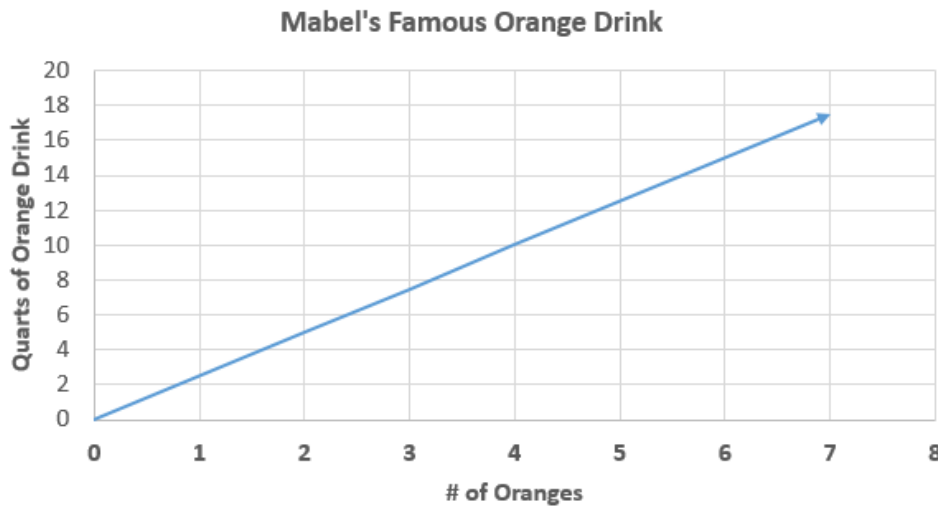
- Is the number of quarts of lemonade proportional to the number of lemons used? Explain why or why not.
 - Yes, because there exists a constant, $\frac{3}{2}$, such that when you multiply the number of lemons you get the corresponding number of quarts.
- Does the graph show the two quantities being proportional to each other? Explain.
 - Yes the points appear to form a straight line and that line goes through the point (0,0).
- What is the unit rate of quarts of lemonade to number of lemons used? What is the meaning of the unit rate to the context of the problem?
 - The unit rate is $\frac{3}{2}$ which means that for every 1 lemon you get 1.5 quarts of lemonade.
- Write an equation that can be used to represent the relationship.
 - $y = \frac{3}{2}x$ where y is the number of quarts and x is the number of lemons.

**Teacher Note
(Essential Question 1):**

When examining a graphical representation of a proportional relationship, students should be able to articulate that the values plotted form a straight line that goes through the origin.

Activity Three: (Productive Group Work)

Place students into groups and provide each group with the handout titled **Mable's Famous Orange Drink** (handout page 64). Students should work in their groups to collaboratively solve the questions on the handout. Below is the answer key.



Standards for Mathematical Practice

- Students make sense of problems and persevere in solving them by interpreting the meaning of a point on the line of a proportional relationship. [SMP 1]
- Students model with mathematics when graphing real world situations. [SMP 4]
- Students look for and make use of structure when they connect the relationship between the unit rate, constant of proportionality, scale factor and multiplicative comparison. [SMP 7]

1. What do the following ordered pairs represent?
 - (0,0)
 - Zero oranges will make zero quarts of Orange Drink
 - (2,5)
 - 2 oranges will make 5 quarts of Orange Drink
 - $(\frac{1}{2}, 1\frac{1}{4})$
 - $\frac{1}{2}$ of an orange will make 1 and $\frac{1}{4}$ quarts of Orange Drink
 - (3,7.5)
 - 3 oranges will make 7.5 quarts of Orange Drink
2. How many quarts of Orange Drink can Mabel make with no oranges? Where is this shown on the graph?
 - 0 quarts
 - (0,0) origin
3. Mable has 1 orange left, how many quarts of Orange Drink can she make?
 - 2.5 quarts of Orange Drink
4. Where is the point, represented in question 3, located on the graph?
 - (1,2.5)
5. What is the unit rate for this proportional relationship?
 - 2.5 or $\frac{5}{2}$
6. Looking at the point (0, 0) and (2, 5) what is the vertical distance and what is the horizontal distance between those two points?
 - The vertical distance is 5 and the horizontal distance is 2.
7. Looking at the point (2, 5) and the point (4, 10) what is the vertical distance and what is the horizontal distance between the two points?
 - The vertical distance is 5 and the horizontal distance is 2.

Directions for Activity Four: (Productive Group Work)

Part I:

- Provide each group a copy of the Activity Four Productive Group Work (handout page 65).
- As a group, create a table and a graph for the provided situation.
- Answer the following questions from the handout:
 - What is one unit rate for this situation that makes sense in the context of the problem?
 - *2 cups of food per 1 dog*
 - Based on your ratio table what is your scale factor (multiplicative comparison)?
 - $\times 2$
 - If you were to graph the quantities, how would you identify the scale factor from the graph? Explain your thinking.
 - *Have students discuss as a group, but do not at this point in time provide the answer. You want students to discover patterns in the relationship.*
 - What happens when your x (**independent**) value is 1?
 - *Have students discuss as a group.*
 - If you wrote your unit rate as a coordinate point? What would it look like?
 - $(1, 2)$
- At this point and time in the lesson, students should be able to answer essential question 2. Showing the relationship between scale factor, unit rate and constant of proportionality, specifically from the answers to the questions provided above (Essential Question 2).

Jackie has four dogs. It takes 8 cups of dry dog food to feed the dogs.



Teacher Note:

In any given proportional relationship, one needs to remember that there are always two unit rates (a unit rate where $y = 1$ and a unit rate where $x = 1$). The key is knowing that the **ONLY** unit rate that is the constant of proportionality is the y value when $x = 1$. (Essential Question 3)

It is important for students to understand what the independent versus the dependent variable in the context of the problem.

In this particular situation, the dependent variable is going to be dog food because the number of dogs is going to determine the amount of food.

Part II: (handout page 65)

The table represents a proportional relationship. Determine the constant of proportionality and explain your reasoning.

X	Y
2	2.5
4	5
6	7.5

Dominique's Work:

The constant of proportionality is 2.5 because $5 - 2.5 = 2.5$ and $7.5 - 5 = 2.5$.

- Explain why Dominique's reasoning is incorrect.
- Explain how you would help Dominique find the correct constant of proportionality.

Sample Student Response:

Dominique's reasoning is incorrect because she used subtraction between only one quantity (the y values) to find the constant of proportionality. Because the table represents a proportional relationship, the ratio between the y and x values will be the same. I found the constant of proportionality to be 1.25.

$$y/x = 2.5/2 = 5/4 = 7.5/6 = 1.25$$

Check for Understanding: (Independent)

Teacher Directions: All students need to be able to complete questions 1, 2 and 3. Question 1 is provided as an entry point for students because it resembles scenarios discussed in class. This question should be completed in a small group with the teacher for those students who struggled through the lesson prior to the students attempting questions 2 and 3 independently.

Questions:	Solutions:										
<div>1. It takes 2.5 cups of dry dog food and $\frac{1}{3}$ cup of wet dog food to feed 1 dog. Create a table and a graph to represent the situation when every dog eats the same amount of dry and wet food.<div>a. What is one unit rate for this situation that makes sense in the context of the problem?</div><div>b. What is your scale factor (multiplicative comparison)? How does this quantity relate to the situation above?</div><div>c. Write the equation that would represent this situation.</div></div>	<div><ul style="list-style-type: none">1 can of wet food for every 7.5 cups of dry food.$\times \frac{2}{15}$$y = \frac{2}{15}x$</div> <table><tr><td>Dry</td><td>2.5</td><td>5</td><td>10</td><td>7.5</td></tr><tr><td>Wet</td><td>$\frac{1}{3}$</td><td>$\frac{2}{3}$</td><td>$1\frac{1}{3}$</td><td>1</td></tr></table>	Dry	2.5	5	10	7.5	Wet	$\frac{1}{3}$	$\frac{2}{3}$	$1\frac{1}{3}$	1
Dry	2.5	5	10	7.5							
Wet	$\frac{1}{3}$	$\frac{2}{3}$	$1\frac{1}{3}$	1							
<div>2. The graph shows the movement of a Lady Bug.<div>Describe all of the key characteristics of the graph such that another classmate could create the graph and the equation showing the relationship between meters traveled and minutes from your description.</div><div></div></div>	<div>Students responses will vary however below are key characteristics that need to be included in their description.</div> <div><ul style="list-style-type: none">The lady bug traveled 0 meters in 0 minutes. (0,0)The graph describes the lady bug traveling meters per minute.After one minute the lady bug travels 0.25 meters.For the lady bug to travel one meter it takes 4 minutes.</div>										
<div>3. Create a visual representation showing the relationship between the constant of proportionality, scale factor, composed unit, multiplicative comparison, and the unit rate.</div>	<div>Student responses will vary.</div> <div>In students responses they should show that the constant of proportionality, scale factor, multiplicative comparison and the unit rate all represent the same value.</div> <div>When the composed units are all simplified down to the same unit, the value is the reciprocal of the multiplicative comparison.</div>										

Closure: (Independent)

Pose the following situation:

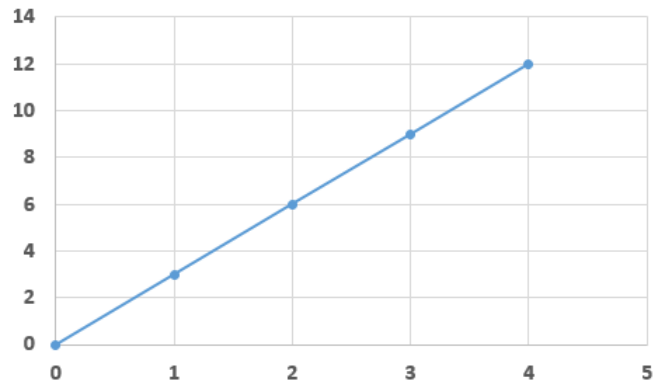
Evan was absent from school today. He needs your help in explaining the important components of today’s lesson. In your own style (words, pictures, graphics, actions, etc. ...) describe two key aspects you learned about the constant of proportionality.

Notice:	Wonder:

Example A:

$$y = 3x$$

Example B:



Example C:

<i>x</i>	1	3	5
<i>y</i>	3	9	15

Multiplicative Comparison

$\times 3$

Example D:

<i>x</i>	5	6	7
<i>y</i>	15	18	21

Scale Factor

3

Example E:

***It cost \$3.00 for every
1 pound of bananas.***

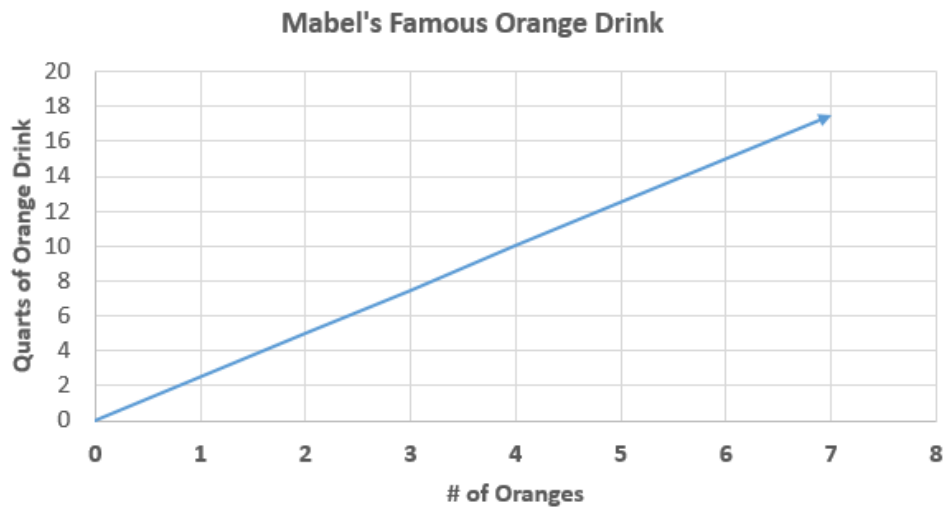
Example F:

(x, y)

$(1, 3)$

Mabel's Famous Orange Drink Handout

Below is a model of the amount of oranges needed to make Mabel's famous Orange Drink.



(1) What do the following ordered pairs represent?

- (0, 0)
- (2, 5)
- (4, 10)
- (3, 7.5)

(2) How many quarts of Orange Drink can Mabel make with no oranges? Where is this shown on the graph?

(3) Mable has 1 orange, how many quarts of Orange Drink can she make?

(4) Where is this point located on the graph?

(5) What is the unit rate for this proportional relationship?

(6) Looking at the point (0,0) and (2,5) what is the vertical distance and what is the horizontal distance between those two points?

(7) Looking at the point (2,5) and the point (4, 10) what is the vertical distance and what is the horizontal distance between the two points?

Activity 4 Productive Group Work Handout:

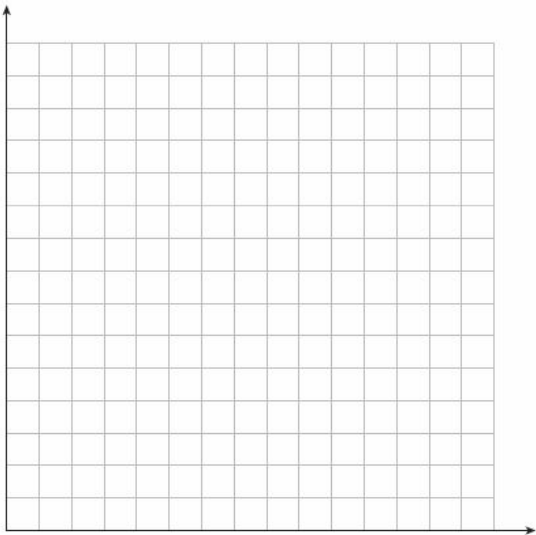
Part I:

Jackie has four dogs. It takes 8 cups of dry dog food to feed the dogs.



Ratio Table:

Coordinate Plane:



What is one unit rate for this situation that makes sense in the context of the problem?

Based on your ratio table, what is your scale factor (multiplicative comparison)?

If you were to graph the quantities, how would you identify the scale factor from the graph? Explain your thinking.

What happens when your x (independent) value is 1?

If you wrote your unit rate as a coordinate point, what would it look like?

Part II:

The table represents a proportional relationship. Determine the constant of proportionality and explain your reasoning.

X	Y
2	2.5
4	5
6	7.5

Dominique’s Work:
The constant of proportionality is 2.5 because $5 - 2.5 = 2.5$ and $7.5 - 5 = 2.5$.

- Explain why Dominique’s reasoning is incorrect.
- Explain how you would help Dominique find the correct constant of proportionality.

Culminating Assessment Part I

Group Assessment

This unit addresses the grade 7 major cluster **7.RP.A.2** recognize and represent proportional relationships between quantities.

- **CCSS.Math.Content.7.RP.A.2a**

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- **CCSS.Math.Content.7.RP.A.2b**

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- **CCSS.Math.Content.7.RP.A.2c**

Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

- **CCSS.Math.Content.7.RP.A.2d**

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Background:

Throughout this unit, students have been asked to determine whether or not relationships that exist in a table, graph, equation, or real world situation, are proportional. This assessment will allow students to determine whether or not the given cards represent a proportional relationship. Furthermore, mathematical concepts have many representations; words, diagrams, algebraic symbols, tables, graphs, and so on. It is important for students to understand these representations in order to translate between them. This culminating assessment encourages students to discuss connections between verbal, numerical, spatial and algebraic representations. For the following assessment, students should work in pairs or triads.

Directions for the Teacher:

- Cut out the set of cards on pages 68 – 71
- Card Set A: Algebraic Equations, Card Set B: Real World Situations, Card Set C: Tables, and Card Set D: Graphs
- Place students into pairs or triads
- Hand out cards to the groups
- Each group will need all cards from sets A-D

Teacher Note:

In phase one of this assessment, students should identify four cards that Do Not represent a proportional relationship. After the four cards have been identified. They need to be removed from the pile prior to starting phase two. The four cards that are not proportional are: T2, G1, E5, R6.

Directions for the Students:

Phase One:

- Sort the cards so that each group of cards has equivalent meaning.
- Explain your reasoning for why one card is or is not equivalent to another as you and your partner or group sort the cards.
- Your goal for phase one of the sorting assessment is to determine whether or not each group of sorted cards is or is not a proportional relationship.
- Once you have determined whether a proportional relationship exists, remove all non-proportional cards.

Phase Two:

- Each “sorted” group should have a real world situation, an algebraic equation, a table, and a graph.
- If the “sorted” group does not have one of each, students will need to construct for themselves any missing representations.

Solution to Card Sort:

T1, G7, E2, R3	T5, G8, E3, R2
T2, G1, E5, R6 NOT Proportional Relationships	T6, G4, E7, R7
T3, G5, E8, R4	T7, G6, E1, R5
T4, G2, E6, R1	T8, G3, E4, R8

E1

$$y = 2x$$

E2

$$y = 45x$$

E3

$$y = \frac{9}{4}x$$

E4

$$y = 1.5x$$

E5

$$y = 5x + 10$$

E6

$$y = \underline{\hspace{1cm}} x$$

E7

$$y = x$$

E8

$$y = \underline{\hspace{1cm}} x$$

R1 A map has a scale of 3 inches = 1.5miles.	R2 At a sporting goods store, 4 cans of tennis balls cost \$9.00.
R3	R4 A professional soccer team is donating money to a local charity for each goal they score. The team scored 12 goals and donated \$720.
R5 To make 1 batch of green paint you will need one cup of blue paint and 2 cups of yellow paint.	R6 Becky sells flower arrangements. She sells arrangements for \$5.00 per flower and \$10 for the vase.
R7 You go shopping at the “Every item is one Dollar” store.	R8

Card Set A: Tables

T1

x	y
3	135
3.5	157.5
4	180
5	225
5.5	247.5
6	270

T2

x	y
2	20
5	35
8	50
11	65
14	80
17	95

T3

x	y
2	120
3	180
4	240
5	300
6	360
7	420

T4

x	y
2	1
3	1.5
4	2
5	2.5
6	3
7	3.5

T5

x	y
1	2.25
2	4.50
3	6.75
4	9.00
5	11.25
6	13.50

T6

x	y

T7

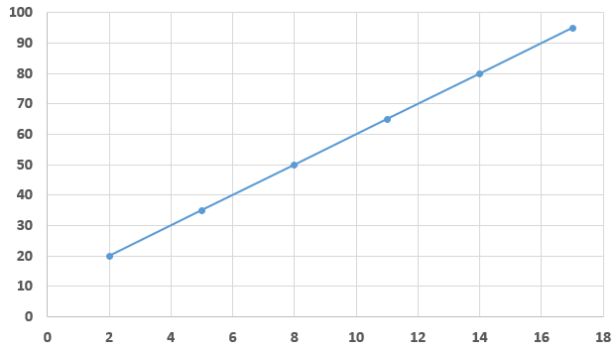
x	y
1	2
2	4
3	6
4	8
5	10
6	12

T8

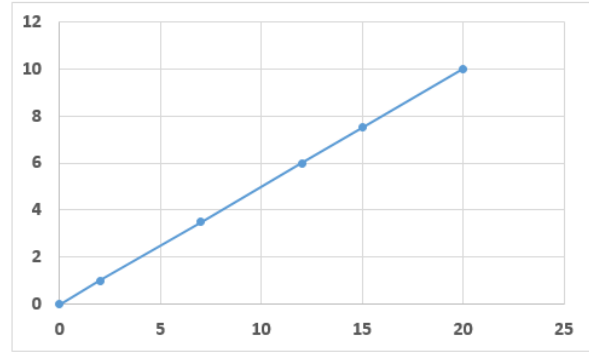
x	y
0	0
1	1.5
3	4.5
5	7.5
7	10.5
9	13.5

Card Set A: Graphs

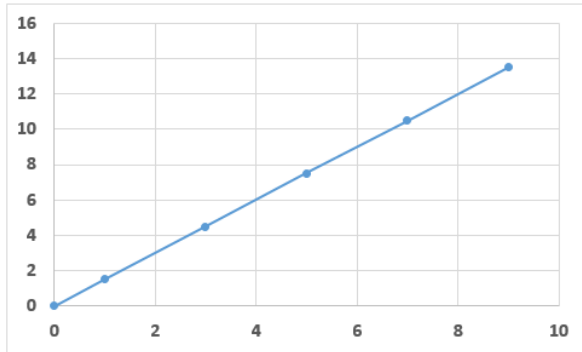
G1



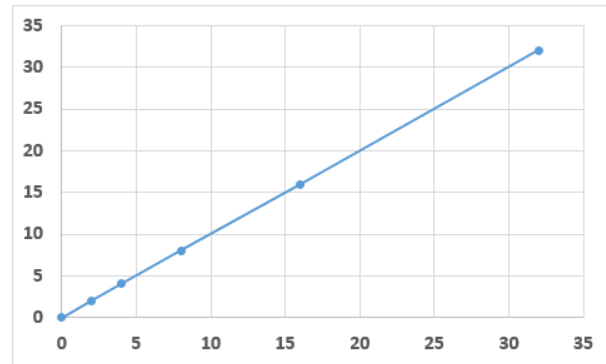
G2



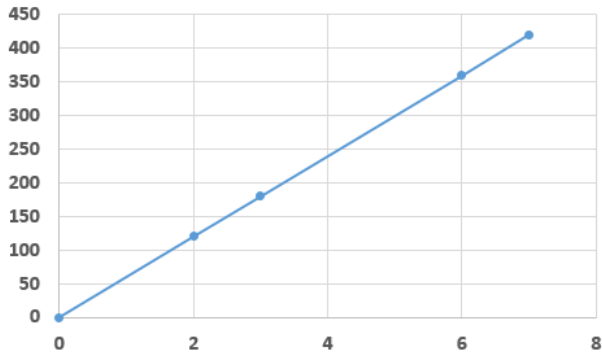
G3



G4



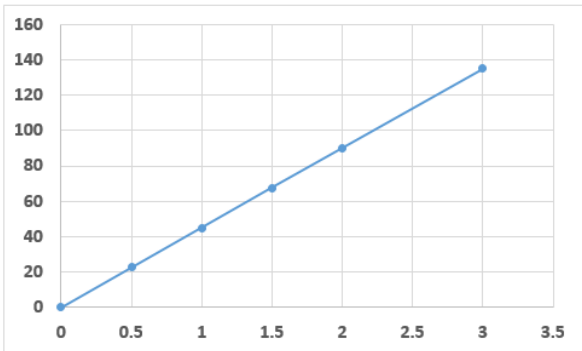
G5



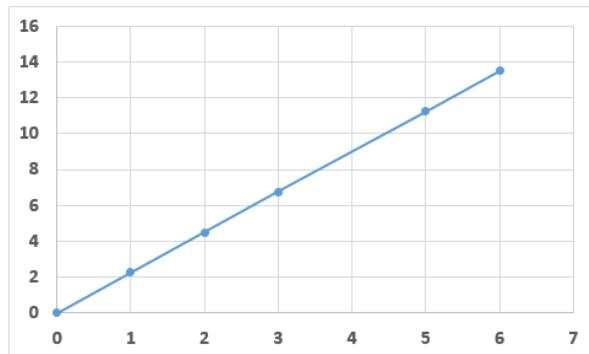
G6



G7



G8



Culminating Assessment Part II

Independent Assessment

This unit addresses the grade 7 major cluster **7.RP.A.2** recognize and represent proportional relationships between quantities.

- **CCSS.Math.Content.7.RP.A.2a**

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- **CCSS.Math.Content.7.RP.A.2b**

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- **CCSS.Math.Content.7.RP.A.2c**

Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

- **CCSS.Math.Content.7.RP.A.2d**

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Name: _____

Date: _____

Final Unit Assessment [7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2c, 7.RP.A.2d]

No Calculator Allowed

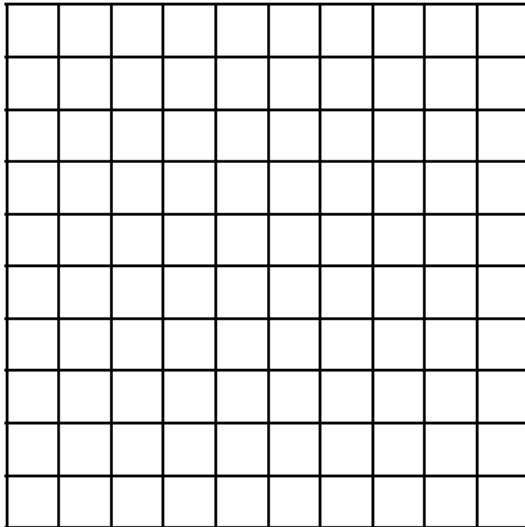
#1

7.RP.A.2a

The coordinates for point A are (4, 2).

The coordinates for point B are (x, 3).

What is the value of x, if points A and B are in a proportional relationship? After finding the value of x, plot points A and B below.



#2

7.RP.A.2a

Dominique kept track of her stats during a basketball game. She recorded the amount of time she played and the number of baskets scored. Determine whether or not the ratios in the given table are in proportion to one another. Explain your reasoning.

Time During Game	3.75 min	12.5 min
Dominique's Basket's Made	6 baskets	20 baskets

Show Your Thinking Below:

#3

7.RP.A.2b

Which equation has a constant of proportionality equal to 9?

- (a) $9y = 9x$
- (b) $9y = 18x$
- (c) $18x = 2y$
- (d) $2x = 18y$

#4

7.RP.A.2b

The table shows a proportional relationship between x and y.

x	y
2	$\frac{1}{4}$
4	$\frac{1}{2}$
6	$\frac{3}{4}$
10	$1\frac{1}{4}$

What is the constant of proportionality between x and y?

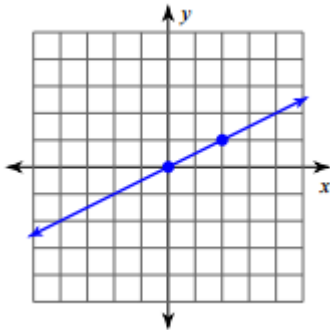
(#5)

7.RP.A.2b

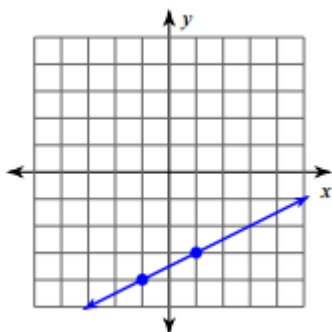
Choose all of the relationships below that have the same constant of proportionality between y and x as in the

equation $y = \frac{1}{2}x$?

(a)



(b)



(c)

x	2.5	3.5	4.5	5.5	6.5
y	5	7	9	11	13

(d)

x	2.5	3.5	4.5	5.5	6.5
y	1.25	1.75	2.25	2.75	3.25

(e)

$$10y = 5x$$

(f)

$$3y = 6x$$

(#6)

7.RP.A.2c

The students in Ms. Barone's Art Class were mixing blue and red paint to make purple paint. Ms. Barone told her students that the mixtures will be the same shade of purple paint if the ratios for the blue and red paint are the same for each batch.

The table below shows the different batches of paint the students made.

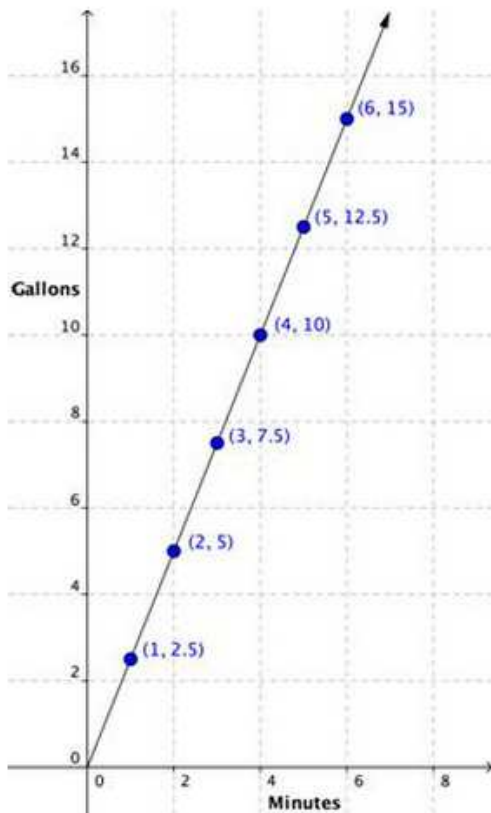
	Blue Paint x	Red Paint y
Batch #1	1.5 cups	3 cups
Batch #2	3 cups	6 cups
Batch #3	6 cups	3 cups
Batch #4	2.5 cups	8.75 cups
Batch #5	3 cups	10.5 cups
Batch #6	4.5 cups	15.75 cups
Batch #7	12 cups	6 cups

(a) How many Different Shades of Purple Paint did the students make?

(b) Write an equation that relates the number of cups of blue paint (x) and the number of cups of red paint (y) for EACH different shade of purple paint.

(#7)**7.RP.A.2d**

This graph shows the relationship between time and the amount of water in a bathtub.



Select **EACH** statement about the graph that is TRUE.

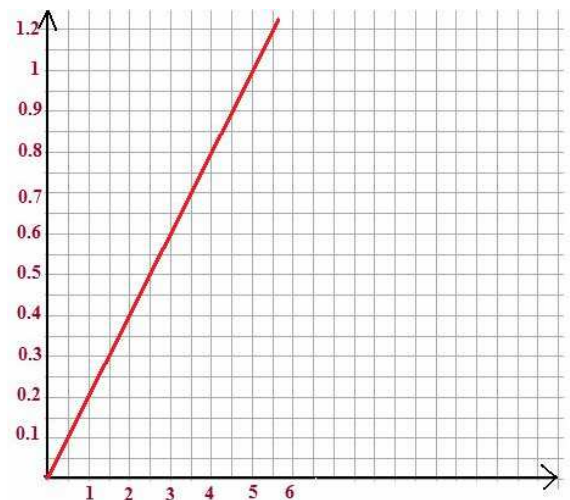
Select **ALL** that apply.

- (a) The point (0, 0) shows that at 0 min there was no water in the tub.
- (b) The point (1, 2.5) shows that after 1 min, there was 2.5 gallons of water in the tub.
- (c) The point (1, 2.5) shows that after 2.5 min, there was 1 gallon of water in the tub.
- (d) The point (2, 5) shows that after 5 min, there was 2 gallons of water in the tub.
- (e) The point (0.4, 1) shows that after 0.4 min there was 1 gallon of water in the tub.

(#8)**7.RP.A.2d**

The graph below has no titles on either axis and needs your help.

- (a) Create a title for the graph.
- (b) Label both the x and y axis on the graph
- (c) Write one sentence explaining the meaning of the point (0, 0) on your graph.
- (d) Identify the point when $x = 1$ and describe what that means in the context you created.



Solutions to Unit Assessment:

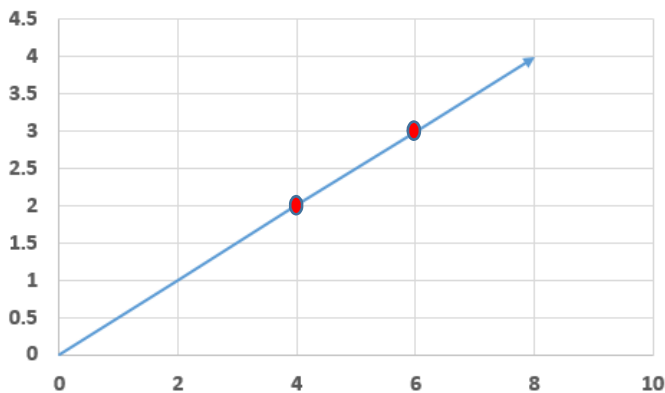
(#1)

7.RP.A.2a

The coordinates for point A are (4, 2).
The coordinates for point B are (x, 3).

What is the value of x, if points A and B are in a proportional relationship? After finding the value of x, plot points A and B below.

X = 6



(#2)

7.RP.A.2a

Dominique kept track of her stats during a basketball game. She recorded the amount of time she played and the number of baskets scored. Determine whether or not the ratios in the given table are in proportion to one another. Explain your reasoning.

Time During Game	3.75 min	12.5 min
Dominique's Basket's Made	6 baskets	20 baskets

Show Your Thinking Below:

Yes they are in proportion to one another.

Time	3.75	7.5	1.25	12.5
Baskets	6	12	2	20

(#3)

7.RP.A.2b

Which equation has a constant of proportionality equal to 9?

- (a) $9y = 9x$
- (b) $9y = 18x$
- (c) $18x = 2y$**
- (d) $2x = 18y$

(#4)

7.RP.A.2b

The table shows a proportional relationship between x and y.

x	y
2	$\frac{1}{4}$
4	$\frac{1}{2}$
6	$\frac{3}{4}$
10	$1\frac{1}{4}$

What is the constant of proportionality between x and y?

The Constant of Proportionality is $\frac{1}{8}$.

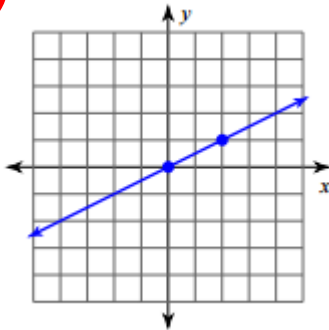
(#5)

7.RP.A.2b

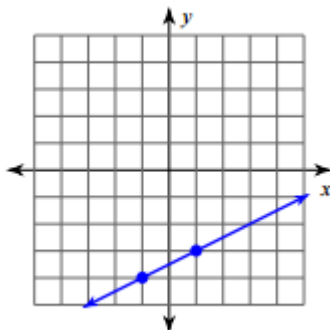
Choose all of the relationships below that have the same constant of proportionality between y and x as in the

equation $y = \frac{1}{2}x$?

(a)



(b)



(c)

x	2.5	3.5	4.5	5.5	6.5
y	5	7	9	11	13

(d)

x	2.5	3.5	4.5	5.5	6.5
y	1.25	1.75	2.25	2.75	3.25

(e)

$$10y = 5x$$

(f)

$$3y = 6x$$

(#6)

7.RP.A.2c

The students in Ms. Barone's Art Class were mixing blue and red paint to make purple paint. Ms. Barone told her students that the mixtures will be the same shade of purple paint if the ratios for the blue and red paint are the same for each batch.

The table below shows the different batches of paint the students made.

	Blue Paint x	Red Paint y
Batch #1	1.5 cups	3 cups
Batch #2	3 cups	6 cups
Batch #3	6 cups	3 cups
Batch #4	2.5 cups	8.75 cups
Batch #5	3 cups	10.5 cups
Batch #6	4.5 cups	15.75 cups
Batch #7	12 cups	6 cups

(c) How many Different Shades of Purple Paint did the students make?

There are 3 different shades of purple.

(d) Write an equation that relates the number of cups of blue paint (x) and the number of cups of red paint (y) for EACH different shade of purple paint.

$$y = 2x$$

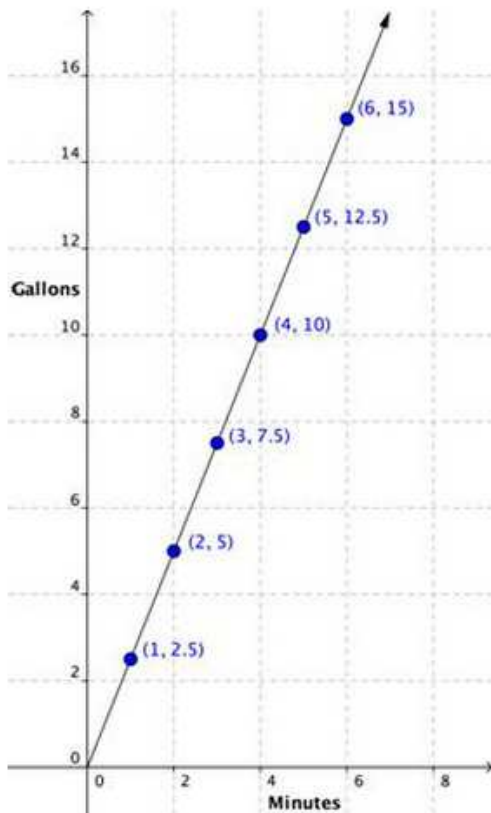
$$y = \frac{1}{2}x$$

$$y = \frac{7}{2}x$$

(#7)

7.RP.A.2d

This graph shows the relationship between time and the amount of water in a bathtub.



Select **EACH** statement about the graph that is TRUE.

Select **ALL** that apply.

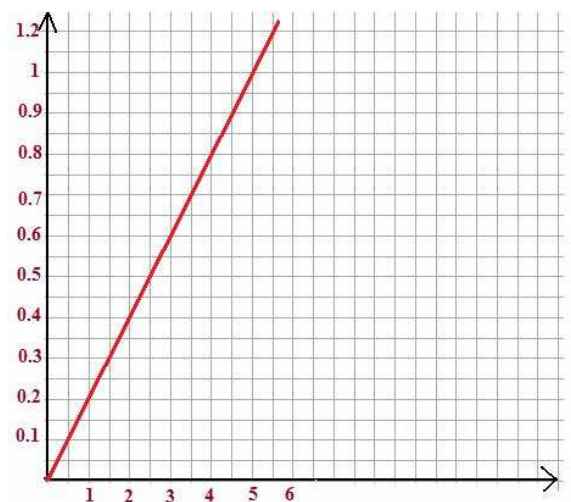
- ☒ (a) The point (0, 0) shows that at 0 min there was no water in the tub.
- ☒ (b) The point (1, 2.5) shows that after 1 min, there was 2.5 gallons of water in the tub.
- ☐ (c) The point (1, 2.5) shows that after 2.5 min, there was 1 gallon of water in the tub.
- ☐ (d) The point (2, 5) shows that after 5 min, there was 2 gallons of water in the tub.
- ☒ (e) The point (0.4, 1) shows that after 0.4 min there was 1 gallon of water in the tub.

(#8)

7.RP.A.2d

The graph below has no titles on either axis and needs your help.

- (a) Create a title for the graph.
- (b) Label both the x and y axis on the graph
- (c) Write one sentence explaining the meaning of the point (0, 0) on your graph.
- (d) Identify the point when $x = 1$ and describe what that means in the context you created.



- (a) Answers will vary.
- (b) Answers will vary
- (c) Students will need to state that at the point (0, 0) there is nothing of one variable and nothing of the other variable.
- (d) Students will need to state that when $x = 1$, $y = 0.2$ and provide an explanation in the context of their problem.

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