Inventory and Corporate Risk Management*

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ABSTRACT

We examine the role of inventory in corporate risk management with a dynamic model of a firm exposed to costly external finance and featuring endogenous default. We find that inventory management allows net worth risk management irrespective of the level of current net worth, because it does not affect the trade off between external finance and risk management. We show that inventory and cash holdings are synergic tools: while the first is a valuable operational hedge against commodity price risk, the second enhances the hedge offered by inventory in the face of costly external finance. Using our model, we rationalize the empirical incidence of inventory and cash holdings in the cross-section of U.S. manufacturing corporations, showing that savings and storage of raw materials are both positively related to financing constraints and cash flow risk.

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Introduction

Inventory management is perhaps the most traditional and commonly used operational hedge by corporations. However, over the last fifty years U.S. corporations have drastically reduced the stock of inventory, owing to high storage costs on one side, and improvements in supply chain efficiency, outsourcing, and reliance on multiple suppliers on the other. Not surprisingly, the very technological and regulatory innovations that allowed a reduction of inventory holdings have exposed firms to new risks.[1] These risks and the historical increase in domestic and global competition have recently rekindled the managers' interest on inventory vis-à-vis other risk management tools, like cash holdings.[2]

Relative to other risk management tools (e.g., derivatives), inventory has been little explored in corporate finance. In this paper, we study the use of inventory for net worth risk management, and therefore we focus on a risk management motive for storing raw materials, whereby firms hold inventory to mitigate the effects of input price shocks. By analyzing the role of inventory in corporate risk management we make also a theoretical contribution to the literature, which has so far been mainly focussed on either noncontingent tools (cash holdings) or collateralized contingent instruments (e.g. derivative contracts and lines of credit). In this respect, inventory is a contingent tool that does not need to be collateralized.[3]

We capture the contribution of inventory to risk management within a dynamic model of investment, in which the firm stores the commodity used in production to manage risks generated by a productivity shock and the commodity price. In our model, net worth management is motivated by external finance costs (Froot, Scharfstein, and Stein 1993), and firms manage risk to reduce costs triggered in states of the world in which they are financially constrained.[4]

In our model, operating cash flow is convex in the price of the commodity, given the flexibility to adjust the level of the commodity used in production. Such a convexity would suggest an incentive for the firm to increase risk. However, the presence of equity issuance

[1] Chen, Frank, and Wu (2005) document the decline of holdings of inventory for U.S. manufacturing firms between 1981 and 2000. However, they argue that firms prefer to hold inventory, in order to manage supply chain risks that just-in-time production or reliance on multiple suppliers cannot completely eliminate. One such a risk is given by prices fluctuations.

[2] Bates, Kahle, and Stulz (2009) find that one of the main factors that caused an increase in cash holdings of American firms has been the reduction of net working capital.

[3] We describe the relevant literature in Section I.

[4] Risk management is motivated also by distress and bankruptcy costs and increased tax payments (Smith and Stulz 1985), the loss of tax shield (Stulz (1996), Leland (1998)), and agency costs (DeMarzo and Duffie 1995).

costs generates a demand for net worth risk management. Therefore, conditional on survival, the firm engages in net worth risk management by shifting net worth across states using inventory. When the price of the commodity is relatively low, the firm invests in storage in view of an increase of the price in the future. In this way, it transfers net worth in states in which the marginal value of net worth is high from states in which it is low.

Differently from contingent financial contracts like derivatives, risk management using inventory is not subject to collateral or margin requirements. This characteristic of inventory management has important implications. For a starter, inventory management is not restricted in firms with low net worth. Even though investment in inventory is increasing in net worth, risk management using storage is not precluded by low net worth. As a result, absence of risk management occurs only when the cost of hedging is too high (i.e., the current price of the commodity is high). Indeed, because there is no trade-off between financing and risk management, the firm invests in capital and inventory at the same time, even when net worth is low.

We find that higher persistence of either of the shocks induces more or less investment in capital depending on the state of the cash flow. In line with Froot, Scharfstein, and Stein (1993), we find that risk management using inventory is reduced when the price of the commodity is persistent. It is also reduced when the productivity shock is more persistent conditional on a current state of high productivity. Conversely, when the firm faces a bad productivity state, higher persistence of productivity leads to more investment in inventory, given the higher value of risk management in this case. More risk, deriving from a higher volatility of either productivity or the price of the commodity, increases risk management and reduces investment in capital, even though we find a positive relation between the volatility of the price of the commodity and investment when the current price is low.

The persistence and standard deviation parameters of the commodity price process play distinctive roles as of the risk management value of inventory. While a higher standard deviation unambiguously increases such value, the persistence parameter has a non-obvious effect: on the one hand it increases the unconditional cash flow volatility and therefore makes inventory more valuable; on the other hand, a high persistence reduces the likelihood of an adverse change in commodity price, thus lowering the importance of inventory. The latter effect prevails, making inventory less valuable to hedge against a highly persistent commodity price.

We offer a novel angle on the traditional comparison between real flexibility (in our case, inventory management) and financial flexibility, and more generally on enterprise risk

management. Although the real flexibility of storage has been widely recognized, at the best of our knowledge we are the first to present a financial dimension of inventory other than simply being it a source of liquidity (as for instance, in Fazzari and Petersen 1993, Carpenter, Fazzari, and Petersen 1994, Kashyap, Lamont, and Stein 1994). In this regard, we study the interaction between cash holdings and hedging by letting the firm take positions on commodity price with inventory and use savings to finance investments without recurring to costly external finance.

We then analyze inventory management in conjunction with cash management. While inventory is a better suited risk management tool in states in which commodity price risk is predominant, cash holdings help manage risk in those states in which inventory is not useful, or may be even detrimental to firm value in the event of an adverse change in the commodity price. While in principle cash management complements inventory management, the contribution of inventory to risk management is actually enhanced by cash holdings: there is a positive synergy between the management of inventory and savings for financially constrained firms. Overall, cash holdings are particularly useful to finance investments in inventory in states in which firms have a strong incentive to manage risk.

The value and level of cash holdings has been extensively studied in relation to the characteristics of a single productivity shock. However, less attention has been devoted to the analysis of multiple shocks affecting the firm's operating cash flow, in particular, from the perspective of the integrated management of inventory and cash holdings.[5] We describe the states of the firm's business in which inventory and cash holdings are relevant for risk management.

To analyze the interactions between storage and savings, we calibrate the model to replicate empirical moments related to the integrated management of inventory and cash holdings of manufacturing firms in the United States. Our empirical analysis supports the idea of a financial dimension to inventory management. When focussing on inventory and cash holdings in a panel of U.S. manufacturing corporations, we find that savings and storage are both positively correlated to financing constraints and cash flow volatility, which together determine the expected cost of external financing. Specifically, we find that more constrained and risky firms hold at the same time more inventory and cash.

These findings are in contrast with the literature in which inventory and cash holdings are both considered as sources of liquidity, and with the literature focused on operational motivations, whereby cash holdings substitute inventory. We interpret these empirical find-

[5]See Gamba and Triantis (2008), and Riddick and Whited (2009), among others.

ings through the lens of our model, and show in which situations the firm relies more on inventory or on cash holdings. Indeed, cash holdings can be considered substitutes of inventory, because a firm can avoid recurring to costly external finance caused by an increase in the commodity price by either storing the commodity or by saving cash. However, we also highlight a complementary relation between the two tools, which explains the empirical findings above. First, cash flow risk is determined by multiple factors, which can hardly be controlled with a single risk management tool. Second, cash holdings are useful for hedging states in which inventory management is not effective. Third, the negative impact of financing constraints on risk management using inventory can be significantly mitigated by internal liquidity.

The rest of this paper continues as follows. In Section I, we review studies closely related to ours, and highlight our contribution to the literature. In Section II, we develop the model and explain the differences with respect to previous literature; present analytical results on firms' optimal policies and the predictions of the model on storage in net worth risk management. In Section III, we describe the numerical solution of the model, and analyze the empirical incidence of inventory and cash holdings in a sample of American manufacturing firms in relation to factors that affect risk management. Section IV concludes.

I. Related literature

This paper is at the intersection of several strands of the literature. First of all, our paper is related to the risk management literature, especially on integrated management of investment and hedging. Mauer and Triantis (1994) show that production flexibility substitutes for the flexibility of adjusting debt level. Mello, Parsons, and Triantis (1995) find that the flexibility of moving production between countries and financial hedging are substitutes.

We study the integrated investment and liquidity policies of the firm, as in Bolton, Chen, and Wang (2011), who recognize the interactions between financial risk management and investment decisions, in their ability to provide liquidity. Gamba and Triantis (2014) find that derivatives are inefficient instruments to hedge real frictions, which can be better managed using cash holdings and production flexibility.

Empirical studies, such as Allayannis, Ihrig, and Weston (2001), Pantzalis, Simkins, and Laux (2001), MacKay (2003), and Hankins (2011) support the imperfect substitution between operational flexibility and financial flexibility. Gézcy, Minton, and Schrand (2006)

provide evidence on the complementarity between storage of natural gas and cash holdings in a sample of gas companies in the United States. In contrast to previous studies, we explore the role of inventory in risk management by focusing on the flexibility of adjusting storage of production inputs in relation to their market prices, and analyze the interactions between real flexibility provided by inventory and financial flexibility provided by cash holdings.

Recently, Rampini and Viswanathan (2010), Rampini and Viswanathan (2013), and Rampini, Sufi, and Viswanathan (2014) study the interaction between risk management and debt financing. Focusing on financing constraints deriving from collateral requirements on debt, they provide a rationale for the observed limited use of derivatives by firms with low pledgeable net worth based on the trade-off between debt financing and risk management. Differently from them, in our model risk management is implemented using inventory, which is subject to storage costs but does not require collateralization. As we abstract from debt financing under limited commitment, we do not tie risk management together with financial policies. Therefore, in our model the firm can manage risk and invest at the same time, even when the net worth is low. In line with the analysis of these papers, Nikolov, Schmid, and Steri (2017) show that financing constraints, in the form of collateral requirements on credit lines and debt financing, determine a motive for holding cash. This incentive is stronger for poorly collateralized firms with good investment opportunities, that exhaust debt capacity and require additional funding from internal liquidity to finance investments. Relative to this work, we study inventory which is an operational hedge that provides liquidity in a contingent manner likewise credit lines. However, inventory does not require collateral. Therefore, our model predicts that inventory and cash are complementary risk management tools mainly in relation to the support that savings provide for investment in inventory, especially when cash flow is scarce.

We bring the economics of inventory into a corporate finance setting. The way we model inventory management is reminiscent of the competitive storage model originated by Gustafson (1958) and analyzed by Deaton and Laroque (1992) among others, and of the theory of storage in general (see Kaldor 1939, Working 1948, Brennan 1958, Telser 1958). Differently from the storage literature, the objective of the present paper is the study of the inventory policy of manufacturing firms within an integrated risk management plan, as opposed to study how commodity prices are formed.

Our model is in part related also to the macroeconomic literature on (s, S) models (see the reviews of Blinder and Maccini 1991, Khan and Thomas 2007), given our focus on holding inventory in relation to the avoidance of costs of purchasing inputs. However, we

significantly depart from this literature, because we concentrate on the market price of inputs as the relevant cost component of purchasing inputs, and ignore fixed ordering costs.

Our work is related to the literature that emphasizes the financial component of inventory management. Fazzari and Petersen (1993), Gertler and Gilchrist (1994), Kashyap, Lamont, and Stein (1994) and Carpenter, Fazzari, and Petersen (1994) provide empirical evidence on inventory as a reserve of liquidity, especially for financially constrained firms. The theoretical premise of these studies is the liquidation value of inventory, given the high degree of reversibility of investment in inventory. Differently from this literature, we highlight a specific channel (hedging of commodity price) through which investment in inventory mitigates negative shocks to cash flows. The inclusion of this motive makes inventory quite different from liquidity and highlights new interactions between inventory and cash holdings.

Finally, we contribute to the literature on the precautionary motive for holding cash. Several papers study precautionary savings in response to cash flow volatility. These works include Kim, Mauer, and Sherman (1998), Opler, Pinkowitz, Stulz, and Williamson (1999), Gamba and Triantis (2008), Riddick and Whited (2009), Bates, Kahle, and Stulz (2009), and Bolton, Chen, and Wang (2011), among others. Previous studies take a general perspective on cash flow uncertainty and savings, while we disentangle specific sources of cash flow volatility, and study how savings respond to each one. Focussing on cash flow risk as a whole hides important insights on risk management. For example, we find that cash holdings are less useful for managing commodity price risk, even though such risk constitutes an important component of cash flow volatility. Relative to Riddick and Whited (2009), our paper analyzes a risk management role of cash, besides the interaction with the investment policy.

Recently, the finance literature has shown renewed interest in inventory.[6] Dasgupta, Li, and Yan (2016) relate financial constraints with inventory investment, finding that financially constrained firms vary the inventory stock more aggressively than their financially unconstrained counterparts, in response to shocks to production costs. Given the assumption of their paper, their results are basically unchanged if the firm is allowed to hold cash, as cash holdings and inventory are (imperfect) substitutes. In contrast, our focus on risk management reveals unexplored interactions between inventory investment and savings, which support complementarity between these risk management tools.

Gao (2017) contends that the reduction of input inventory stock and the contemporaneous increase in cash holdings for U.S. corporations in the last decades can be explained by

[6]Belo and Lin (2012) and Jones and Tuzel (2013) propose asset pricing models that predict a negative correlation between investment in inventory and stock return risk, consistently with empirical data.

the adoption of just-in-time production by the majority of U.S. firms and the consequent need of cash for transactional purposes. Kulchania and Thomas (2017) provide evidence on the causal relation going from deregulation in the trucking industry and the adoption of innovations in supply chain management to the secular decrease of input inventory holdings. They find that higher expected costs of supply chain disruptions induce an increase of cash holdings, given the increased exposure to supply chain risks caused by the reduction of inventory.

Even though a transactional motive for hoarding cash is present also in our model, differently from these works, we concentrate on a risk management perspective and find a complementary relation between inventory and cash holdings at the cross-sectional level. Cash holding targets the risk of incurring external financing costs and complements the management of commodity price risk in states in which storage cannot be used. Furthermore, cash holdings are crucial for financing risk management using inventory for financially constrained firms. Empirically, the variable used to scale inventory and cash holdings in regressions may confound the actual relation between these risk management tools. Using capital as scaling variable, we find that inventory and cash holdings are positively related especially for financially constrained firms and firms with high cash flow volatility.

II. Model

We study a dynamic model of firm investment, production choices, and risk management. The firm is exposed to commodity price risk for an input used in production, and to productivity shocks. In the model, the firm can reduce the impact of input price risk by storing the commodity. The need of costly external finance is the risk management motive in the model.

A. Firm policies

We consider an all-equity financed firm that faces external financing costs and has production based on fixed capital and a commodity as inputs. The manager of the firm, who acts on behalf of shareholders, maximizes firm value. We model the decisions of the firm in a discrete-time infinite-horizon setting and assume that agents are risk neutral.

We assume that the firm has a Cobb-Douglas production function, $g(z_t, k_t, u_t) = z_t k_t^{\theta} u_t^{\gamma}$, in which $k_t \geq 0$ is the stock of capital, $u_t \geq 0$ is the amount of the commodity used in production, and $z_t > 0$ is the total factor productivity of the technology at time t.[7] The parameters $\theta \in]0,1[$ and $\gamma \in]0,1[$ gauge the productivity of capital and commodity, respectively. We assume decreasing returns to scale, so that $\theta + \gamma < 1$. Therefore, $g(z_t, k_t, u_t)$ is increasing and strictly concave in the last two arguments (i.e., the partial derivatives are $\partial_k g > 0$, $\partial_u g > 0$, and the associated Hessian matrix is negative definite).

The commodity has price $p_t > 0$ at time t. The two random variables of the model, z_t and p_t, together define the exogenous state of the firm. They have compact supports, respectively $[z_\ell, z_u]$ and $[p_\ell, p_u]$. We assume the joint process (z_t, p_t) is a Markov chain.

At time t, in anticipation of a price increase in the coming period, the manager can store away an amount $n_{t+1} \geq 0$ of commodity.[8] Storage costs of warehousing, deterioration, damages, and theft are $h(n_{t+1})$ over the period, where $h(0) = 0$, $\partial_n h(n_{t+1}) \geq 0$, and $\partial_{nn} h(n_{t+1}) > 0$. The strict convexity of $h(n_t)$ captures limited storage capacity. In our model, the convenience value of inventory results from building the stock of commodity when the price is low, and from using the stored commodity in production when its price is high.[9] The avoidance of stockouts, which can be costly if the firm is forced to purchase the commodity at a high price, is part of this convenience value.[10] Overall, at time t, the firm purchases at p_t an amount $u_t + i_t^n$ of commodity, where $i_t^n = n_{t+1} - n_t$ and n_t is the current stock of the commodity.

At t, the manager can adjust the capital stock in response to changes in productivity and commodity price by investing an amount $i_t^k = k_{t+1} - (1-\delta)k_t$. We assume that capital has constant unit price. For the moment, we exclude adjustment costs on capital stock. Finally, the manager can build financial slack over the coming period by saving c_{t+1} in the

[7] We abstract from labor, or equivalently we assume that it has already been optimized upon in the current period.

[8] Rampini, Sufi, and Viswanathan (2014) develop a model with a similar production function and a commodity as production factor. Our model differs from theirs in the way the firm hedges the commodity price risk: they focus on hedging using derivatives and do not consider the possibility to store the commodity.

[9] We make an important deviation from traditional inventory models (Kydland and Prescott 1982) and follow Humphreys, Maccini, and Schuh (2001) by not including n_t in the production function, in order to isolate the role of inventory in risk management. Our way of modelling the management of inventory derives from the theory of storage (Kaldor 1939, Working 1948, Brennan 1958). Because we assume risk neutral agents, we ignore the risk premium of investing in the commodity. Finally, the convenience yield of the commodity is the marginal productivity of the amount of inventory that the firm decides to use.

[10] In the literature on inventory, (opportunity) costs of stockout are generally given by lost production. Producing at a high cost gives an equivalent effect of reducing the operating cash flow.

cash account. Savings are penalized, as they yield a return, r, lower than the one earned in the market by risk-free securities, $r < 1/\beta - 1$.[11]

Given the current state of the firm and the above described policies, the dividend at t is

$$d_t = z_t k_t^{\theta} u_t^{\gamma} - \psi - p_t \left(u_t + i_t^n\right) - h(n_t) - i_t^k + (1+r)c_t - c_{t+1}, \tag{1}$$

where $\psi \geq 0$ is a overhang of fixed costs. For simplicity, we will remain agnostic about the nature of these costs, which can be thought of as either fixed production costs, or debt servicing, or any other senior obligations, by assuming that the decisions made by the manager maximize the value of the firm. The instantaneously optimal amount of commodity,

$$\hat{u}_t = \left(\frac{\gamma z_t k_t^{\theta}}{p_t}\right)^{\frac{1}{1-\gamma}}$$

is decreasing in p_t.[12] Therefore, production is negatively affected by a positive shock on commodity price. Replacing the expression of $\hat{u}$ in (1), we can rewrite the dividend as

$$d_t = \pi(z_t, p_t) k_t^{\alpha} - \psi - p_t i_t^n - h(n_t) - i_t^k + (1+r)c_t - c_{t+1}, \tag{2}$$

where $\alpha = \theta/(1-\gamma) < 1$, and

$$\pi(z_t, p_t) = \gamma^{\frac{1}{1-\gamma}} \left(\frac{1}{\gamma} - 1\right) \left(\frac{z_t}{p_t^{\gamma}}\right)^{\frac{1}{1-\gamma}}. \tag{3}$$

Under the assumptions we made on the parameters of the model, the function π is non-negative, increasing and convex in z_t, and decreasing and convex in p_t.

Although in the dynamic optimization the decision on optimal inventory and optimal production occur at the same date t, we assume that inventory management is a two-stage process, in which the manager first optimizes current production and finds $\hat{u}_t$, and next she optimally chooses n_{t+1}, given the optimal cash flow from operations. This simplification is possible because the first order condition on u_t is independent of n_t. To see that, because in (1) z_t, k_t, n_t, and c_t are known at time t, consider the optimization of d_t with respect to

[11] The reduced form approach to a penalty on savings is motivated by a wedge between taxation of returns on firm's savings and taxation of returns on shareholders' savings. Motivations like free cash flow agency issues or a debt-equity cashing out issue would be inconsistent with the other assumptions of the model.

[12] The first-order condition related to the optimization of d_t with respect to u_t is $\gamma z_t k_t^{\theta} u_t^{\gamma-1} = p_t$. Rearranging, we find $\hat{u}_t$.

u_t. Although the firm purchases $u_t - n_t$ to produce the output good, the optimal level of production is independent of n_t and depends only on commodity price, p_t.[13]

B. Firm value optimization

Let V denote firm value as a function of the state $(z_t, p_t, k_t, n_t, c_t)$. It results from the maximization of the present value of future dividends with respect to the control variables:

$$V(z_t, p_t, k_t, n_t, c_t) = \max_{(k_{j+1}, n_{j+1}, c_{j+1}), T} \mathbb{E}_t \sum_{j=t}^{T} \beta^{j-t} \left[(1+\lambda) d_t^- + d_t^+ \right], \tag{4}$$

where T is the stochastic default time, d_t is defined in (2), $d^+ = \max\{d, 0\}$, and $d^- = \min\{d, 0\}$. The cost on external equity, gauged by $\lambda > 0$, is the reason why the firm manages risk, in line with Froot, Scharfstein, and Stein (1993): when $d_t < 0$, the firm issues equity at a cost λd_t.[14]

In (4), the default time is chosen based on current information to maximize shareholders' value. This implements the limited liability option of the equity holders: when the value is negative, the firm is sold. Because we do not specify the nature of the fixed costs, we can ignore the liquidation procedure.

We will now turn to a recursive description of the model of the firm. Primed variables will indicate values at time $t+1$, and non-primed variables will be values at time t. For convenience, we will denote the exogenous state variables with $s = (z, p) \in \mathcal{S}$, and the decision variables with $x = (k, n, c)$. Similarly to Cooley and Quadrini (2001) (and also Hennessy and Whited, 2007 and Rampini, Sufi, and Viswanathan, 2014), we introduce the realized net worth at s, denoted $w(s, x)$, which is a sufficient statistics of the state of the firm. Differently from Rampini, Sufi, and Viswanathan (2014), who assume non-negative net worth and non-negative dividends, we explicitly model strategic default, in the sense that although the firm has negative net worth and can have negative dividends, default does not occur until cum-dividend value of equity is zero.[15]

[13] Notice that the optimal quantity of commodity used in production $\hat{u}_t$ is unchanged by the introduction of external financing costs, λ, as we do later on. In other words, optimizing d_t with respect to u_t yields the same optimal $\hat{u}_t$, because both the marginal product and cost of u_t are multiplied by the same factor $(1+\lambda)$ when the dividend is negative. In addition, in equilibrium, the cash flow obtained with the marginal product of u_t is exactly offset by the expense $p_t u_t$, so that there is no effect on cash flow.

[14] The external finance cost can be motivated by taxation, adverse selection, and transaction fees, as summarized by Fazzari, Hubbard, and Petersen (1988).

[15] An assumption of non-negative dividends corresponds to assuming infinitely costly equity financing, which is per se a risk management motive, on top of limited enforcement. The net worth can be restricted

According to the *principle of optimality*, we can solve the firm's program using a recursive approach

$$V(s,w) = \max\left\{0, \max_{d,x',\mathbf{w}'}(1+\lambda)d_t^- + d_t^+ + \beta\int V(s',w')\mu(ds'|s)\right\}, \tag{5}$$

where $\mathbf{w}' = (w(s',x'), s' \in \mathcal{S})$, subject to

$$w' = \pi(s')(k')^\alpha - \psi + p(s')n' - h(n') + (1-\delta)k' + (1+r)c' \quad \text{for all } s' \in \mathcal{S} \tag{6}$$

$$w = d + pn' + k' + c', \tag{7}$$

and $k' \geq 0$, $n' \geq 0$, and $c' \geq 0$. In (5), $\mu(\cdot|s)$ is the probability distribution of $s' = (z', p')$, conditional on state s. We assume that μ has the Feller property.

The following propositions, which are instrumental to the analysis of corporate risk management in the next section, characterize the value function $V(s,w)$ and the optimal policy function. For the moment, we restrict our attention to the case $x = (k,n)$. We will discuss the cash policy later on. The proofs are in Appendix A.

Proposition 1. *Under the assumption that the distribution μ has the Feller property, the value function, V, of the program* (5)-(7) *exists, is unique, and coincides with the value function attained solving the program in* (4).

Proposition 2. *For each s, there exist $w_d(s) < \underline{w}(s) < \overline{w}(s)$ such that the firm defaults if the realized net worth is $w \leq w_d(s)$, the dividend is $w - \underline{w}(s)$ (i.e., the firm raises an amount $\underline{w}(s) - w$ of equity) if $w_d(s) < w < \underline{w}(s)$, and pays a dividend $w - \overline{w}(s)$ if $w > \overline{w}(s)$.*

Proposition 3. *The optimal investment policy in capital and inventory as a function of (s,w), is unique and continuous. Given s, for each $w \leq \underline{w}(s)$ the optimal policy will be $(\underline{k}(s), \underline{n}(s))$, and for each $w \geq \overline{w}(s)$ it will be $(\overline{k}(s), \overline{n}(s))$.*

Proposition 4. *The cum dividend value function $V(s,w)$ is concave and strictly concave for $w \in [\underline{w}, \overline{w}]$, and continuously differentiable in w.*

Proposition 5. *The optimal investment in capital and optimal investment in inventory satisfy condition*

$$\underline{\int \partial_w V(s',w(s'))\left[\frac{p(s')-h'(n)}{p(s)}\right]\mu(ds'|s)} = \int \partial_w V(s',w(s'))\left[\pi(s')\alpha k^{\alpha-1} + (1-\delta)\right]\mu(ds'|s).$$

to non-negative values by a debt covenant. Relative to Rampini, Sufi, and Viswanathan (2014), we analyze corporate risk management in the absence of both the non-negative dividends restriction and the net worth covenant. This is a non-trivial extension, as a non-negative net worth covenant reduces shareholders' value, thereby reducing the incentive to manage risk.

From Proposition 4, the presence of external finance costs ($\lambda > 0$) determines the concavity of the value function with respect to net worth for low levels of current net worth. While the conclusion is the same as in Rampini, Sufi, and Viswanathan (2014), our motivation is different as in their case collateral constraints are the motive for managing risk. In our model, financing constraints in the form of external financing costs as in Froot, Scharfstein, and Stein (1993) induce a demand for risk management as if the firm was *risk averse*.

From Proposition 2, the payout policy of the firm is quite simple. Assume the firm is currently solvent (i.e., $w > w_d(s)$). If $w < \underline{w}(s)$ then the firm issues equity to raise $d = w - \underline{w}(s)$ at a cost of λd per unit in order to achieve an adjusted net worth $\underline{w}(s)$, and based on Proposition 3 to invest $(\underline{k}(s), \underline{n}(s))$, which depends on s only. For the firm, it makes sense to raise costly external equity because in this region the marginal value of net worth, calculated on an ex dividend basis, is relatively high. If $w > \overline{w}(s)$, then the firm has excessive net worth, and pays a dividend $d = w - \overline{w}(s)$. This is because the marginal value of net worth, calculated on an ex dividend basis, is lower than the marginal value of dividend to the shareholders. After achieving the adjusted net worth $\overline{w}(s)$, the firm invests $(\overline{k}(s), \overline{n}(s))$, which depends on s only. Finally, if $\underline{w}(s) < w < \overline{w}(s)$, the firm is self-sufficient, because the marginal value of net worth calculated on an ex dividend basis is lower than $1 + \lambda$ and higher than 1. In this region, the investment policy in capital and inventory fully depends on (s, w). To summarize, given the concavity of the value function, the marginal value of net worth is a crucial determinant of the payout policy and of the investment policy of the firm.

From Proposition 5, the optimal risk management policy using inventory is such that the expected marginal value of inventory is equal to the expected marginal productivity of capital. The investment in capital and in risk management compete with one the other in the budget constraint. While the marginal productivity of capital depends on both z' and p', the marginal value of inventory depends only on p', and is given by the ratio of price in the future state s', net of the marginal storage cost, over the current price of inventory. Notably, if the current price of the commodity is sufficiently low, the incentive to invest in inventory for risk management purposes is high. On the other hand, if p is relative high, given a convex storage cost the firm finds it optimal to reduce inventory. This is based on current commodity price, irrespective of net worth.

The results in Propositions 1-5 can be illustrated by a numerical solution of program (5)-(7), for a version of the model in which we consider investment in capital and inventory, and

exclude cash holdings for simplicity.[16] Figure 1 presents the optimal policy as a function of current net worth, w, at two different states s. In Panel A we consider a state characterized by a relative low commodity price, whereas in Panel B p is high relative to z. The payout policy, described by the optimal dividend, confirms Proposition 2, with a dividend increasing linearly with respect to w in $]w_d, \underline{w}[$ and in $[\overline{w}, \infty[$, and no payout for $[\underline{w}, \overline{w}]$. Comparing the state with low price (Panel A) to the state with high price (Panel B), we see that investment in capital always dominates investment in inventory, and if p is relatively high the firm does not invest in inventory, regardless of the current level of net worth.

This is different from the case of risk management using derivatives or other state contingent contracts, which need to be collateralized, typically in the form of margin requirements. In that case, as proved by Rampini, Sufi, and Viswanathan (2014), if current net worth is large enough, the marginal value of net worth is the same in all future states and equal to the marginal value of current net worth. It is only if current net worth is low that it is optimal for the firm *not* to hedge some future states, even if they are potentially insurable. In a model in which risk management is based on inventory, because inventory does not require any collateral,[17] the optimality condition is that the average marginal productivity of capital equals the average marginal value of inventory. This has two implications. First, the marginal value of net worth in the future states can be different under the optimal policy in Proposition-5, even when current net worth is large. Second, if the marginal benefit of inventory is too small, which occurs if current price is high, the firm decides optimally not to use inventory as an operational hedge, irrespective of current net worth. Hence, absence of risk management is related to the cost of the hedging instrument, rather than to the net worth of the firm.

C. Inventory management as risk management

The operating cash flow is convex in p, as we showed in Section II.A. This would suggest a positive incentive of the firm to increase risk because of the higher expected cash flow that would follow. However, from Proposition 4, costly external finance causes the concavity of the value function with respect to w. For low levels of net worth, the curvature of the value function increases. Therefore, when firm value maximization (i.e., when the intertemporal effect of higher risk) is considered, the firm behaves in a risk averse manner and there is demand for risk management.

[16] The numerical methods are detailed in Appendix B. Although the parameters used in this simulation are from Section B, the properties described here are general.

[17] Of course, inventory is subject to other technical restrictions, like storage capacity or storage costs.

Managing risk is value improving because the firm wants to transfer net worth in those states in which the marginal value of net worth is higher (i.e., net worth is lower). In our model, risk management is implemented using inventory, which allows to transfer net worth from current to future unfavorable states, therefore reducing the dispersion of w. Formally, given two policies $x_1' = (k_1', n_1')$ and $x_2' = (k_2', n_2')$ that induce the same payout, $d(s, w, x_1') = d(s, w, x_2')$, then x_1' is preferred to x_2' if and only if $\mathbb{E}\left[V(s', w(s', x_1'))|s\right] \geq \mathbb{E}\left[V(s', w(x_2'))|s\right]$. From Jensen's inequality, given the concavity of $V(s, \cdot)$, this occurs only if the dispersion of $w(s', x_1')$ is not higher than the dispersion of $w(s', x_2')$.

Inventory management can be used to reduce the dispersion of net worth. The intuition is as follows. A high input price reduces production and net worth, and because the firm is risk averse with respect to net worth, the manager insures against future states in which the commodity price is high relative to total factor productivity, so that net worth is preserved in those states. Specifically, when p is low relative to z the firm subtracts resources from investment in capital stock and dividend payments and invests these resources in risk management at a cost pn'. The effect of this decision is to increase the net worth by $p(s')n' - h(n')$ in state s'. This effect will be larger exactly in those future states in which production, $\pi(s')(k')^{\alpha}$, is lower because $p(s')$ is higher.[18]

The intertemporal mechanism of risk management in our model can be understood from the optimality condition of investment in inventory. We denote $\mathcal{D} = \{d < 0\}$ the event of issuing equity at date t, $\mathcal{D}'$ the same event at $t+1$, $\chi_{\mathcal{D}}$ the indicator function of $\mathcal{D}$, and $\chi_{\mathcal{D}'}$ the indicator function of $\mathcal{D}'$. The optimality condition for the inventory decision n' is

$$p\left(1 + \lambda\chi_{\mathcal{D}}\right) = \beta\mathbb{E}\left[\partial_w V(s', w')\partial_n w'|s\right] = \beta\mathbb{E}\left[\left(p' - \partial_{n'}h(n')\right)\left(1 + \lambda\chi_{\mathcal{D}'}\right)|s\right], \tag{8}$$

[18]Considering their frictions, inventory and derivatives are different risk management instruments. While inventory requires an investment in the current period and generates storage costs, derivatives are subject to basis risk and limits imposed by the firm's counterparty risk such as collateral constraints, margin requirements, or premiums on the price paid by the firm. However, inventory and derivatives are similar in that they both are contingent risk management tools. An example can make the similarity more clear. A firm using inventory invests an amount pn' in the current period and will obtain a payoff $p'n' - h(n')$ in the next period. Using a futures, the firm typically does not pay anything at inception of the contract and will receive a payoff $m'(p' - p_f)$ in the next period, where m' is the amount invested in the futures and p_f is the futures price agreed in the current period. Assuming for simplicity $\beta = 1$ (which is legitimate if the time interval is reasonably short), we can express the payoff from investing in inventory net of investment costs as $p'n' - h(n') - pn' = n'[p' - p - h(n')/n']$, where $h(n')/n'$ is the cost of storage per unit of commodity. No arbitrage in the commodity market would guarantee that $p_f = p + h(n')/n'$. Substituting for p_f into the payoff of the futures, we obtain $m'[p' - p - h(n')/n']$, which equals $n'[p' - p - h(n')/n']$ for an amount $n' = m'$ of resources allocated to risk management. Therefore, the payoff of the futures contract is equivalent to that obtained with inventory management.

where $\partial_n w' = p' - \partial_n h(n')$ and from the envelope condition $\partial_w V' = \partial_w d' = 1 + \lambda \chi_{\mathcal{D}'}$. In (8), the current cost to change inventory, p, augmented by the possible equity issuance cost on the left-hand side, equals the expected net marginal benefit of holding inventory on the right-hand side, $p' - \partial_n h(n')$. Ultimately, the value contributed by risk management is the reduction of the expected equity issuance costs at $t+1$, obtained by transferring net worth from states in which the marginal value of net worth is low to states in which it is high.

Inventory management allows the firm to reduce the volatility of investment in capital stock by smoothing the cash flow that can be used to finance investment. In this sense Froot, Scharfstein, and Stein (1993) argue that the effectiveness of risk management depends on the correlation between cash flow and investment opportunities. When cash flow and investment opportunities are driven by the same factors, highly persistent shocks increase the probability of low cash flows when investment opportunities are worse, thereby reducing the need of hedging against low states.

In Figure 2, we show investment in capital and storage for different levels of the persistence of total factor productivity, ϕ_z, and of the commodity price, ϕ_p, for different current states. For the comparative static on ϕ_z, we set p lower than the unconditional mean (so that $i^n > 0$) and choose z either below or above its unconditional mean. Similarly, for the comparative static on ϕ_p we set z at its unconditional mean and choose p either below or above its unconditional mean. In Panels A and C, we observe that, when $\pi(s)$ is more likely to persist in the same state because of a higher autocorrelation of either z or p, investment in capital is higher in the high state (high z or low p) and lower in the low state (low z and high p). This is because the future $\pi(s')$ is more predictable when ϕ_z or ϕ_p are higher. Therefore, higher persistence sustains investment.

Panel B of Figure 2 presents perhaps the most interesting effect. The firm engages more in risk management when productivity is low and persistence is high because of the higher likelihood of future states in which net worth will be valuable. On the other hand, if productivity is high there is less need of risk management and a higher persistence of the productivity shock reduces the investment in inventory even further. According to Froot, Scharfstein, and Stein (1993), risk management should be less relevant with higher ϕ_z, as high persistence provides a natural hedge of investment opportunities. Panel B shows that this is true if the current state is favorable, but not if the current productivity is low.

Finally, Panel D presents a sensitivity on ϕ_p. Clearly, when $p = 1.29$, there is no investment in inventory for the simple reason the net marginal benefit of inventory is too low, as illustrated before. If the price is sufficiently low, $p = 0.78$, high persistence of commodity

price risk makes $\pi(s)$ more persistent and has a negative impact on risk management, in accordance to what Froot, Scharfstein, and Stein (1993) predicted.

We examine the effect of risk on investment and storage conducting comparative statics of optimal policies on σ_z and σ_p.[19] In Figure 3, Panel A, we observe that an increase in risk has the effect of reducing investment in capital because the firm is reluctant to invest when there is a high probability of a future state with low return. Higher risk induced by σ_z increases risk management, as we can see in Panel B. As for the effect of an increase in σ_p, the sign of the relation between the volatility of p and investment depends on the current state, given the persistence of p. In particular, for low p, a high σ_p increases the probability of a large decrease of the commodity price. In Panel C, we choose a low p and observe that k' is increasing in σ_p.[20] Finally, in Panel D a higher volatility of the price of the commodity induces more risk management for any level of the net worth.

D. Inventory and cash holdings

So far we have excluded cash holdings from our analysis. We now analyze the interaction between inventory management and cash management for risk management purposes. The main difference between cash holdings and inventory is that the latter is a contingent risk management tool, whereas the former is noncontingent. The optimality condition for the cash holdings decision of the firm is

$$1 + \lambda\chi_{\mathcal{D}} = \beta\mathbb{E}\left[\partial_w V(s', w')\partial_c w(s', k', n', c')|s\right] = \beta\left(1 + r\right)\mathbb{E}\left[\left(1 + \lambda\chi_{\mathcal{D}'}\right)|s\right], \tag{9}$$

where $\partial_c w(s', k', n', c') = (1 + r)$, and using the envelope condition $\partial_w V = 1+\lambda\chi_{\mathcal{D}'}$ as before. On the left-hand side, the cost of saving increases possible external financing costs, if the firm is at the equity issuance margin. The marginal benefit provided by cash on the right-hand side of (9) depends on earned interest, which reduces the likelihood of incurring external finance costs in the next period.

Inventory and cash holdings are complementary risk management tools for several reasons.[21] First of all, inventory is better suited for managing systematic (i.e., variability of

[19] Varying either one of the two parameters induces a variation of both risk and unconditional average of $\pi(s)$. To compensate for this effect, we adjust the average values of z or p, respectively. We keep the unconditional average of $\pi(s)$ constant, rather than the average net worth, to examine the impact of risk on optimal policies before the effect of risk management on net worth.

[20] For a high p we would observe the opposite effect.

[21] Similarly, Bolton, Chen, and Wang (2011) and Gamba and Triantis (2014) conclude that hedging with derivatives and cash holdings are complementary risk management tools.

the commodity price) and firm specific risks (e.g., supply chain disruptions), while cash holdings are useful to avoid costly external finance and other types of firm specific costs (e.g., investment adjustment costs). Second, for a deterministic cost, $1/\beta - (1 + r)$, cash holdings yield a deterministic return that contributes to reducing the probability of equity issuance in all states s'. Because of this, savings are inefficiently held in states of the world with high net worth, which is an opportunity cost of holding cash. However, such cost is partially offset by the reduction of the probability of equity issuance also in states in which the benefits from inventory management are low or absent (i.e., states s' with low $p(s')$ and high $z(s')$). Underinvestment in inventory induced by financing constraints can be relieved by cash holdings. Hence, the third source of complementarity between inventory and cash holdings is the incremental risk management capacity (using inventory) of constrained firms that have internal liquidity. In this way, cash holdings sustain risk management.

To describe the interaction between investment in inventory and cash holdings, we examine how the firm's policies change according to the state s, and the related $p(s)$ and $\pi(s)$. When $\pi(s)$ is high and $p(s)$ is low, the firm stores more, given the relatively high probability of an increase of the commodity price in the future. In the same scenario, while the firm has an incentive to save because of a non-zero probability of a bad state next period (i.e., low $z(s')$ or high $p(s')$, which implies a lower $\pi(s')$), a substantial part of internal financial resources is used to invest in capital given the relatively high $\pi(s)$. To finance investments in capital and in storage, constrained firms prefer to draw on cash balance rather than to tap external markets.

Conversely, when $\pi(s)$ is low and $p(s)$ is high, the firm has an incentive to minimize inventory. In addition, the possible reduction of capital adds to the stock of cash, because of a low value of $\pi(s)$. Given the persistence of the processes, in this state of low cash flow, the precautionary value of cash holdings is sizeable, because it may be useful to finance future investments in inventory, should the future commodity price be sufficiently low, when cash flow cannot be used because $\pi(s')$ persists in a bad state (i.e., a low $z(s')$).

In states in which high $\pi(s)$ and $p(s)$ are observed, the firm uses cash holdings to fund investment in capital, because of the increase of the marginal productivity of capital. This happens when $z(s)$ is sufficiently high relative to $p(s)$. More importantly, the firm does not invest in inventory because the commodity price may decrease later on. This scenario shows a non-trivial interaction between saving and storage: owing to persistence of the commodity price, a high $p(s')$ is likely also in the next period. Hence, *the firm has an incentive to save and the value of cash is increased by the prospect of purchasing commodity in the next period.*

Finally, when low $\pi(s)$ and $p(s)$ are observed, the firm has a weak incentive to invest in capital. At the same time, it invests in inventory to take advantage of the low commodity price, and dissaves in order to finance such investment. Also in this case the model delivers a non-trivial complementarity between cash and inventory management: *cash holdings allow to implement, in periods of low cash flows, a risk management policy based on inventory to hedge against commodity price risk.*

The analysis above reveals important interactions between inventory and cash holdings. Overall, the demand for investments is the main determinant of savings for constrained firms. Riddick and Whited (2009) find that savings have prevalently a negative correlation with the state of the business, after controlling for investment opportunities which are positively correlated with cash flows. In our model, which shares a similar production technology to the one in Riddick and Whited (2009), the second most important determinant of savings is the firm's incentive to manage risk in an integrated manner with storage.

Although useful to build the intuition of the interaction between inventory and cash holdings, the analysis in this section is hardly conclusive as for whether the two instruments are complement or substitute. In what follows, we resort to numerical analysis to quantify the synergy between inventory and cash holdings in terms of enterprise value, providing insights on the interactions between risk management and financing constraints. In addition, we will shed light on how such a synergistic interaction determines the incidence of inventory and cash holdings in empirical data and in data obtained simulating the model.

III. Empirical analysis

In this section, we first calibrate the model so that it replicates empirical moments of a sample of Compustat manufacturing firms. Next, we use a simulated sample obtained from the calibrated model to formulate empirical predictions as for the integrated management of inventory and cash holdings. Finally, we contrast the theoretical predictions obtained with the model to empirical findings.

A. Calibration

We first choose the base case values of the parameters by contrasting moments of variables obtained from a Monte Carlo simulation of the model (see Appendix B for details) with their

empirical counterparts in the U.S. economy. We construct the empirical sample starting from all firms in the Compustat North America database in the manufacturing industry (SIC codes 2000-3999), which is the suitable industry given our assumptions on firm's technology. We exclude firms with less than two observations, firms-year observations with negative values of total assets, sales, and book equity, and firms whose book items did not comply with standard accounting identities. The final sample is an unbalanced panel of firms observed between 1969 and 2014 with at least 1367 observations per year.

We compute the investment rate in capital stock as capital expenditures (capx in Compustat) less sale of capital (sppe) scaled by beginning-of-period capital (ppegt). Investment in inventory is the change of the stock of raw materials (invrm) between two consecutive years scaled by beginning-of-period capital. The inventory ratio is computed as inventory scaled by the beginning-of-period capital, and the cash ratio as cash and short-term securities (che) scaled by total assets (at). The market-to-book ratio is computed as the sum of total assets and the market value of equity (prccf x csho) minus the book value of equity (ceq) and deferred taxes (txdb), scaled by total assets. Equity issuance is net sale of common and preferred stock (sstk - prstkc).

We winsorize all variables at 1% to mitigate the impact of outliers. In addition, we compute the probability of divestment of inventory considering only observations of negative investment in inventory larger than 1% of the beginning of period capital stock. The probability of equity issuance refers to net sales of stock larger than 1% of total assets. Such thresholds are used to reduce the impact of measurement errors. This motivation is particularly relevant for investment in inventory, which is typically a small fraction of the size of the firms, and therefore more easily affected by accounting errors or misreporting.

The base case parameters values are summarized in Table I, while moments are reported in Table II. We calibrate our model in annual frequency, setting the discount factor at $\beta = 1/1.05$ and the interest rate on cash holding r at 0.0462.

The productivity has dynamics $\log z' = \phi_z \log z + \sigma_z \varepsilon_z'$, where $|\phi_z| < 1$, $\sigma_z > 0$, and ε_z' are i.i.d. shocks with truncated standard Normal distribution. The commodity price follows the process $\log p' = \phi_p \log p + \sigma_p \varepsilon_p'$, where $|\phi_p| < 1$, $\sigma_p > 0$, and ε_p' are i.i.d. shocks with truncated Normal distribution. The shock ε_z is contemporaneously correlated with ε_p so that $\mathbb{E}[\varepsilon_z \varepsilon_p] = \rho$ and $\mathbb{E}[\varepsilon_{z,t} \varepsilon_{p,s}] = 0$ for $t \neq s$.[22] We assume that z and p have

[22]In commodity markets, prices tend to revert to the average marginal cost of production (see Schwartz 1997). Also productivity shocks are typically modelled as autoregressive processes in the financial economics literature (e.g. Gomes 2001, Hennessy and Whited 2005, Zhang 2005). The support of z and p must be compact to ensure that the dynamic program we described has solution, and we achieve this by truncating

an idiosyncratic component, besides a common systematic factor, and capture this fact by letting the correlation, ρ, be between 0 and 1. As for p, while the systematic component is related to the market risk of the commodity, the idiosyncratic component can be interpreted as a supply chain shock. We set ρ to 0 for the base case. Assuming z is systematic, this corresponds to the assumption that the shocks to the commodity price are entirely firm-specific. Given the importance for our results of the systematic component of the stochastic evolution of p, later on we will provide comparative statics on ρ.

The autoregression coefficient of z, ϕ_z, is set to 0.62 and the volatility σ_z to 0.20, in line with values selected by Gomes (2001) and with the estimates of Hennessy and Whited (2005). The autoregression ϕ_p and the volatility σ_p of the commodity price process are also set at 0.62 and 0.20 respectively, in order to have a marginal distribution of p comparable to that of z, so that the relevance of risk management using inventory is not overstated. To have a comparison with real data, we take the time series of the main commodity price indexes from the World Bank GEM Commodities database. Our choice for the value of σ_p is in line with the volatilities of indexes returns. The volatility of agricultural and metal indexes returns is respectively 0.10 and 0.16, whereas for the energy index, it averages at around 0.35. Because we do not consider a specific commodity, a value of 0.20 is reasonable, and in line with values reported in Geman (2005).

In a sample of U.S. industries, Basu (1996) reports an empirical average of 0.60 for the share of materials (our "commodity") in total costs of production, while the remaining share is split between capital and labor. We set the overall return to scale in production to 0.90, and assign a share of productivity of $\gamma = 0.54 = 0.90 \times 0.60$ to the commodity, and of $\theta = 0.36 = 0.90 \times 0.40$ to capital.[23] The way we assign productivity shares to factors reflects a production function specified with commodity and value added as production factors, as in Basu (1996).[24] The Cobb-Douglas specification we adopt can be thought of as a special case of the more general functional form presented by Basu (1996).

We set the capital depreciation rate δ to 0.12 (e.g., see Gomes 2001) to match the empirical average capital investment rate. In order to obtain a volatility of the capital investment rate and a probability of negative investment in capital close to the respective empirical

the distribution of z and p within three times the unconditional standard deviation around the unconditional average. More details are given in Appendix B.

[23]The value selected for the total return to scale is based on the estimates of Basu and Fernald (1997).

[24]For simplicity, we consider a production function after labor has been optimized out. We take the productivity of labor into account implicitly by selecting appropriate values for the productivity parameters of capital and commodity.

counterparts, we introduce adjustment costs for capital of the form adopted in literature on q theory of investment.[25] When the firm varies capital, it incurs adjustment costs

$$a(k, k') = \frac{\xi}{2k}\left(i^k\right)^2, \qquad \text{with} \qquad \xi = \overline{\xi}\chi_{\mathcal{I}^+} + \underline{\xi}\chi_{\mathcal{I}^-},$$

where $\mathcal{I}^+ = \{i^k > 0\}$ is the event of investment, $\mathcal{I}^- = \{i^k < 0\}$ is the event of disinvestment, and $\overline{\xi} < \underline{\xi}$ to gauge partial irreversibility, as in Zhang (2005).[26] The function $a(k, k')$ is zero for $i^k = 0$, it is twice continuously differentiable except in points such that $i^k = 0$, it is strictly convex in k' and convex in k. We set $\overline{\xi} = 0.75$ and $\underline{\xi} = 7.5$, so that $\overline{\xi}/\underline{\xi} = 1/10$ as in Zhang (2005).

We specify the storage cost function as $h(n) = (\eta/2)n^2$, setting $\eta = 0.034$ to match the average and volatility of inventory scaled by the beginning-of-period capital stock, given the values selected for the parameters of the process of the commodity price.[27] The ratio of inventory to capital is an appropriate benchmark in relation to inventory management, given the purpose of storing the commodity related to the mitigation of the commodity price risk that affects cash flow, which is determined by the size of the capital stock.

Given the values selected for the parameters in the production function, we set $\psi = 0.03$, and $\lambda = 0.05$ to approximately match the average market-to-book ratio and the empirical probability of equity issuance.[28] Finally, the joint distribution selected for the productivity shock and for the price of the commodity and the values assigned to λ and r help approximate the average and the volatility of cash scaled by total assets in empirical data.

B. Integrated risk management with inventory and cash holdings

We analyze inventory and cash holdings in relation to cash flow risk and financing constraints, which together gauge the probability of a firm's need to resort to external finance. We will show that the model predicts that risky and financially constrained firms have a bigger incentive to manage commodity price risk using inventory. For this purpose, cash holdings

[25] See Hayashi (1982).

[26] Partial irreversibility can be motivated by adverse selection as in Arrow (1968), limited assets redeployability as in Williamson (1988), or leverage of potential buyers as in Shleifer and Vishny (1992), and Asquith, Gertner, and Scharfstein (1994).

[27] Our storage cost function is similar to the one used by Blinder (1986), with the exception that we restrict to zero the linear and fixed cost components.

[28] We select a value of λ very close to the estimate (0.058) of Hennessy and Whited (2005), who adopt a linear specification for the equity issuance costs function like ours.

are used to finance risk management using inventory, which would otherwise be reduced if operating cash flow is not sufficient, given the financing frictions.

We illustrate the prediction by first analyzing the states in which cash management (together with inventory) adds value. In Figure 4, we compute the value due to cash management as $V^*/V^n - 1$, where $V^* = V - c$ is enterprise value and V^n is firm value of a firm not allowed to hold cash. The value of cash holdings by construction is related to the presence of positive external finance costs. What is remarkable is that such value is mainly given by the synergy between inventory and cash holdings: the latter allow to avoid external financing costs when there is a big incentive to use inventory as an operational hedge.

Figure 4 shows the value of the synergy between inventory and cash holdings as a function of the productivity shock z and the commodity price p, in a state in which capital k, inventory n, and cash holdings c are at the unconditional average computed as described in Appendix B. Such value is positive and it increases in the commodity price for high values of z, while it decreases with p when z is low. This is because, when productivity is high, also the net worth is high. For low p, the incentive to invest in inventory is high, but the firm can finance risk management using the cash flow, and so cash holdings are less important. For high p, cash holdings allows investment in inventory also for a relatively high price, without incurring external financing costs. When productivity is low, savings are valuable when $\pi(s)$ is low (i.e., a high p), given the high probability of negative cash flows and of raising costly external finance in this state. More importantly, savings significantly contributes to enterprise value for low z and low p, a state in which cash flow can be low because of a low capital productivity but the commodity price is sufficiently low to spur investment in risk management.

Next, to explain how inventory and cash policies are determined by cash flow volatility conditional on financing constraints, we show in which circumstances cash holdings provide the greatest benefit when combined with risk management using inventory. We conduct comparative statics on the optimal inventory and cash holdings with respect to cash flow risk. In Figure 5, we compare the unconditional average inventory (n/k) and cash (c/k) from a simulated economy for different values of σ_z (Panels A and B), σ_p (Panels C and D), and ρ (Panels E and F), compensating for the variation of the unconditional average of $\pi(s)$ due to the change in the relevant parameter. All the other parameters are as in Table I. For comparison, in the figure we show also the inventory ratio computed from a simulation of the model with dynamic inventory only (i.e, excluding cash management).

In Panel A, the relation between average inventory and σ_z is either flat or slightly decreasing. The reduction of inventory is due to the fact that commodity price risk becomes relatively smaller compared to productivity risk for high σ_z. Also, when cash flow is less volatile (low σ_z and/or high ρ), the firm more likely funds investment in inventory using cash flow, and so less cash holdings are needed on average, as can be seen in Panel B. Remarkably, Panel A show that, with savings, inventory is higher for all σ_z. This is consistent with the synergic role of cash described above, which is more evident when commodity price risk has a prevalent effect on cash flow risk (i.e., when σ_z is low).

While inventory is not very sensitive to productivity risk, the effect of σ_p on inventory and cash holdings is strong, and as expected, average n/k is always monotonically increasing in σ_p, in Panel C. For $\sigma_p \leq 0.35$, cash holdings help finance risk management using storage, as the inventory ratio is higher when the firm is allowed to save. For higher σ_p, inventory is higher in the case with no savings because internal liquidity becomes relatively more effective than inventory as a risk management tool when there is a bigger benefit of avoiding external financing costs.

The model predicts a strong complementarity between inventory and savings due to financing constraints and cash flow risk driven by commodity price risk. As evidence of the positive synergy between inventory and cash, in Panel D, cash holdings are monotonically increasing in commodity price risk. The higher sensitivity of cash holdings to σ_p is a reflection of the sensitivity of inventory to commodity price risk, given the higher need of financing inventory investment with internal resources if σ_p is high.

As for Panel E, inventory increases with ρ. This result may seem counterintuitive because a lower correlation increases risk and should lead to more risk management. However, as in the case with low σ_z, near perfect correlation makes more likely to have sufficient cash flow for financing investment in storage. Indeed, the positive relation between ρ and n/k is reinforced exactly when investment in inventory takes place.[29] At the same time, average cash holdings are decreasing in ρ, as shown in Panel F, because the firm is able to finance investments with cash flow. This effect is not pronounced because the firm benefits of funding investment without recurring to external finance also for high values of correlation. The outcome is that c/k is not greatly influenced by ρ.

In summary, we find that savings are a valuable source of financing for investment in inventory, especially when cash flow risk is driven by commodity price risk. However, savings

[29]For example, in the base case, taking the average n/k conditional on all values of p lower than the unconditional mean, we find that for $\rho = 0$ $n/k = 0.43$ while for $\rho = 1$ $n/k = 0.62$.

plays an important role in supporting inventory investment even when the firm benefits from a less volatile cash flow: we find that, when cash flow risk is reduced (lower productivity risk or high correlation), the firm holds a significant amount of cash holdings.

We contrast the prediction from the model on the interaction between cash holdings and inventory to empirical data, by reporting the actual use of inventory and cash holdings for financially constrained and unconstrained firms with different levels of cash flow volatility. In Table III, we report holdings of inventory and cash (scaled by capital) in our Compustat sample of manufacturing firms, by double sorting firms on cash flow volatility and on the tightness of financing constraints.

Specifically, we compute the firm's cash flow volatility as the standard deviation of the firm's operating cash flow scaled by total assets over the entire time span in which a firm is observed and sort firms in two subsets of cash flow volatility divided by the median cash flow volatility.[30] We measure the tightness of financing constraints using three proxies: size (natural log of total assets), a dividend payment dummy, and the Whited and Wu (2006) index (WW).[31] We classify firms in each year as financially constrained (unconstrained) if they belong to the first (fourth) quartile of the size distribution, they do not pay (do pay) dividends, their WW index is in the fourth (first) quartile of the WW index distribution.[32]

In Table III, we observe that financially constrained firms hold more inventory and cash than their financially unconstrained counterparts, regardless of the proxy used to measure financing constraints.[33] White, Pearson, and Wilson (1999) find that smaller firms typically rely more on inventory, as larger firms can afford more efficient production systems (e.g., just-

[30] Opler, Pinkowitz, Stulz, and Williamson (1999) compute the firm's cash flow volatility over the previous 20 years for each firm-year observation in their sample. However, their method is not suitable for our sample, because it would drastically reduce the number of observations. To test the robustness of our conclusions, we computed cash flow volatility using a five years window and obtained results (available upon request) very similar to those reported.

[31] The index of financing constraints estimated by Whited and Wu (2006) is

$$WW_{it} = -0.091CF_{it} - 0.062DIVPOS_{it} + 0.021TLTD_{it} - 0.044LNTA_{it} + 0.102ISG_{it} - 0.035SG_{it},$$

a linear combination of: the ratio of long-term debt to total assets (TLTD), the dividend indicator (DIVPOS), size (LNTA), the ratio of cash flow over total assets (CF), the firm's sales growth (SG), the firm's three digit industry sales growth (ISG). The index can take either sign and directly measures financial constraints (i.e., the more financially constrained a firm is, the higher the WW index).

[32] A popular proxy of financing constraints is the Kaplan and Zingales (1997) index. However, it cannot be used in our setup, because it is computed using the cash ratio, which is endogenous in our analysis.

[33] For both inventory and cash holdings, we test the null hypothesis of equal means between groups of firms with equal degree of financing constraints but different cash flow volatilities, and between groups with equal cash flow volatility but different tightness of financing constraints. We reject the null at 1% level in each test, except in the one testing equal means of inventory between groups with different cash flow volatility classified as constrained using the WW index, where the null is rejected at 5% level.

in-time). However, Chen, Frank, and Wu (2005) find that, although a more efficient supply chain undoubtedly reduces the need to hold inventory of raw materials, manufacturing firms prefer to store raw materials to cope with risks not eliminated by advanced techniques in supply chain management. One such a risk is given by prices fluctuations, as remarked by Chen, Frank, and Wu (2005) who find a significant positive relation between inflation and raw materials holdings, showing that manufacturing firms are actually sensitive to price risk.

Also, in Table III we see that firms exposed to higher cash flow volatility, whether they are constrained or unconstrained, increase the holdings of inventory and cash. Unconstrained firms would not need to hoard liquidity. However, because they are likely to become constrained in the near future, they hold cash, especially when cash flow volatility is high.

More importantly, we find that cash flow risk and financing constraints positively impact the incidence of *both* inventory and cash holdings in the cross-section of manufacturing firms. This result can only be rationalized by the positive synergy between inventory and cash predicted by our model. This marks an important distinction with respect to the previous literature on inventory and financing constraints, which more or less implicitly assumes near perfect substitutability between cash holdings and inventory, as the latter is seen as a reserve of liquidity.

IV. Conclusion

We examined the contribution of inventory to corporate risk management in the context of a dynamic model in which the firm invests in capital, manages commodity price risk using storage, and saves in the presence of costly external finance and endogenous default.

In our model, risk management using inventory is implemented by firms with different levels of net worth. Remarkably, firms with low net worth engage in risk management. This is because inventory does not require collateral or margins, which would impose a trade off between external financing and risk management for a low level of net worth and the cost of risk management using inventory is determined by the current price of the commodity.

The risk management value of inventory crucially depends on the joint distribution of the shocks to cash flow here considered, a productivity shock and a commodity price. Disentangling the effects of each source of cash flow volatility allows to understand in which circumstances firms should rely on inventory as a risk management tool.

We propose a risk management explanation for the incidence in the use of inventory and cash holdings in the cross-section of U.S. manufacturing corporations. In this regard, we find that cash flow risk and financing constraints are important factors that lead manufacturing firms to hold both inventory and cash.

Inventory and cash holdings are typically considered substitutes in operations and in generating liquidity. The intensity of the substitution between inventory and cash is enhanced by technological and regulatory innovations taking place over time. In our paper, we have shown that inventory and cash holdings can be complementary risk management tool, and that the intensity of their synergy is crucially determined by cash flow volatility and by financing constraints.

In summary, the specificity of inventory and cash holdings in managing different risks, the support of cash to investment in inventory, and the possibility of using internal liquidity as a buffer against cash flow shocks when inventory would be ineffective make inventory and cash holdings complementary tools in risk management. Such positive synergy is supported by empirical data from the manufacturing industry, where cash flow risk and financing constraints increase the incidence of both inventory and cash holdings.

Appendix

A. Proof of propositions

Although the manager's decisions on dividends, investment in capital stock, inventory, and cash holdings occur at the same date t, we can separate them in two stages, as in Cooley and Quadrini (2001): in the first stage a default/dividend decision is made; in the second stage the firm decides investment in capital and inventory. We will present the proof for the model based on capital and inventory. The extension to the case with also cash holdings is straightforward.

The two-stage model of the firm is as follows. Proceeding backwards, we define first the *ex dividend* value of equity as

$$\begin{aligned} v(s,e) &= \max_{x'} \beta \int V\left(s', w(s',x')\right) \mu(ds'|s) \\ \text{s.t.} \qquad & e = pn' + k', \end{aligned} \tag{10}$$

where e is the *adjusted net worth* following a payout/default decision of the firm after the current state s has been observed (given past decision variables k and n). The *realized net worth* can be written as

$$w(s,x) = \begin{cases} w_d & \text{if } s \in \mathcal{S}_d \\ w_d + \left[\pi(s) - \pi(s_d)\right] k^\alpha + \left[p(s) - p(s_d)\right] n & \text{if } s \in \mathcal{S}_d^c. \end{cases} \tag{11}$$

In (11), w_d is the default threshold at s, to be defined later on. Given w_d, we define the set

$$\mathcal{S}_d = \{s : w_d \geq \pi(s)k^\alpha - \psi + p(s)n - h(n) + (1-\delta)k\}$$

of the states in which the realized net worth is lower than the default threshold, and $\mathcal{S}_d^c = \mathcal{S} \setminus \mathcal{S}_d$.[34] The value function, which is the *cum dividend* value of equity, is

$$V(s,w) = \max_e \ (1+\lambda)(w-e)^- + (w-e)^+ + v(s,e). \tag{12}$$

[34]Because the default threshold is a contour in the (z,p) space, there can be $s_d^1 = (z_d^1, p_d^1)$ and $s_d^2 = (z_d^2, p_d^2)$ such that $s_d^1 \neq s_d^2$ and $w(s_d^1, x) = w(s_d^2, x)$. This is not a problem as far as the representation of $w(s,x)$ in equation (11) is concerned, because from $w(s_d^1, x) = w(s_d^2, x)$ we have $\pi(s_d^1)k^\alpha + p(s_d^1)n = \pi(s_d^2)k^\alpha + p(s_d^2)n$ and therefore, the right-hand side of (11) in the case $s \in \mathcal{S}_d^c$ is the same using either s_d^1 or s_d^2.

where $a^+ = \max\{a, 0\}$ and $a^- = \min\{a, 0\}$. $V(s, \cdot)$ is a function of the *realized net worth*, w. From (12), e results from w by a dividend decision $d = w - e$, which takes into account the implications on the ensuing investment decisions through $v(s, e)$. However, if the optimal value at the right-hand side of (12) is negative, shareholders prefer to default and set their value, $V(s, w)$, at zero (limited liability). We define the default threshold on realized net worth w_d by condition

$$V(s, w_d) = 0. \tag{13}$$

This last condition closes the model.

First of all, we show that the above description of the model is equivalent to the one in (5)-(7). In particular, (11) is derived as follows. From

$$\begin{cases} 0 & \text{if } w \leq w_d \\ V(s, w) & \text{if } w > w_d \end{cases}$$

we can rewrite the realized net worth as

$$w(s, x) = \begin{cases} w_d & \text{if } s \in \mathcal{S}_d \\ \pi(s)k^\alpha - \psi + p(s)n - h(n) + (1 - \delta)k & \text{if } s \in \mathcal{S}_d^c. \end{cases}$$

Using the definition of the default threshold in the (z, p) space, we set

$$w_d = \pi(s_d)k^\alpha - \psi + p(s_d)n - h(n) + (1 - \delta)k,$$

from which we can derive the second line in (11). To show that the optimal program is the same as in (5)-(7), it suffices to replace $v(s, e)$ from (10) in the right hand side of (12), and consider that the decision $x' = (k', n')$ determines also the dividend, $d = w - e = w - pn' - k'$.

Proof of Proposition 1. Following Cooley and Quadrini (2001), we show that the solution of the program (10)-(13) exists and is unique. We conjecture the existence of a lower bound $\underline{w}$ below which equity capital is raised and an upper bound $\overline{w}$ above which dividends are paid, with $\underline{w} < \overline{w}$. We will prove later on that this is indeed warranted. Based on this conjecture, we can restrict $e \in [\underline{w}, \overline{w}]$.

Given decreasing returns to scale, there is an upper bound k_u such that $k > k_u$ would not be economically profitable and would never be chosen in equilibrium. For similar reasons,

given convex storage costs $h(n)$, there is an upper bound n_u such that $n > n_u$ would never be chosen. Because the domains of e, k, n, and p are bounded, the correspondence

$$\mathcal{F}(s,e) = \{(k',n') : k' \in [0,k_u], n' \in [0,n_u], e = pn' + k'\}$$

that defines the feasible set of the program in (10) is continuous, compact, and convex valued.

In problem (12), the payoff is continuous and strictly increasing in w. Then also $V(s,\cdot)$ is strictly increasing in w, and we can properly define w_d in (13). Using the same argument as in Proposition 5 of Hennessy and Whited (2007), w_d is continuous and non-increasing.

From (10) we define the Bellman operator

$$(Tv)(s,e) = \max_{x' \in \mathcal{F}(s,e)} \beta \int V\left(s', w(s',x')\right) \mu(ds'|s).$$

We now show that this operator maps the set of bounded and continuous functions into itself. Using the same argument as in Cooley and Quadrini (2001), this is because if v is continuous and bounded, then also V is continuous and bounded. The boundedness and continuity of $\int w(s',x)\mu(ds'|s)$ and of V imply, together with the Feller property of μ, that the objective function (10) is continuous and bounded. Because the correspondence $\mathcal{F}$ is continuous, compact, and convex valued, the maximum exists and v is continuous (see Theorem 3.6 in Stokey and Lucas 1989). The resulting function Tv is unique because the operator T is a contraction. The proof of this claim is straightforward showing that T satisfies Blackwell's sufficient conditions, following p. 1739 in Hennessy and Whited (2007). ■

Lemma 1. *The ex dividend value function v in* (10) *is strictly increasing, strictly concave, and differentiable with respect to e.*

Proof of Lemma 1. The argument follows the same logic as in Cooley and Quadrini (2001), so we refer the reader to their paper and report here the parts that are specific to our model. If v is concave and $v(0) \geq 0$, then V is strictly increasing and concave because the dividend $(w-e)^+ + (w-e)^-(1+\lambda)$ is strictly increasing and concave. As w is strictly increasing, then the compound function $V \circ w$ is strictly increasing. Therefore, Tv is strictly increasing.

To show that v is strictly concave, Cooley and Quadrini (2001), on pp 1306-1307, impose restrictions on the conditional distribution $\mu(ds'|s)$. Under these restrictions, to establish strict concavity of $V \circ w$ with respect to $x = (k,n)$ it is sufficient to show that $\int w(s',x)\mu(ds'|s)$ is strictly concave with respect to x. Because we adopt the same distributional assumption on s' as in Cooley and Quadrini (2001), in particular we assume that

the joint conditional distribution of $(\log(z'), \log(p'))$ is Normal, the argument is valid also in our case. In particular, we show that $\int w(s', x)\mu(ds'|s)$ is strictly concave with respect to x. From a direct calculation, we have

$$\begin{aligned}
\int w(s', x)\mu(ds'|s) &= \\
&= \int_{\mathcal{S}_d} w_d \; \mu(ds'|s) + \int_{\mathcal{S}_d^c} \{w_d + [\pi(s) - \pi(s_d)]\, k^\alpha + [p(s) - p(s_d)]\, n\} \, \mu(ds'|s) \\
&\qquad = (1-\delta)k - \psi - h(n) + \mathbb{E}\left[\pi(s')|s\right] k^\alpha + \mathbb{E}\left[p(s')|s\right] n \\
&\qquad\qquad + k^\alpha \int_{\mathcal{S}_d} [\pi(s_d) - \pi(s')]\, \mu(ds'|s) + n \int_{\mathcal{S}_d} [p(s_d) - p(s')]\, \mu(ds'|s).
\end{aligned}$$

The first part, $(1-\delta)k - \psi - h(n) + \mathbb{E}\left[\pi(s')|s\right] k^\alpha + \mathbb{E}\left[p(s')|s\right] n$, is strictly concave in (k, n). The second part, $\int_{\mathcal{S}_d} \{[\pi(s_d) - \pi(s')]\, k^\alpha + [p(s_d) - p(s')]\, n\} \, \mu(ds'|s)$, under the distributional assumptions, is not very sensitive to changes in (k, n), as in Lemma 1 in Cooley and Quadrini (2001). Therefore, the dominating part of $\int w(z', x)\mu(ds'|s)$ is strictly concave, which is what we need.

Finally, differentiability of v with respect to e is a consequence of Theorem 9.10 in Stokey and Lucas (1989), ■

Proof of Proposition 2. From Lemma 1, v is strictly concave and differentiable with respect to e. Therefore, $\partial_e v(s, \cdot)$ is strictly decreasing. From the first order conditions for the optimal e in (12), we can determine $\underline{w}$ from condition $1 + \lambda = \partial_e v(s, \underline{w})$ and $\overline{w}$ from $1 = \partial_e v(s, \overline{w})$. Because $\partial_e v(s, e) > 1 + \lambda$ for $e < \underline{w}$, the optimal dividend in this case is $w - \underline{w} < 0$, which takes the adjusted net worth at $\underline{w}$. On the other hand, from $\partial_e v(s, e) < 1$ for $e > \underline{w}$, the optimal dividend in this case is $w - \overline{w} > 0$, and the resulting adjusted net worth is $\overline{w}$.

Proof of Proposition 3. Lemma 1 establishes strict monotonicity and concavity of v. Hence, the correspondence of the optimal policy is single-valued (i.e., for each (s, w) there is only one $x' = (k', n')$ that maximizes (10)). From Proposition 2, the net worth is adjusted to stay within $[\underline{w}, \overline{w}]$, so the optimal policy from program (10) coincides with the one at $e = \underline{w}$ for all $w < \underline{w}$, and with the one at $e = \overline{w}$ for all $w > \overline{w}$. ■

Proof of Proposition 4. We can establish differentiability of $V(s, \cdot)$ from differentiability of $v(s, \cdot)$, see Lemma 1, and the fact that the payoff function of problem (12) is differentiable for values of $e \neq w$. When the dividend is zero, $e = w$, which occurs for $w \in [\underline{w}, \overline{w}]$, we have $V(s, w) = v(s, w)$. Therefore, differentiability of $V(s, \cdot)$ in this case is a direct consequence

of differentiability of $v(s,\cdot)$. The function V is equal to v for $w \in [\underline{w}, \overline{w}]$, so it is strictly concave in w in that region, and it is linear in w out of that region. ■

Proof of Proposition 5. The Lagrangian function of program (5)-(7) if the firm is currently solvent (i.e. $w > w_d$) is

$$\mathcal{L}(d, k', n', \mathbf{w}') = d\left(1 + \lambda\chi_{\{d>0\}}\right) + \beta \int V(s', w')\mu(ds'|s) - \nu\left[d + pn' + k' - w\right]$$
$$- \beta \int \nu(s')\left[w' - \pi(s')(k')^{\alpha} - \psi - p(s')n' + h(n') - (1-\delta)k'\right]\mu(ds'|s).$$

where ν is the Lagrange multiplier of constraint (7), and $\beta\mu(s'|s)\nu(s')$ is the multiplier of (6). The first order conditions with respect to the decision variables give

$$\nu = 1 + \lambda\chi_{\{d>0\}},$$

$$\nu = \beta \int \partial_w V(s', w(s'))\left[\pi(s')\alpha(k')^{\alpha-1} + (1-\delta)\right]\mu(ds'|s),$$

$$\nu p = \beta \int \partial_w V(s', w(s'))\left[p(s') - h'(n)\right]\mu(ds'|s),$$

and

$$\nu(s') = V_w(s', w(s')) \qquad \text{for all } s \in \mathcal{S}.$$

From these, Proposition 5 follows immediately. ■

B. Numerical methods

Given the properties of the value function, we solve (5)-(7) using a successive approximations method to find V and the optimal policies for capital, inventory, and cash holdings. We discretize the capital set in 61 points chosen as $k_u(1-\delta)^j/2$, for $j = 1, \ldots, 61$. The sets of inventory and cash holdings are discretized in $[0, n_u]$ and $[0, c_u]$ with 61 equally spaced points. The exogenous variables z and p define a reduced-form vector autoregression that we approximate through a discrete-state Markov chain with 9 points for each variable with truncated support in $[-3\sigma_j^u, 3\sigma_j^u]$, $j = p, z$, where $\sigma_j^u = \sigma_j/\sqrt{1-\phi_j^2}$ is the unconditional standard deviation for $j = p, z$. The discrete abscissae and the risk-neutral Markov transition probabilities are computed according to the method proposed by Terry and Knotek (2011), which is based on the Gauss-Hermite quadrature rule, as in Tauchen (1986), but allows for non-zero correlation.

A Monte Carlo simulation is used to generate a sample path for the firm following an optimal policy. We generate a sequence of one million independent draws from a truncated bivariate Normal distribution and generated a path for p and z using the VAR(1) specification. Starting from an initial condition (k_0, n_0, c_0), we apply the optimal policy from the program (5) and generate a simulated path for the firm. We drop the first 1,000 observations, to exclude any influence of the initial condition.

To keep the number of firms in the economy constant, in the event of default, a new company enters the market in place of the old one. The new company is endowed with a level of capital equal to the intermediate value of the grid of k, with no inventory, $n = 0$, and with no cash holdings, $c = 0$. This choice allows the new firm to be considered as a relatively small unhedged company.

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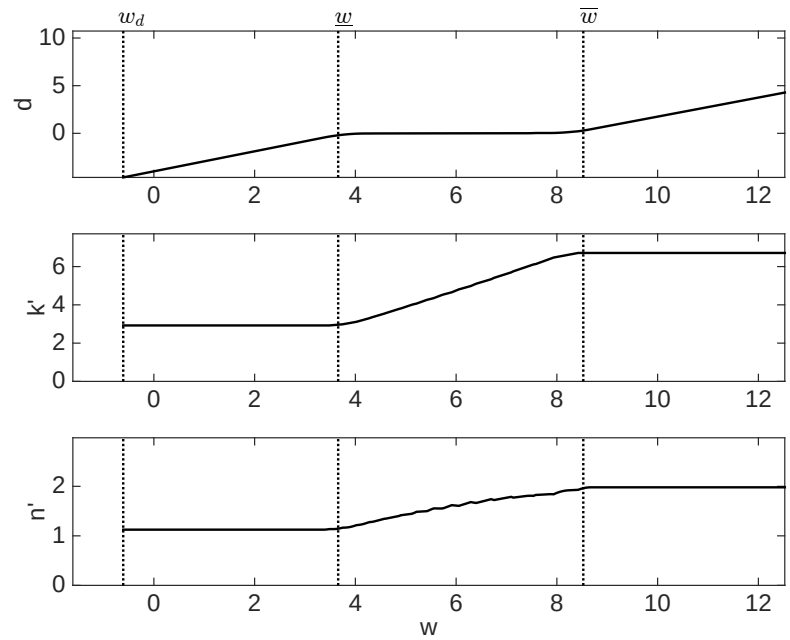
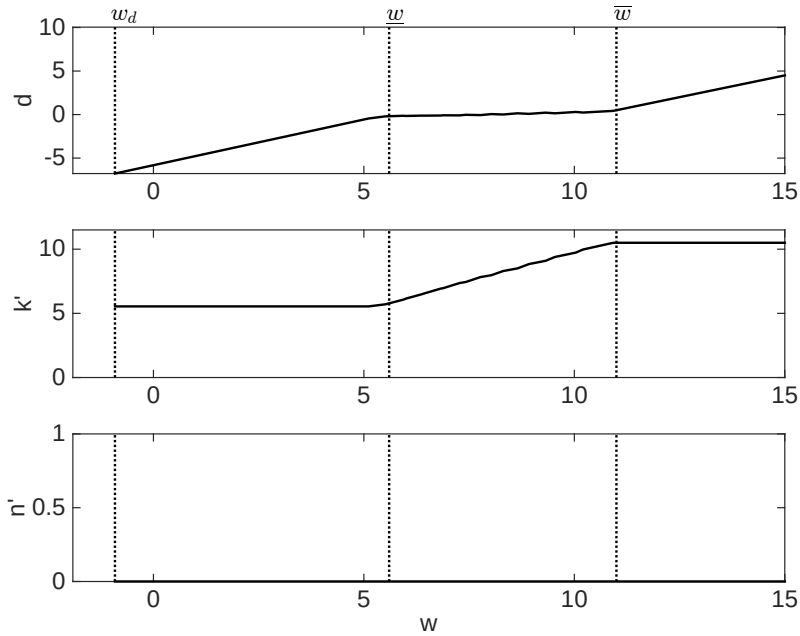


(A)



(B)

Figure 1: **Optimal policies**. We plot optimal policies of dividends d, investment in capital k' and investment in inventory n' against net worth w, at a predetermined state, $s = (z, p)$. In Panel A, $z = 1$ and $p = 0.78$, while in Panel B $z = 1.29$ and $p = 1$. In this figure, w_d, $\underline{w}$, and $\overline{w}$ are defined in Proposition 2. Parameters are as in Table I.

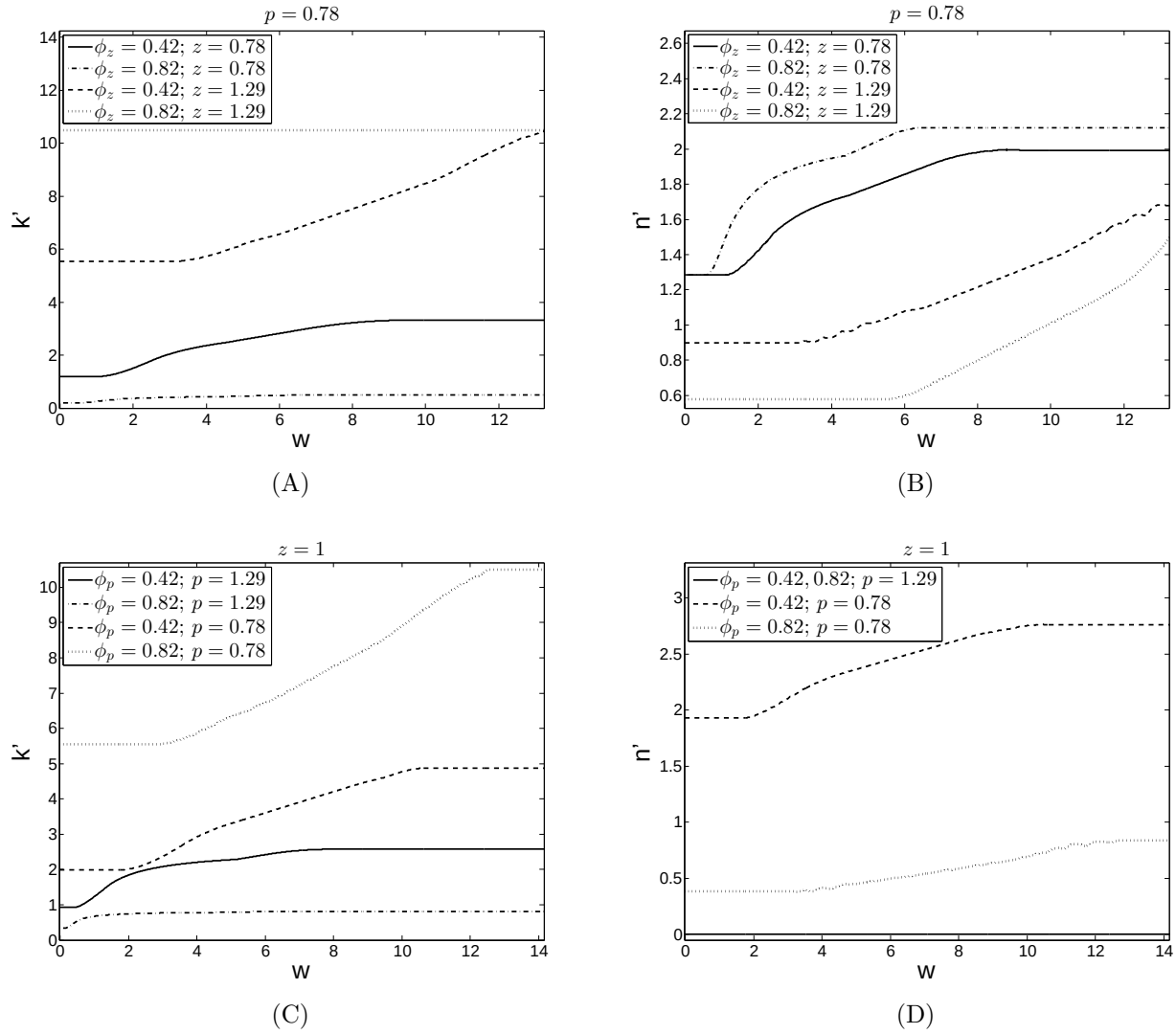


Figure 2: **The effect of persistence**. We plot optimal policies of investment in capital k' and inventory n' against net worth w, for different values of ϕ_z (Panels A and B) and ϕ_p (Panels C and D), at different current states. In Panel A and B, $p = 0.78$ and the productivity shock can be either $z = 0.78$ or $z = 1.29$. In Panel C and D, $z = 1$, and the price can be either $p = 0.78$ or $p = 1.29$. All the other parameters are as in Table I.

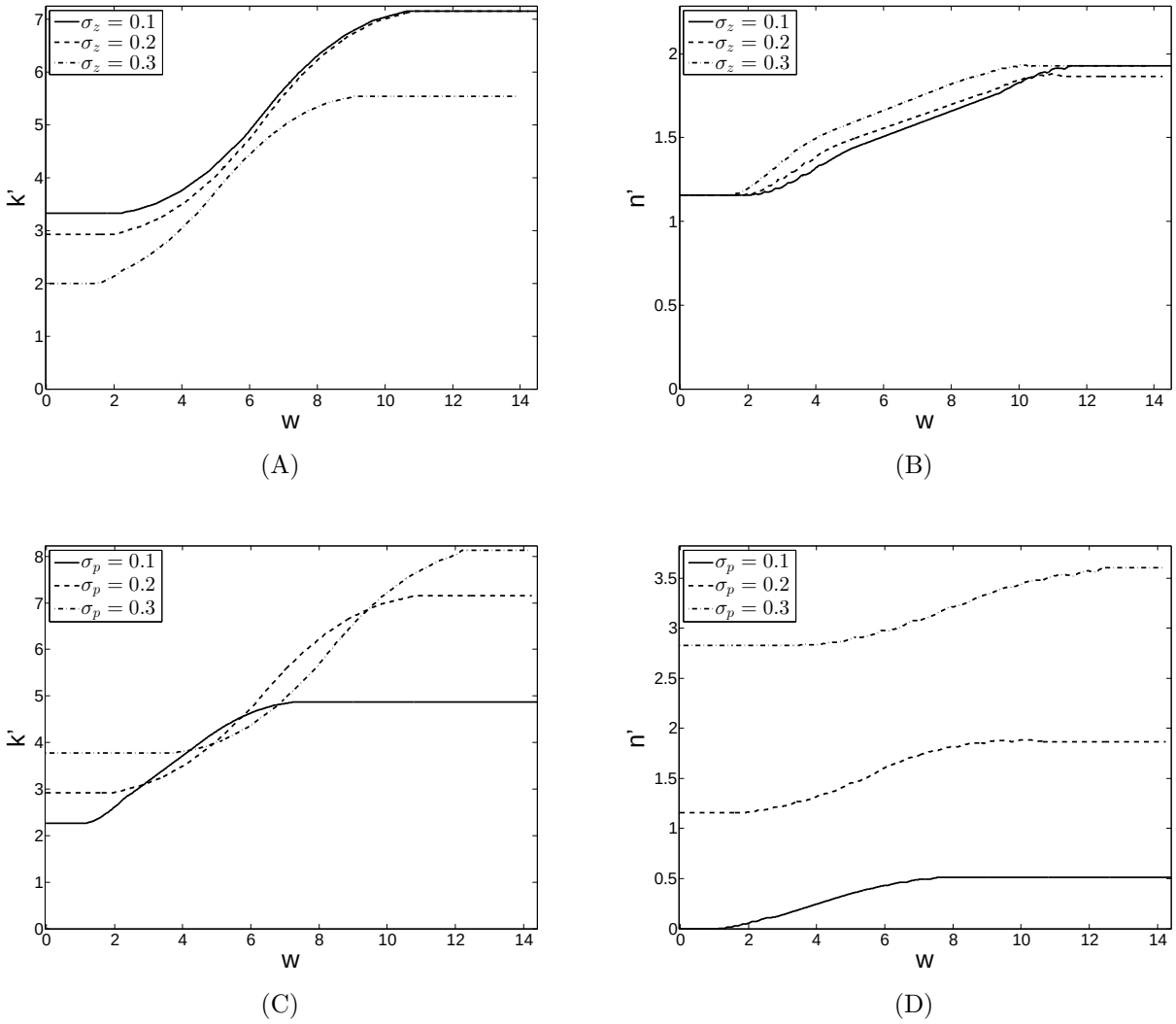


Figure 3: **The effect of volatility**. We show a sensitivity of optimal investment and risk management policies for different values of σ_z (Panels A and B), and of σ_p (Panels C and D). In each case, we compensate for the increase in the average level of $\pi(s)$ due to a higher volatility of either z or p by adjusting the unconditional average of z or p, respectively. All the other parameters are as in Table I.

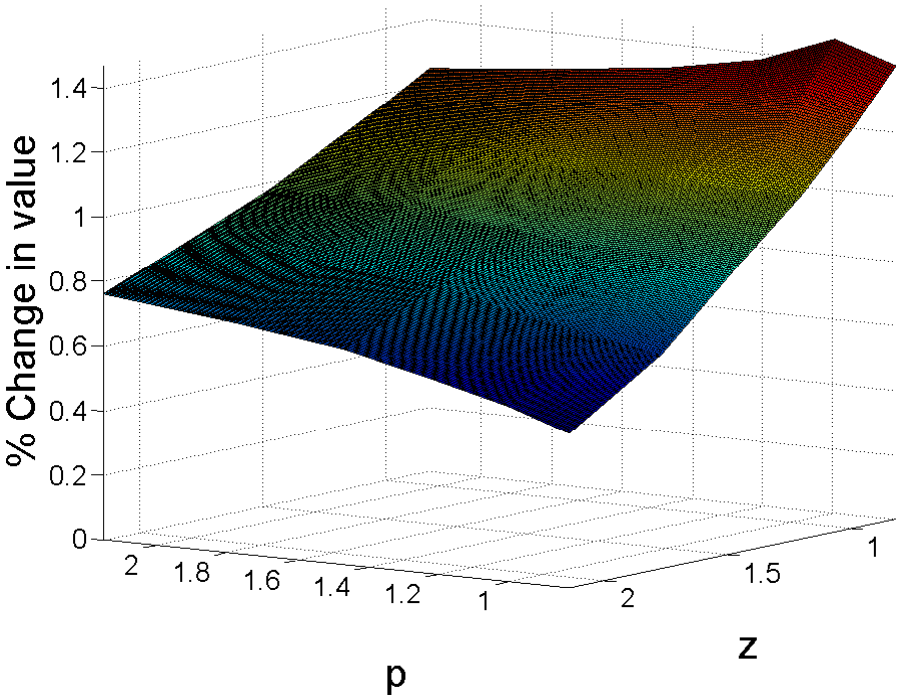


Figure 4: **Interaction between inventory and cash holdings**. We plot against z and p the value created by the interaction between inventory and cash holdings, given by $V^*/V^n - 1$, where $V^* = V - c$ is the enterprise value in the baseline model, and V^n is the value with inventory but no cash holdings. The plot is based on choosing the current k, n, and c equal to the unconditional averages calculated from a simulated sample, using the methodology described in Appendix B.

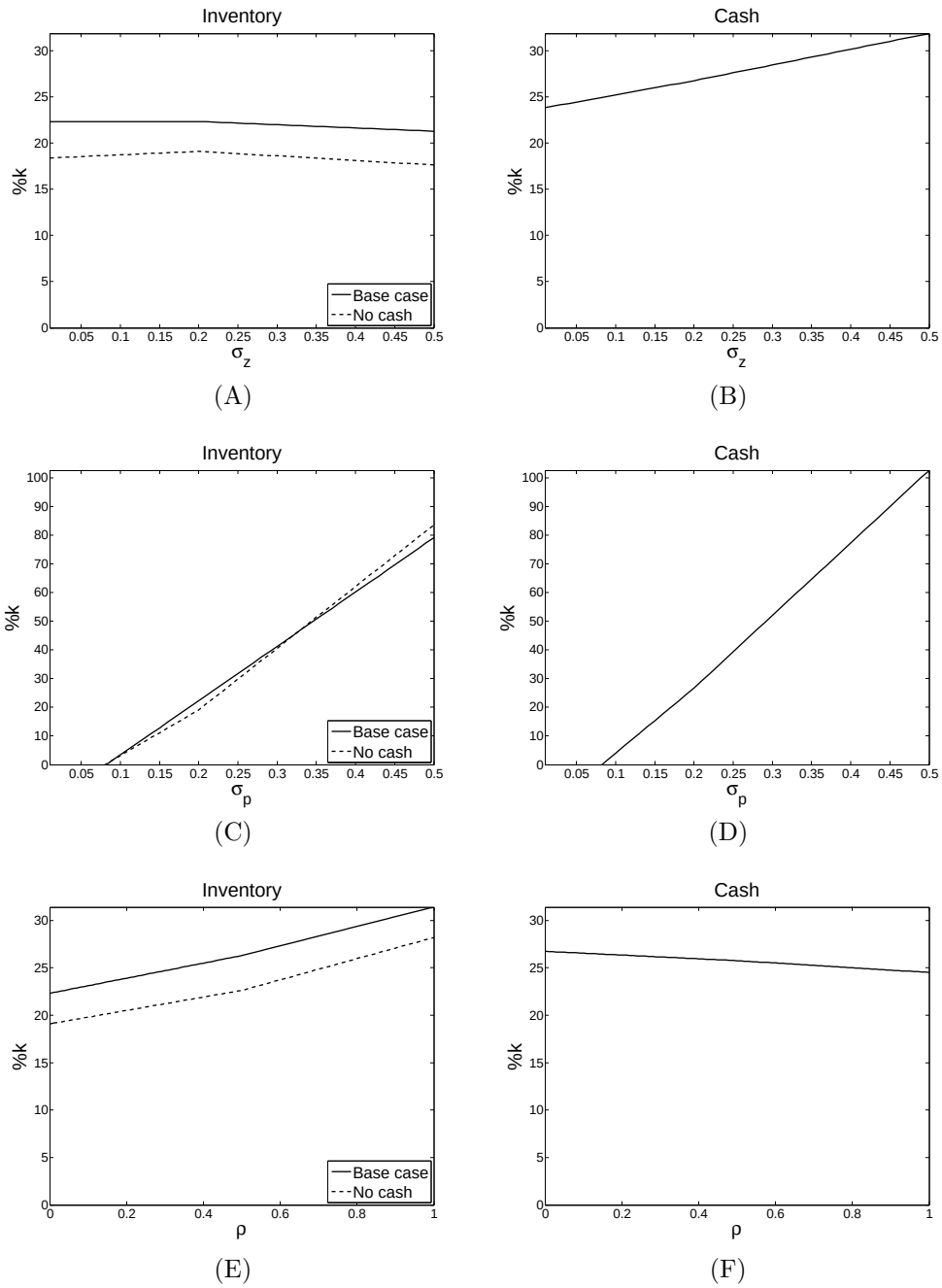


Figure 5: **Cash flow risk, inventory, and cash holdings**. We plot against σ_z in Panels A and B, against σ_p in Panels C and D, and against ρ in Panels E and F, the unconditional averages of inventory, n/k, and cash holdings, c/k, computed from simulated economies of the model. We also show n/k for the model with dynamic inventory but no cash holdings. In all cases, all the other parameters are at the base case value. The numerical methods are described in Appendix B.

Parameter	*Symbol*	*Value*
Capital productivity	θ	0.36
Commodity productivity	γ	0.54
Capital depreciation rate	δ	0.12
Inventory storage costs	η	0.034
Capital adjustment costs	$\overline{\xi}/\underline{\xi}$	0.75/7.5
Fixed operating costs	ψ	0.03
Autoregression of log of z_t	ϕ_z	0.62
Conditional standard deviation of log of z_t	σ_z	0.20
Autoregression of log of p_t	ϕ_p	0.62
Conditional standard deviation of log of p_t	σ_p	0.20
Correlation between logs of z_t and p_t	ρ	0.00
Equity issuance costs	λ	0.05
Discount factor	β	1/1.05
Return on cash holdings	r	0.0462

Table I: **Base case parameters**. Parameters from the calibration of the model to empirical data.

	Empirical	Model
Capital investment/capital, mean	0.13	0.13
Capital investment/capital, std.dev.	0.13	0.13
Probability of negative capital investment	0.03	0.06
Inventory investment/capital, mean	0.02	0.01
Inventory investment/capital, std.dev.	0.11	0.36
Probability of negative inventory investment	0.25	0.19
Inventory/assets, mean	0.08	0.12
Inventory/capital, mean	0.23	0.22
Inventory/capital, std.dev.	0.31	0.36
Cash/assets ratio, mean	0.17	0.17
Cash/assets ratio, std.dev.	0.21	0.15
Market/book, mean	1.84	1.84
Probability of equity issuance	0.25	0.14

Table II: **Calibration**. This table presents the moments used to calibrate the model. Empirical data are a sample of firms in the Compustat North America database in the manufacturing industry (SIC codes 2000-3999) observed between 1969 and 2014. The investment rate in capital stock is calculated as capital expenditures (capx in Compustat) less sale of capital (sppe) scaled by beginning-of-period capital (ppegt). Investment in inventory is the change of the stock of raw materials (invrm) between two consecutive years scaled by beginning-of-period capital. The inventory ratio is inventory scaled by the beginning-of-period capital, and the cash ratio is cash and short-term securities (che) scaled by total assets (at). The market-to-book ratio is the sum of total assets and the market value of equity (prccf x csho) minus the book value of equity (ceq) and deferred taxes (txdb), scaled by total assets. Equity issuance is net sale of common and preferred stock (sstk - prstkc). The moments from the model are based on a Monte Carlo simulation of the model. See Appendix B for details.

Volatility	Size			
	Constrained		Unconstrained	
	Inventory	Cash	Inventory	Cash
Low	0.32	0.66	0.11	0.32
	(0.35)	(2.11)	(0.13)	(0.83)
	Obs = 4524		15559	
High	0.35	1.53	0.13	1.22
	(0.43)	(3.47)	(0.19)	(2.54)
	Obs = 17539		4358	
	Dividends			
	Constrained		Unconstrained	
	Inventory	Cash	Inventory	Cash
Low	0.23	0.88	0.16	0.25
	(0.28)	(2.41)	(0.18)	(0.69)
	Obs = 14763		26694	
High	0.30	1.94	0.23	0.92
	(0.39)	(3.83)	(0.29)	(2.45)
	Obs = 32184		12376	
	WW Index			
	Constrained		Unconstrained	
	Inventory	Cash	Inventory	Cash
Low	0.31	0.68	0.11	0.26
	(0.34)	(2.37)	(0.12)	(0.58)
	Obs = 3514		14308	
High	0.33	1.45	0.13	0.89
	(0.40)	(3.18)	(0.17)	(2.14)
	Obs = 16680		3125	

Table III: **Inventory and Cash Holdings**. We report average raw materials inventory and cash holdings scaled by capital (ppegt in Compustat) for a sample of manufacturing firms from Compustat described in Section B. Firms are sorted in two subsets divided by the median of cash flow volatility, computed as the historical standard deviation of the firm's operating income (oibdp) scaled by total assets for each firm, and on the tightness of financing constraints. We use three measures for this task: size (the natural log of total assets), dividend payments (indicator variable equal to one if the firm pays cash dividends), and the Whited and Wu (2006) index. A firm is classified as financially constrained (unconstrained) if: it belongs to the first (fourth) quartile of the size distribution, it does not pay (does pay) dividends, it belongs to the fourth (first) quartile of the WW index distribution. Standard deviations are in brackets. The null hypothesis of equal means for the same variable across subgroups is tested using a standard t-test. We reject the null hypothesis for all pairs of groups at 1%, except in the test comparing the means of inventory for constrained firms in the WW column with different cash flow volatility, for which the null is rejected at 5% level.