CHAPTER 5: VARIABLE CONTROL CHARTS

Outline

- Construction of variable control charts
- Some statistical tests
- Economic design

Control Charts

- Take periodic samples from a process
- Plot the sample points on a control chart
- Determine if the process is within limits
- Correct the process before defects occur

Types of Data

- Variable data
  - Product characteristic that can be measured
    - Length, size, weight, height, time, velocity
- Attribute data
  - Product characteristic evaluated with a discrete choice
    - Good/bad, yes/no

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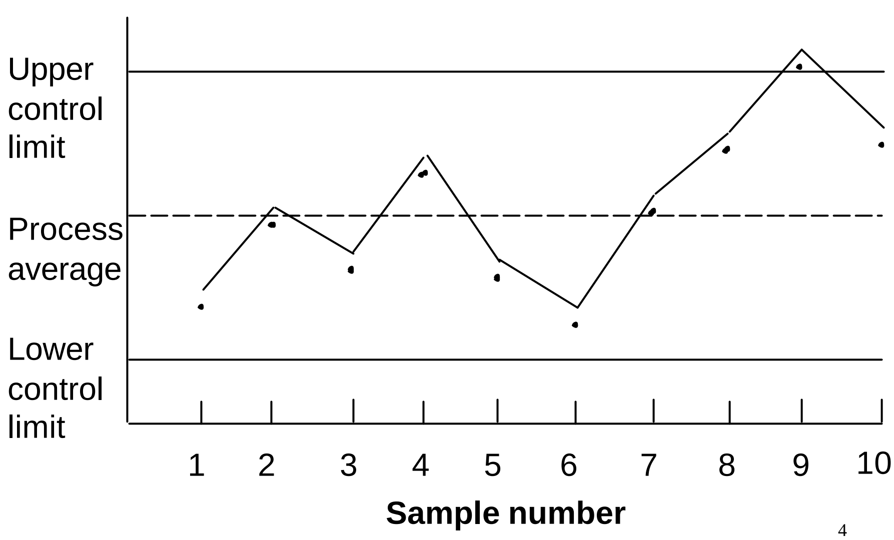
Process Control Chart



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Variation

- Several types of variation are tracked with statistical methods. These include:

1. Within piece variation
2. Piece-to-piece variation (at the same time)
3. Time-to-time variation

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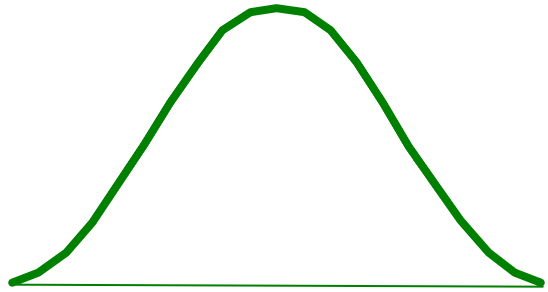
Common Causes

Chance, or common, causes are small random changes in the process that cannot be avoided. When this type of variation is suspected, production process continues as usual.



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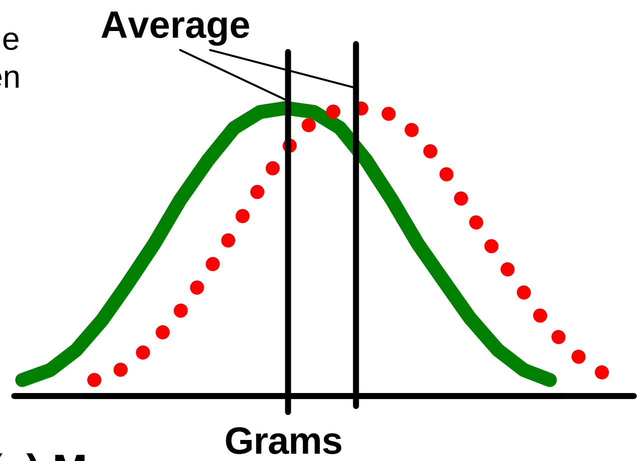
Assignable Causes

Assignable causes are large variations. When this type of variation is suspected, production process is stopped and a reason for variation is sought.

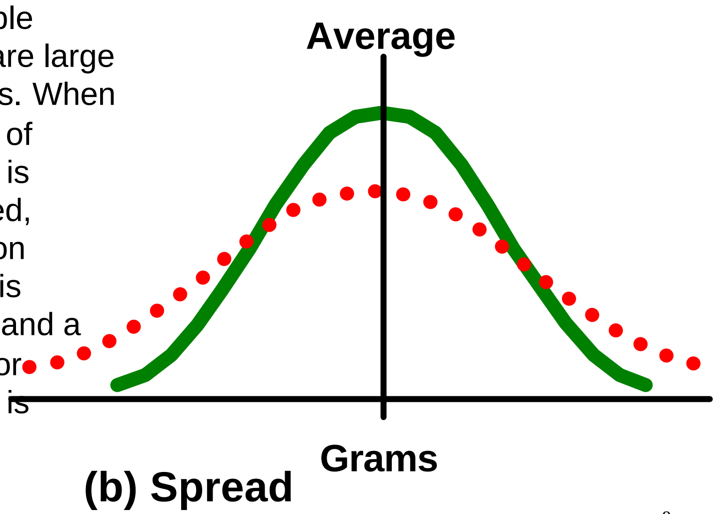


(a) Mean

7

Assignable Causes

Assignable causes are large variations. When this type of variation is suspected, production process is stopped and a reason for variation is sought.

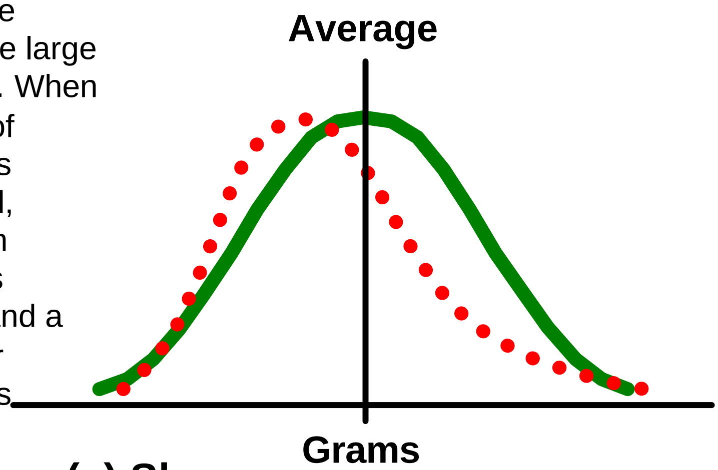


(b) Spread

8

Assignable Causes

Assignable causes are large variations. When this type of variation is suspected, production process is stopped and a reason for variation is sought.



(c) Shape

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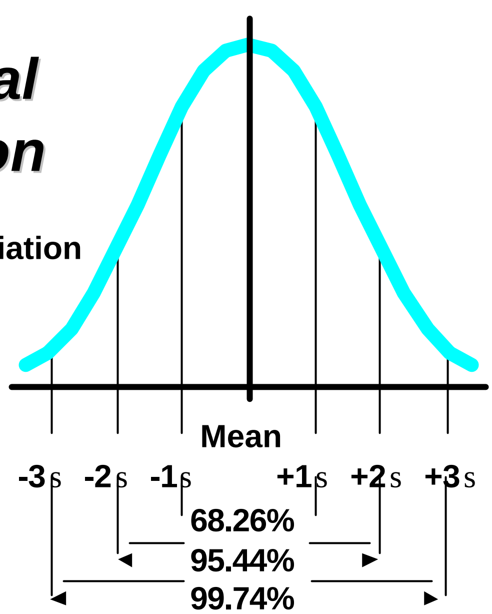
The Normal Distribution

s = Standard deviation



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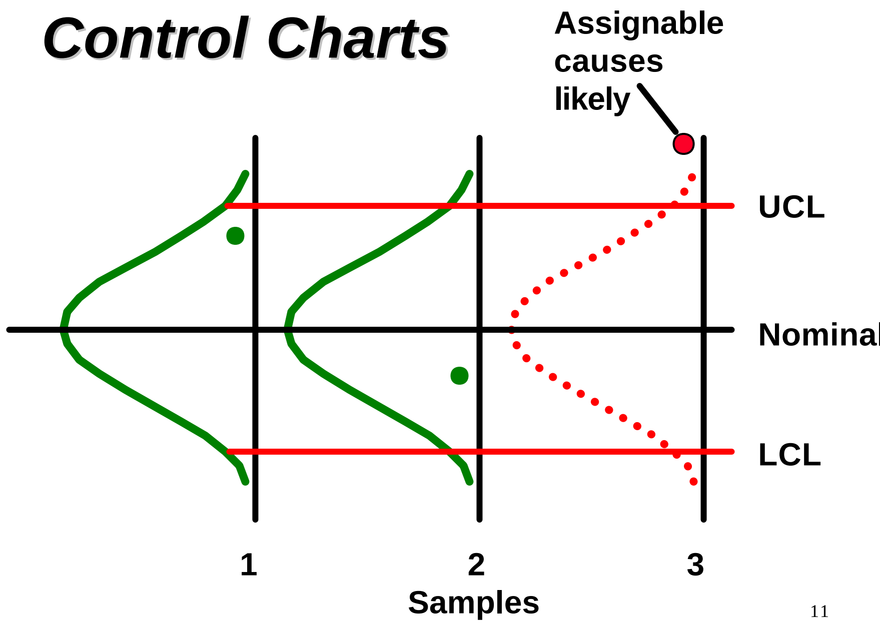
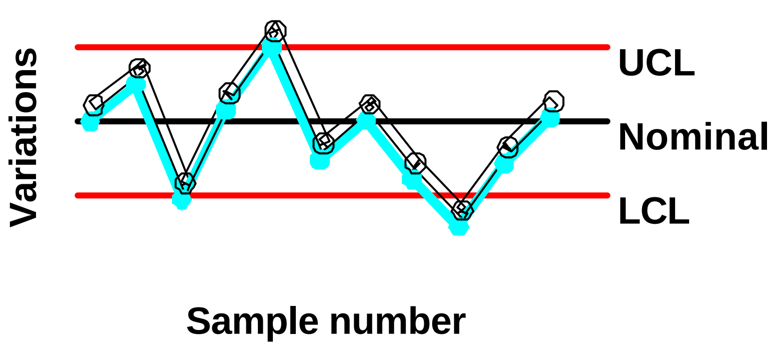
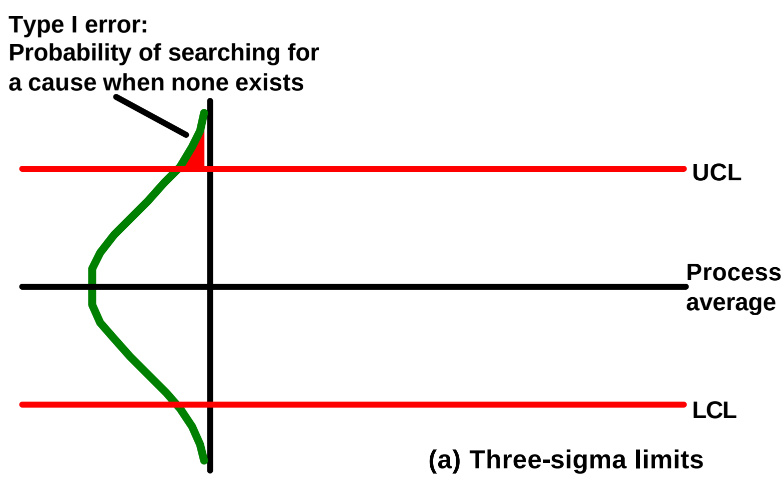
Control Charts



Control Chart Examples



Control Limits and Errors



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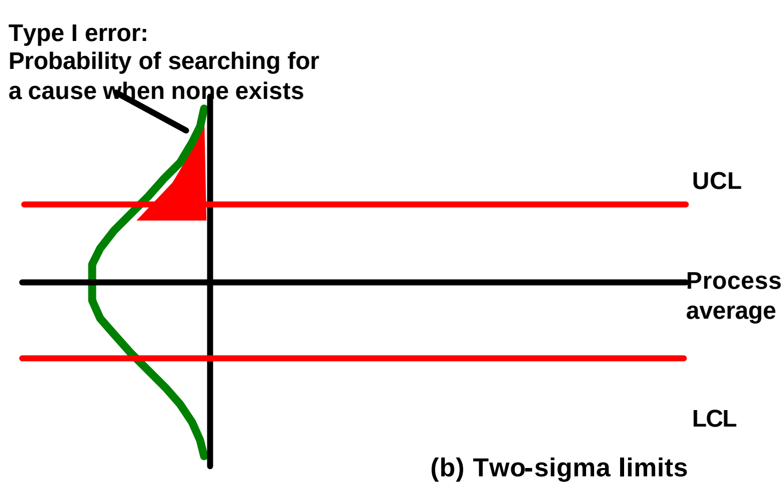
Control Limits and Errors



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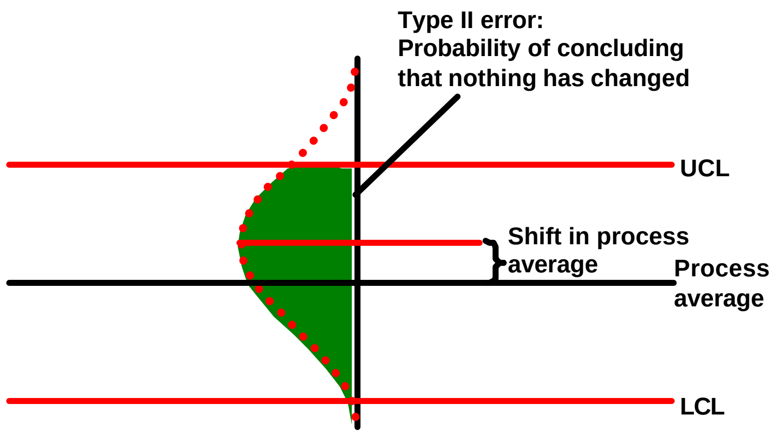
Control Limits and Errors



(a) Three-sigma limits

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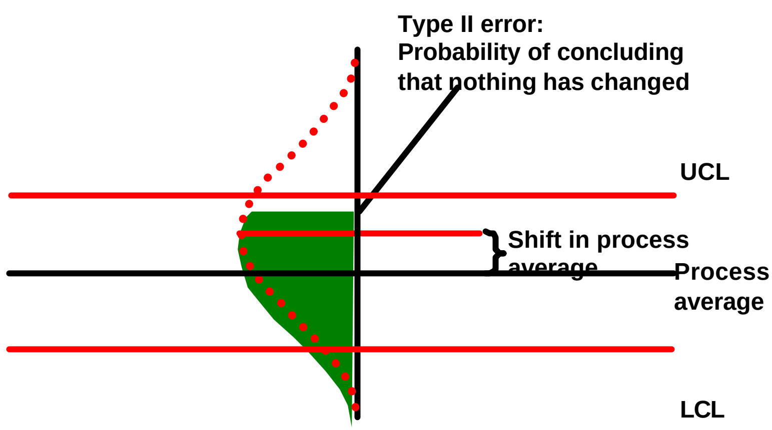
Control Limits and Errors



(b) Two-sigma limits

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Control Charts For Variables

- Mean chart ($\overline{X}$ Chart)
  - –Measures central tendency of a sample
- Range chart (R-Chart)
  - –Measures amount of dispersion in a sample
- Each chart measures the process differently. Both the process average and process variability must be in control for the process to be in control.

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Constructing a Control Chart for Variables

1. Define the problem
2. Select the quality characteristics to be measured
3. Choose a rational subgroup size to be sampled
4. Collect the data
5. Determine the trial centerline for the $\overline{X}$ chart
6. Determine the trial control limits for the $\overline{X}$ chart
7. Determine the trial control limits for the R chart
8. Examine the process: control chart interpretation
9. Revise the charts
10. Achieve the purpose

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Example: Control Charts for Variables

Sample	Slip Ring Diameter (cm) 1	2	3	4	5	$\bar{X}$	R
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Normal Distribution Review

- If the diameters are normally distributed with a mean of 5.01 cm and a standard deviation of 0.05 cm, find the probability that the sample means are smaller than 4.98 cm or bigger than 5.02 cm.

Normal Distribution Review

- If the diameters are normally distributed with a mean of 5.01 cm and a standard deviation of 0.05 cm, find the 97% confidence interval estimator of the mean (a lower value and an upper value of the sample means such that 97% sample means are between the lower and upper values).

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Normal Distribution Review

- Define the 3-sigma limits for sample means as follows:

$$\text{Upper Limit} = m + \frac{3s}{\sqrt{n}} = 5.01 + \frac{3(0.05)}{\sqrt{5}} = 5.077$$

$$\text{Lower Limit} = m - \frac{3s}{\sqrt{n}} = 5.01 - \frac{3(0.05)}{\sqrt{5}} = 4.943$$

- What is the probability that the sample means will lie outside 3-sigma limits?

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Normal Distribution Review

- Note that the 3-sigma limits for sample means are different from natural tolerances which are at $m \pm 3s$

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Determine the Trial Centerline for the $\overline{X}$ Chart

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{m} \overline{X}_i}{m} = \frac{50.09}{10} = 5.01$$

where $m =$ number of subgroups $= 10$

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Determine the Trial Control Limits for the $\overline{X}$ Chart

$$\overline{R} = \frac{\sum_{i=1}^{m} R_i}{m} = \frac{1.15}{10} = 0.115$$

$$\text{UCL}_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} = (5.01) + 0.58(0.115) = 5.077$$

$$\text{LCL}_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} = (5.01) - 0.58(0.115) = 4.943$$

See p. 27 or Text Appendix 2 for the value of A_2

Note: The control limits are only preliminary with 10 samples. It is desirable to have at least 25 samples.

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Determine the Trial Control Limits for the *R* Chart

$$\text{UCL}_R = D_4\overline{R} = (2.11)(0.115) = 2.43$$

$$\text{LCL}_R = D_3\overline{R} = (0)(0.115) = 0$$

See p. 27 or Text Appendix 2 for the values of D_3, D_4

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3-Sigma Control Chart Factors

Sample size	$\bar{X}$-chart	R-chart	
n	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86

Examine the Process Control-Chart Interpretation

- Decide if the variation is random (chance causes) or unusual (assignable causes).
- A process is considered to be in a state of control, or under control, when the performance of the process falls within the statistically calculated control limits and exhibits only chance, or common, causes.

Examine the Process
Control-Chart Interpretation

- A control chart exhibits a state of control when:

1. Two-thirds of the points are near the center value.
2. A few of the points are close to the center value.
3. The points float back and forth across the centerline.
4. The points are balanced on both sides of the centerline.
5. There no points beyond the centerline.
6. There are no patterns or trends on the chart.
    - Upward/downward, oscillating trend
    - Change, jump, or shift in level
    - Runs
    - Recurring cycles

Revise the Charts

A. Interpret the original charts
B. Isolate the cause
C. Take corrective action
D. Revise the chart: remove any points from the calculations that have been corrected. Revise the control charts with the remaining points

Revise the Charts

$$\overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{m - m_d}$$

$$\overline{R}_{new} = \frac{\sum R - R_d}{m - m_d}$$

Where

$\overline{X}_d$ = discarded subgroup averages

m_d = number of discarded subgroups

R_d = discarded subgroup ranges

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Revise the Charts

The formula for the revised limits are:

$$\overline{\overline{X}}_0 = \overline{X}_{new}, R_0 = \overline{R}_{new}$$

$$\sigma_0 = \frac{R_0}{d_2}$$

$$\text{UCL}_{\overline{X}} = \overline{X}_0 + A\sigma_0$$

$$\text{LCL}_{\overline{X}} = \overline{X}_0 - A\sigma_0$$

$$\text{UCL}_R = D_2\sigma_0$$

$$\text{LCL}_R = D_1\sigma_0$$

where, A, D_1, and D_2 are obtained from Appendix 2.

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Reading and Exercises

- Chapter 5:
  - – Reading pp. 192-236, Problems 11-13 (2nd ed.)
  - – Reading pp. 198-240, Problems 11-13 (3rd ed.)

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CHAPTER 5: CHI-SQUARE TEST

- Control chart is constructed using periodic samples from a process
- It is assumed that the subgroup means are normally distributed
- Chi-Square test can be used to verify if the above assumption

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Chi-Square Test

- The Chi-Square statistic is calculated as follows:

$$\chi^2 = \sum^{k} \frac{(f_0 - f_e)^2}{f_e}$$

- Where,

  k = number of classes or intervals

  f_0 = observed frequency for each class or interval

  f_e = expected frequency for each class or interval

  $\sum^{k}$ = sum over all classes or intervals

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Chi-Square Test

- If $\chi^2 = 0$, then the observed and theoretical distributions match exactly.
- The larger the value of χ^2, the greater the discrepancy between the observed and expected frequencies.
- The χ^2 statistic is computed and compared with the tabulated, critical values recorded in a χ^2 table. The critical values of χ^2 are tabulated by degrees of freedom, ν vs. the level of significance, α

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Chi-Square Test

- The null hypothesis, H_0 is that there is no significant difference between the observed and the specified theoretical distribution.
- If the computed χ^2 test statistic is greater than the tabulated critical χ^2 value, then the H_0 is rejected and it is concluded that there is enough statistical evidence to infer that the observed distribution is significantly different from the specified theoretical distribution.

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Chi-Square Test

- If the computed χ^2 test statistic is not greater than the tabulated critical χ^2 value, then the H_0 is not rejected and it is concluded that there is not enough statistical evidence to infer that the observed distribution is significantly different from the specified theoretical distribution.

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Chi-Square Test

- The degrees of freedom, ν is obtained as follows

$$\nu = k - 1 - p$$

- Where,

  k = number of classes or intervals

  p = the number of population parameters estimated from the sample. For example, if both the mean and standard of population date are unknown and are estimated using the sample date, then p = 2
- A note: When using the Chi-Square test, there must be a frequency or count of at least 5 in each class.

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Example: Chi-Square Test

- 25 subgroups are collected each of size 5. For each subgroup, an average is computed and the averages are as follows:

104.98722	99.716159	92.127891	93.79888
97.004707	102.4385	99.61934	101.8301
99.54862	95.82537	95.85889	100.2662
92.82253	100.5916	99.67996	99.66757
100.5447	105.8182	95.63521	97.52268
100.2008	104.3002	102.5233	103.5716
112.0867			

- Verify if the subgroup data are normally distributed. Consider $\alpha = 0.05$

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Example: Chi-Square Test

- Step 1: Estimate the population parameters:

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{n} \overline{x}_i}{n} = \frac{\sum_{i=1}^{25} \overline{x}_i}{25} = 99.92$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} \left(\overline{\overline{X}} - \overline{x}_i\right)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{25} \left(\overline{\overline{X}} - \overline{x}_i\right)^2}{25-1}} = 4.44$$

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Example: Chi-Square Test

- Step 2: Set up the null and alternate hypotheses:

- Null hypotheses, H_o: The average measurements of subgroups with size 5 are normally distributed with mean = 99.92 and standard deviation = 4.44

- Alternate hypotheses, H_A: The average measurements of subgroups with size 5 are not normally distributed with mean = 99.92 and standard deviation = 4.44

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Example: Chi-Square Test

- Step 3: Consider the following classes (left inclusive) and for each class compute the observed frequency:

Class Interval	Observed Frequency f_o
0 - 97	
97 - 100	
100 -103	
103 - ∞	

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Example: Chi-Square Test

- Step 3: Consider the following classes (left inclusive) and for each class compute the observed frequency:

Class Interval	Observed Frequency f_o
0 - 97	6
97 - 100	7
100 -103	7
103 - ∞	5

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Example: Chi-Square Test

- Step 4: Compute the expected frequency in each class

Class Interval	$Z = \frac{\left(x - \bar{\bar{x}}\right)}{s}$	$P(x \le Z)$	p_i	Expected Frequency f_e
0 - 97				
97 - 100				
100 -103				
103 - ∞				

- The Z-values are computed at the upper limit of the class

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Example: Chi-Square Test

- Step 4: Compute the expected frequency in each class

Class Interval	$Z = \frac{\left(x - \bar{\bar{x}}\right)}{s}$	$P(x \le Z)$	p_i	Expected Frequency f_e
0 - 97	-0.657	0.2546	0.2546	6.365
97 - 100	0.018	0.5080	0.2534	6.335
100 -103	0.693	0.7549	0.2469	6.1725
103 - ∞	-------	1.0000	0.2451	6.1275

- The Z-values are computed at the upper limit of the class

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Example: Chi-Square Test

- Sample computation for Step 4:
- Class interval 0-97

$Z = (97 - 99.92)/4.44 = -0.657 \approx -0.66$

For $z = -0.66$ cumulative area on the left,

$P(x \le Z) = 0.2546$ (See Appendix 1)

Hence, $p_1 = P\left(0 \le \overline{x_i} \le 97\right) = 0.2546$

$f_e = n \times p_1 = 25 \times 0.2546 = 6.365$

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Example: Chi-Square Test

- Sample computation for Step 4:
- Class interval 97-100

$Z = (100 - 99.92)/4.44 = 0.018 \approx 0.02$

For $z = 0.02$ cumulative area on the left,

$P(x \le Z) = 0.5080$ (See Appendix 1)

Hence, $p_2 = P\left(97 \le \overline{x_i} \le 100\right) = 0.5080 - 0.2546 = 0.2534$

$f_e = n \times p_2 = 25 \times 0.2534 = 6.335$

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Example: Chi-Square Test

- Sample computation for Step 4:
- Class interval 100 -103

$Z = (103 - 99.92)/4.44 = 0.693 \approx 0.69$

For $z = 0.69$ cumulative area on the left,

$P(x \le Z) = 0.7549$ (See Appendix 1)

Hence, $p_3 = P\left(100 \le \overline{x_i} \le 103\right) = 0.7549 - 0.5080 = 0.2469$

$f_e = n \times p_3 = 25 \times 0.2469 = 6.1725$

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Example: Chi-Square Test

- Step 5: Compute the Chi-Square test statistic:

$$\chi^2 = \sum^{k} \frac{(f_0 - f_e)^2}{f_e}$$

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Example: Chi-Square Test

- Step 5: Compute the Chi-Square test statistic:

$$\chi^2 = \sum^{k} \frac{(f_0 - f_e)^2}{f_e}$$

$$= \frac{(6-6.365)^2}{6.365} + \frac{(7-6.335)^2}{6.335} + \frac{(7-6.1725)^2}{6.1725} + \frac{(5-6.1275)^2}{6.1275}$$

$$= 0.0209 + 0.0698 + 0.1109 + 0.2075$$

$$= 0.4091$$

Example: Chi-Square Test

- Step 6: Compute the degrees of freedom, ν and the critical χ^2 value
  - – There are 4 classes, so k = 4
  - – Two population parameters, mean and standard deviation are estimated, so p = 2
  - – Degrees of freedom,

$$\nu = k - 1 - p =$$

  - – From χ^2 Table, the critical

$$\chi^2 =$$

Example: Chi-Square Test

- Step 6: Compute the degrees of freedom, ν and the critical χ^2 value
  - There are 4 classes, so $k = 4$
  - Two population parameters, mean and standard deviation are estimated, so $p = 2$
  - Degrees of freedom,

$$\nu = k - 1 - p = 4 - 1 - 2 = 1$$

  - From χ^2 Table, the critical

$$\chi^2 = 3.84$$

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Example: Chi-Square Test

- Step 7:
  - Conclusion:

$$\chi^2_{test} = 0.4091 < \chi^2_{critical} = 3.84$$

Do not reject the H_0

  - Interpretation:

There is not enough statistical evidence to infer at the 5% level of significance that the average measurements of subgroups with size 5 are not normally distributed with $\mu = 99.92$ and $\sigma_{\bar{x}} = 4.44$

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Reading and Exercises

- Chapter 5:
  - Reading handout pp. 50-53

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Economic Design of $\overline{X}$ Control Chart

- Design involves determination of:
  - Interval between samples (determined from considerations other than cost).
  - Size of the sample (*n* = ?)
  - Upper and lower control limits (*k* =?)

$$\text{LCL} = m - \frac{ks}{\sqrt{n}} \qquad \text{UCL} = m + \frac{ks}{\sqrt{n}}$$

- Determine *n* and *k* to minimize total costs related to quality control

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Relevant Costs for $\overline{X}$ Control Chart Design

- Sampling cost
  - Personnel cost, equipment cost, cost of item etc
  - Assume a cost of a_1 per item sampled. Sampling cost = $a_1 n$

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Relevant Costs for $\overline{X}$ Control Chart Design

- Search cost (when an out-of-control condition is signaled, an assignable cause is sought)
  - Cost of shutting down the facility, personnel cost for the search, and cost of fixing the problem, if any
  - Assume a cost of a_2 each time a search is required
- Question: Does this cost increase or decrease with the increase of k?

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Relevant Costs for $\overline{X}$ Control Chart Design

- Cost of operating out of control
  - Scrap cost or repair cost
  - A defective item may become a part of a larger subassembly, which may need to be disassembled or scrapped at some cost
  - Costs of warranty claims, liability suits, and overall customer dissatisfaction
  - Assume a cost of a_3 each period that the process is operated in an out-of-control condition
- Question: Does this cost increase or decrease with the increase of *k*?

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Procedure for Finding *n* and *k* for Economic Design of $\overline{X}$ Control Chart

Inputs

- a_1 cost of sampling each unit
- a_2 expected cost of each search
- a_3 per period cost of operating in an out-of-control state
- π probability that the process shifts from an in-control state to an out-of-control state in one period
- δ average number of standard deviations by which the mean shifts whenever the process is out-of-control. In other words, the mean shifts from μ to $\mu \pm \delta\sigma$ whenever the process is out-of-control.

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Procedure for Finding *n* and *k* for Economic Design of $\bar{X}$ Control Chart

The Key Step

- A trial and error procedure may be followed
- The minimum cost pair of *n* and *k* is sought
- For a given pair of *n* and *k* the average per period cost is

$$a_1 n + \frac{a_2(1-b)[p + a(1-p)] + a_3 p}{1 - b(1-p)}$$

where

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Procedure for Finding *n* and *k* for Economic Design of $\bar{X}$ Control Chart

α is the type I error = $2\Phi(-k)$

β is the type two error = $\Phi(k - d\sqrt{n}) - \Phi(-k - d\sqrt{n})$ and

$\Phi(z)$ is the cumulative standard normal distribution function

Approximately,

$$\Phi(z) = 0.500232 - 0.212159\, z^{2.08388} + 0.5170198\, z^{1.068529} + 0.041111 z^{2.82894}$$

$\Phi(z)$ may also be obtained from Table A1/A4
or Excel function NORMSDIST

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Procedure for Finding *n* and *k* for Economic Design of $\overline{X}$ Control Chart

A Trial and Error Procedure using Excel Solver

- Consider some trial values of *n*
- For each trial value of *n*, the best value of *k* may be obtained by using Excel Solver:
  - Write the formulae for α, β and cost
  - Set up Excel Solver to minimize cost by changing *k* and assuming *k* non-negative

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Notes

- Expected number of periods that the system remains in control (there may be several false alarms during this period) following an adjustment

$$E(T) = \frac{1-p}{p}$$

- Expected number of periods that the system remains out of control until a detection is made

$$E(S) = \frac{1}{1-b}$$

- Expected number of periods in a cycle, $E(C) = E(T)+E(S)$

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Notes

- Expected cost of sampling per cycle = $a_1 n E(C)$
- Expected cost of searching per cycle = $a_2[1 + aE(T)]$
- Expected cost of operating in an out-of-control state = $a_3 E(S)$ per cycle
- To get the expected costs per period divide expected costs per cycle by *E*(*C*)

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Problem 10-23 (Handout): A quality control engineer is considering the optimal design of an $\overline{X}$ chart. Based on his experience with the production process, there is a probability of 0.03 that the process shifts from an in-control to an out-of-control state in any period. When the process shifts out of control, it can be attributed to a single assignable cause; the magnitude of the shift is 2σ. Samples of *n* items are made hourly, and each sampling costs $0.50 per unit. The cost of searching for the assignable cause is $25 and the cost of operating the process in an out-of-control state $300 per hour.

a. Determine the hourly cost of the system when *n*=6 and *k*=2.5.

b. Estimate the optimal value of *k* for the case *n*=6.

c. determine the optimal pair of *n* and *k*.

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We have

$\pi = 0.03, \delta = 2$

$a_1 = 0.50$ / unit, $a_2 = 25$ / search, $a_3 = 300$ / hour

Given, $n = 6$, $k = 2.5$

The Type I error, $\alpha = 2\Phi(-k) = 2\Phi(-2.5) = 2(1-\Phi(2.5))$

$= 2(1-0.9938)$ from Table A - 4

$= 0.0124$

For $\delta = 2$, the Type II error, $\beta = \Phi(k - \delta\sqrt{n}) - \Phi(-k - \delta\sqrt{n})$

$= \Phi(2.5 - 2\sqrt{6}) - \Phi(-2.5 - 2\sqrt{6})$

$= \Phi(-2.40) - \Phi(-7.40) = \Phi(-2.40) - 0 = \Phi(-2.40)$

$= 1 - \Phi(2.40)$

$= 1 - 0.9918$ from Table A - 4

$= 0.0082$

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$$E(T) = \frac{1-\pi}{\pi} = \frac{1-0.03}{0.03} = 32.33$$

$$E(S) = \frac{1}{1-\beta} = \frac{1}{1-0.0082} = 1.0083$$

$$E(C) = E(T) + E(S) = 32.33 + 1.0083 = 33.34$$

Cost of sampling per cycle

$= a_1 n E(C) = 0.50(6)(33.34) = 100.02$

Cost of searching per cycle

$= a_2[1 + \alpha E(T)] = 25[1 + 0.0124(32.33)] = 35.02$

Cost of operating in out - of - control condition per cycle

$= a_3 E(S) = 300(1.0083) = 302.49$

Cost per period

$$= \frac{100.02 + 35.02 + 302.49}{E(C)} = \frac{437.53}{33.34} = 13.12$$

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Economic design of X-bar control chart				
Inputs				
Sampling cost, a_1				per item
Search cost, a_2				per search
Cost of operating out of control, a_3				per period
Prob(out-of-control in one period), π				
Av. shift of mean in out-of-control, δ				sigma
n	*k*	α	β	Cost
1				
2				
3				
4				
5				
6				

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Economic design of X-bar control chart				
Inputs				
Sampling cost, a_1			0.5	per item
Search cost, a_2			25	per search
Cost of operating out of control, a_3			300	per period
Prob(out-of-control in one period), π			0.03	
Av. shift of mean in out-of-control, δ			2	sigma
n	*k*	α	β	Cost
1	1.50072829	0.133426	0.30856	17.277317
2	1.89014638	0.058738	0.17405	13.988819
3	2.17004146	0.030004	0.09782	12.916314
4	2.40183205	0.016313	0.055	12.650835
5	2.60636807	0.009151	0.03104	12.750338
6	2.79299499	0.005222	0.0176	13.032513

Click on the above spreadsheet to edit it

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Reading

- Reading: Nahmias, S. "Productions and Operations Analysis," 4th Edition, McGraw-Hill, pp. 660-667.

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$\overline{X}$ and *s* Chart

- The $\overline{X}$ chart shows the center of the measurements and the R chart the spread of the data.
- An alternative combination is the $\overline{X}$ and s chart. The $\overline{X}$ chart shows the central tendency and the s chart the dispersion of the data.

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$\overline{X}$ and *s* Chart

- Why s chart instead of R chart?
  - Range is computed with only two values, the maximum and the minimum. However, s is computed using all the measurements corresponding to a sample.
  - So, an R chart is easier to compute, but s is a better estimator of standard deviation specially for large subgroups

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$\overline{X}$ and *s* Chart

- Previously, the value of σ has been estimated as: $\dfrac{\overline{R}}{d_2}$
- The value of σ may also be estimated as: $\dfrac{\overline{s}}{c_4}$

  where, $\overline{s}$ is the sample standard deviation and c_4 is as obtained from Appendix 2
- Control limits may be different with different estimators of σ (i.e., $\overline{R}$ and $\overline{s}$)

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$\overline{X}$ and *s* Chart

- The control limits of $\overline{X}$ chart are

$$\overline{\overline{X}} \pm 3\frac{\sigma}{\sqrt{n}} = \overline{\overline{X}} \pm 3\frac{\overline{s}}{c_4\sqrt{n}}$$

- The above limits can also be written as $\overline{\overline{X}} \pm A\sigma$ or, $\overline{\overline{X}} \pm A_3\overline{s}$
Where

$$A = \frac{3}{\sqrt{n}}, A_3 = \frac{3}{c_4\sqrt{n}}$$

so, $A = A_3 c_4$ (Appendix 2 gives the values of A and A_3, check)

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$\overline{X}$ and *s* Chart: Trial Control Limits

- The trial control limits for $\overline{X}$ and s charts are:

$\text{UCL}_{\overline{X}} = \overline{\overline{X}} + A_3\overline{s}$ $\qquad$ $\text{UCL}_S = B_4\overline{s}$

$\text{LCL}_{\overline{X}} = \overline{\overline{X}} - A_3\overline{s}$ $\qquad$ $\text{LCL}_s = B_3\overline{s}$

Where, the values of A_3, B_3 and B_4 are as obtained from Appendix 2

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{m} \overline{X}_i}{m}, \overline{s} = \frac{\sum_{i=1}^{m} s_i}{m}$$

$m =$ the number of subgroups

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$\overline{X}$ and *s* Chart: Trial Control Limits

- For **large** samples:

$$c_4 \approx 1, A_3 \approx \frac{3}{\sqrt{n}}$$

$$B_3 \approx 1 - \frac{3}{\sqrt{2n}}, B_4 \approx 1 + \frac{3}{\sqrt{2n}}$$

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$\overline{X}$ and *s* Chart: Revised Control Limits

- The control limits are revised using the following formula:

$$\overline{X}_0 = \overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{m - m_d}$$

$$s_0 = \overline{s}_{new} = \frac{\sum s - s_d}{m - m_d}$$

Where

$\overline{X}_d$ = discarded subgroup averages

m_d = number of discarded subgroups

R_d = discarded subgroup ranges

Continued…

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$\overline{X}$ and *s* Chart: Revised Control Limits

and

$$\sigma_0 = \frac{s_0}{c_4}$$

$$\text{UCL}_{\overline{X}} = \overline{X}_0 + A\sigma_0$$

$$\text{LCL}_{\overline{X}} = \overline{X}_0 - A\sigma_0$$

$$\text{UCL}_s = B_6\sigma_0$$

$$\text{LCL}_s = B_5\sigma_0$$

where, A, B_5, and B_6 are obtained from Appendix 2.

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Example 1

- A total of 25 subgroups are collected, each with size 4. The $\overline{X}$ and s values are as follows:

  $\overline{X}$

  6.36, 6.40, 6.36, 6.65, 6.39, 6.40, 6.43, 6.37, 6,46, 6.42, 6.39, 6.38, 6.40, 6.41, 6.45, 6.34, 6.36, 6.42, 6.38, 6.51, 6.40, 6.39, 6.39, 6.38, 6.41

  $$\sum_{i=1}^{25} \overline{X}_i = 160.25$$

  s

  0.034, 0.045, 0.028, 0.045, 0.042, 0.041, 0.024, 0.034, 0.018, 0.045, 0.014, 0.020, 0.051, 0.032, 0.036, 0.042, 0.056, 0.125, 0.025, 0.054, 0.036, 0.029, 0.024, 0.036, 0.029

  $$\sum_{i=1}^{25} s_i = 0.965$$

  Compute the trial control limits of the $\overline{X}$ and s chart

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Example 2

- Compute the revised control limits of the $\overline{X}$ and s chart obtained in Example 1.

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Reading and Exercises

- Chapter 5
  - Reading pp. 236-242, Exercises 15, 16 (2^{nd} ed.)
  - Reading pp. 240-247, Exercises 15, 16 (3^{rd} ed.)

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Reading and Exercises

- Chapter 6
  - Reading pp. 280-303, Exercises 3, 4, 11, 13 (2nd ed.)
  - Reading pp. 286-309, Exercises 3, 4, 9, 13 (3rd ed.)