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A NEW PRODUCT ANALYSIS AND DECISION MODEL*

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The decision to add, or to reject, or to investigate more fully a new product proposal is one of the most important problems faced by businessmen. The factors surrounding the decision can be mathematically considered by four sub-models in the areas of demand, cost, profit, and uncertainty. The demand model is structured to consider life cycle, industry, competitive and product interdependency effects, and will admit non-linear and discontinuous functions. A cost minimization model is joined to the demand model to formulate a constrained profit maximization problem. The optimization is accomplished by the use of dynamic programming. The final decision is based on the businessman's criterion in combining uncertainty and the rate of return on investment. A practical application of the model is presented to demonstrate the usefulness and problems of a quantitative approach to new product decisions.

Mathematical models and quantitative techniques have found an increasing number of applications as tools for management decision making. They are most useful to management in areas where a high degree of complexity forces an almost complete reliance upon subjective reasoning. One of the most difficult and complex decisions businessmen face is the new product decision. At some stage in a new product's development, the executive must decide if the product is to be introduced, if it is to be rejected, or if more study is needed before a decision can be reached. A nebuli of complex factors relating to sales, costs, investment, and uncertainty surround the decision. This paper develops a mathematical model which considers the significant factors surrounding the new product decision and then, based on empirical data and manager's business judgment, recommends the adoption, the rejection, or further investigation of the product. The total model is based on four sub-models in the areas of demand, cost, profit, and uncertainty. After the models have been developed and the decision environment has been accurately described, dynamic programming is utilized as the basis of the solution method. The central emphasis in the model building is the creation of a realistic model that can be used by businessmen as a tool for new product decision making.

Modeling the Demand for a New Product

The modeling of the demand for the new product can be begun by considering the estimated quantity to be sold in each year of some time period. This estimate of the quantity to be sold in each year is called the life cycle of the product. Estimates of the life cycle sales levels probably would be based on manager's subjective estimates of the product's performance. This complete reliance on business judgment and the executive's understanding of the market may be relaxed under some conditions. If the new product has been test marketed, the

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preliminary sales results may be used as prediction guides. If the new product is analogous to products offered by the firm or its competitors, the growth rate of these products may aid in estimating the life cycle for the new product. The best estimate of the industry sales levels of the new product in future periods is the basic input to the demand model. Since this is essentially a subjective estimate, a distribution of the sales about the expected values should be estimated for each year. The variance of the distributions is used in assessing the level of uncertainty associated with the product.

A question may be raised as to whether the industry sales of the new product or the brand sales of the firm introducing the new product should be estimated. In this model development, industry sales will be utilized as the life cycle estimate; the competitive brand effects will be considered in a separate sub-model.

The estimated life cycle is dependent upon a number of marketing factors. The estimate would be different if a different market price existed for the new product. In fact, the life cycle estimate is supplied with a complete marketing program of price level, advertising expenditure, and distribution effort in mind. This basic program is called the reference marketing program and the corresponding estimate of the quantities to be sold over time is called the reference life cycle.

If the reference price level of the new product were changed, the estimate of the quantity to be sold would change. These changes might be noted by a term of the form:[1]

$$(1) \quad X_{It} = k\bar{X}_{1t}P_{1t}^{EP}$$

X_{It} = industry sales of product one in year "t"

$\bar{X}_{1t}$ = reference industry life cycle sales estimate for product one in year "t"

P_{1t} = average price of product one in year "t"

EP = price elasticity

k = scale constant

This form requires that the price elasticity be constant and lends itself to econometric estimation procedures. Taking logarithms of equation one would produce a linear equation, and properly designed regressions of test market sales data or the sales patterns of analogous products could supply estimates of the price elasticity. Using the exponential form and requiring the elasticity to be constant implies a hyperbolic demand function which may not be reasonable in actual practice. Allowing the elasticity to be a function of price is a tempting alternative, but it results in an inconsistent representation of the demand.[2]

[1] See [13, p. B-106] for models utilizing this form.

[2] Given:

$$X_1 = \bar{X}_{1t}(P_1^{EP})$$

Taking logs:

$$\ln X_1 = EP(\ln P_1) + \bar{X}_{1t}$$

A general form which considers non-linear and discontinuous price-quantity relationships can be formulated. This is based on the concept of a "response function." The response function measures the *proportionate* changes in the level of the reference estimate as a result of an *absolute* change in the variable's level.

For example, price response input might be executive estimates of the sales levels associated with different price levels. These estimates could be plotted on a price-sales graph and fitted by a mathematical equation. Suppose the price-quantity relationship is:

(2) $$X_{It} = a - bP_{1t}^2 + cP_{1t}^3$$

X_{It} = quantity sold of product one in year "t"

a, b, c = constants in year "t"

P_{1t} = price of product one in year "t"

then the price response function for product one in year "t" is:

(3) $$PR_{1t} = a/\bar{X}_{1t} - bP_{1t}^2/\bar{X}_{1t} + cP_{1t}^3/\bar{X}_{1t}$$

$\bar{X}_{1t}$ = reference sales estimate for product one in year "t"

The price response function explicitly describes the proportionate changes in sales that result from changing the price. It always equals one when the price equals the reference price level. The quantity sold in any year is

(4) $$X_{It} = \bar{X}_{1t}PR_{1t}\,.$$

Although this relationship appears to be self-evident, the strength of the formulation lies in the fact that it can be extended to include advertising and distribution responses. For example:

(5) $$X_{It} = \bar{X}_{1t}PR_{1t}AR_{1t}DR_{1t}$$

$\bar{X}_{1t}$ = reference quantity of product one in year "t"

PR_{1t} = price response function for product one in year "t"

AR_{1t} = advertising response function for product one in year "t"

DR_{1t} = distribution response function for product one in year "t"

This equation reflects the changes in the industry sales of the product as a result

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Taking the total differential:

$$dX_1/X_1 = (EP)dP_1/P_1 + (\ln P_1)d(EP)$$

It is now evident that the ($\ln P_1$) $d(EP)$ term does not represent proportionate changes in X. The expression

$$(\ln P_1)\ d[(dX_1/X_1)/(dP_1/P_1)] \neq dX_1/X_1\,.$$

The ($\ln P_1$) $d(EP)$ term results in an inconsistent representation of demand. This is analogously true for all X_{EX} forms when EX varies since then EX is not an elasticity.

of changes in the average industry price, total industry advertising, or total industry distribution level, but does not require these relationships to be defined by any particular mathematical form. Additional demand variables could be added to equation (5). For example, special promotions could be added as a variable if the effects were summarized in a response function that represented the proportionate sales in the product produced by changing the level of special promotion activity.

The estimation of the response functions can be accomplished in three basic ways. The first method is experimentation. Establishing different levels for the variables in each segment of the test market can yield data that are amenable to statistical analysis (see [11]). Laboratory experiments could also be used to generate response data (see [19]). The experimental results could be the basis of defining response functions. Equation (5) does not specify a particular functional form for responses. The form should be the result of the generated data and not vice versa. Given a mathematical representation of the specific data, a specific response function can be formed as demonstrated in equations (2) and (3). The second method of defining the response functions is to analyze historical data generated by similar products. Statistical analyses may produce valid results, but they must be carefully applied to avoid the pitfalls of econometric procedures (see [20]). Both of these methods may be infeasible in given new product situations because the product has not been test marketed and similar historical data does not exist. In this situation, subjective input must be relied upon, but confidence intervals should be established for the estimates so that the overall uncertainty associated with the project can be determined.

The level of the variables is important, but the total response to change in a variable in a given year may depend on the level and sequence of the values of the variable in the previous years. To account for the effects of various sequences, lagged response functions can be added to the equation. The lagged response functions measure the proportionate changes in the reference quantity sold in a year as a result of the absolute level of the variable in previous years. The nature of these lags could be estimated by the application of the distributed lag model of econometrics to sales data of products that have similarity to the expected new product behavior (see [18]). When no existing products are similar to the new product, subjective estimates of the dynamic effects should be used if it is felt that the lagged effects will be significant in evaluating the proposed new product.

In addition to the dynamic effects of sequences of variables, another cumulative effect may be a consideration in new product marketing. This is the effect of introductory campaigns. These initial spurts of promotion are designed to increase the rate of diffusion of the new product innovation. This can be considered in the demand equation by specifying a shift in the reference life cycle. For example, if a Gompertz curve is a reasonable approximation to the life cycle, the dynamics could be incorporated by the equation:

$$\bar{X}_{1t} = G^{B^{t+t_s}}, \text{ where } G,B = \text{constants.} \tag{6}$$

t_s is the shift in the life cycle and it is a function of the size of the initial promotional campaign. The magnitude of the shift could be estimated from the examination of the results of initial promotions for other products. Ideally one would like data resulting from past experiments with initial promotions, but without it, the shift could be approximated from new product life cycle estimates based on different given initial promotions. The aggregate industry demand for the new product can now be described as:

$$X_{It} = \bar{X}_{1t} PR_{1t} LPR_{1t} LLPR_{1t} AR_{1t} LAR_{1t} LLAR_{1t} DR_{1t} LDR_{1t} \cdot LLDR_{1t} \tag{7}$$

PR_{1t} = industry price response function for product one in year "t"

LPR_{1t} = one year lagged price response function for product one in year "t"

$LLPR_{1t}$ = two year lagged price response function for product one in year "t"

AR_{1t} = advertising response function for product one in year "t"

LAR_{1t} = one year lagged advertising response function for product one in year "t"

$LLAR_{1t}$ = two year lagged advertising response function for product one in year "t"

DR_{1t} = distribution response function for product one in year "t"

LDR_{1t} = one year lagged distribution response function for product one in year "t"

$LLDR_{1t}$ = two year lagged distribution response function for product one in year "t"

The total industry sales described in equation (7) are divided among the companies in the industry on the basis of the competitive behavior of the firms. It seems reasonable to assume that the market is split on the basis of the relative marketing effectiveness of each firm in the industry. If all firms entered at the same time, the market share for firm one is:[3]

$$MS_{11t} = \frac{PR_{11t} AR_{11t} DR_{11t}}{\sum_{i=1}^{m} PR_{i1t} AR_{i1t} DR_{i1t}} \tag{8}$$

MS_{11t} = market share for firm one in product market one in year "t"

PR_{i1t} = price response function for firm "i" and product one in year "t"

[3] This is similar to Kotler [13].

AR_{i1t} = advertising response function for firm "i" and product one in year "t"

DR_{i1t} = distribution response function for firm "i" and product one in year "t"

m = number of firms in the industry

The parameters of the firm's and its competitors' response functions could be empirically estimated if test market data were available. The parameter estimates could be found by an iterative search routine to minimize the total variation between the observed and expected market shares.

If competitors enter at different times, equation (8) would not be reasonable because it would not account for any competitive lead the introductory firm may have developed. To account for the competitive advantages gained by early entry, the market share can be expressed as:

$$MS_{11t} = \frac{\sum_{T=t-c}^{t} e_{1T} PR_{11T} AR_{11T} DR_{11T}}{\sum_{T=t-c}^{t} \sum_{i=1}^{m} e_{iT} PR_{i1T} AR_{i1T} DR_{i1T}} \tag{9}$$

e_{iT} = effiiciency of firm "i" in year "T"

c = number of years which are cumulated

The summation is over some period of years and e_{iT} reflects the efficiency of each firm's marketing effort in a given year. This equation indicates that the introductory firm has a time lead and that if the competitor matches his marketing program, he will not receive a full proportion of the market until he has achieved full efficiency. The inefficiency in early years reflects an unwillingness of customers to recognize the new firm's product as equal to the original product. The rate at which e_{iT} approaches one will depend in part on the rate of diffusion of innovation in this product class. The values of e_{iT} can be inferred from estimates of the rate at which competitors will penetrate the market given the reference program for the product. Equation (9) must be fitted to the reference market shares; e_{iT} and c are the parameters in this estimation.

The market share a firm receives also depends upon its competitive strategy. The introductory firm may have a non-adaptive strategy as in the case of price leader or it may follow an adaptive strategy based on sales, market share, or profits. These alternate strategies and counterstrategies can be used to generate a matrix of rewards. Game theory could be utilized to select the best strategy if all other firms were grouped into one competitor [13]. The reward matrix will probably be non-zero sum, but game theory may still yield information concerning convergence and equilibria which is valuable in selecting competitive strategies (see [21]).

The sales of the new product will be affected by interaction between competitors, but the new product may also be affected by other products offered in the firm's product line. These demand interdependencies may be significant. The new product may reduce the sales of other products or it may increase the demand for other products. The interaction effects may be based on price, advertising, or sales effort interdependencies. These can be incorporated into the model by again utilizing the concept of response functions, but now "cross response functions" could be utilized. These measure the proportionate change in the reference quantity of one product as a result of an absolute change in the level of a parameter of another product and could be explicitly estimated by the same procedures utilized in specifying direct response functions. The cross response relationships can be added to the chain of response functions to specify the demand for the new product.

The complete equation for the new product is:

$$X_{ijt} = \bar{X}_{jt}[PR_{jt}LPR_{jt}LLPR_{jt}AR_{jt}LAR_{jt}LLAR_{jt} \cdot DR_{jt}LDR_{jt}LLDR_{jt}] \cdot \left[\frac{\sum_{T=t-c}^{t} e_{ijT}\, PR_{ijT}\, AR_{ijT}\, DR_{ijT}}{\sum_{T=t-c}^{t} \sum_{i=1}^{m} e_{ijT}\, PR_{ijT}\, AR_{ijT}\, DR_{ijT}}\right] \cdot \prod_{k=1;\, k\neq j}^{N} CPR_{ijkt}\, CAR_{ijkt}\, CDR_{ijkt} \tag{10}$$

X_{ijt} = quantity of good "j" sold by firm "i" in period "t"

$\bar{X}_{jt}$ = reference level of industry sales for product "j" in year "t"

PR_{jt} = industry price response function for product "j" in year "t"

LPR_{jt} = one year lagged price response function for product "j" in year "t"

$LLPR_{jt}$ = two year lagged price response function for product "j" in year "t"

AR_{jt} = industry advertising response function for product "j" in year "t"

LAR_{jt} = one year lagged advertising response function for product "j" in year "t"

$LLAR_{jt}$ = two year lagged advertising response function for project "j" in year "t"

DR_{jt} = industry distribution response function for product "j" in year "t"

LDR_{jt} = one year lagged distribution response function for product "j" in year "t"

$LLDR_{jt}$ = two year lagged distribution response function for product "j" in year "t"

PR_{ijt} = price response function for firm "i" on good "j" at time "t"

DR_{ijt} = distribution response function for firm "i" on good "j" at time "t"

e_{ijt} = efficiency of firm "i's" marketing program for product "j" in year "t"

AR_{ijt} = advertising response function for firm "i" on good "j" at time "t"
CPR_{ijkt} = cross price response of product "k's" price on product "j" in firm "i" in period "t"
CDR_{ijkt} = cross distribution response of product "k's" price on product "j" in firm "i" in period "t"
CAR_{ijkt} = cross advertising response of product "k's" advertising on product "j" in firm "i" in period "t"
N = number of interdependent products

Similar equations could be specified for the other products in the firm's product line. When the optimum levels of the demand variables are determined, the maximum total profit generated by these products can be calculated and the new line profit is specified. If the profits of the product line without the new product are estimated and deducted from the new line profits, the change in total line profits is generated. This change is called the "differential profit" and it is a measure of the profits generated by adding the new product when demand interdependencies are considered.

Modeling the Cost Structure for a New Product

If a new product is produced and distributed in a system independent of other products, its cost function may be directly specified in a single equation. When the product shares common production facilities with other products in the line, the cost structure is more complex. When cost interdependencies are present the problem is to minimize the cost of producing a specified product line. Given production requirements for each product in the line, the problem is to minimize:

$$\sum_{j=1}^{n} c_j I_j \tag{11}$$

subject to

$$\sum_{j=1}^{n} a_{ij} I_j \geqq X_i \quad \text{and} \quad \sum_{j=1}^{n} \tilde{a}_{kj} I_j \leqq q_k \quad \text{and} \quad I_j \geqq 0$$

where

c_j = cost per unit of input factor "j"
I_j = amount of input factor "j" utilized
X_i = minimum quantity of good "i" to be produced
a_{ij} = technical production relationships
$\tilde{a}_{kj} = 0$ if $k \neq j$
$\tilde{a}_{kj} = 1$ if $k = j$
q_k = constraint or input factor availability
n = number of input factors

If the unit input costs are constant, linear programming computational routines can be used to solve the problem. If the unit input costs are not constant, piecewise linear programming may be used to solve the non-linear problem.

By specifying various production requirements in terms of the minimum amounts of the new and old product to be produced, the program will calculate the minimum variable cost. If the number of interdependent products and the number of input factors are large, finding the solution to the L.P. may be time consuming. In order to reduce the number of times the program must be repeated in finding the optimum profit, a regression of the linear or log-linear total cost function against the output of the L.P. for a given set of product requirements might be a reasonable approximation to the cost structure. This precludes the necessity of solving the L.P. for each possible mix of product line product requirements. The output of the cost model is the minimum cost of producing the quantities of each product specified by the demand model.

Modeling the Profit for the New Product

The demand model and cost model can be combined to specify the differential profit. Assuming that profit maximization is the objective of the firm in introducing this product, the problem is to maximize the differential profits generated by the new product subject to the constraints on the product and the firm. Constraints on the profit maximization will exist in each year. The productive plant capacity, the size of the sales force, the advertising budget, or the number of trained personnel may be some of the limitations in each year of the planning period.

The maximization of the total differential profit over the firm's planning period for new products can be visualized as a discrete multistage decision process. In each year product variables must be specified, and based on these variables, a differential profit for that year can be specified by the combination of the demand and cost models. The total revenue and total costs for the new line are calculated given the product parameters. After the variables have been tested to see that the firm's constraints are satisfied, the old line profits are deducted from the new line profits to determine the differential profits. The differential profit for each year is discounted at the corporation rate of return and the total differential profit is gained by summing the yearly rewards.

The differential profit in year "t" is expressed by:

$$DP_t = f(P_t, A_t, D_t, S_t) = f(m_t, S_t) \tag{12}$$

where m_t are combinations of the marketing variables in year t and where S_t are combinations of past marketing variables, $S_t = G(m_{t-1}, S_{t-1})$. The function described by equation (12) is specified in equations (6) to (11). Dynamic programming is suited to the analysis of the optimization. The recursion relationship for the maximization of the total discounted differential profit is:

$$TDDP_t(S_t) = \text{Max}_{m_t \varepsilon M_t} [f(m_t, S_t) + TDDP_{t+1}(G(m_t, S_t))] \tag{13}$$

and

$$TDDP_{pp}(S_{pp}) = \text{Max}_{m_{pp} \varepsilon M_{pp}} [f(m_{pp}, S_{pp})]$$

where

pp = last year in the planning period

$TDDP_1(S_1)$ = maximum total discounted differential profit over the planning period

$TDDP_t(S_t)$ = total discounted differential profit earned in year "t" and following years in the planning period

$TDDP_{pp}(S_{pp})$ = discounted differential profit accrued in the last year of the planning period

M_t = set of marketing variables to be considered in year "t"

This deterministic process can be solved by the upstream algorithm of dynamic programming.

The computational limitations of this formulation may be significant. If the number of price, advertising, and distribution mixes to be tested (M_t) and the number of combinations of past mixes (S_t) are large, even high speed computers may be overburdened. The problem is one of restricting the number of decision alternatives to test in each period (M_t) and the state space (S_t). The potential decision set can be reduced by carrying out a grid search procedure in each period. For example, if the quartiles of the relevant intervals of price, advertising, and distribution are tested in all combinations, the best rough estimation of the mix is found. Then the interquartile range can be searched to find a better and more exact specification of the mix. This procedure could be continued until the desired accuracy is obtained. The iterative procedure suggested here is especially useful with an online time-sharing system, since a manager can be used to guide the search. The state space (S_t) could be reduced by assuming all the variables experienced the same lagged effects. Then S_t would be one aggregate variable instead of combinations of the three separate mix elements and the number of S_t would equal M_{t-1}.

The output of the search program is the optimum levels and sequences for the new product parameters of price, advertising, and distribution. The optimization could be re-run with various levels of production constraints and alternate competitive strategies to determine the opportunity costs of specified policies and constraints.

Modeling the Uncertainty Associated with a New Product

The maximum differential profit of new products is an important parameter in the new product decision, but it must be balanced against the uncertainty associated with the product proposal. Since the new product probably will amplify or compensate for profit fluctuations in the existing products offered by the firm, the uncertainty interdependencies should be considered in the decision process. This interdependency can be approached by considering the "differential uncertainty" connected with the new product. The differential uncertainty is the change in the total line uncertainty. Using the variances of the new and old line profits as surrogates for uncertainty, the differential uncertainty could be measured by the standard deviation of the differential profit distribution.

(14)
$$DU^2 = V' + V - 2\,(\text{COV}\,(Pr, Pr'))$$
$$\begin{aligned} DU &= \text{differential uncertainty} \\ V' &= \text{variance of new line profits} \\ V &= \text{variance of old line profits} \\ \text{COV}\,(Pr, Pr') &= \text{covariance of new and old line profits} \\ &= E[(Pr - E(Pr))\cdot(Pr' - E(Pr'))] \\ Pr &= \text{old line profits} \\ Pr' &= \text{new line profits} \\ E &= \text{expected value operator} \end{aligned}$$

The covariance term will be significant since the new line includes all or some of the old line products. The total variance (V) of a group of items can be shown by a variance-covariance matrix:

$$V = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2n} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3n} \\ \vdots & \vdots & & \vdots & \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \cdots & \sigma_{nn} \end{vmatrix}$$

The total variance of the total profit of "n" products is:

(15)
$$V = \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}$$

$$\sigma_{ij} = \text{covariance of "}i\text{" and "}j\text{"} = E[(y_i - E(y_i))\cdot(y_j - E(y_j))]$$

If each product's profit is normally distributed, the variance can be expressed as [17]:

$$V = \textstyle\sum_{i=1}^{n} \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1, i\neq j}^{n} \sigma_{ij}$$

The determination of the direct variance of profits (σ_i^2) can be analytically determined by combining the uncertainty of the quantity and cost estimates. The variance of the distribution of the profit is the joint distribution of the expression:

(16)
$$\text{PROFIT} = P\cdot X - C\cdot X$$
$$\begin{aligned} P &= \text{price} \\ C &= \text{cost per unit} \\ X &= \text{quantity sold} \end{aligned}$$

The variance of this joint profit distribution is:

(17)
$$\sigma^2_{\text{Profit}} = \sigma^2_{Px} + \sigma^2_{xC} - 2\,\text{COV}\,(Px, xC)$$
$$\begin{aligned} \sigma^2_{Px} &= \text{variance of distribution of price times quantity} \\ \sigma^2_{xC} &= \text{variance of distribution of cost times quantity} \\ \text{COV}\,(Px, xC) &= \text{covariance of the two distributions of price times} \\ &\quad\ \text{quantity and cost times quantity} \\ \text{COV}\,(Px, xC) &= E[(Px - E(Px))\cdot(xC - E(xC))] \end{aligned}$$

The mean of the cost distribution is the expected value of the distribution of cost times quantity.

$$E(xC) = E(x)E(C) + \text{COV}(x, C) \tag{18}$$

E = expected value operator

COV (x, C) = the covariance of x and C, which is $E[(x - E(x)) \cdot (C - E(C))]$

If the quantity estimates and unit cost estimates are independent, the covariance term is zero and the mean of the joint distribution is simply the product of the individual means. They would not be independent unless the cost function were linear in the relevant range. The mean of the total revenue distribution (Px) necessary for the calculation of the covariance in equation (17) is simply $P \cdot E(x)$, since price is specified and treated as certain.

The variance of the joint cost distribution for independence of unit cost and quantity is:[4]

$$\sigma_{xC}^2 = \sigma_x^2\sigma_C^2 + [E(x)]^2\sigma_C^2 + [E(C)]^2\sigma_x^2. \tag{19}$$

Substituting price for cost in this formula and remembering that price is considered certain (i.e. $\sigma_p^2 = 0$), the variance for the total revenue distribution is $\sigma_{Px}^2 = p^2 \cdot \sigma_x^2$. These variances can now be substituted into the joint profit equation to calculate the profit variance in a given time period. Once the means and variances of profit are determined for each year, they must be combined to yield an overall mean and variance of the total profit for the period under consideration. The sum of the means of each year when discounted will reflect the best estimate of total profit if the yearly profits are assumed to be serially independent. In dealing with the variances in the demand model, complications are introduced

[4] This is derived from the basic computational formula for variance. The variance of the distribution of the cost times the quantity sold is noted as σ_{xC}.

$$\sigma_{xC}^2 = E(xC)^2 - [E(xC)]^2, \qquad E = \text{expected value operator}$$

since $[E(xC)] = E(x)E(C)$ if x and C are independent (i.e., COV $(x, C) = 0$)

$$\sigma_{xC}^2 = E(x^2C^2) - [E(x)E(C)]^2$$

since $E(x^2C^2) = E(x^2)E(C^2)$ if x^2 and C^2 are assumed to be independent,

$$\sigma_{xC}^2 = E(x^2)E(C^2) - [E(x)E(C)]^2$$

But

$$E(x^2) = \sigma_x^2 + E(x)^2 \quad \text{and} \quad E(C^2) = \sigma_C^2 + E(C)^2$$

so

$$\sigma_{xC}^2 = (\sigma_x^2 + E(x)^2)(\sigma_C^2 + E(C)^2) - E(x)^2E(C)^2$$

or

$$\sigma_{xC}^2 = \sigma_x^2\sigma_C^2 + E(x)^2\sigma_C^2 + E(C)^2\sigma_x^2 + E(x)^2E(C)^2 - E(C)^2E(x)^2$$

or finally, the variance of the total cost distribution is:

$$\sigma_{xC}^2 = \sigma_x^2\sigma_C^2 + E(x)^2\sigma_C^2 + E(C)^2\sigma_x^2$$

by the fact that the entrance of competition is distributed along the time dimension. The combined variance must be calculated for each possible competitive entrance time. These combined variances when weighted by the probability of competition entering at each specific time will give the aggregate total variance of profit. The combined variance, given a specific entrance time for competition and assuming independence of variances, is the sum of the individual yearly variances.

The covariances (σ_{ij}) are as important as the variances. These can be determined by using the procedure suggested by Harry Markowitz [16] or by other subjective methods. After the specification of the variances and covariances has been accomplished, the differential uncertainty and the probabilities associated with the specified deviations from the mean can be calculated as suggested above when given normal or lognormal distributions for all parameters. If the normal or lognormal distributions are not reasonable approximations of the input distributions, a Montè Carlo analysis could be carried out to determine the distribution of differential profits about the mean estimate of differential profit.

Modeling the Decision for the New Product

The differential profit and differential uncertainty must be combined to indicate whether the new product should be introduced (GO decision), should be rejected (NO decision), or should be investigated more fully (ON decision). The risk and return plane must be divided into GO, ON, and NO areas. The GO, ON, and NO areas can be defined by two methods:

(1) Define the total risk-return utility preference map and then by specifying a minimum utility for GO and maximum for NO divide the map into three areas.

(2) Define constraints on the decision process that can be represented on the risk-return plane to divide the areas. These constraints need not be in terms of utility, but some other measure (e.g. profits).

The first approach is very difficult to carry out in practice, since determining a utility map for an individual is difficult and almost impossible for a corporation. There could be a question as to whether a corporation utility function actually exists. The second approach has been formalized by A. Charnes, *et al.* [7]. They propose two constraints to divide the GO, ON, and NO areas. The constraints are based on a probability of the investment making a specified payback and a minimum dollar profit. These constraints can be adapted and utilized for the model proposed in the previous sections.

The constraints chosen to divide GO, ON, and NO areas for this model are:

(1) For a GO decision the probability of obtaining a target discounted rate of return must be greater than a specified level.

(2) For a NO decision the probability of obtaining a target discounted rate of return must be less than a specified level.

These constraints can be expressed in terms of the differential profit and differential uncertainty. For the GO decision the constraint is

$$P\left(\frac{TDDP}{I} \geqq 1\right) \geqq A_G \tag{20}$$

and for the NO decision

$$P\left(\frac{TDDP}{I} \geqq 1\right) \leqq A_N \tag{21}$$

where

A_N = maximum probability for a NO decision

A_G = minimum probability for a GO decision

P = probability operator

I = total investment in new product (assumed to be known)

$TDDP$ = total discounted differential profit, discounted at the target rate of return. This return is achieved when $TDDP/I = 1$.

If the distribution of differential profit is normal, the decision criteria can be expressed in a convenient graphical form.[5]

Equation (20) can be expressed as

$$P(TDDP \geqq I) \geqq A_G$$

or

$$P\left(\frac{TDDP - E(TDDP)}{DU} \geqq \frac{I - E(TDDP)}{DU}\right) \geqq A_G,$$

where

DU = differential uncertainty and $DU > 0$.

Assuming $TDDP$ is normally distributed, $(TDDP - E(TDDP))/DU$ is normally distributed with a mean of zero and a variance of one. The equation can be restated in an equivalent form as $[(I - E(TDDP))/DU] \leqq t_{GO}$, where t_{GO} is the fractile of $(TDDP - E(TDDP))/DU$ associated with A_G. In Figure 1, the shaded area represents the probability required for a GO decision. If $A_G > .5$, then $t_{GO} < 0$, so let $t_{GO} = -|t_{GO}|$, then

$$E(TDDP) \geqq |t_{GO}|\,DU + I \tag{22}$$

is the equation for the GO constraint level of probability of achieving the specified rate of return.

Similarly for a NO decision the constraint is:

$$E(TDDP) \leqq |t_{NO}|\,DU + I, \quad \text{if} \quad A_N > .5 \tag{23}$$

A_N = maximum probability for NO decision

t_{NO} = fractile corresponding to A_N in $N(0, 1)$

[5] These derivations of the graphical properties of the criteria are not identical to, but are based on, proofs by A. Charnes, et al., *DEMON*. The proofs presented here differ in three respects. First they are related to a profit-risk plane of total discounted differential profit—variance of differential profit rather than cash flow profits—variance of quantity sold. Second, this proof is for the normal rather than the lognormal distribution. Third, the constraint is based on a probability of making a specified rate of return rather than or a payback requirement.

If $A_N < .5$, $t_{NO} > 0$, the equation for the NO decision is:

$$E(TDDP) \leqq -t_{NO}DU + I.$$

These constraints can be plotted as straight lines on the certainty equivalence plane and the decision areas can be specified. (See Figure 2.)

A decision is specified when the total discounted differential profit generated

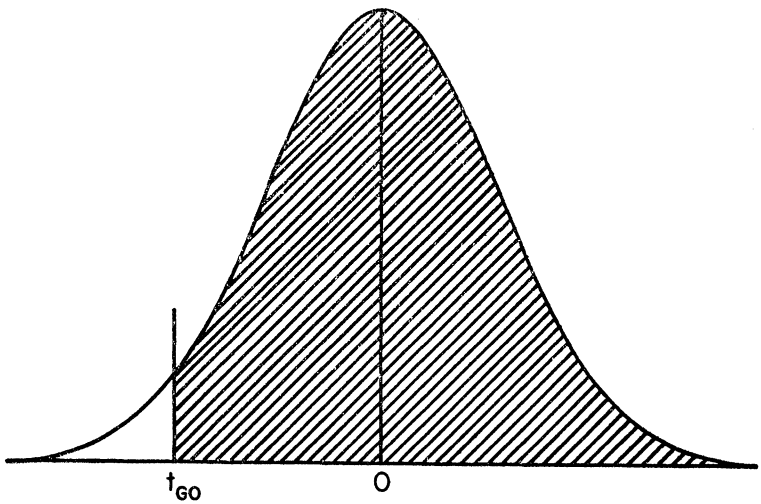


FIGURE 1. GO decision fractile

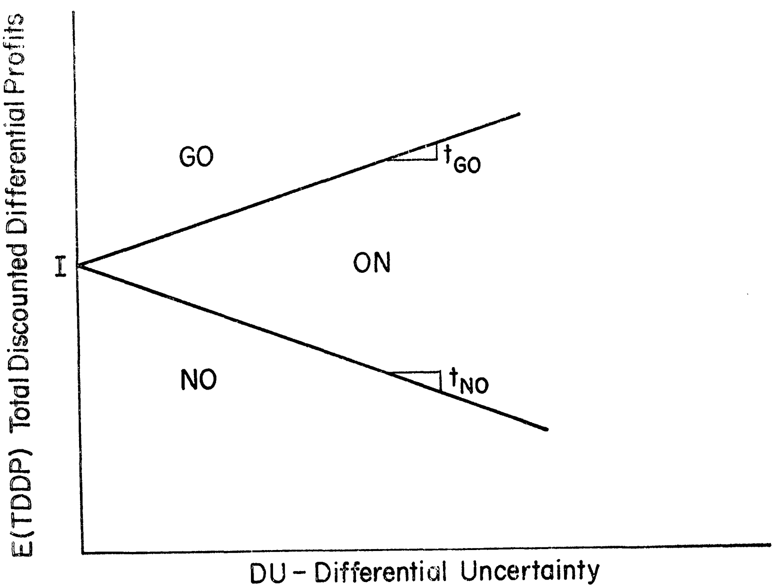
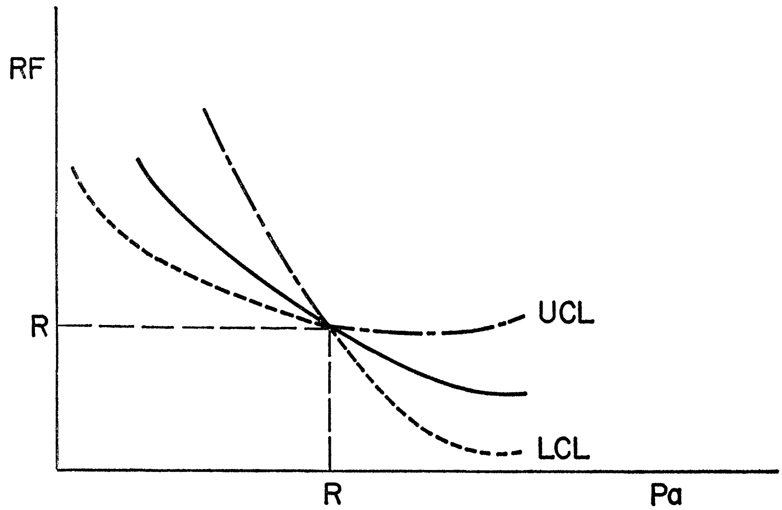


FIGURE 2. Decision Quadrant for $A_G > 50\%$ and $A_N < 50\%$

by the dynamic programming routine (see equation 13) and the differential uncertainty (see equation 14) are plotted on the certainty equivalence plane. This decision format assumes that the project has a single measure of uncertainty. The project may not have the same uncertainty at different commitments when the uncertainty is measured by the variance of the estimated discounted differential profit. As different prices are established, the profit variance may change even if the quantity variance is constant. In fact, the estimates of quantity variance may be different for different levels of price. At the reference quantity all uncertainty is reflected in the distribution about the life cycle estimate, but the price-quantity relationships may be subject to additional estimation uncertainty. This is because the reference estimate is to be the decision maker's best estimate. This may be based on a market test or on past studies relating to the response relationship. If there is additional uncertainty connected with values other than the reference value, this would cause the variance of the differential profit to vary as different price levels are established. For example, the confidence limits may be as in Figure 3. The fact that the uncertainty will vary with different prices, advertising, and distribution poses a problem for the decision model, since now multiple points will be plotted rather than one *TDDP-DU* point. The points will represent different combinations of mean estimates of discounted differential profit and variance based on a different set of trial values of the input variables. (See Figure 4.)



RF = response function
Pa = parameter
LCL = lower confidence limit = - - - - - - - - - - -
UCL = upper confidence limit = —·——·——·—
R = reference level

FIGURE 3. Confidence in response function

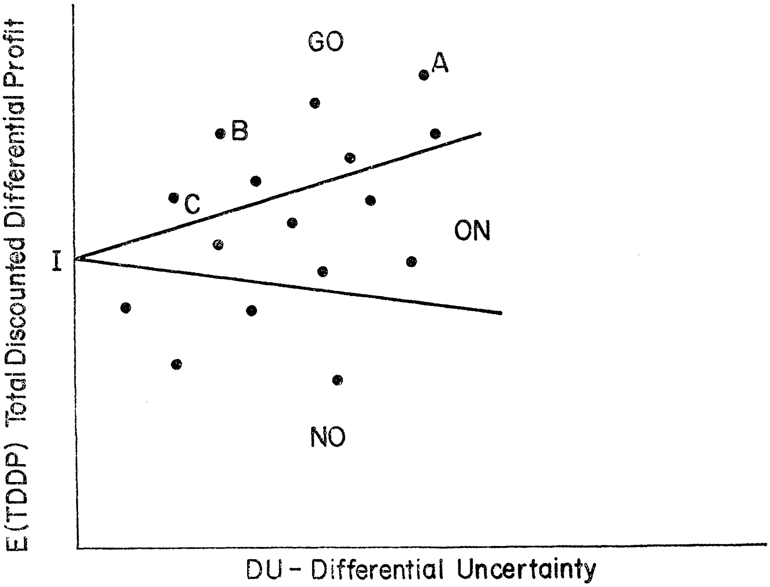


FIGURE 4. Decision quadrant

This complication can be handled in a number of ways. First, it is presumed that the GO area is preferred to the ON area and the ON area is preferred to the NO area whenever possible. This means if any points fall in GO, the decision will be GO and the remaining question is which point in the GO area is to be chosen. If no points are in the GO area and some fall in the ON area, the decision is ON, and the selection of the exact point can be deferred until after the next "best" study. If no points fall in the GO or ON area, a NO decision is reached. The most difficult problem is the choice of the best point if any points lie in the GO area.

If only one point lies in the GO area, the problem will not appear. When more than one point is in the GO area, the selection of the "optimum" point is important since each point represents a different level of commitment to the product and a different marketing mix for the product. This problem can be approached in several ways. The most obvious is a preference approach. The executive could specify the points in increasing order of preference and choose the most preferred as the optimum. This could be a lengthy process if many points were present, but it would be possible.

Another solution is a chance constrained programming approach. Chance constrained programming attempts to solve the problem:

$$\text{(24)} \qquad \begin{aligned} &\text{optimize: } f(c, x) \\ &\text{subject to: } P(Ax \geqq b) \geqq \alpha \end{aligned}$$

where A, b, c are random variables and $P(B) \geqq \alpha$ indicates that the probability of B occurring must be greater than α.[6]

[6] For an explanation of chance constrained programming, see [5].

Although the analytic algorithm of chance constrained programming cannot be applied to the new product model proposed here because of the complex nature of the objective function, it is useful conceptually. If $f(c, x)$ is defined as discounted differential profit ($TDDP$) and if the constraint represents the probability of making the minimum rate of return required at the GO level, the logic of the chance constrained approach is applicable. In this case the problem is:

$$\text{(25)} \qquad \begin{aligned} &\text{maximize: } E(TDDP) \\ &\text{subject to: } P\left(\frac{TDDP}{I} \geqq 1\right) \geqq A_G \end{aligned}$$

$$\begin{aligned} A_G &= \text{minimum GO probability} \\ TDDP &= \text{discounted differential profit} \\ I &= \text{total investment} \\ E &= \text{expected value operation} \\ P &= \text{probability operator} \end{aligned}$$

This formulation is called the "E" model. The "E" model can be solved from the plot of points on the $E(TDDP)$-DU quadrant. For example, if the points are plotted as in Figure 4, the point "A" would be the solution to the single stage chance constrained "E" model. Point "A" has the greatest expected profit level in the GO area. The use of expected value of profit is only one choice of several objective functions. The decision maker may wish to minimize risk. Then the problem is:

$$\text{(26)} \qquad \begin{aligned} &\text{minimize: } DU \\ &\text{subject to: } P\left(\frac{TDDP}{I} \geqq 1\right) \geqq A_G \end{aligned}$$

$$DU = \text{differential uncertainty}$$

This is called the "V" model or more fittingly here, the "DU" model and the solution would be point "C" for this example. (See Figure 4). Point "C" has the lowest value of the differential uncertainty in the GO area. If the businessman were interested in a "satisficing" solution, the problem would be to maximize the probability of achieving the minimum rate of return. For the GO decision criterion this would be:

$$\text{(27)} \qquad \begin{aligned} &\text{maximize: } P\left(\frac{TDDP}{I} \geqq 1\right) \\ &\text{subject to: } P\left(\frac{TDDP}{I} \geqq 1\right) \geqq A_G \end{aligned}$$

This is called the "P" model and the solution to this model in this example would be point "B". (See Figure 4.) Point "B" is the farthest radial distance from the probability constraint line and therefore is associated with the highest probability of any of the points in this example.

The solution of the decision model depends upon the criterion the businessman chooses to use to determine the "optimum." Perhaps the profit maximization model would be the one most commonly used. When this is true, the choice of the points in the GO area is made on the basis of the plotting of the maximum total discounted differential profit (as generated by the dynamic program routine) and the differential uncertainty associated with this program. If the decision maker does not choose profit maximization as the criterion for "optimum," the use of the preference approach or the "P" or "V" chance constrained models would be appropriate and many trial value points would be plotted.

If a GO decision is reached, a commitment to market the product is made. If a NO decision is reached, the product is rejected. If a GO or NO decision is not specified, an ON decision is made since the three decisions form a mutually exclusive and collectively exhaustive set. When an ON decision is specified, an information gathering study is carried out. After the study, the project would be re-evaluated and a new GO, ON or NO decision would be made. In this way, the project proceeds down an information network which eventually ends in a terminal GO or NO decision. The decision maker might, however, find it instructive to look at the plot of *TDDP-DU* and see how much improvement must be made before a GO decision can be reached. If he can see no possible way of achieving the information necessary to reach the GO area, or if he can not justify the funds for an additional study, he may feel a preemptive NO decision is in order.[7]

The decision approach outlined in this paper is analogous to the sequential procedures prepared by A. Wald [25]. He suggested that information may be compiled bit by bit and that a decision be made as soon as the cumulative evidence is sufficient. Much of this analysis deals with specific distributions with known means or variances, but his proof of optimality for sequential testing is general.

Wald showed that the minimization of risk is achieved by a sequential testing procedure and that it produces a smaller expected number of trials than any other method [26]. This means that if costs of studies are greater than zero, the cost of a sequential procedure is less than any other testing method.

To apply Wald's proof to the new product decision model proposed here, one more factor must be considered. The statistical test Wald proposes assumes homogeneous tests at each decision. In fact, however, the model presented here assumes the "best" test will be carried out at each ON step. This further strengthens the optimality characteristics. Based on Wald's proofs, it can be reasonably concluded that the decision model proposed here for new product decisions will produce the minimum number of studies on the average for new product decisions. Since the studies are undertaken in order of decreasing desirability,

[7] See [3] for a Bayesian analysis that might be useful in determining if a pre-emptive NO decision is in order. This approach is also useful in identifying "best" study to carry out if the ON decision is appropriate.

i.e., the best test first, the return on research funds will be maximized. This implies that the optimum use of research funds will be made by a long run application of the proposed decision model.

An Application of the Model to a New Product Problem

The modeling concepts developed in the previous sections have been applied to an actual new product decision problem. A description of this case will clarify the models and will demonstrate the potential advantages of utilizing the quantitative approach to new product decisions.

The case study was carried out in the chemical industry in a company which will be called "Chemi." Chemi had developed a new plastic product and it wanted to know if it should introduce the product, collect more information and study the proposal more extensively, or reject the product. At the initiation of the quantitative analysis, the firm's executive board made a conditional GO decision for the product. This decision was based on an estimate of future sales and the resulting profits. The investment was eight million dollars and their estimate of about 18.5 million dollars of undiscounted cash flow profits was sufficient for them to warrant a conditional acceptance.

Input to Models

The input for the quantitative modeling analysis began with the data underlying the conditional decision and was supplemented almost entirely by subjective business judgments since the product had not been test marketed and was not similar to existing products. In generating the input, it became obvious that the executives had a wealth of information that they had not been able to integrate into their simple analysis. The quantitative approach attempted to utilize all this existing information in the integrated new product decision model.

The most basic input to the model was the estimate of the reference life cycle sales level in each year in the firm's established ten year planning period. The graph of these sales estimates was approximated by an exponential function for the first four years and a Gompertz curve for the last six years. Underlying this sales prediction was a specific marketing program. The price of the product was to be \$350/carton for the first three years and \$250/carton for the last seven years. One per cent of the sales force was to be allocated to the product and an advertising level of \$10,000 per year was to be established. Competition was expected to enter five years after introduction and follow Chemi's price changes, but at a level five per cent below them. The competition was expected to be non-adaptive with respect to Chemi's advertising and sales effort. Questioning of the managers revealed that the reference sales estimate could be significantly shifted if prices in the first three years were lowered to speed the diffusion of new product innovation. Alternate life cycle estimates were generated for each level of initial price and a minimum variation fitting procedure indicated that the shift function was:

$$t_{st} = [((P_t - 280)^2/ -8000) + .61]K_t \tag{28}$$
$$K_1 = .7$$
$$K_2 = .2$$
$$K_3 = .1$$
$$K_t = 0 \quad \text{for} \quad t > 3$$
$$P_t = \text{price of new product in year “}t\text{”}$$

This equation implies that if prices were reduced from \$350/carton to \$280/carton for the first three years, the reference life cycle estimate would occur about seven months earlier. The complete life cycle equation was

$$\bar{X}_{1t} = H(4 - (t + t_{st}))(100e^{1.23(t+t_{st}+1)}) + H(t + t_{st} - 5)\ (1000(10.07)^{1.08(t+t_{st}-5)}) \tag{29}$$

where

$$H(4 - (t + t_{st}))\begin{cases} = 1 & \text{if} \quad t + t_{st} \leqq 4 \\ = 0 & \text{if} \quad t + t_{st} > 4 \end{cases}$$

$$H(t + t_{st} - 5)\begin{cases} = 1 & \text{if} \quad t + t_{st} \geqq 5 \\ = 0 & \text{if} \quad t + t_{st} < 5 \end{cases}$$

In addition to the dynamic effects of low prices in the diffusion process, changes in the market variables in each year would cause the sales level to change. The response functions that monitor these effects were estimated from subjective data supplied by executives concerning the sales results of establishing different price, advertising or distribution levels. For example, the price response function was found to be described by $PR = [268/(P_1 - 104.5)] - .844$. This implies that sales at a price of \$200/carton would be about twice the reference sales level. Similar response functions were specified for the other marketing variables. The response forms changed in each year to reflect the changing sensitivity in each year of the life cycle, but no significant lagged effects were present. The response functions were specified for industry response and were based on the average industry price and total advertising and distribution.

The competitive response function was established by a similar subjective estimation and fitting procedure. In the first four years, there was no competition so the industry response function was Chemi's response function and Chemi had a market share of one hundred per cent. When competition entered, their marketing program was expected to be similar to Chemi's, but it would not be as effective in generating sales since Chemi had an introductory time lead. This competitive efficiency is included in the model's equation (9). The equation was fitted to the reference market shares by a trial and error procedure to minimize the variation between the forecasted market shares and the shares predicted by the equation. The efficiency values for the competitors were $e_{25} = .1$, $e_{26} = .6$, and $e_{2t} = 1.0$ for $t > 6$, and the effects were cumulated over three years. (See equation 9.)

The final demand consideration was the interdependency between the new product and Chemi's existing products. Two other products would feel substitution effects by introduction of the new product. The magnitude of this sales substitution was estimated for the reference program and interaction response functions were specified to reflect the proportionate changes in this penetration if the new product price was changed. These responses were approximated by linear functions for the difference between the new and interdependent product variables. This consideration of interactions between the new and old line completed the input specification for the demand relationship of the model. (See equation 10.)

The costs for the new product were described by a single equation and in fact, it was estimated that constant marginal cost would be appropriate for the product in the relevant range of sales.

The total differential profit for the product was specified when the forecasted old line profits were deducted from the total profits generated by the new line of products. In the case study only the interdependent old products were included in the old line. The new line included these two products and the new product. Since the task for the model is to optimize the differential profit, the constraints on the optimization must be specified. The constraints that were relevant were (1) the maximum advertising budget for the new and old products was $23,000; (2) the maximum total selling commitment for the two old and the new products was two per cent of the sales force time; (3) production of the new product was limited to 2000 units in the first three years, 30,000 units for years four to six, and 60,000 units for the last four years of the ten year planning period; and (4) the quality of technical service was to be maintained.

The technical service constraint was an interesting one. It was the company's policy not to accept a reduction in the quality of service. This constraint was approximated by the amount of the product one technician could service, where the growth in the number of technicians was limited in each year. The constraint expressed in terms of units of x sold was

$$x_{1t} \leqq 36{,}000 \text{ units for } t = 1, 2, 3 \tag{30}$$

$$x_{1t} \leqq (.43(t - 3) + 3)\ 12{,}000 \text{ for } t = 4 \text{ to } 10.$$

The uncertainties in the estimation of the demand and cost inputs were obtained from questioning designed to specify the confidence intervals about the best estimates. All points on the response function in the relevant range were known with equal certainty, so the divergence in the response function confidence intervals as shown in Figure 3 did not occur in the case study. The variances of the estimates were determined and combined as shown in equations (14) to (19). The final distribution of the new product was approximated by a normal distribution. Although there was some skewness in the distribution, the executives felt that it was a reasonable approximation.[8] The covariances between

[8] If a normal or lognormal distribution had not been appropriate, a Monte Carlo simulation would have been required to determine the probabilities associated with the distribution. In this case, the graphical representation in Figure 2 would not be appropriate, but the decision criteria described in the equations (21) and (20) could still be utilized.

the new and old products were estimated by Markowitz's procedure [16]. The procedure prescribes COV $(X, Y) = S_x S_y$ VAR (I) where S_x and S_y are the changes in sales of product x and y for a per cent change in the index (I). In this case the index was the total industry sales of the class of products. The VAR (I) is the per cent variance of the index. The covariances between the new and old products were both positive. The covariance between the new product and the first old product was ten times as great as the covariance with the second old product.

The final inputs to the model were the decision criteria. The firm set a target rate of return on new product investment of fifteen per cent and required a ninety per cent probability of reaching this goal before a GO decision would be made. A NO decision would be made only if there was less than fifty per cent chance of making the target rate of return on investment.

Execution

The profit model described earlier in this section was designed to maximize the discounted differential profit attributable to the product. This maximization would specify the optimum marketing mix for the product in each year of the product's life cycle. In this case study the search was simplified by the fact that there were no lagged price, advertising or distribution response functions, so the size of the dynamic programming formulation was reduced. The only significant dynamic effects to be considered were the effects of the first three years price in shifting the reference life cycle (see equation 29). The profit rewards used in the recursion (see equation 13) were developed by a computer program called SPRINTER.[9] SPRINTER examined a given range of prices, advertising, and distribution levels in all combinations in each year of the life cycle. The generation took place in two stages. First the quartiles of the ranges were considered. After the dynamic program had identified the best combinations of quartiles, the interquartile ranges were searched to find a more exact level for the marketing variables. The test procedure began by selecting values for the marketing mix in each year. The demand for the products in the new line was calculated (see equation 10) and the costs of producing these quantities were deducted to obtain new line profits. If a constraint was violated, the profit was reduced to the level specified by the constraint. Then the estimated old line profits were deducted from the new line profits for the year. This procedure was continued until the set of trial values was exhausted. The search was repeated for each year and the maximum discounted differential profits were determined by the recursion specified in equation (13). The two-stage search program produced estimates of the desired accuracy in about ninety minutes of computer time. It should be pointed out that this computational burden was small because the L.P. cost formulation was not required and no lagged responses existed in the case study. Some simplifications should be expected or may have to be

[9] SPRINTER is an abbreviation for *S*pecification of *PR*ofits with *IN*teraction under *T*rial and *E*rror *R*esponse.

made in each actual application to make the computational requirements of the model reasonable.

Output

The first output was an evaluation of the reference program for the product. If the product was considered independent of other products, 18.5 million dollars of undiscounted profit and 8.35 million dollars of discounted cash flow profits were generated. The total investment for the new product was eight million dollars, and if the product were considered alone, and if certainty were assumed, it might have been accepted since the target rate of return would have been achieved.[10] The new product was not independent, however; significant interdependencies were present. The total discounted differential profit for the new product was only six million dollars. The loss in profits of the two old interdependent products accounted for a reduction in the profit. Given this level of profit, a NO decision would be specified since there was less than fifty per cent probability of making fifteen per cent rate of return on the eight million dollars when only six million dollars of total discounted differential profit was generated.

Although the project would have been rejected at the reference level, this did not have to be true for all marketing mixes over the life cycle. The SPRINTER search routine generated an optimum marketing mix based on lower prices as shown in Table 1 and a discounted differential profit of $10,833,000. The fluctuation in price is due to expanded plant capacity in years four and seven. If this flexibility is not present in given cases, the search could be constrained to prevent cyclical patterns.

The decision with the new price levels based on a differential uncertainty of $2.77 million was ON. The ON decision was specified since there was less than a ninety-per cent chance of achieving the target rate of return on investment, but more than a fifty per cent chance of achieving the investment goal.

Chemi was also interested in the effects of relaxing the constraints on the decision. The search program was re-run with a larger plant capacity and a larger sales force. The results are shown in Table One. The doubling of plant capacity throughout the planning period increased profits to $11,561,000. The new capacity affected the optimum pricing. The new profit maximizing prices were lower than with the smaller plant. The greater profits were accompanied by a greater investment and the probability of achieving the fifteen per cent rate of return on investment was less than the GO criterion value of ninety per cent. The larger sales force also produced greater profits. The relaxing of the sales constraint by increasing the sales force by one half of one per cent increased profits to $12,219,000. The additional sales effort was distributed between the products in the firm's offering and the new product received 1.3 per cent of the total sales effort or .3 per cent of the additional .5 per cent sales capacity force.

[10] The conditional decision made by Chemi did not explicitly consider uncertainty, present values, or product interdependency. The model agreed with the conditional decision when these factors are not considered.

TABLE 1
Case Study Results

	Reference Program	Search Program	Larger Plant	Larger Sales Force
Discounted Differential Profit	6,000,000	10,833,000	11,561,000	12,219,000
Investment	8 mm	8 mm	8.3 mm	8.4 mm
Probability of Achieving Target Rate of Return	less than 50%	86%	88%	91%
Advertising Level Per Year	10,000	10,000	10,000	10,000
Percent of Sales Effort Per Year	1.0	1.0	1.0	1.3
Price in Year				
1	350	250	250	250
2	350	250	250	250
3	350	250	250	250
4	250	180	170	190
5	250	200	160	200
6	250	210	170	210
7	250	170	170	180
8	250	180	180	180
9	250	180	180	190
10	250	190	190	200
Recommended Decision	NO	ON	ON	GO

The increased sales effort changed the optimum prices. The new prices were higher than the prices associated with the regular productive facility. A substitution in the marketing mix between sales effort and price had occurred. The new profit investment uncertainty values indicated a ninety-one per cent probability of achieving the target rate of return, so a GO decision could be made if the sales force constraint were relaxed.

The results of the application of the quantitative analysis suggested that additional study be carried out on the product. A preliminary sensitivity analysis indicated that the new product price response should be investigated in the region of lower prices. Additional study funds would also be usefully allocated to investigation of the possibility of relaxing the sales constraint. The predicted effects of altering the sales force also suggested investigating the total sales force situation in the firm.

The investigation was fruitful to the executives since it indicated a direction of improvement for the new product. Specifically it appeared that lower prices might be advisable since they increased estimated profits about 4.8 million dollars. It was also evident that product interdependencies were important in this case. The quantitative approach had shown the managers implications of their subjective inputs that had not been previously apparent. The structure of the model integrated the complex number of inputs and factors into a meaningful decision form.

Summary

The proposed new product decision model explicitly analyzes demand, cost, allocation, and uncertainty interactions and determines whether a new product should be added (GO decision), should be rejected (NO decision), or should be investigated further (ON decision). The model is capable of analyzing the complex input functions that may be generated by subjective estimates or test market data. Competitive strategies and cumulative competitive effects can be specified and analyzed in the model. The dynamic effects of diffusion of the new product innovation and lagged responses can be comprehended. Input distributions can be non-normal and different estimates of differential uncertainty are allowed at various levels of commitment to the project. The marketing mix effects are mathematically considered, so that a maximizing combination of market parameters will be generated for the new product.

The use of the decision model tells the decision maker when to leave the information network, and if the "best" study is chosen at each ON step, the procedure results in the optimum allocation of research funds in the long run. The output of the model in the GO state is the optimum price, advertising, and distribution marketing mix over the life cycle of the new product.

The new product decision model formulated in this paper assumes that products are proposed and analyzed sequentially. This may not always be the case. Sometimes the problem is one of selecting the best projects from a set of proposed projects. In this case the decision quadrant would have a number of points plotted on it and each point would represent a project. The selection procedures outlined in equations (25) to (27) would be an appropriate method of selection if all the projects were at the same point in the information network. If the projects were at different points in the information network, an "information discounting" scheme would be required to impute the value of the information that would be gained by an optimal transversal of the information network (see [24]).

In addition to this theoretical limitation, a question concerned with the feasibility of the approach could be raised. The model rests on the premise that quantitative inputs can be supplied. These inputs are presumed to be based on empirical data or quantifications of subjective estimates. If neither of these approaches seems practical for an input requirement, the factor being estimated could be dropped from the model; but if the factor is felt to be important, experiments and studies should be undertaken to generate the data needed for reasonable estimates. The estimates need not be known with certainty. The confidence intervals about the test estimates become the input to the uncertainty model and an ON decision will be reached if there is not enough information. The second feasibility question is related to cost. The computation cost and input gathering costs may be large. The scope of the model must be restricted to reasonable limits to overcome this problem. For example, in the case study, the L.P. cost model was replaced by a single equation cost function. The case study indicated that even with these feasibility restrictions, valuable results could be generated. The case study application cost, in total, was about ten thousand dollars. This is a reasonable cost when compared to the eight million dollar investment and four million dollars of estimated additional profits. The feasibility question must be answered in each situation, but keeping in mind the vast sums of money invested in new products, and the high failure rate,[11] it would seem that the proposed quantitative approach would be useful in many practical decision situations.

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CRITIQUE OF:

"A NEW PRODUCT ANALYSIS AND DECISION MODEL"

BENJAMIN LIPSTEIN

Sullivan, Stauffer, Colwell & Bayles, Inc.

Dr. Urban's paper is an important addition to the growing literature on new product decision making. The systematic grouping of the elements involved in new product decisions into four key areas of demand, cost, profit and uncertainty is a very natural and useful arrangement. The further identification of the relevant variables within these four problem areas is also a step forward in sound business decisions. The structuring of the decision processes outlined in Urban's paper can provide a useful framework for establishing an information system within a firm. Its use should trigger the accumulation of data necessary for many of these decision processes.

The generality of the paper is perhaps its major shortcoming. The equations suggested by Urban are what I often refer to as "word equations." These equations specify the variables but do not give their functional form. While this generalized structure permits the inclusion of functional relationships in whatever form seems appropriate for the analysis, it makes the paper less useful for the practitioner since individual investigators still have the very substantial job of deciding on the proper functional relationships of the variables. By way of example, the demand function very casually passes over some very substantial difficulties. Consider singularly the advertising response function. In the real world one must consider that the advertising response function is related to the level of advertising dollars for the new product, the levels of advertising of competitive brands, the awareness levels that exist and that can be achieved, decay effects, the subjective impact of unique copy, trial and repeat rates and a host of lesser issues. One could readily see included in the advertising response function of the demand equation a model of the form described by Amstutz in his recent book, *Computer Simulation of Competitive Market Response*, Chapter 8, "A Model of Consumer Behavior." The same criticism applies to a lesser degree to other variables represented in functional form.

In the business world the new product decision issue is almost always approached sequentially. One proceeds from rough estimates of demand, cost and profit potential to successively more refined estimates. Each experiment ideally contributes to refinement of the estimates. At each step in the process, more resources are committed. In the consumer field the final verification of the demand estimate is evaluated through test marketing.

Dr. Urban alludes to this sequential decision process when referring to A. Wald's testing procedure but does not amplify how his model would specifically be used in this sequential process.

The sequential network decision approach to new product planning follows the usual business firm's approach to the problem. This serves the purpose of conserving executive time and funds and provides for an early "no go" decision for uninteresting products at the development stage. However, there is always the risk that the experimental work has not proceeded far enough to properly evaluate the full potential or profitable alternatives.

This is the advantage of Urban's approach which attempts a more encompassing model for decision making. In practice the decision maker must combine both approaches, the network sequence for developmental purposes, and at key junctures, evaluate the total structure. A wide error range is used in the beginning which is narrowed as more funds are to be committed. Serious omissions in Urban's model are the developmental costs and evaluation procedures. This activity is outside of his model and only enters as a function of the parameters which go into the model. A sub-model is needed for the allocation of research funds.

While the paper implies that sensitivity analyses may be appropriate, I would have hoped that this area had been more extensively developed. One of the most useful aspects of modelling is that it permits sensitivity analyses which guide the economic desirability of conducting additional original research. The model calls for vastly more data than are likely to be available. Research funds, even for the most promising products, will be inadequate to quantify all the parameters. Sensitivity analyses could guide the directions of research expenditures to those parameters which would affect the decision.

Lastly, the author suggests, and rightly so, that through a heuristic approach satisfactory solutions can be obtained from very large and complex systems. One suspects that many organizations shy away from the use of complex models because of a failure to appreciate the effectiveness of deriving heuristic solutions without exorbitant programming and computer time.

In spite of the criticisms, the author provides a framework which other investigators can extend.

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[Footnotes]

[1] **Competitive Strategies for New Product Marketing over the Life Cycle**
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