MGS 8770
Service Operations Management

Data Envelopment Analysis

GeorgiaStateUniversity

Learning Objectives

- Understanding the basics of data envelopment analysis (DEA)
- Solve simple DEA problems

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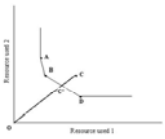
Data Envelopment Analysis (DEA)



- DEA is a linear programming method for comparing the relative productivity (or efficiency) of multiple service units
  - Difficulties in evaluating the relative performance of organizational units
    - Service businesses typically have a very large number of facilities or units that perform similar tasks.
    - There are multiple inputs and outputs.
    - Accounting/financial ratios are often not sufficient.

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Service Companies with Multi-Site Operations

Company	Number of Sites	Company	Number of Sites
Auto-Related		McDonald's	31,129
AAMCO Transmissions	> 700	Subway	17,500
Budget Rent-a-Car	3,240		
Jiffy Lube	2,156	*Hair Styling*	
Meineke Car Care	891	Fantastic Sam's	1,350
Midas (brake/muffler repair)	2,777	Supercuts	1,778
Novus Auto Glass Repair	> 2,200		
		Lodging	
Banks		Choice Hotels	> 5,000
Bank of America	4,495	Marriott	2,600
Wachovia Bank	2,626	Super 8 Motels	> 2,000
Cleaning		*Other*	
Coverall Cleaning Concepts	7,085	Carlson Wagonlit Travel Associates	1,121
Jani-King	> 9,500	Dollar General (discount stores)	6,700
Mitex Indoor Hygenics	> 4,000	GNC (nutrition)	4,811
		Heel Quik! (shoe repair)	732
Desserts		Kumon Math and Reading Centers	1,272
Baskin-Robbins (ice cream)	3,460	Kwik Kurb (concrete services)	> 1,250
Dunkin' Donuts	4,736	Merle Norman Cosmetics Studios	2,006
Tim Horton's (doughnuts)	1,893	Miracle Ear (hearing aids)	1,103
TCBY (frozen yogurt)	2,006	Pearl Vision (eyecare)	811
Yogen Fruz Worldwide, Inc.	> 5,000	Radio Shack	7,113
		Snap-On (tools)	4,680
Fast Food		Wal-Mart (discount stores)	4,800
Burger King	8,246		
Domino's Pizza	5,996		

Issues with Simple Ratios

1. Accounting ratio: *cost per teller transaction*

- Higher ratio would be inefficient relative to others, but could be explained by: (1) Mix of outputs transactions (selling CD's vs simple deposits); (2) Mix of inputs (use of ATM's plus live tellers)

2. Broad based measure: *return on investment*

- Is not sufficient to evaluate operating efficiency of individual decision making unit (DMU). For example: A profitable branch could be the result of higher-than-average proportion of revenue generating transactions rather than cost-efficient use of resources.

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Advantages of DEA

- DEA can handle multiple inputs and multiple outputs. Inputs and outputs can have very different units.
- DEA provides a single comprehensive measure of performance.
- It doesn't assume any specific functional form relating inputs to outputs.
- DEA provides valuable information for less efficient DMUs on how to improve. For each inefficient DMU, DEA also provides a peer best-practice group for reference purposes.

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Efficiency Measure in DEA

$$\text{Efficiency} = \frac{\text{Results Obtained}}{\text{Resources Used}} = \frac{\text{Weighted Sum of Outputs}}{\text{Weighted Sum of Inputs}}$$

$$= \frac{u_1 O_{1j} + u_2 O_{2j} + \cdots}{v_1 I_{1j} + v_2 I_{2j} + \cdots}$$

Where u_1 = weight given to output 1,
O_{1j} = amount of output 1 from unit j
v_1 = weight given to input 1
I_{1j} = amount of input 1 used by unit j

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The DEA Model —Fractional Form

Objective Function $\max E_e = \dfrac{u_1 O_{1e} + u_2 O_{2e} + \cdots + u_M O_{Me}}{v_1 I_{1e} + v_2 I_{2e} + \cdots + v_N I_{Ne}}$

Constraints $\dfrac{u_1 O_{1k} + u_2 O_{2k} + \cdots + u_M O_{Mk}}{v_1 I_{1k} + v_2 I_{2k} + \cdots + v_N I_{Nk}} \le 1.0 \qquad k = 1, 2, \ldots, K$

$u_j \ge 0 \quad j = 1, 2, \ldots, M$

$v_i \ge 0 \quad i = 1, 2, \ldots, N$

E_k = efficiency ratio of unit k, k=1,2,…,K, K = total # of units
u_j = weight given to output j, j=1,2,…,M, M = total # of outputs
v_i = weight given to input i, i=1,2,…,N, N = total # of inputs
O_{jk} = amount of output j from service unit k
I_{ik} = amount of input i used by service unit k

e = index of the unit being evaluated

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Scaling inputs to sum to 1.0

DEA in Standard LP Form

Objective Function:

$$\max \quad E_e = u_1 O_{1e} + u_2 O_{2e} + \cdots + u_M O_{Me}$$

Subject to:

$$u_1 O_{1k} + u_2 O_{2k} + \cdots + u_M O_{Mk} - v_1 I_{1k} - v_2 I_{2k} - \cdots - v_N I_{Nk} \le 0 \qquad k = 1, 2, \ldots, K$$

$$v_1 I_{1e} + v_2 I_{2e} + \cdots + v_N I_{Ne} = 1$$

$$u_j \ge 0 \quad j = 1, 2, \ldots, M$$

$$v_i \ge 0 \quad i = 1, 2, \ldots, N$$

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Example

Burger Palace

Burger Palace is a drive-in-only chain that has six units. Only standard meal (burger, fries, and drink) is available. The inputs are labor-hours and material dollars and the output is 100 meals sold.

Service unit	Meals sold	Labor-hours	Material dollars
1	100	2	200
2	100	4	150
3	100	4	100
4	100	6	100
5	100	8	80
6	100	10	50

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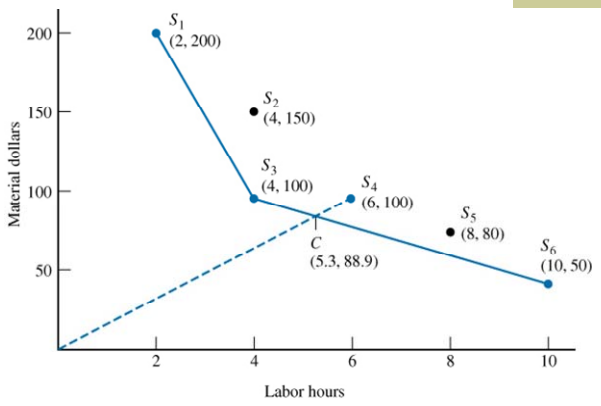
Productivity Frontier



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LP Model for Service Unit 1

Objective Function:

$$\max E(S_1) = u_1 100$$

Subject to:

$$\begin{aligned}
u_1 100 - v_1 2 - v_2 200 &\le 0 \\
u_1 100 - v_1 4 - v_2 150 &\le 0 \\
u_1 100 - v_1 4 - v_2 100 &\le 0 \\
u_1 100 - v_1 6 - v_2 100 &\le 0 \\
u_1 100 - v_1 8 - v_2 80 &\le 0 \\
u_1 100 - v_1 10 - v_2 50 &\le 0 \\
v_1 2 + v_2 200 &= 1 \\
u_1, v_1, v_2 &\ge 0
\end{aligned}$$

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LP Model for Service Unit 2

Objective Function:

Subject to:

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Excel Formulation for Unit 1

	A	B	C	D	E	F	G
1	**Burger Palace**						
2							
3	Variables	U1	V1	V2			
4	Value	1	1	1			
5							
6	Objective						
7	Function	100					
8							
9	Constraints				LHS		RHS
10	Unit 1	100	-2	-200	-102	<=	0
11	Unit 2	100	-4	-150	-54	<=	0
12	Unit 3	100	-4	-100	-4	<=	0
13	Unit 4	100	-6	-100	-6	<=	0
14	Unit 5	100	-8	-80	12	<=	0
15	Unit 6	100	-10	-50	40	<=	0
16	Inputs	0	2	200	202	=	1

=100*B4

=SUMPRODUCT ($B$4:$D$4, B10:D10)
(copy to E11 through E16)

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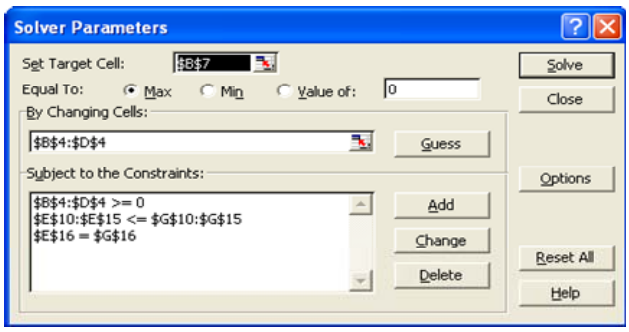
Excel Solver Setup



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Solver Solution for Unit 1

	A	B	C	D	E	F	G
1	**Burger Palace**						
2							
3	Variables	U1	V1	V2			
4	Value	0.01	0.166667	0.003333			
5							
6	Objective						
7	Function	1					
8							
9	Constraints				LHS		RHS
10	Unit 1	100	-2	-200	-1.07E-10	<=	0
11	Unit 2	100	-4	-150	-0.166667	<=	0
12	Unit 3	100	-4	-100	7.44E-12	<=	0
13	Unit 4	100	-6	-100	-0.333333	<=	0
14	Unit 5	100	-8	-80	-0.6	<=	0
15	Unit 6	100	-10	-50	-0.833333	<=	0
16	Inputs	0	2	200	1	=	1

Unit 1 is efficient because its efficiency rating is 1

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Solver Sensitivity Report (Unit 1)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Value U1	0.01	0	100	1E+30	100
$C$4	Value V1	0.166666667	0	0	0	3
$D$4	Value V2	0.003333333	0	0	300	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$10	Unit 1 LHS	-1.0662E-10	1	0	1	0.384615385
$E$11	Unit 2 LHS	-0.166666667	0	0	1E+30	0.166666667
$E$12	Unit 3 LHS	7.43983E-12	0	0	0.2	1
$E$13	Unit 4 LHS	-0.333333333	0	0	1E+30	0.333333333
$E$14	Unit 5 LHS	-0.6	0	0	1E+30	0.6
$E$15	Unit 6 LHS	-0.833333333	0	0	1E+30	0.833333333
$E$16	Inputs LHS	1	1	1	1E+30	1

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Excel Formulation for Unit 2

	A	B	C	D	E	F	G
1	**Burger Palace**						
2							
3	Variables	U1	V1	V2			
4	Value	1	1	1			
5							
6	Objective						
7	Function	100					
8							
9	Constraints				LHS		RHS
10	Unit 1	100	-2	-200	-102	<=	0
11	Unit 2	100	-4	-150	-54	<=	0
12	Unit 3	100	-4	-100	-4	<=	0
13	Unit 4	100	-6	-100	-6	<=	0
14	Unit 5	100	-8	-80	12	<=	0
15	Unit 6	100	-10	-50	40	<=	0
16	Inputs	0	4	150	154	=	1

This is the only part that is changed

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Solver Solution for Unit 2

	A	B	C	D	E	F	G
1	**Burger Palace**						
2							
3	Variables	U1	V1	V2			
4	Value	0.008571	0.142857	0.002857			
5							
6	Objective						
7	Function	0.857143					
8							
9	Constraints				LHS		RHS
10	Unit 1	100	-2	-200	-1.08E-10	<=	0
11	Unit 2	100	-4	-150	-0.142857	<=	0
12	Unit 3	100	-4	-100	7.69E-12	<=	0
13	Unit 4	100	-6	-100	-0.285714	<=	0
14	Unit 5	100	-8	-80	-0.514286	<=	0
15	Unit 6	100	-10	-50	-0.714286	<=	0
16	Inputs	0	4	150	1	=	1

Unit 2 is inefficient because its efficiency rating is less than 1

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Solver Sensitivity Report (Unit 2)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Value U1	0.008571429	0	100	1E+30	100
$C$4	Value V1	0.142857143	0	0	3.333333333	1.333333333
$D$4	Value V2	0.002857143	0	0	49.99999999	125

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$10	Unit 1 LHS	-1.08209E-10	0.285714286	0	0.5	0.454545454
$E$11	Unit 2 LHS	-0.142857143	0	0	1E+30	0.142857143
$E$12	Unit 3 LHS	7.69118E-12	0.714285714	0	0.2	0.5
$E$13	Unit 4 LHS	-0.285714286	0	0	1E+30	0.285714286
$E$14	Unit 5 LHS	-0.514285714	0	0	1E+30	0.514285714
$E$15	Unit 6 LHS	-0.714285714	0	0	1E+30	0.714285714
$E$16	Inputs LHS	1	0.857142857	1	1E+30	1

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Efficiency Reference Set and The Hypothetical Composite Unit

- Each inefficient unit has a set of efficiency reference units
  - Unit 1 (S_1) and Unit 3 (S_3) form the reference set for Unit 2
- A linear combination of these efficiency reference units forms a hypothetical composite unit (CU) with shadow prices as the weights

  $CU = 0.2857*S_1 + 0.7143*S_3$

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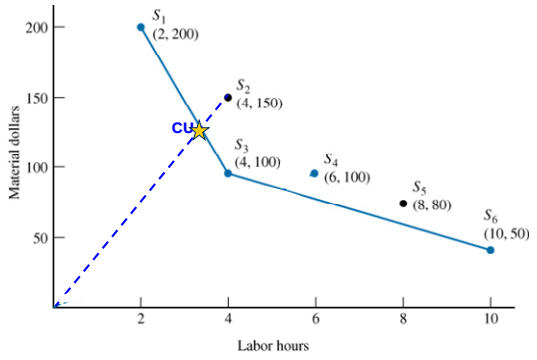
Graphical Illustration of the Composite Unit (CU)



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Meanings of V_1 and V_2

- These weights reflect the relative increase in efficiency with each unit reduction of the corresponding input value
  - V_1 = 0.1429 means that each unit reduction or decrease in labor hours will result in an efficiency increase of 0.1429
  - V_2=0.0029 indicates that each unit ($) reduction of materials used will lead to an increase of efficiency of 0.0029.

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Improving Unit 2's Efficiency

For Unit S_2 to be efficient, it must increase its efficiency rating by 0.1429 (=1-0.8571). This can be achieved by

1. reducing labor hours by 0.1429/0.1429=1,
2. reducing materials by 0.1429/0.0029≈$50, or
3. any linear combination of S_1 and S_3
   CU = 0.2857*S_1 + 0.7143*S_3
   (see next page)

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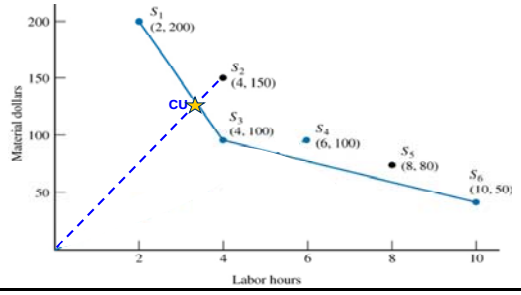
Inputs Used by CU and Unit S_2

Outputs & Inputs	Reference Set S1	Reference Set S3	Composite Reference Unit	S2	Excess Inputs Used
Labor-hours	(0.2857) x 2 + (0.7143) x 4 =		3.43	4	0.57
Material ($)	(0.2857) x 200 + (0.7143) x 100 =		128.57	150	21.43



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Summary of DEA Results

Service unit	Efficiency rating (*E*)	Efficiency reference set	Relative labor-hour value (V1)	Relative material value (V2)
S1	1.000	N.A.	0.1667	0.0033
S2	0.857	S1 (.2857) S3 (.7143)	0.1428	0.0028
S3	1.000	N.A.	0.0625	0.0075
S4	0.889	S3 (.7778) S6 (.2222)	0.0555	0.0067
S5	0.901	S3 (.4545) S6 (.5454)	0.0568	0.0068
S6	1.000	N.A.	0.0625	0.0075

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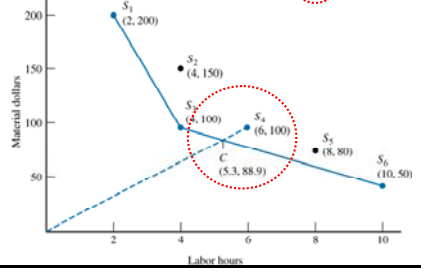
Excess Inputs Used by Unit S_4

Outputs & Inputs	Reference Set S3	Reference Set S6	Composite Reference Unit C	S4	Excess Inputs Used
Meals	(0.7778) x 100 + (0.2222) x 100 =		100	100	0
Labor-hours	(0.7778) x 4 + (0.2222) x 10 =		5.3	6	0.7
Material ($)	(0.7778) x 100 + (0.2222) x 50 =		88.9	100	11.1



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Suggest Efficiency Improvement Alternatives for Unit S_5

- For Unit S_5 to be efficient, it must increase its efficiency rating by 0.099 (=1-0.901). This can be achieved by

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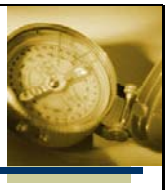
DEA and Strategic Planning



Profit	Low Efficiency	High Efficiency
High	Under-performing potential stars	Benchmark group
Low	Problem Branches	Candidates for divestiture

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