Quantitative Business Analysis (QBA)

I. Lindner and J.R. van den Brink

Quantitative Business Analysis

Dates lectures:

October 31, November 7, 14, 21, 28, December 5

Time: 11:00-12:45,

Location: WN-KC 137

Quantitative Business Analysis

- Tutorials: Tuesdays and Wednesdays
- Prepare exercises beforehand!
- Program for the first two weeks:
  Exercises from chapter "Topic E1 – Exercises Decision Analysis":

  week 1: 1.1, 1.2, 1.3, 1.5, 1.8

  week 2: 1.4, 1.6, 1.7, 1.9

Quantitative Business Analysis

Contents of the Course

Week 1-2 (Ines Lindner)

1. *Decision Analysis* using Decision Trees

Week 3-6 (René van den Brink)

2. *Strategic Thinking* - Noncooperative Games

Quantitative Business Analysis

- Book “Quantitative Business Analyses” by C. van Montfort and J.R. van den Brink: chapter T1, T2, E1 and E2;
- Book is available at Aureus
- Sheets of lectures (on Blackboard)
- Relevant sections for decision theory: 8.1-8.3, 8.5 (except “Using Excel...”), 8.6, 8.8-8.10
- We don’t discuss software applications in this course!

Quantitative Business Analysis

Two efficient strategies to pass the exam!

Quantitative Business Analysis

Facts:

- You have to read the text material in order to pass the exam.
- Rule of thumb: A lecture is only fun if you already know 50 percent.

Conclusion: Read text material *before* lecture and take lecture as revision.

Quantitative Business Analysis

Facts:

- It is very tempting to just sit passively in the tutorials and watch discussion of exercises.
- Problem: Passive understanding is not enough for exam.

Conclusion: Try to do the exercises yourself and take tutorial as a feedback on your performance.

Quantitative Business Analysis

→ Answers to exercises decision theory will be available at the end of week 2.

Decision Theory

Central question: What is the best decision to take?

Assumptions:

- We have some information.
- We are able to compute with perfect accuracy.
- We are fully rational.

What kinds of decisions need a theory?

Optimization – examples

- How can we produce at lowest costs?
- What is the best product mix?
- What is an optimal way to spend my money (intertemporal choice)?

What kinds of decisions need a theory?

Choice under risk – examples

- Should I play the lottery?
- What kind of insurrence should I buy?
- How should I invest my money?

What kinds of decisions need a theory?

Interacting decision makers (game theory) – examples

- The telephone conversation broke down – shall I wait or call back myself (depends on what other person does)?
- As a firm, shall we enter a new market (depends on competing firms)?
- What is the best strategy to get promoted (depends on your boss)?

What kinds of decisions need a theory?

Game theory:

- Additional difficulty: the need to take into account how *other* people in the situation will act.
- Presence of several "players" (strategically acting agents).
- Requires *strategic* analysis (week 3-6).

Decision Theory

Central question: What is the best decision to take?

- Absence of strategic considerations.
- Can be seen as a one-player game.

Decision Analysis using Decision Trees

Dilemma: organize party indoors or in garden? What if it rains?

Events and Results

Choices	***Rain***	***Sunshine***
In Garden	Disaster	Real comfort
Indoors	Mild discomfort but content	Regrets

Decision Analysis using Decision Trees

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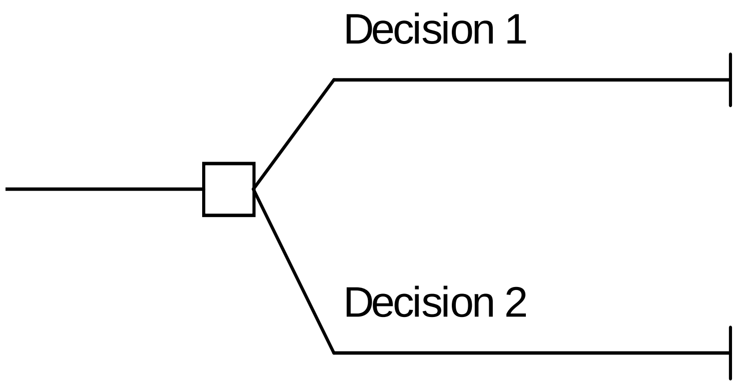
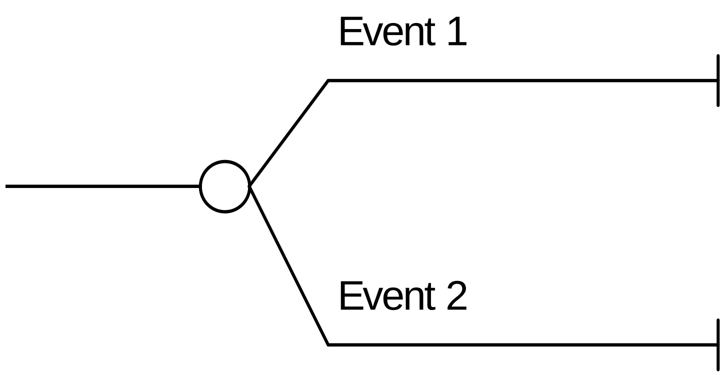
Decision Tree Components

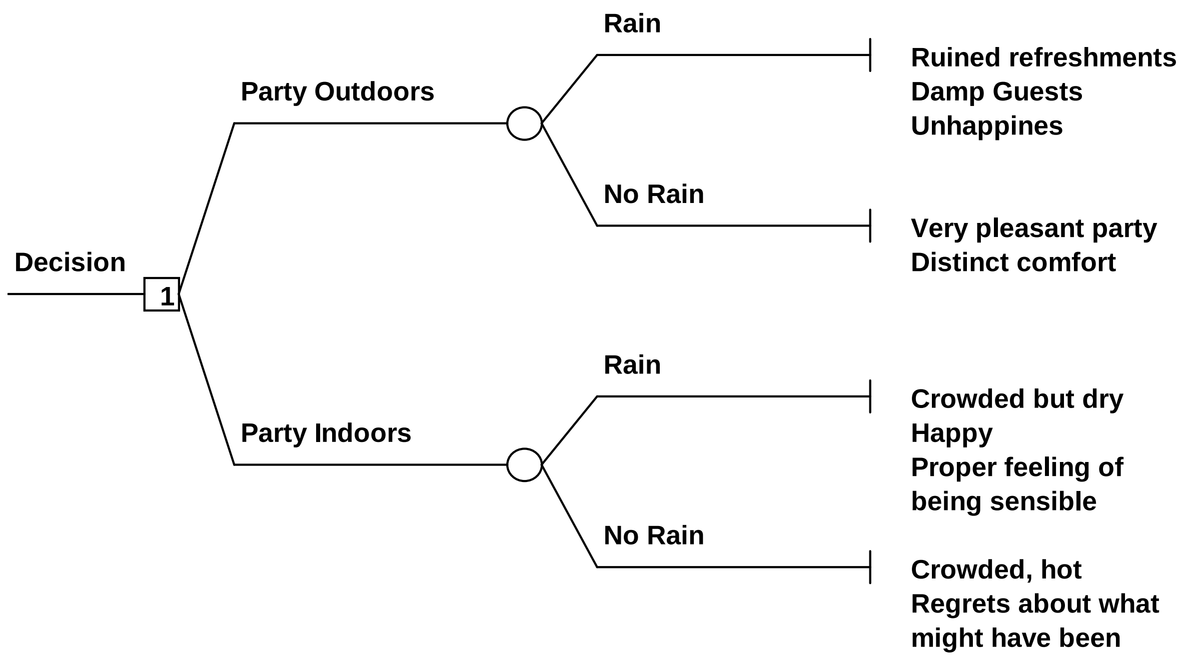


Decision node
or *Decision fork*

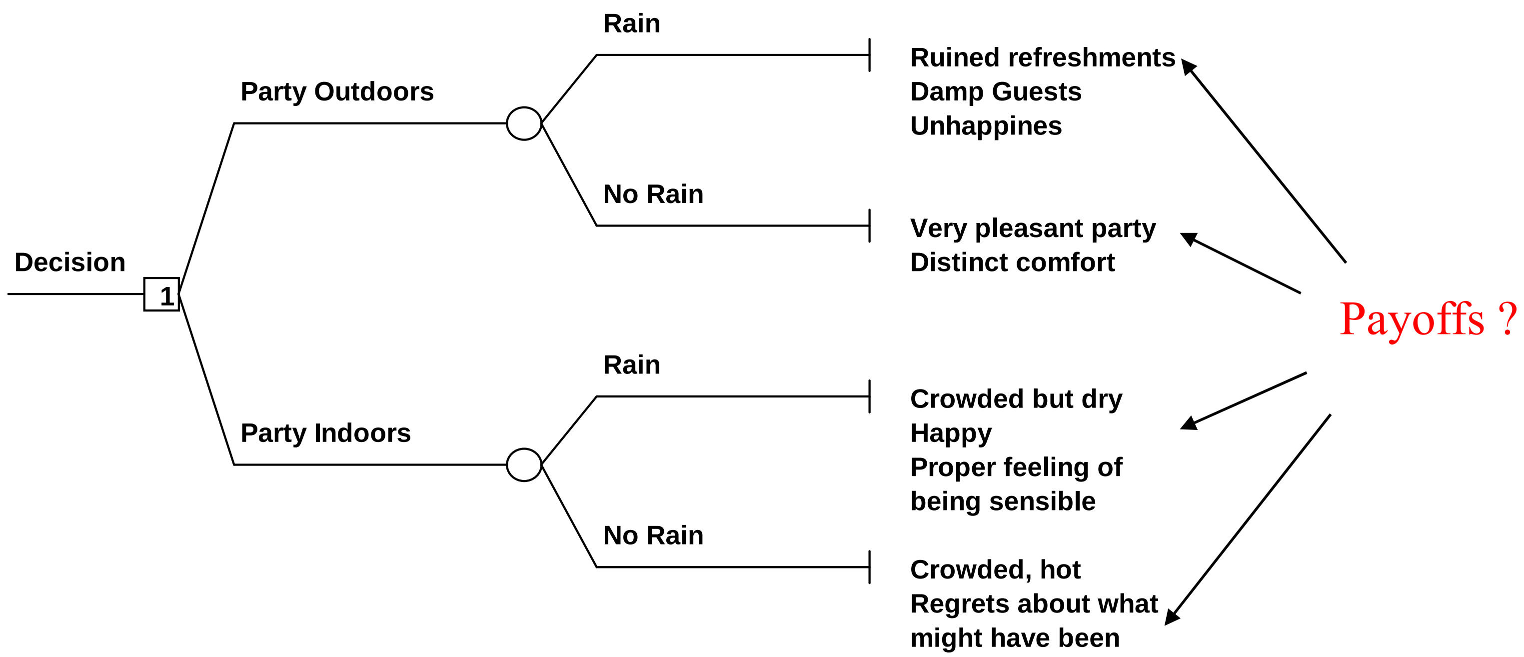


Event node or
Uncertainty fork

Decision Tree



Decision Tree



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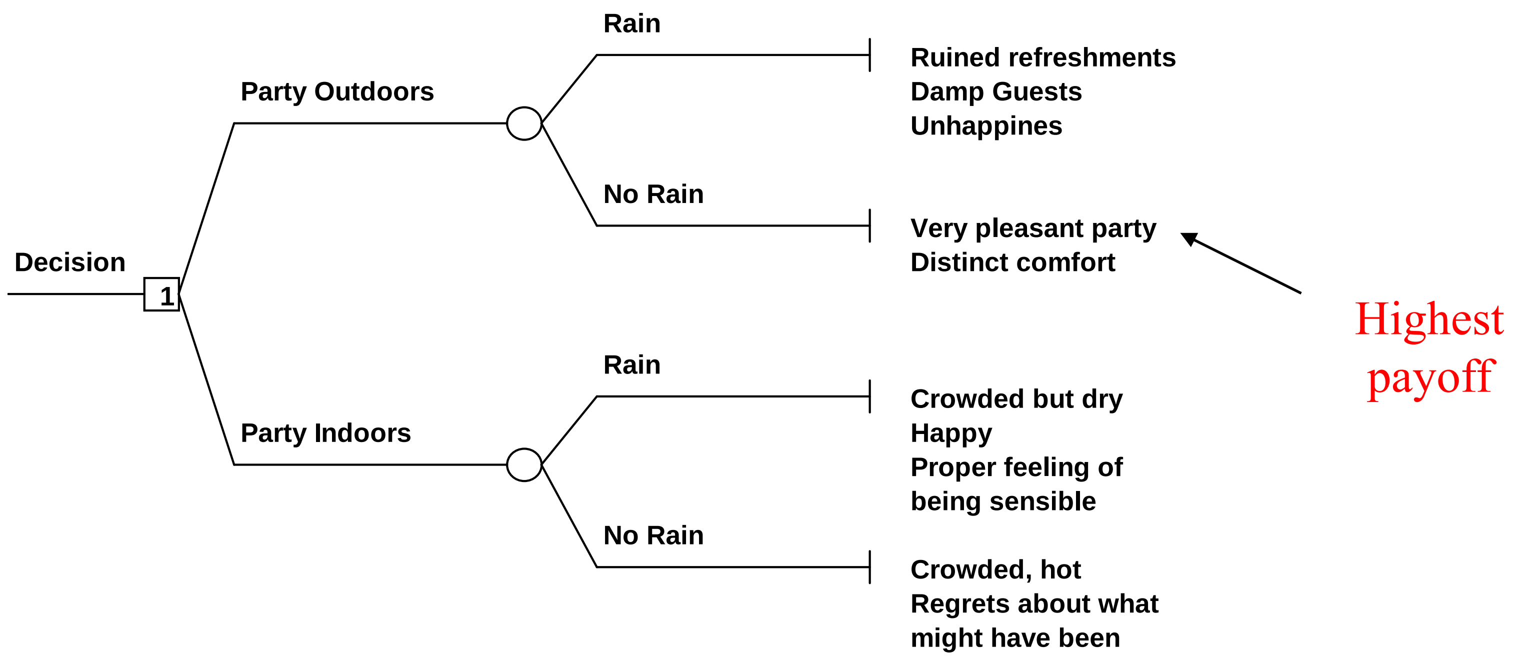
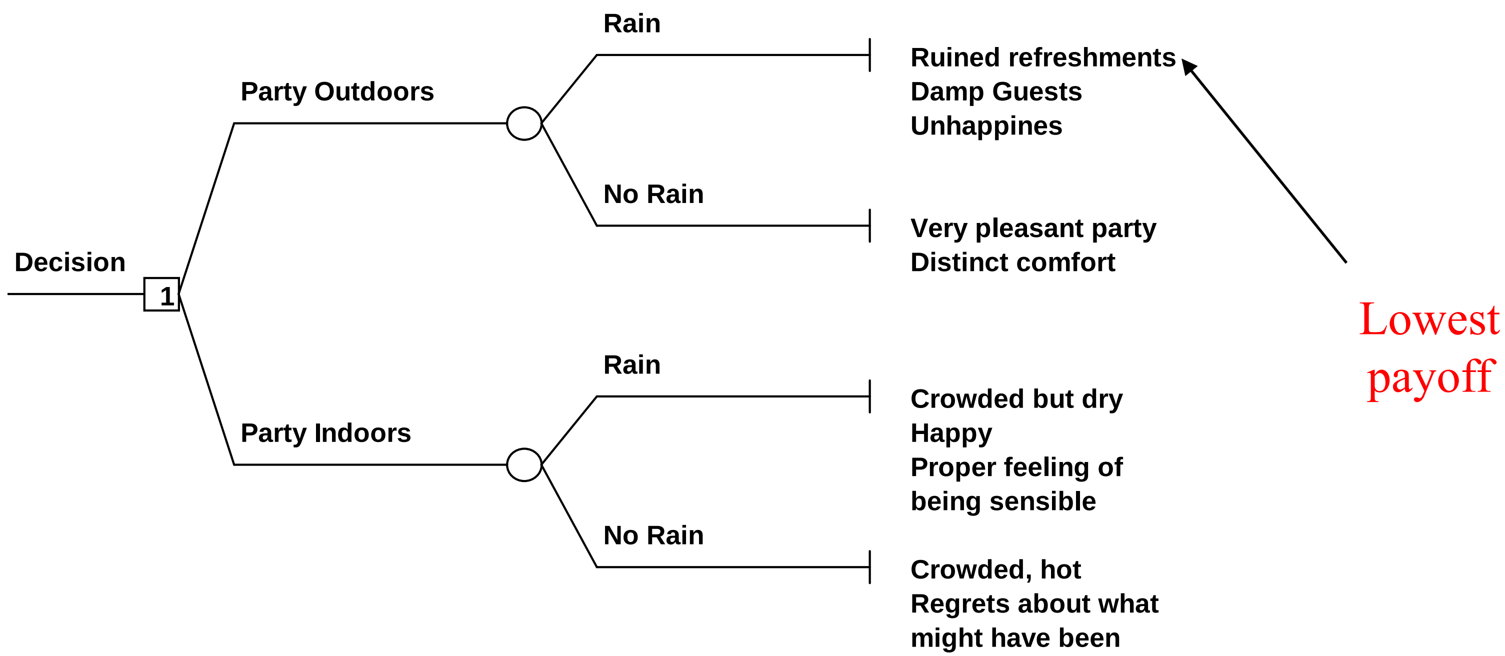
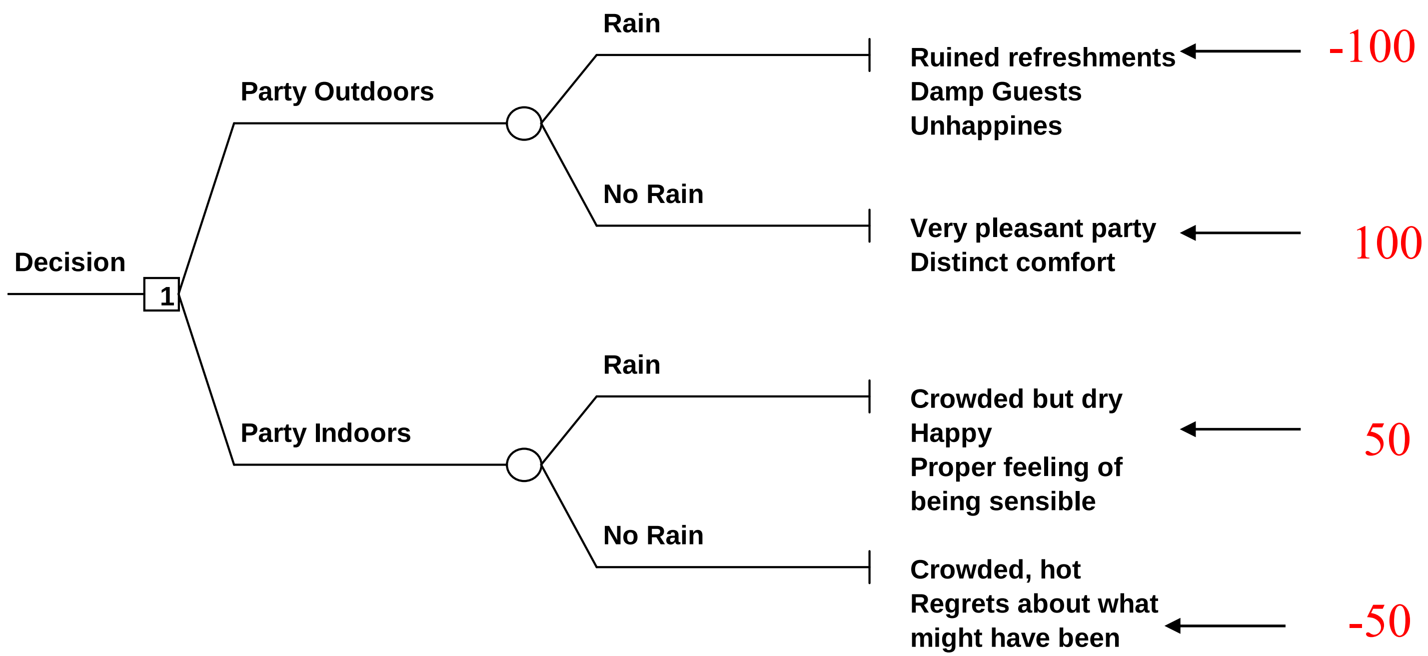


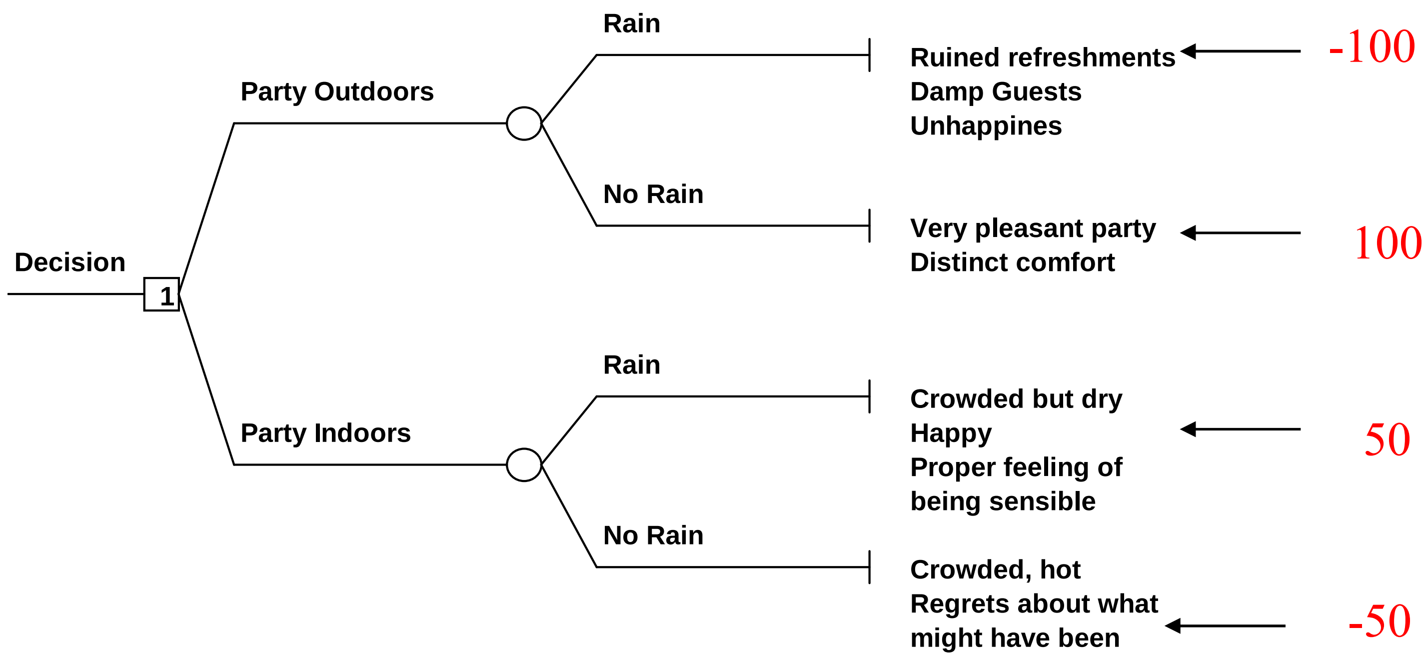
Decision Tree



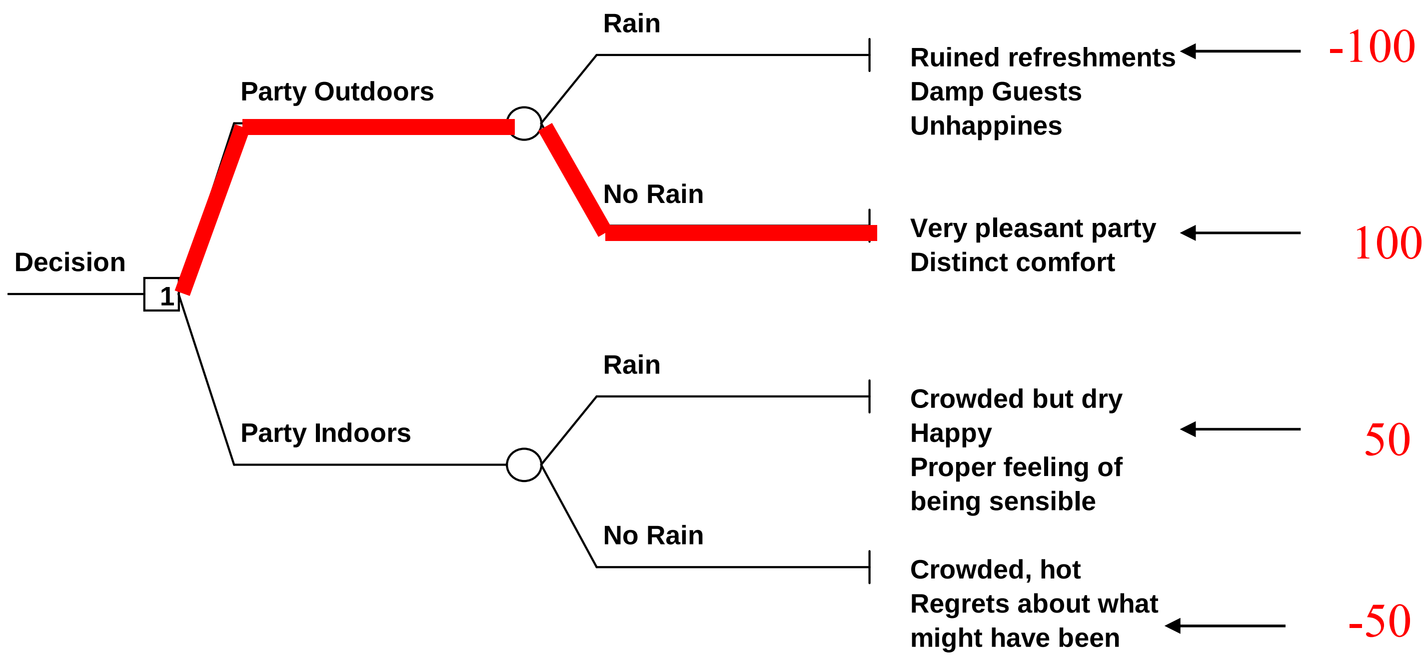
Decision Tree



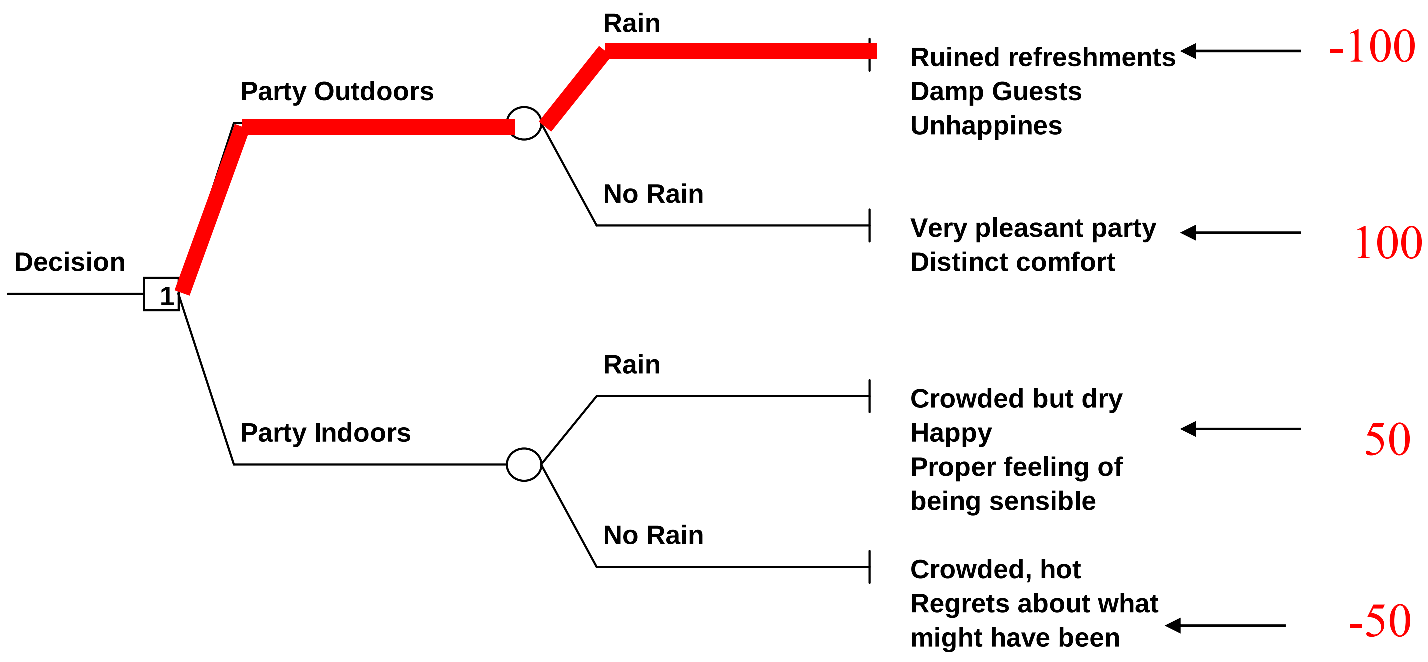
Possible Interpretation of Payoffs: How much is this outcome worth to me?



Example: Having a party outdoors with no rain is worth 100 (Euros) to me,
i.e. this is my maximum willingness to pay for it if it was possible to buy it.



Example: Having a party outdoors *with* rain is worth -100 (Euros) to me,
i.e. a hypothetical someone would have to *pay me* at least 100 Euros to accept this situation.



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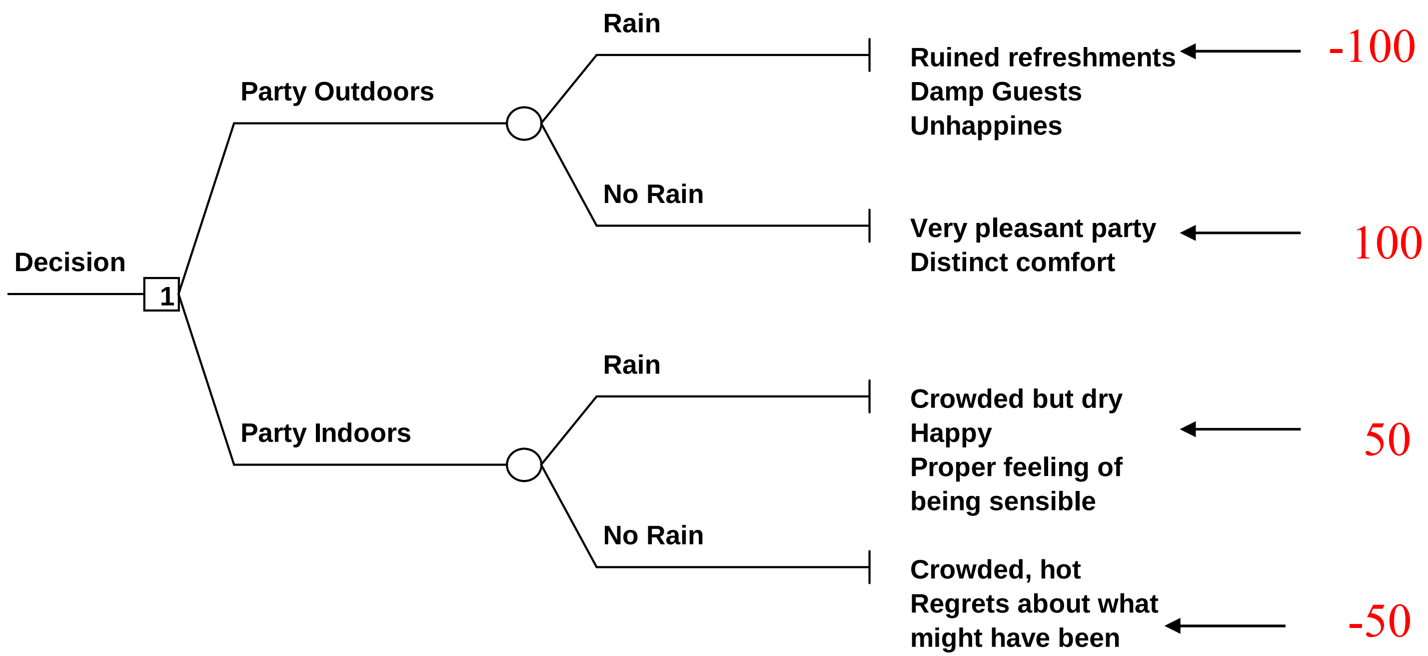

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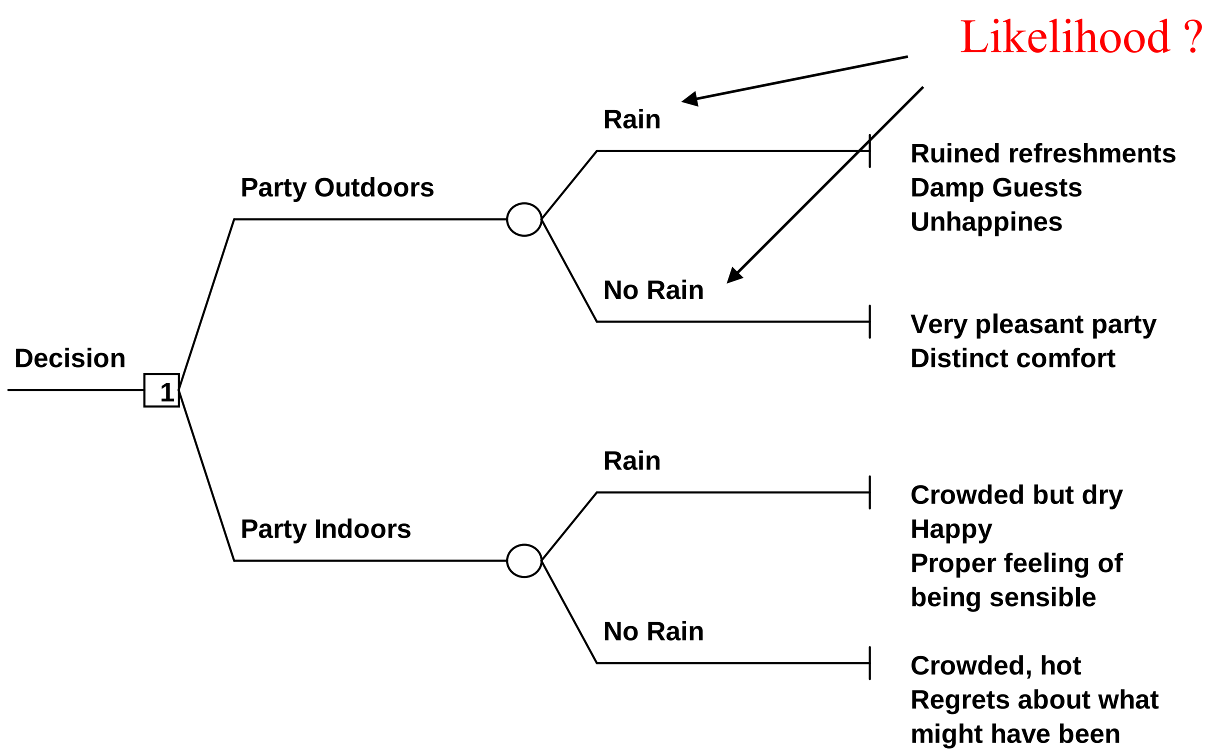

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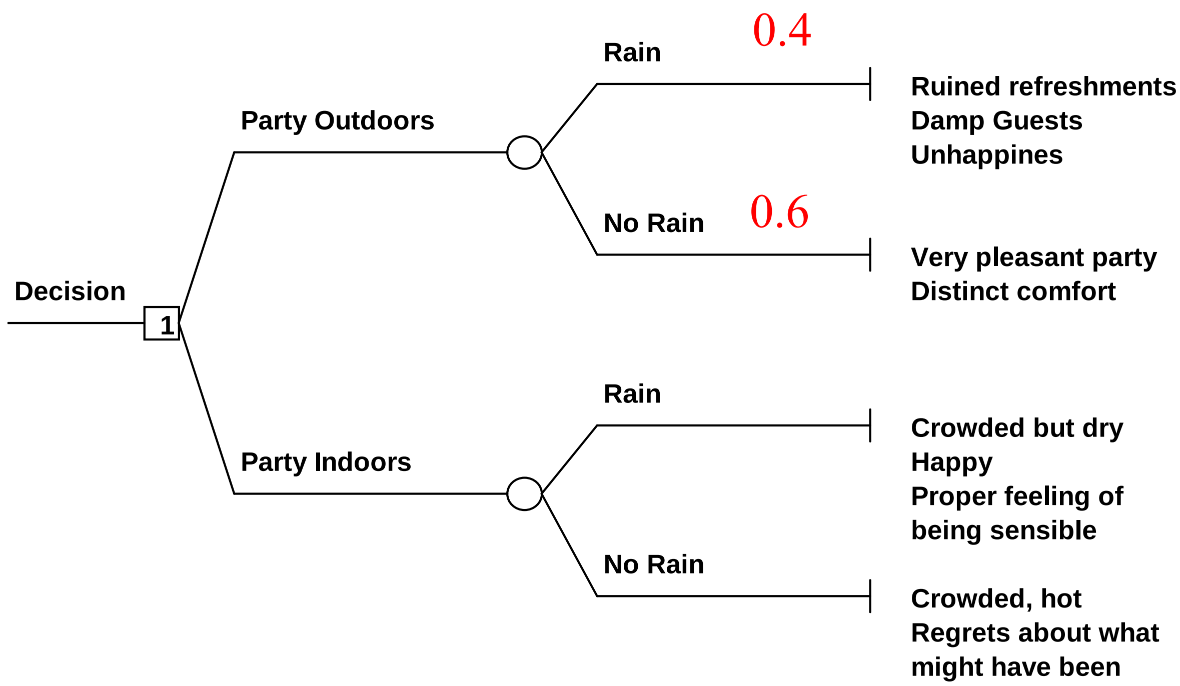

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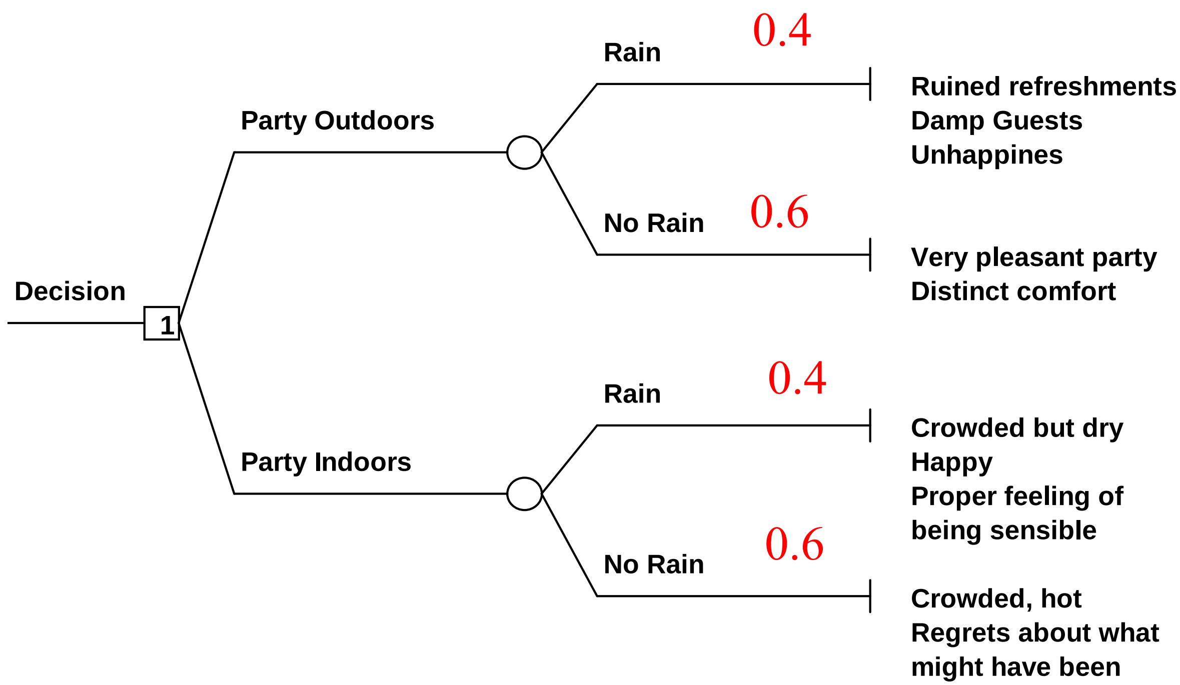

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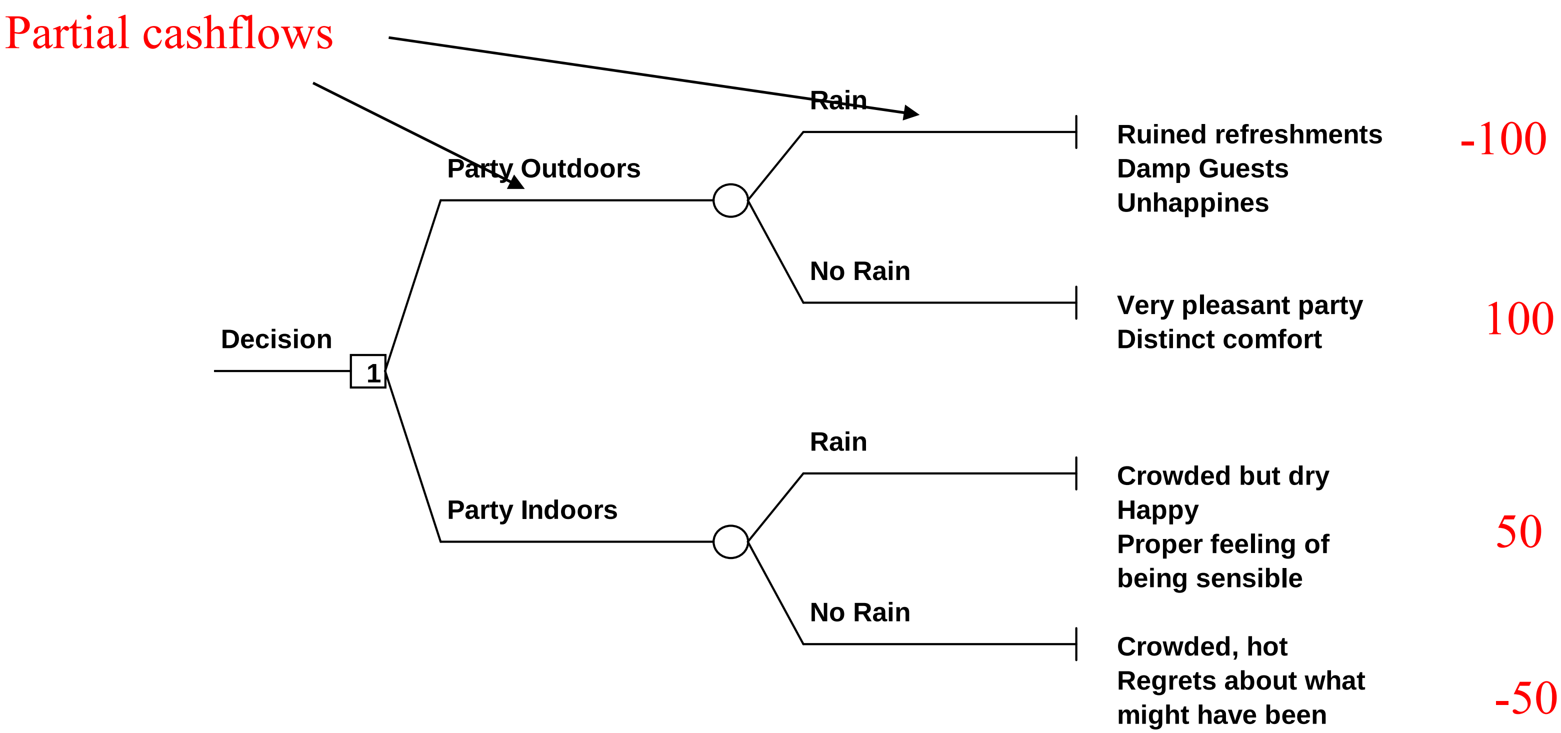

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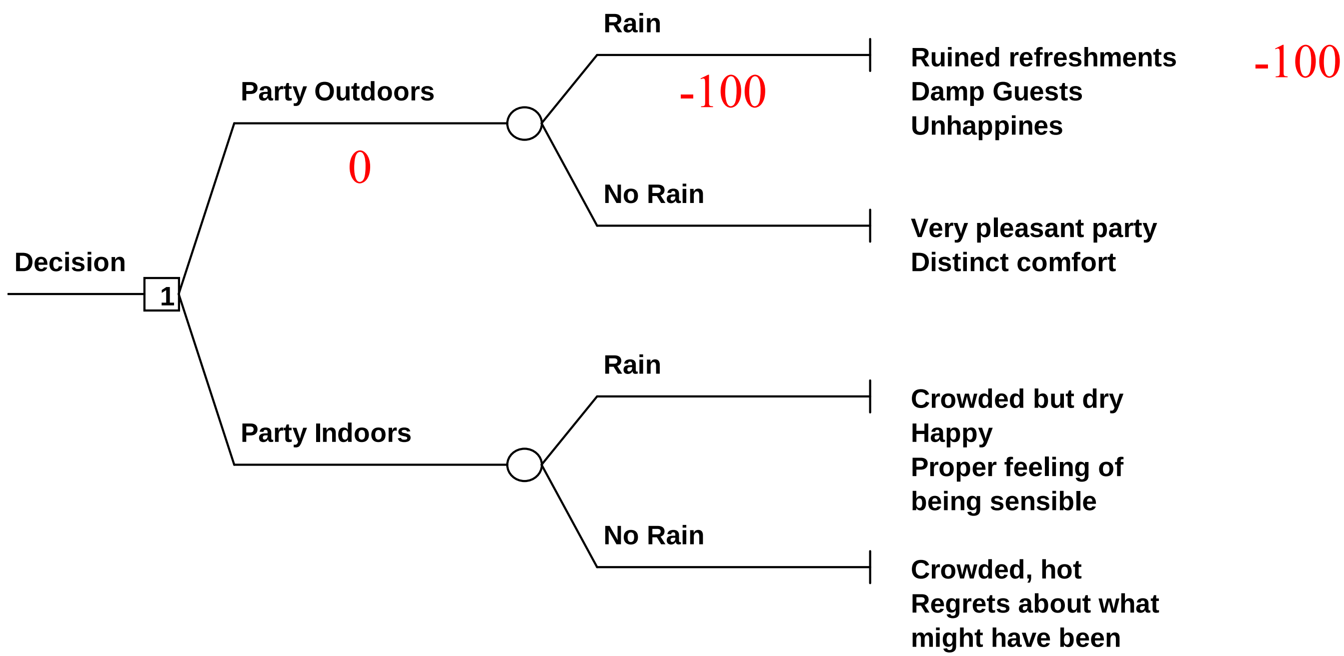

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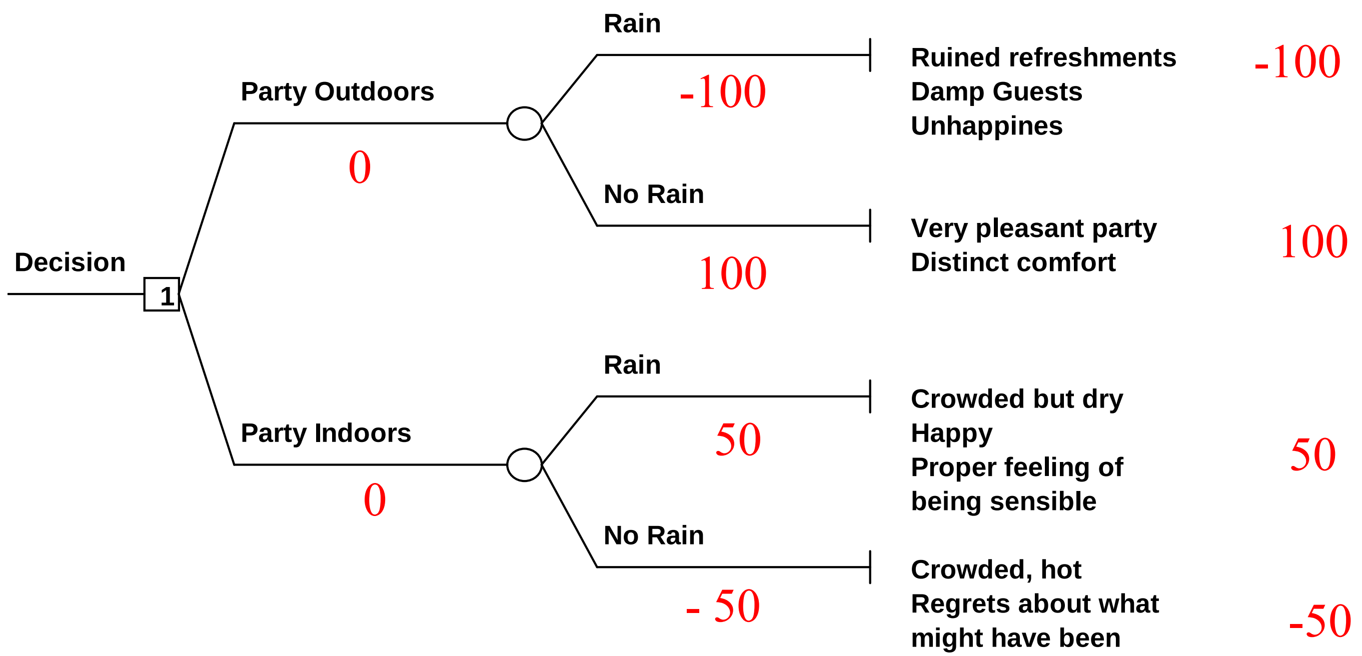
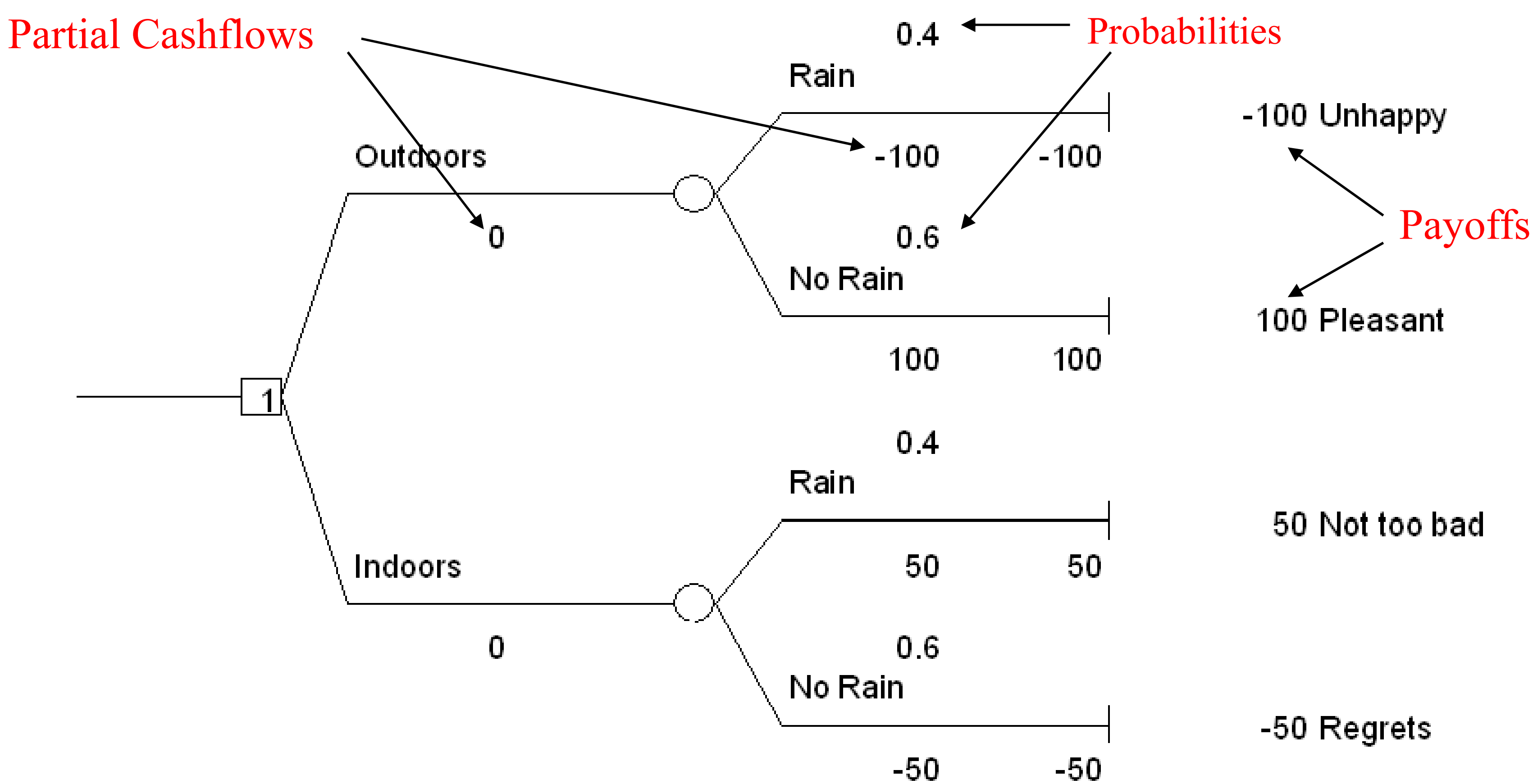


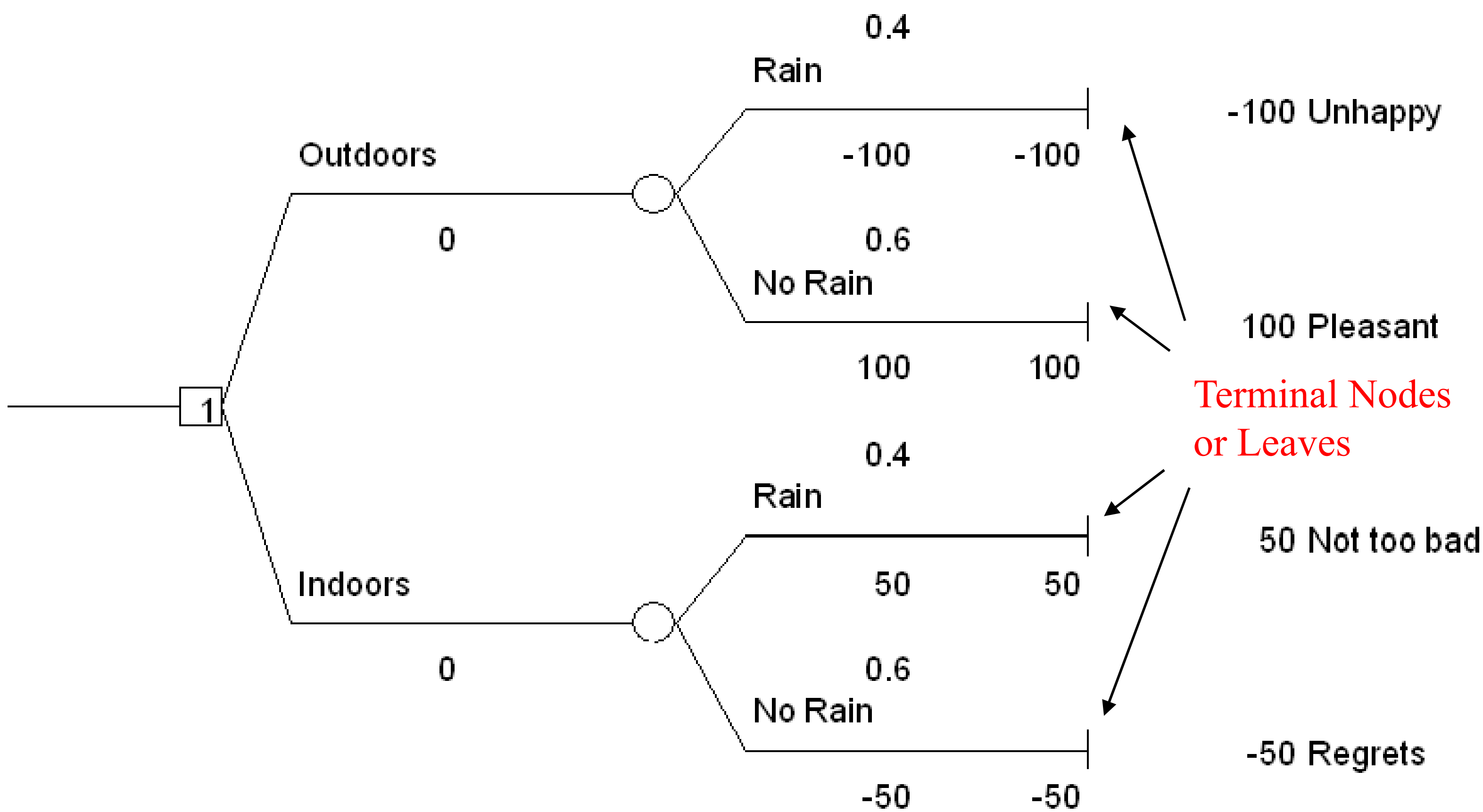
Decision Tree

Partial Cashflows, Payoffs and Probabilities



Decision Tree

Partial Cashflows, Payoffs and Probabilities



Solving a Decision Tree

What is the best decision?

Solving a Decision Tree

Consider decision “Outdoors”

Solving a Decision Tree

Consider decision "Outdoors"

Use Expected Value:

Probability(Rain) = 40%   Value = -100

Probability(No Rain) = 60% Value = 100

Solving a Decision Tree

Consider decision "Outdoors"

Use Expected Value:

Probability(Rain) = 40%   Value = -100

Probability(No Rain) = 60% Value = 100

Expected or average value:

$0.4 \times (-100) + 0.6 \times 100 = 20$

Expected Value

- Event node with n mutually exclusive events (exactly one of the events will occur).

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- Value of event i is V_i

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Expected Value

- Event node with n mutually exclusive events (exactly one of the events will occur).
- Value of event i is V_i
- probability of event i is p_i
- Then the expected value of the event node is $EV = p_1 \times V_1 + p_2 \times V_2 .... + p_n \times V_n$

Expected Value

Sometimes you see the “Gamma” symbol for the sum:

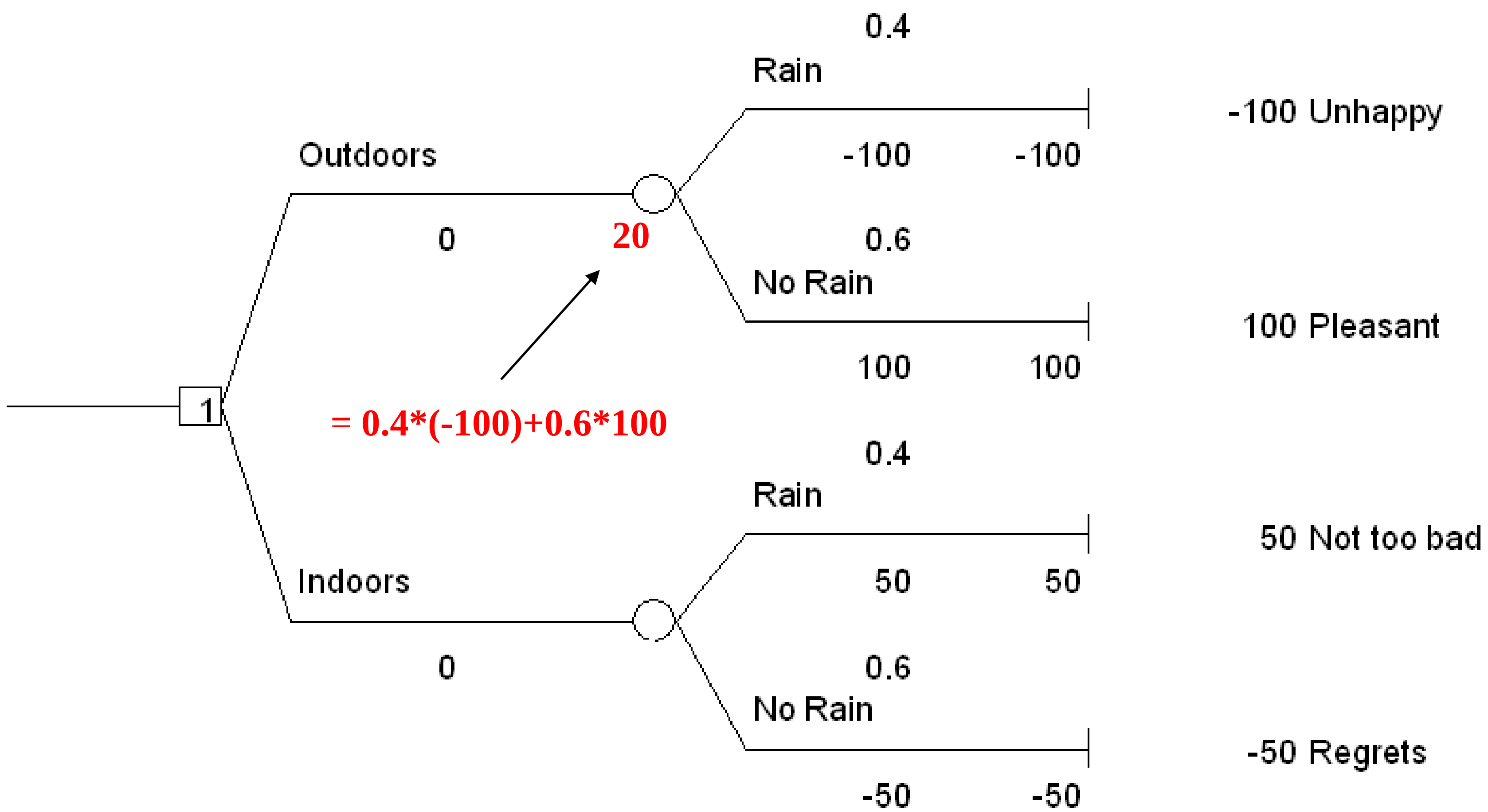
$$p_1 \times V_1 + p_2 \times V_2 + \ldots + p_n \times V_n = \sum_{i=1}^{n} p_i \times V_i$$

Expected Value

Note: $0 \leq p_i \leq 1 \qquad (i=1,2,\ldots,n)$

and $p_1 + \ldots + p_n = \sum p_i = 1$

Decision Tree



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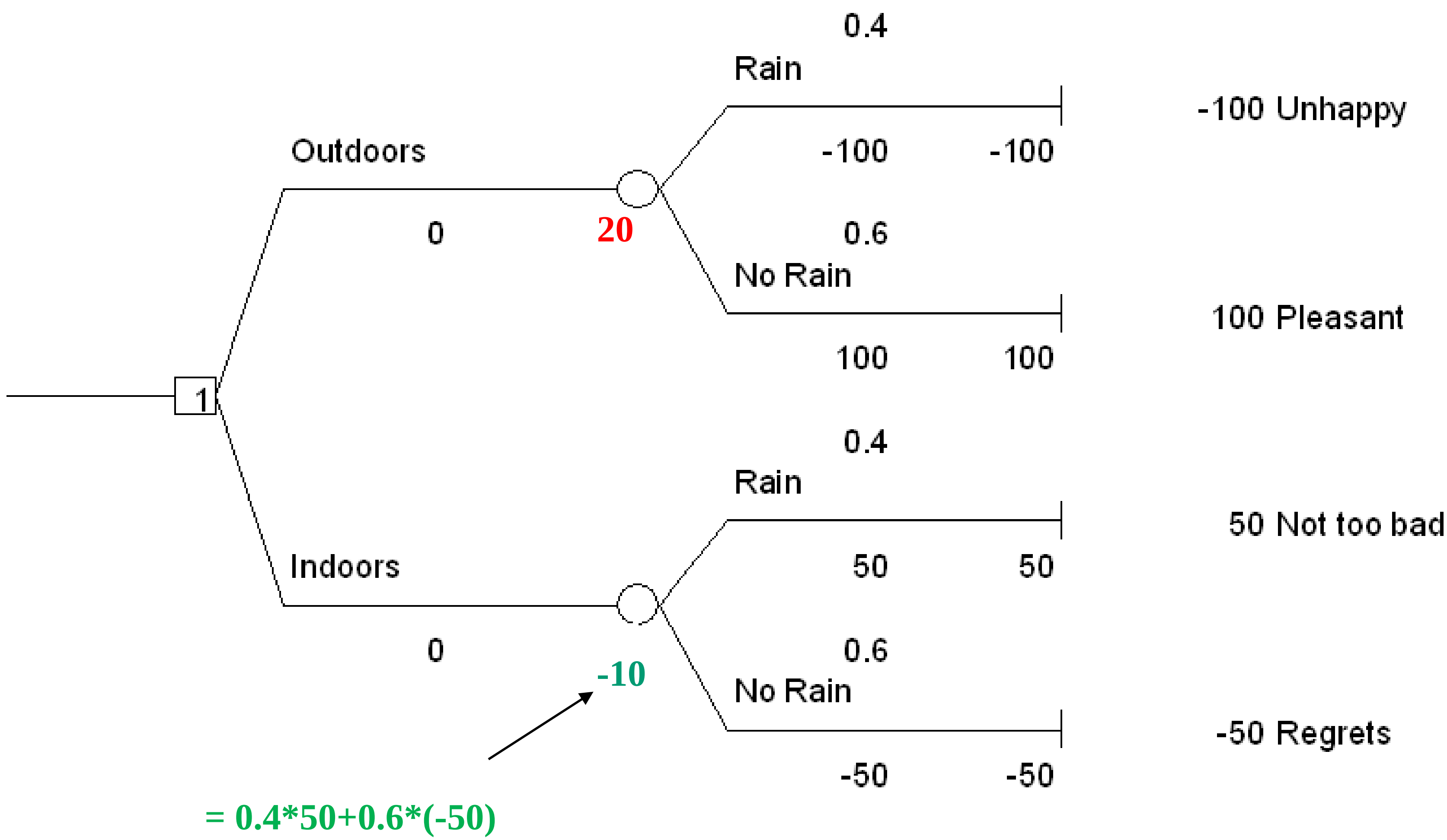

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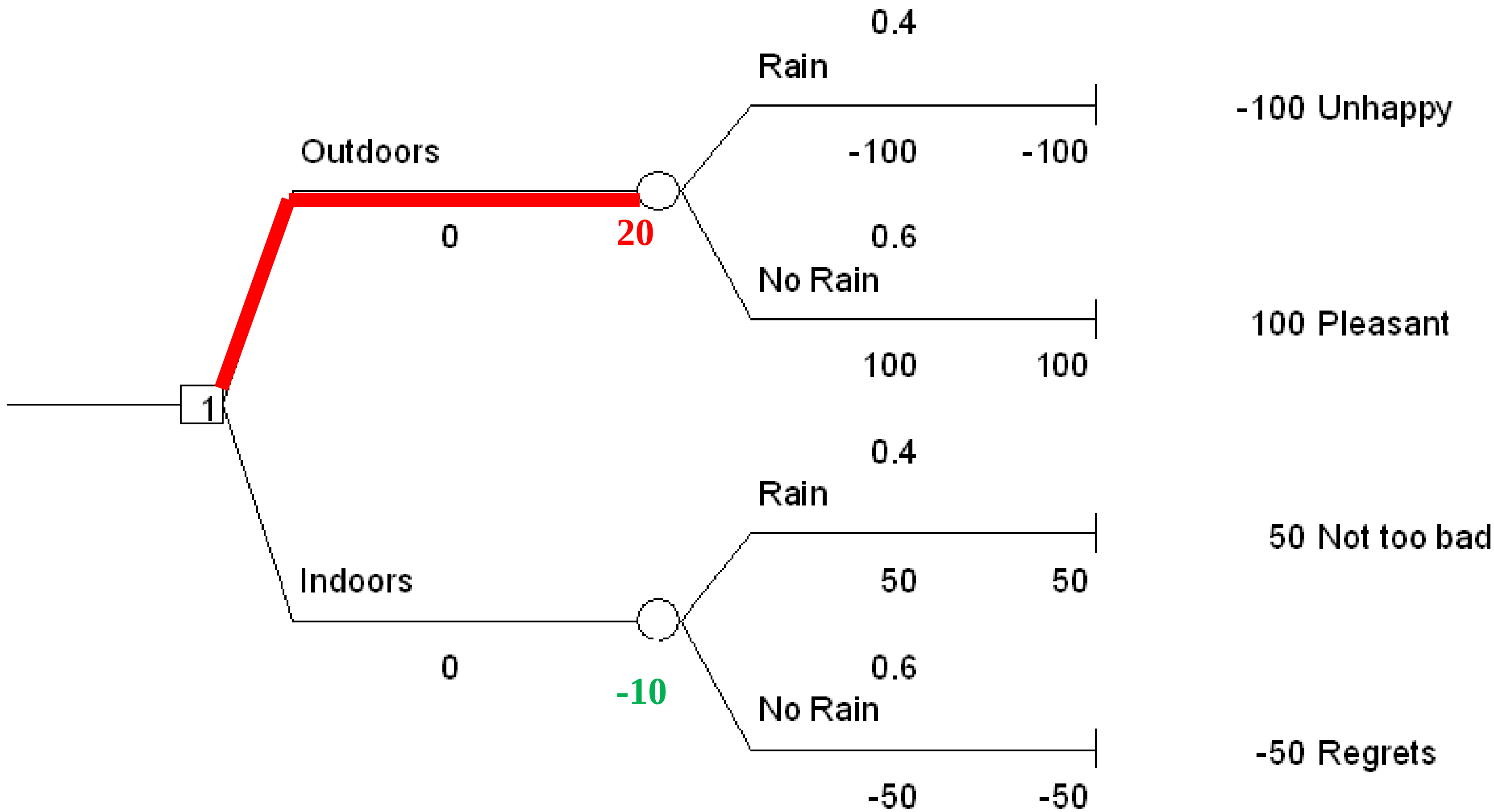

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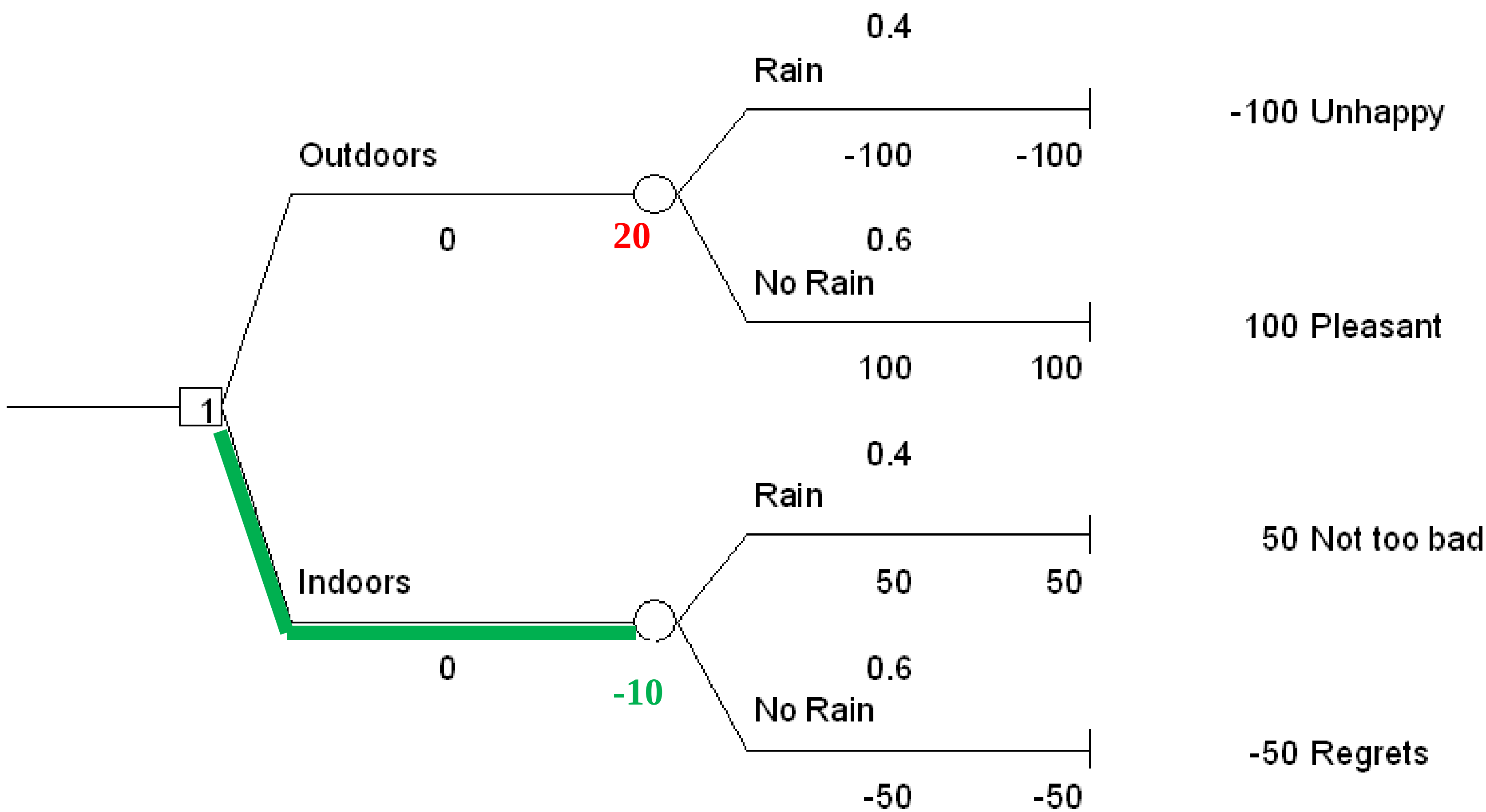
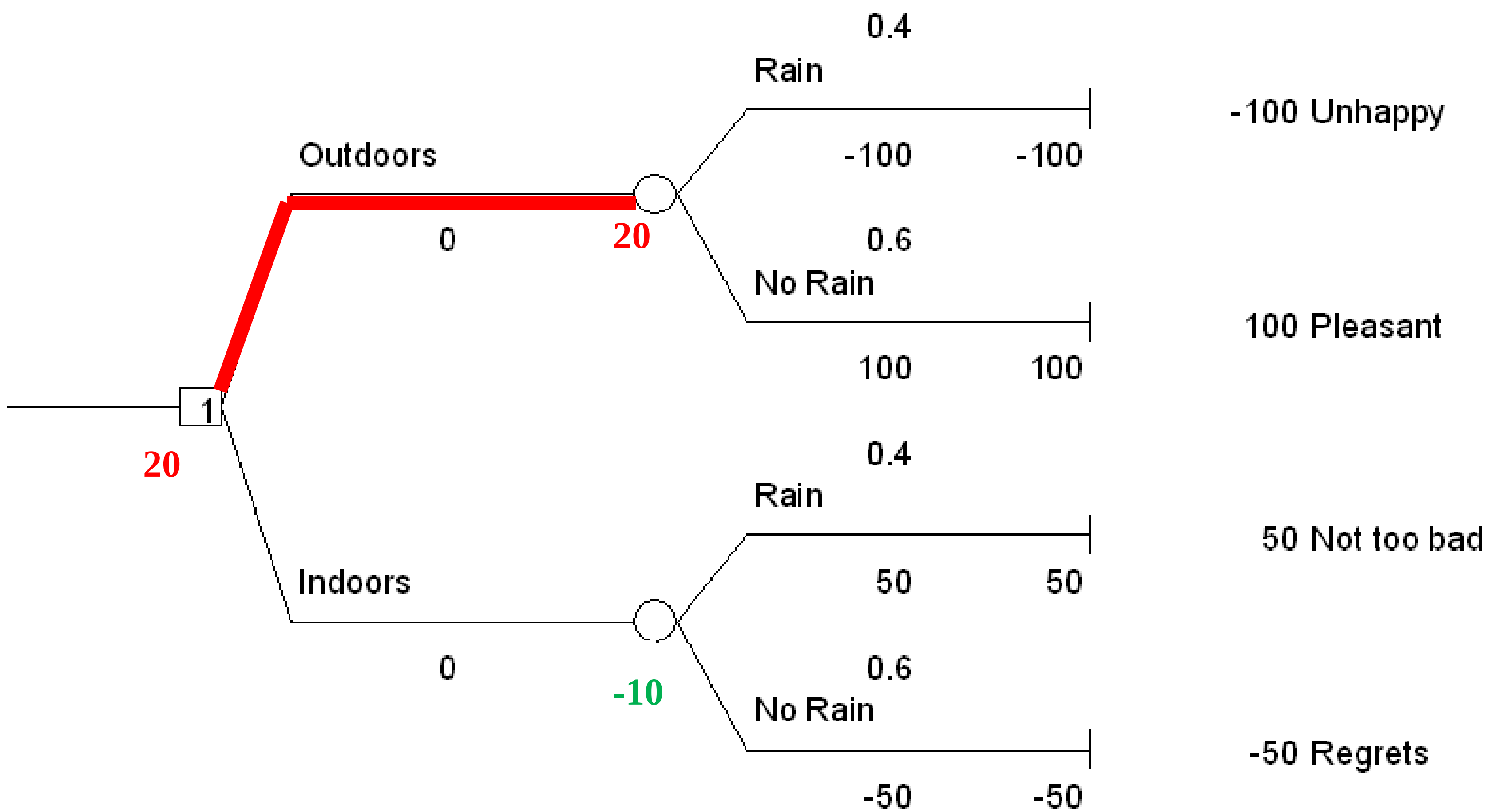


EV(Outdoors) > EV(Indoors)



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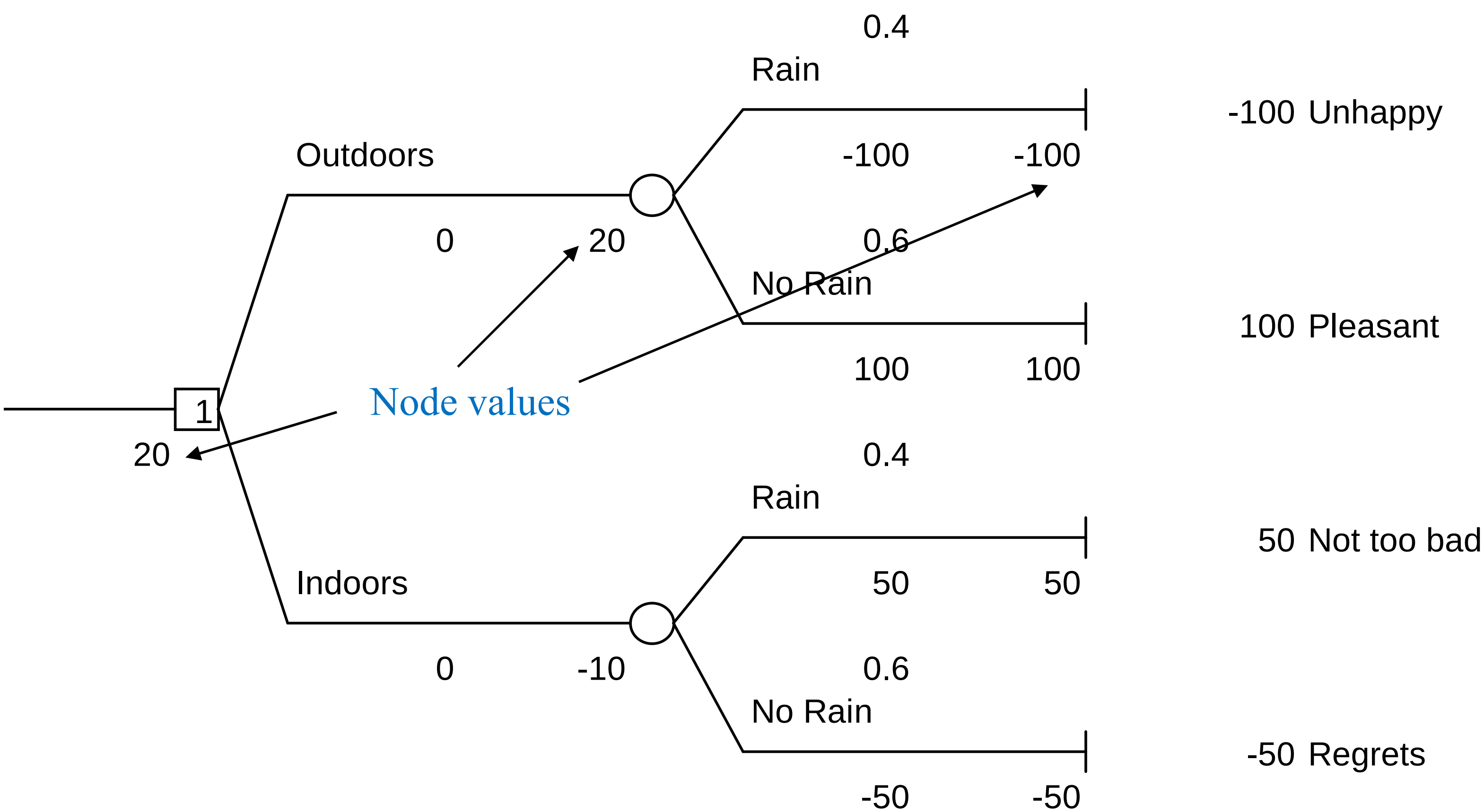
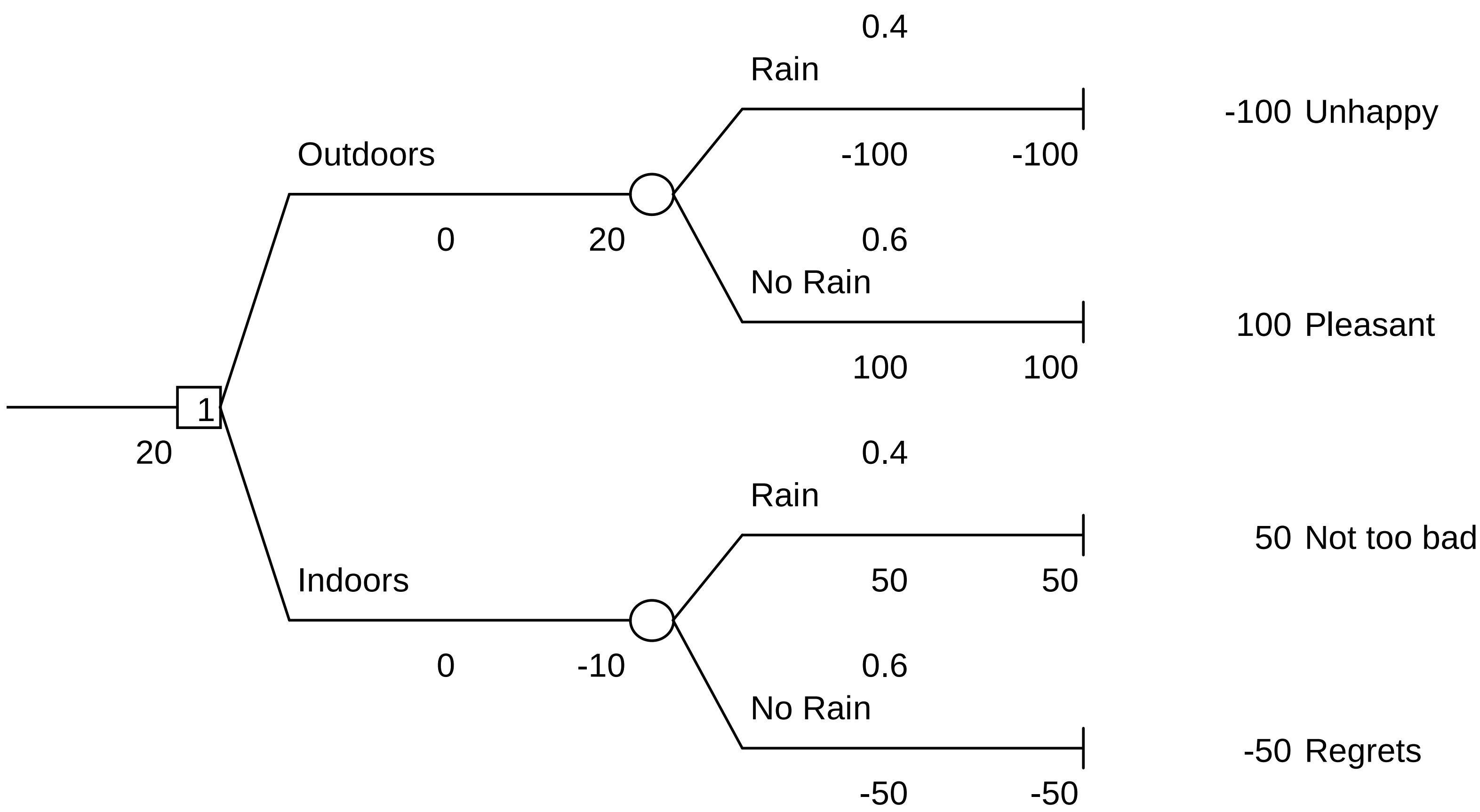


Evaluation of Decision Tree



Evaluation of Decision Tree

Roll back method: start at terminal nodes and work from right to left.

Compute Expected Value of Event Nodes

Choose Event Node with maximum (or minimum) EV in Decision Nodes

The value of the tree is the EV in the root of the tree (if values are money it is called the EMV or *Expected Money Value*)

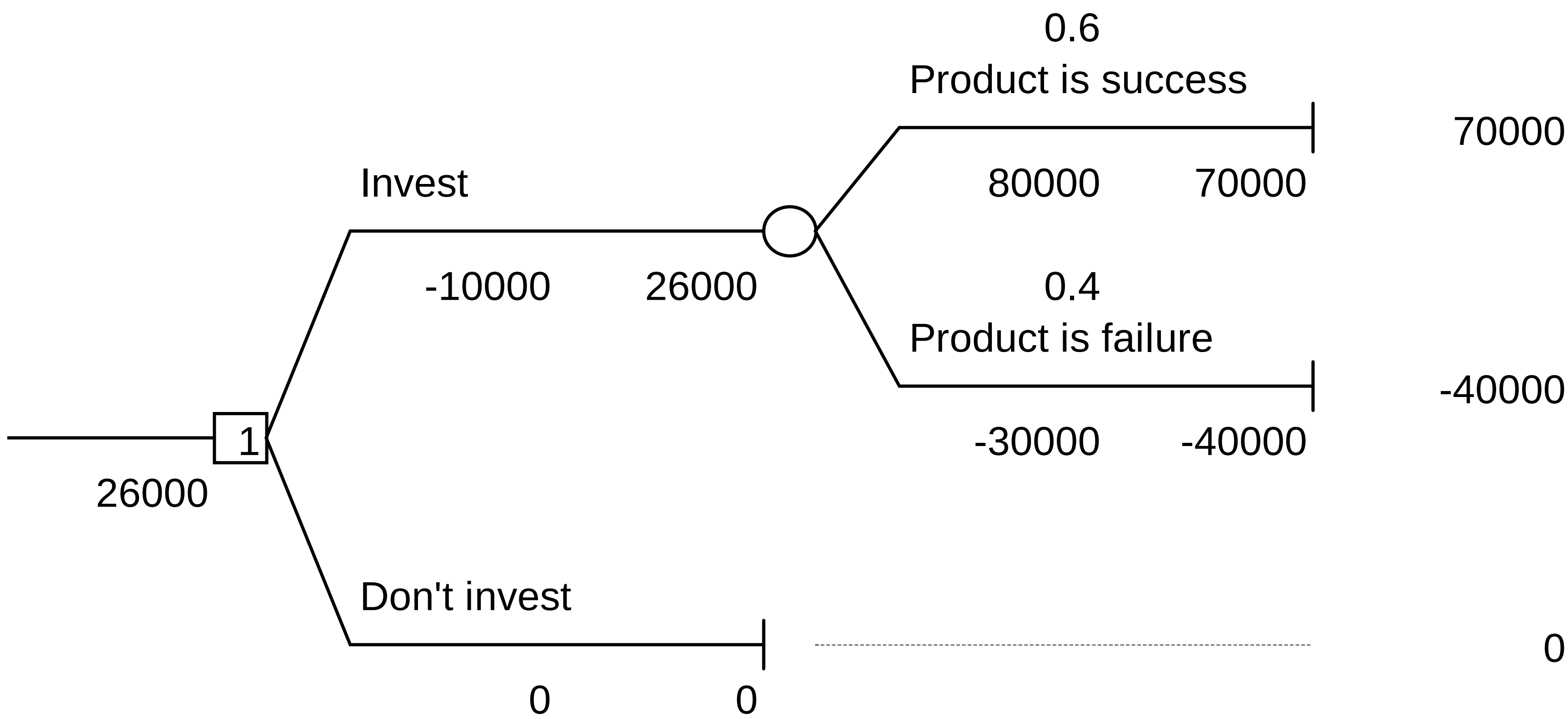
Exercise 1.1: Investment in a new product?

- Investment costs €10.000
- Market research

  => 60% success, €80.000 profit

  40% failure, - €30.000 loss
- Investment cost not yet included in profit.

Exercise 1.1: Investment in a new product?



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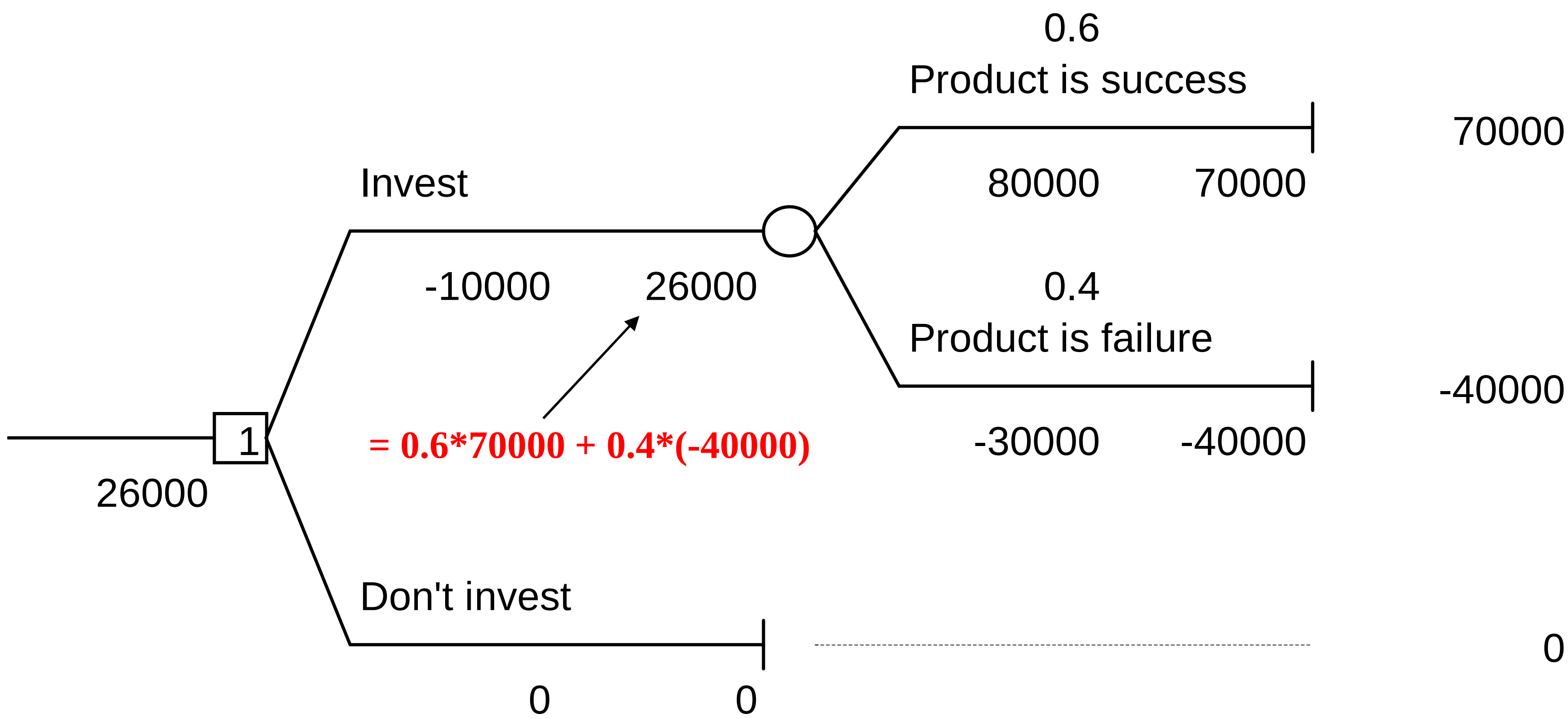
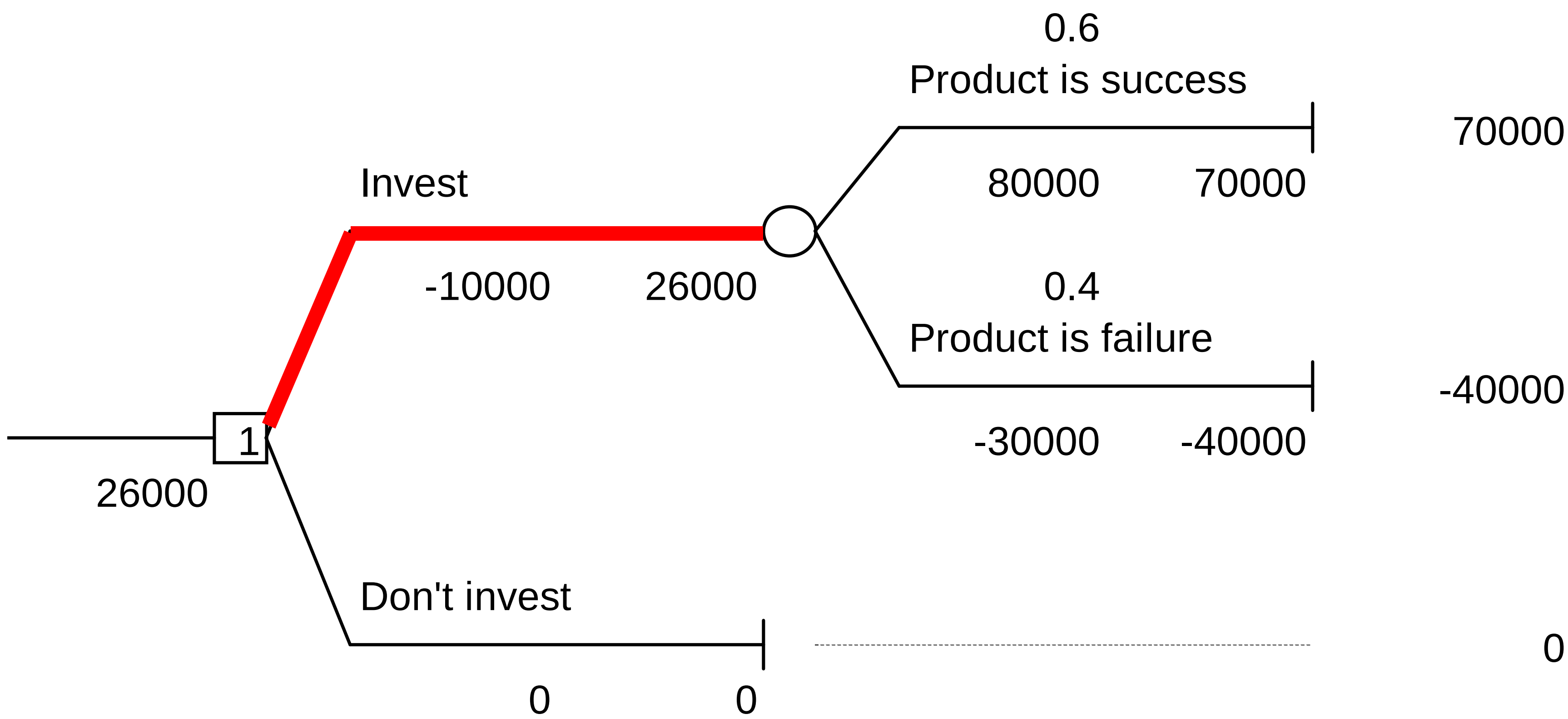
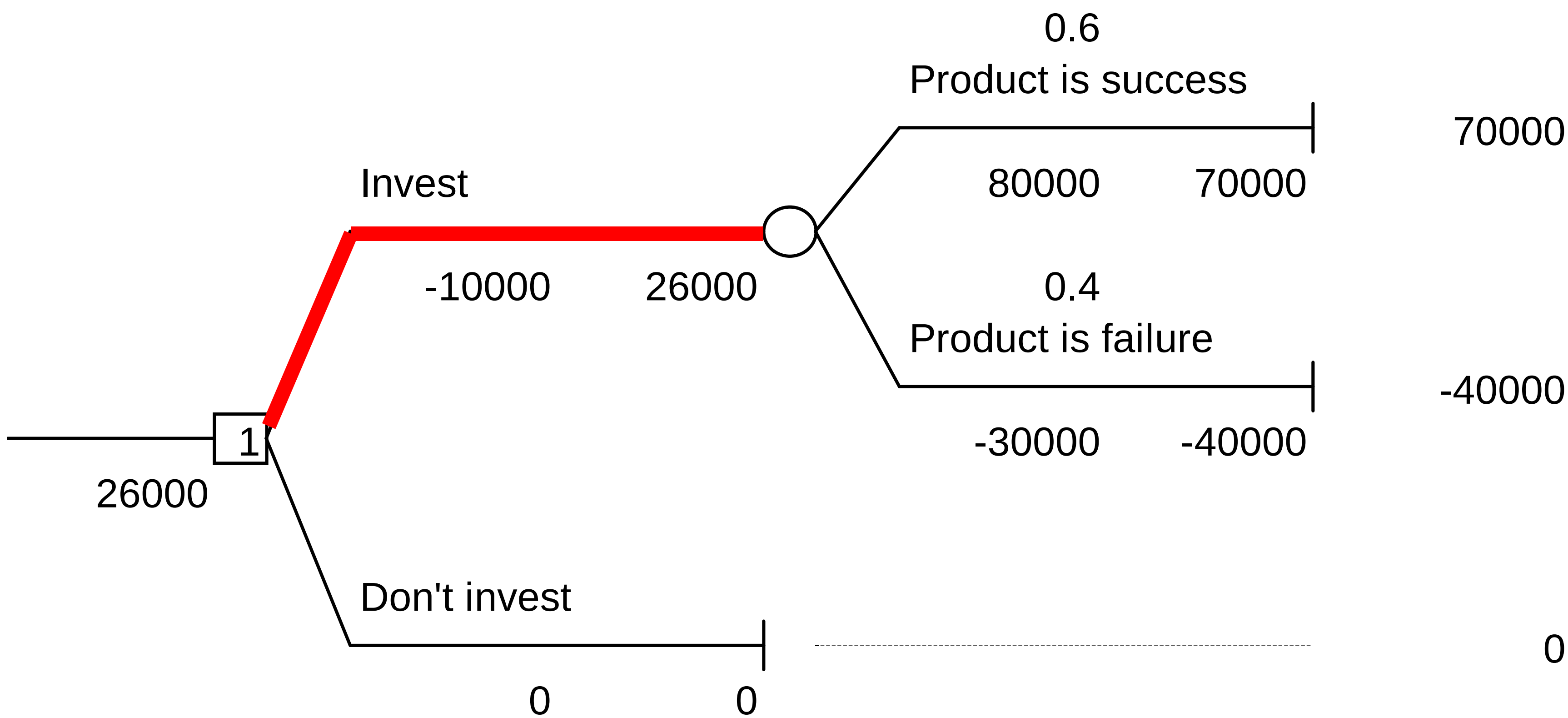


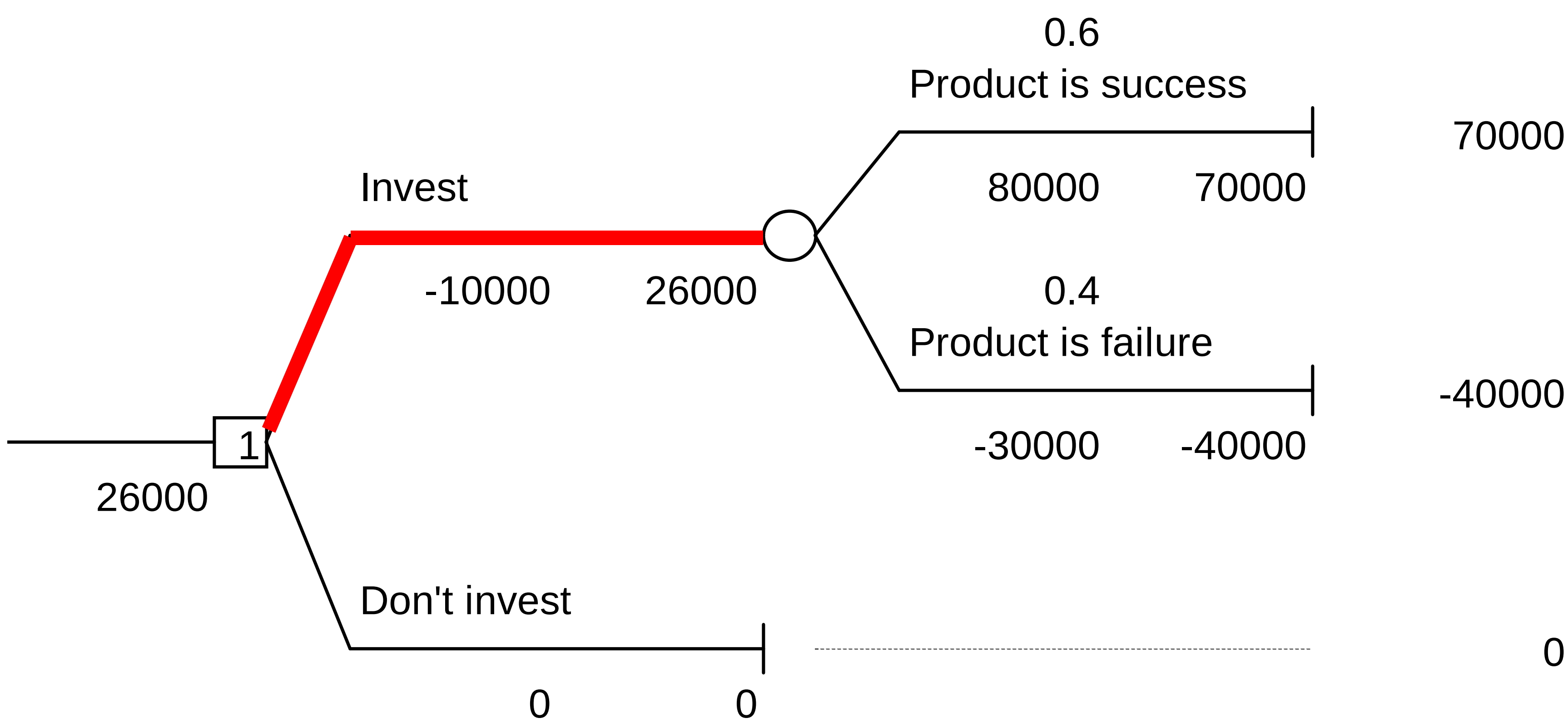
Exercise 1.1: Investment in a new product?



- Note that we have not yet discussed *risk attitude*!
- Investing is risky because we could lose 40000 with a 40% chance.

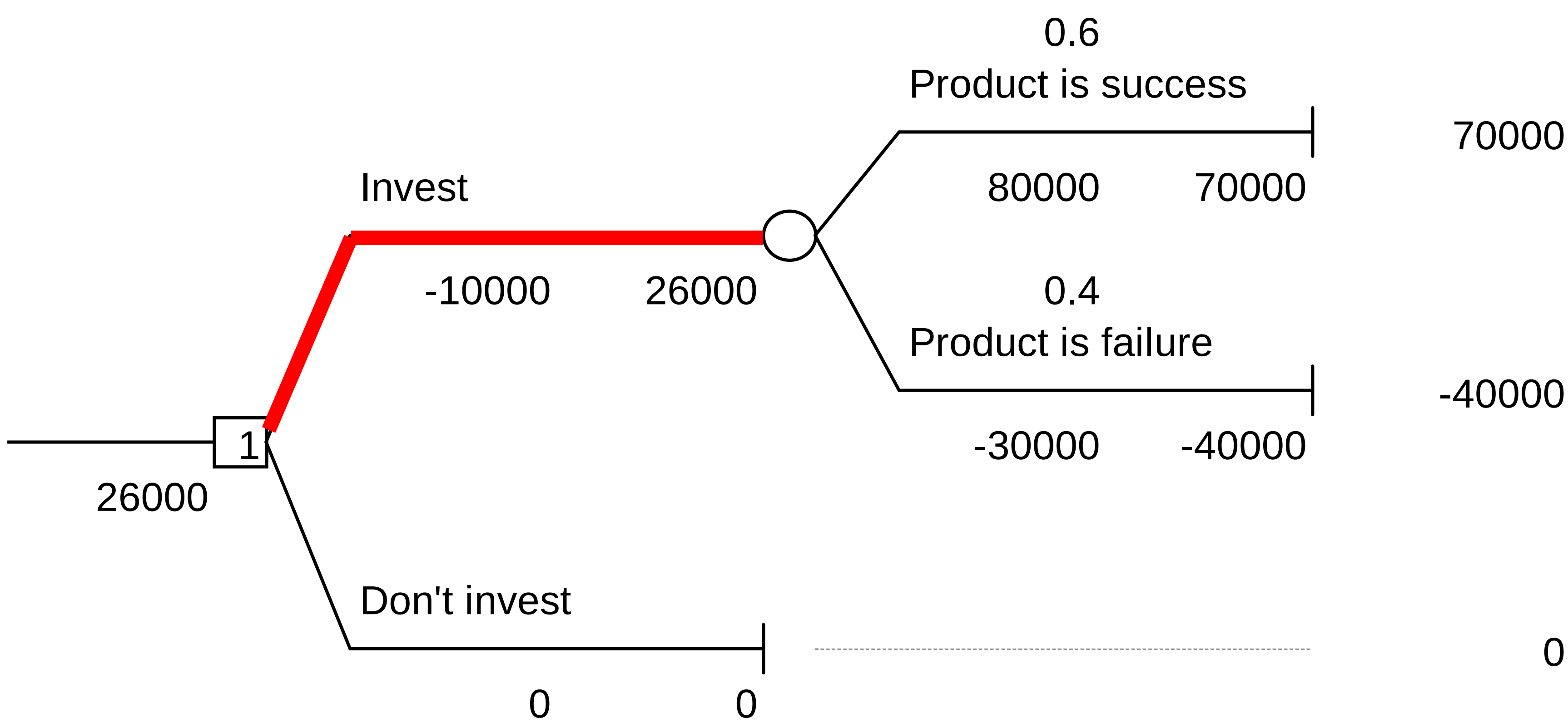


We will talk about risk attitude (risk averse – risk loving) when we discuss the concept of "utility".



Implicit assumption in analysis below:

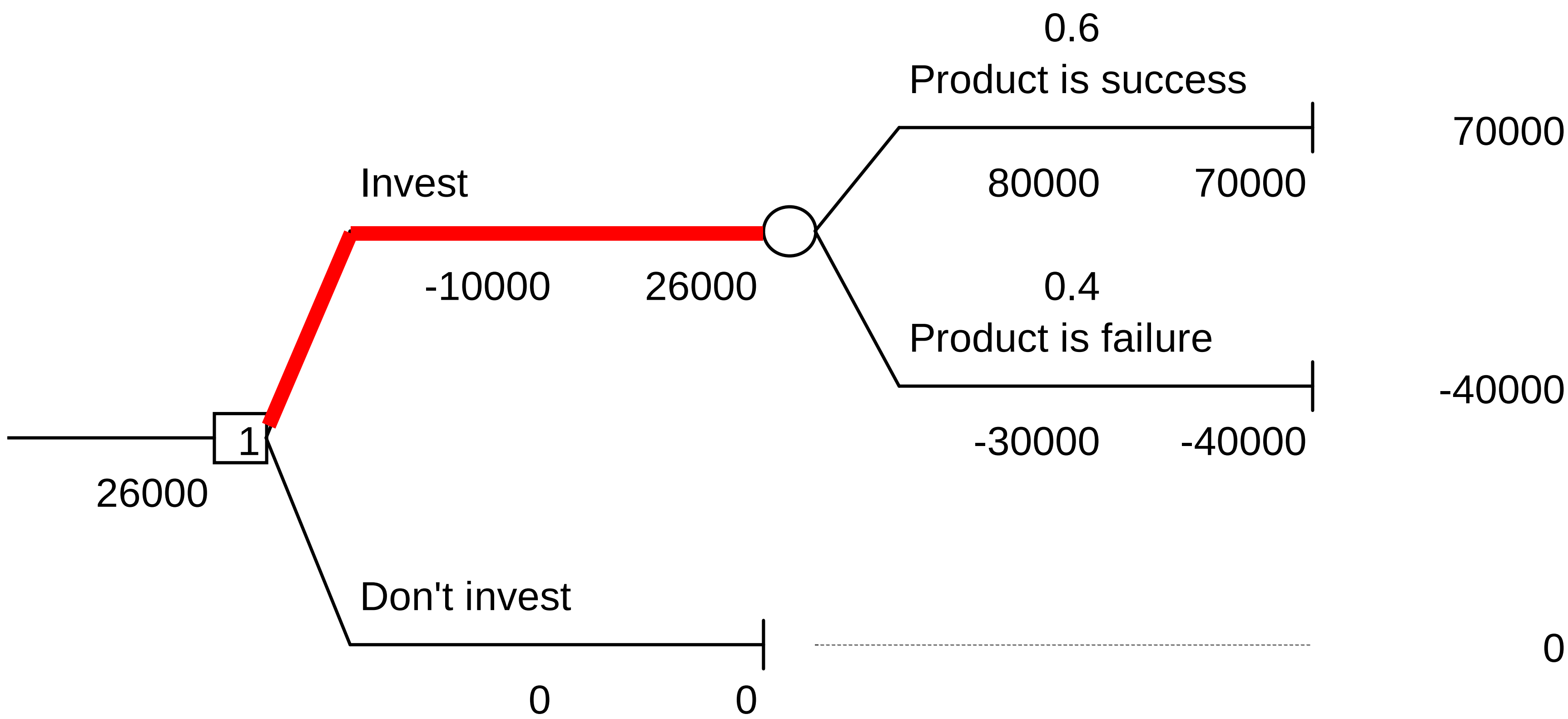
we are *risk indifferent*.



Possible Justifications:

(1) We *are* risk indifferent.

(2) We make these kinds of decisions a lot.



Multistage Decisions

- So far: *single*-stage problems

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- Choose one decision alternative => outcome

Multistage Decisions

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- Choose one decision alternative => outcome
- *Multistage* problem: involves sequence of decision alternatives and outcomes

Multistage Decisions - Example

- Pharmaceutical company concerned with product development on cancer treatment

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Multistage Decisions - Example

- Pharmaceutical company concerned with product development on cancer treatment
- Shall we submit a proposal for government grant of €85,000?
- Problem: Writing the proposal costs €5,000

Multistage Decisions - Example

- Next decision: *If* we get the grant, should we prepare product 1,2 or 3?

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- Best case: low development costs

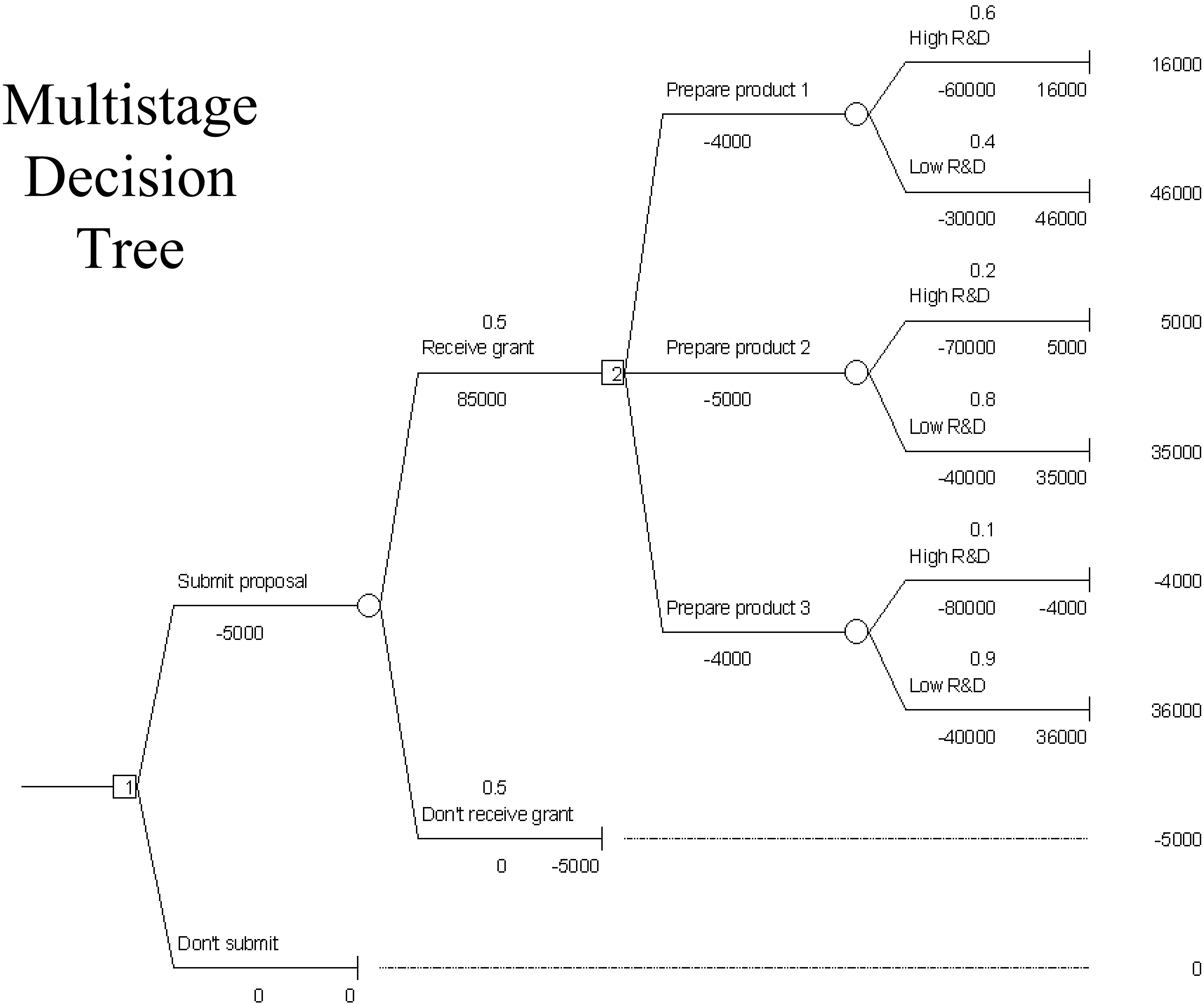
Multistage Decisions - Example

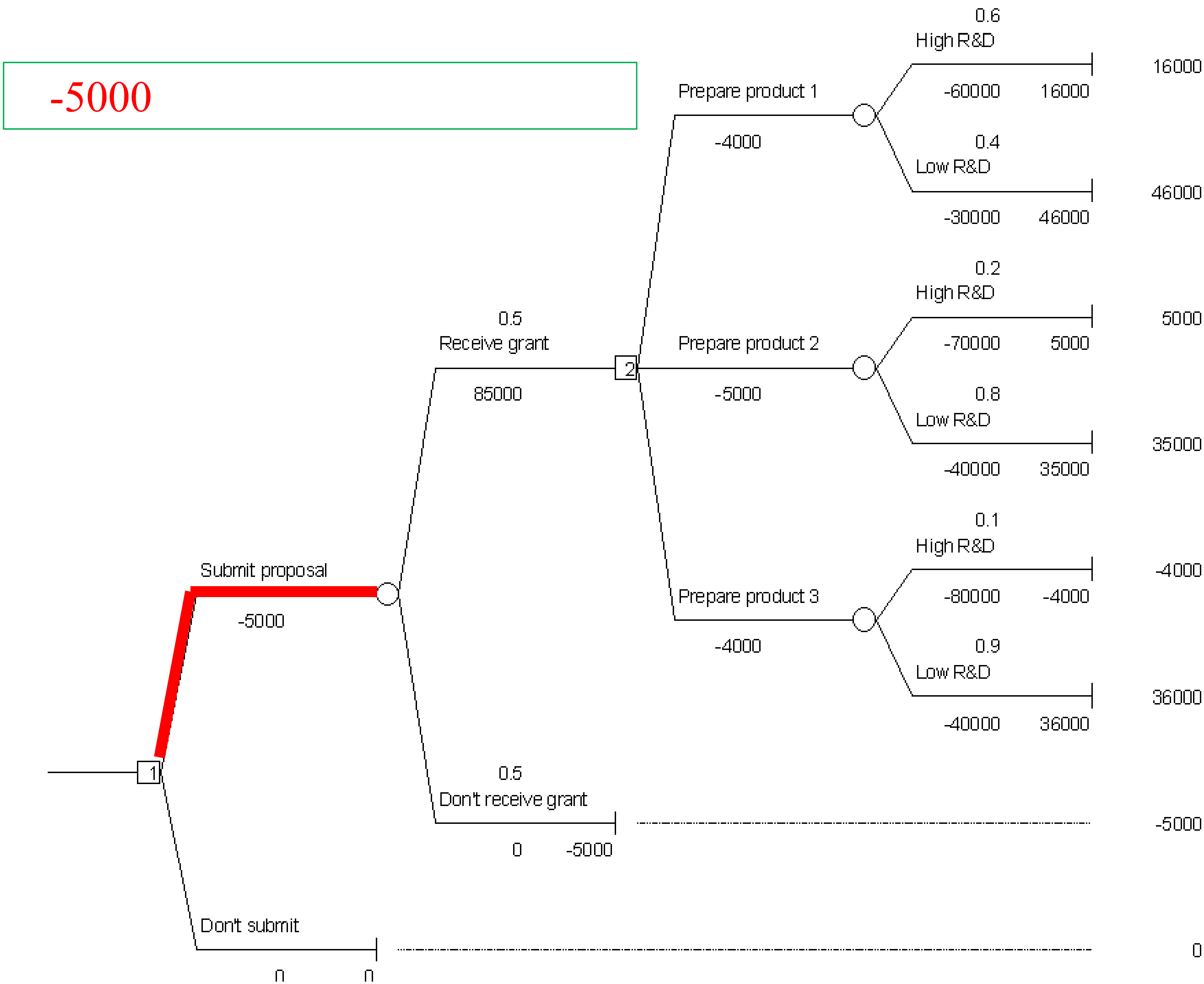
- Next decision: *If* we get the grant, should we prepare product 1,2 or 3?
- Products induce different equipment costs.
- Research and Development (R&D) costs cannot be fully predicted.
- Best case: low development costs
- Worst case: high development costs

Multistage Decisions - Example

		Best Case		Worst Case	
Product	**Equipment costs**	**R&D costs**	**Prob-ability**	**R&D costs**	**Prob-ability**
1	4,000	30,000	0.4	60,000	0.6
2	5,000	40,000	0.8	70,000	0.2
3	4,000	40,000	0.9	80,000	0.1

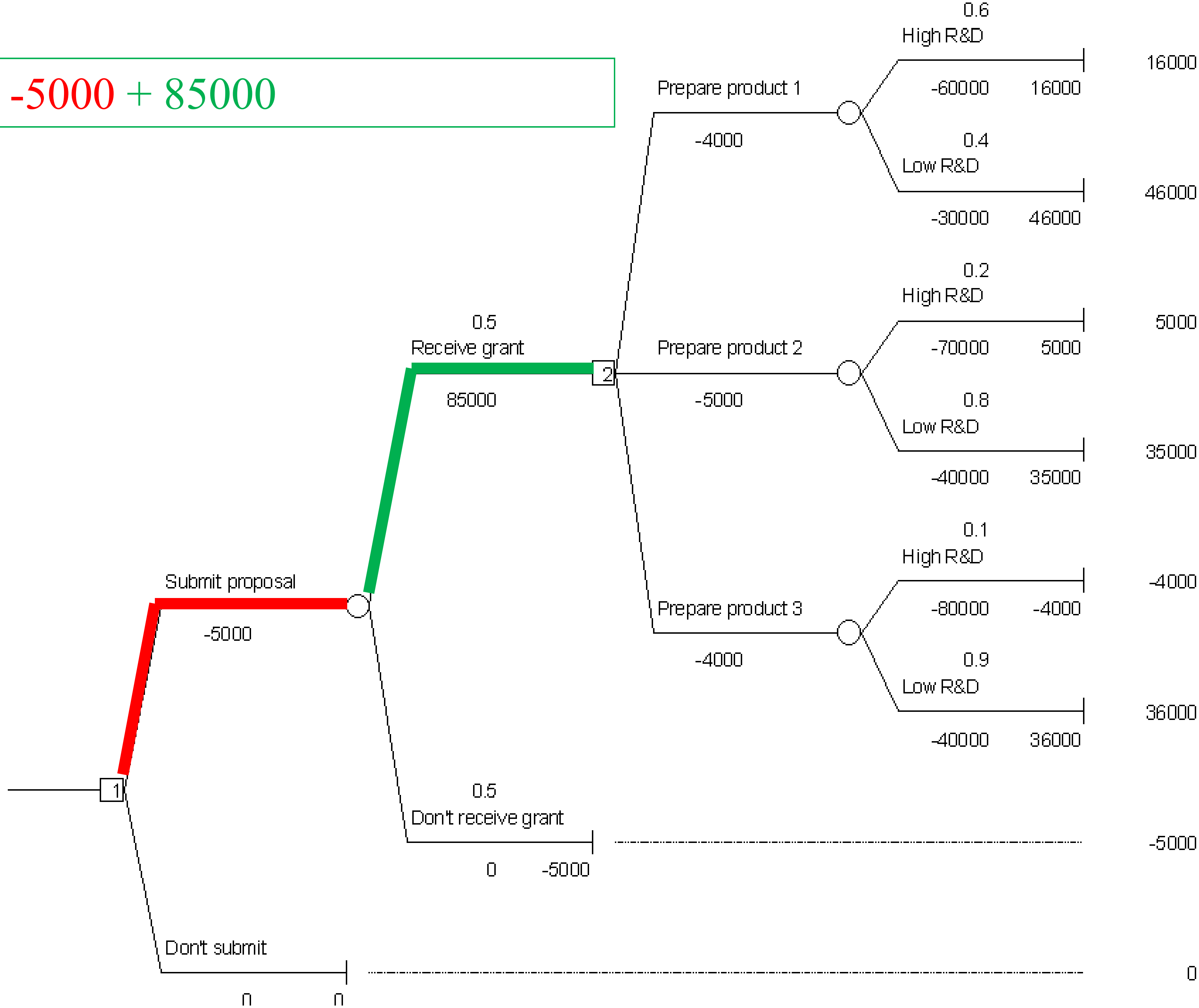
Multistage Decision Tree





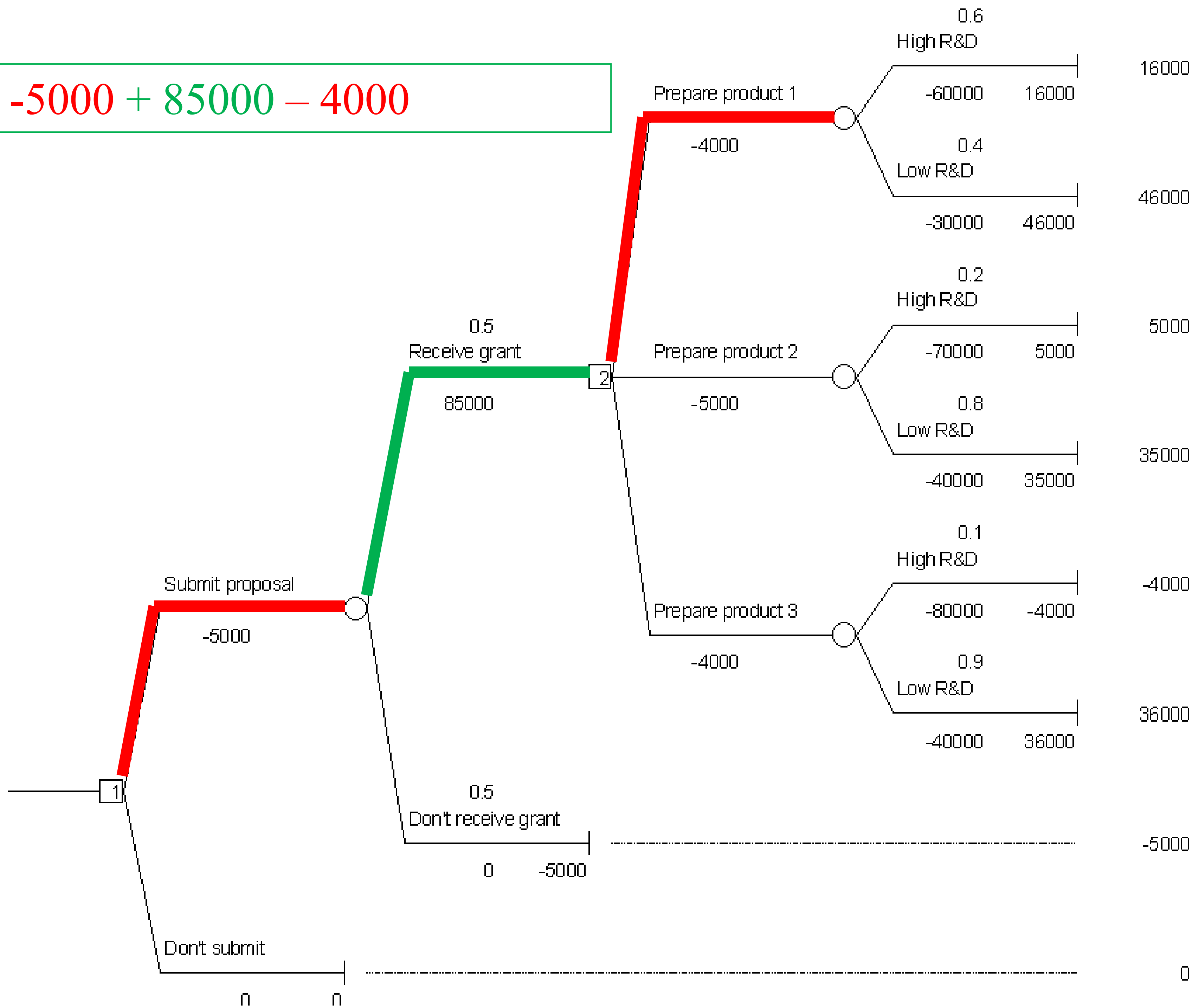
-5000
0.6
High R&D
16000
-60000
16000
Prepare product 1
-4000
0.4
Low R&D
46000
-30000
46000
0.2
High R&D
5000
-70000
5000
0.5
Receive grant
Prepare product 2
2
85000
-5000
0.8
Low R&D
35000
-40000
35000
0.1
High R&D
-4000
Submit proposal
-80000
-4000
Prepare product 3
-5000
-4000
0.9
Low R&D
36000
-40000
36000
1
0.5
Don't receive grant
-5000
0
-5000
Don't submit
0
0
0

-5000 + 85000

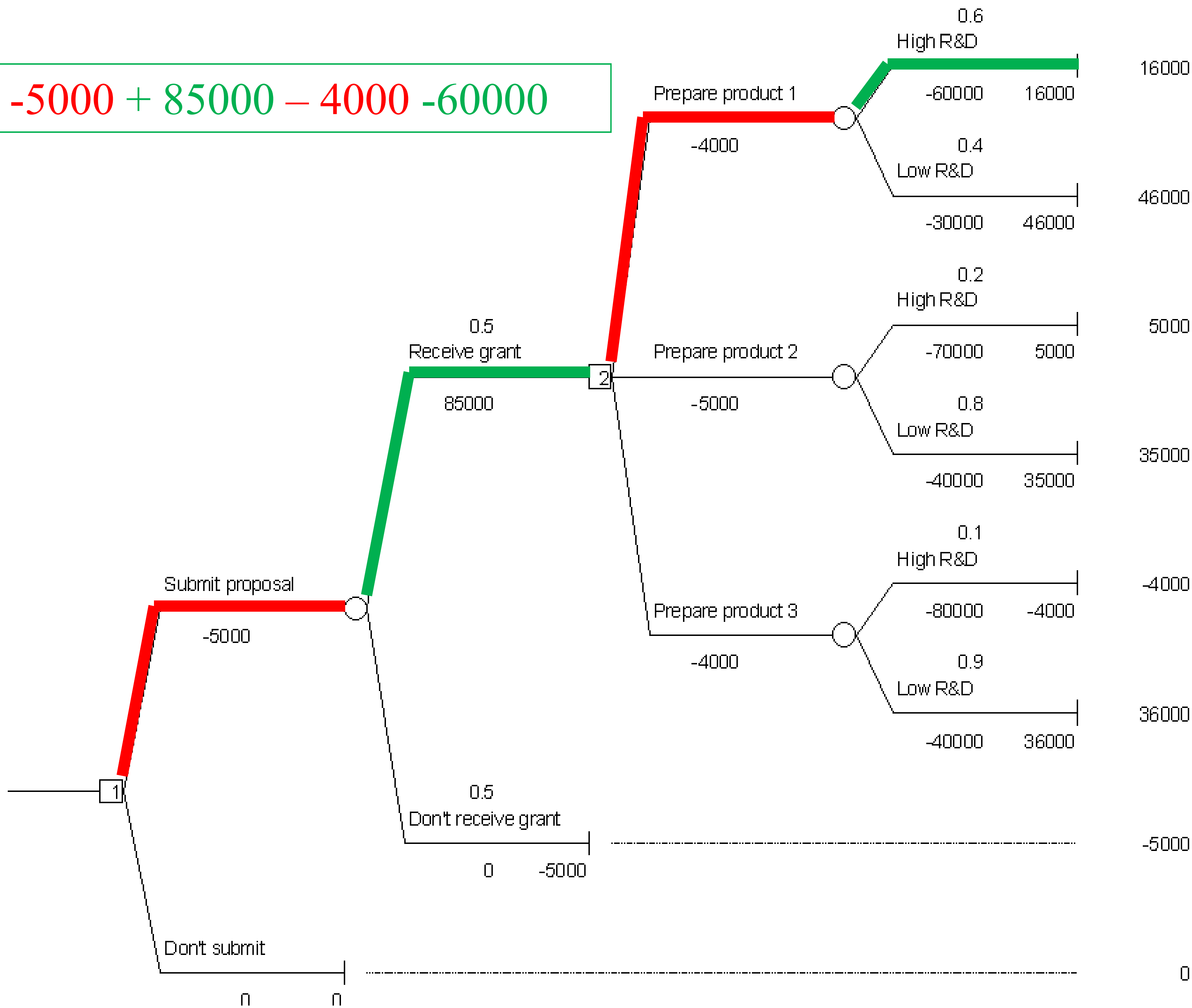


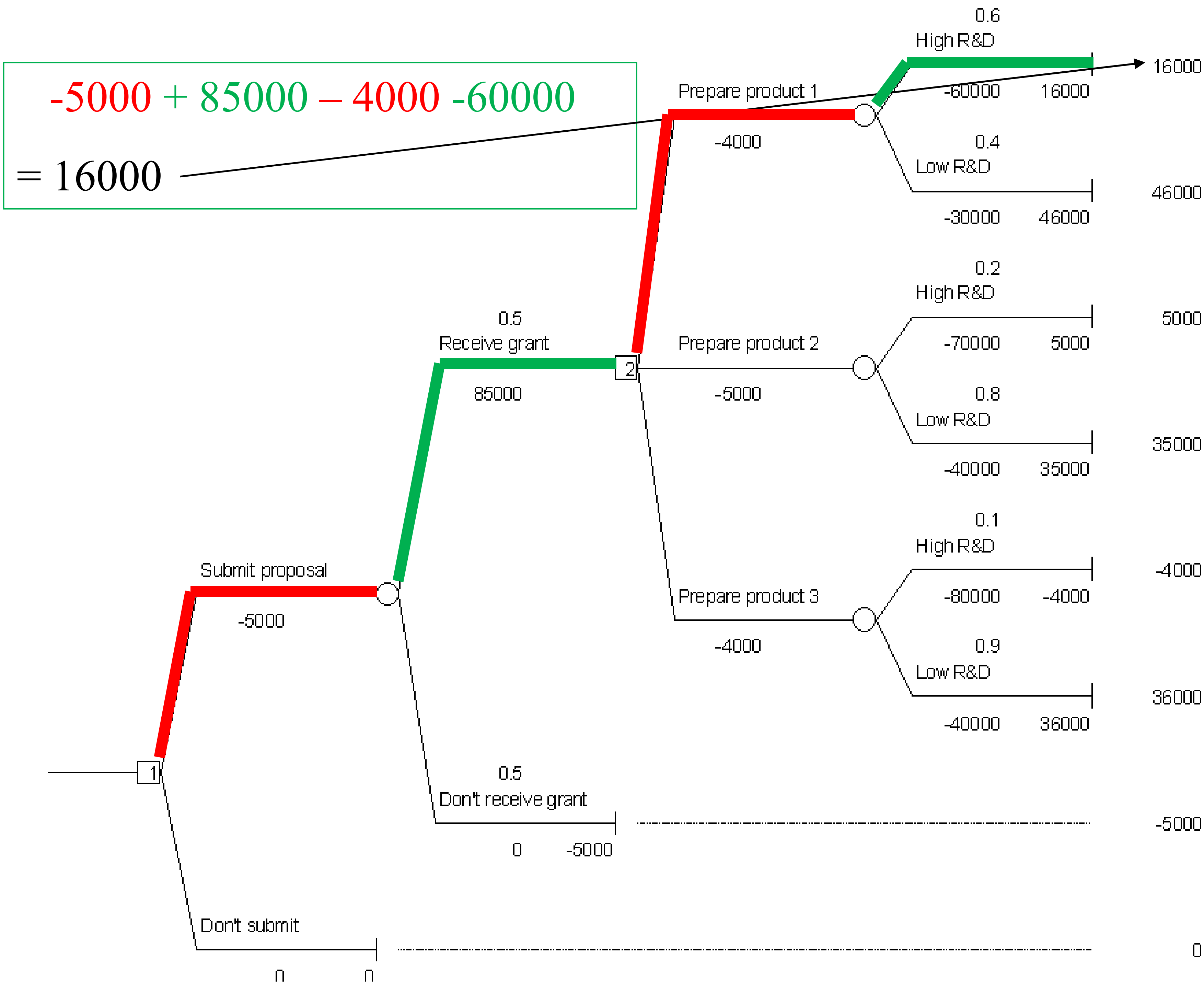
0.6
High R&D
16000
-60000
16000
Prepare product 1
-4000
0.4
Low R&D
46000
-30000
46000
0.2
High R&D
5000
-70000
5000
0.5
Receive grant
Prepare product 2
2
85000
-5000
0.8
Low R&D
35000
-40000
35000
0.1
High R&D
-4000
Submit proposal
-80000
-4000
Prepare product 3
-5000
-4000
0.9
Low R&D
36000
-40000
36000
0.5
1
Don't receive grant
-5000
0
-5000
Don't submit
0
0
0

$$-5000 + 85000 - 4000$$



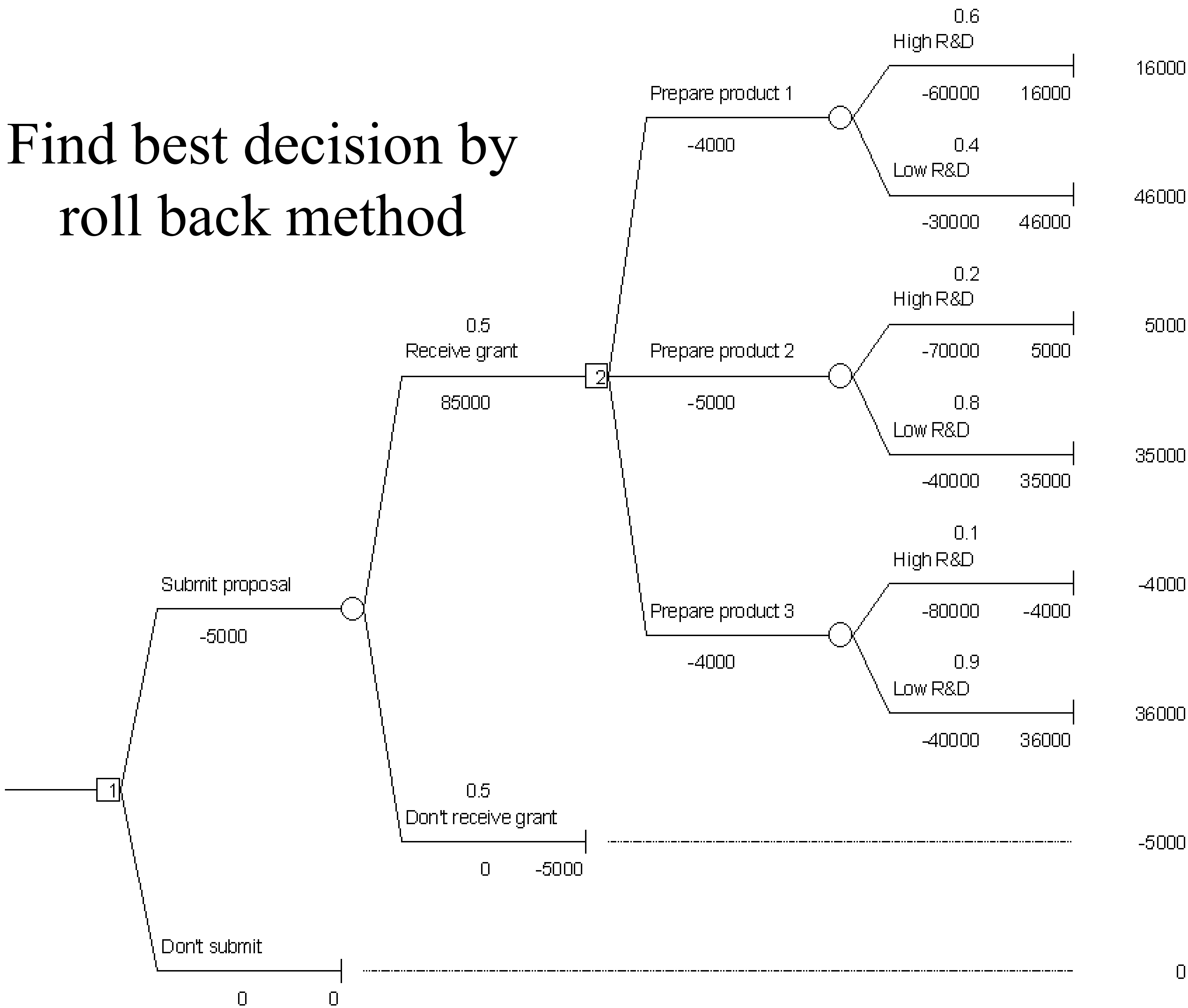
$-5000 + 85000 - 4000 - 60000$



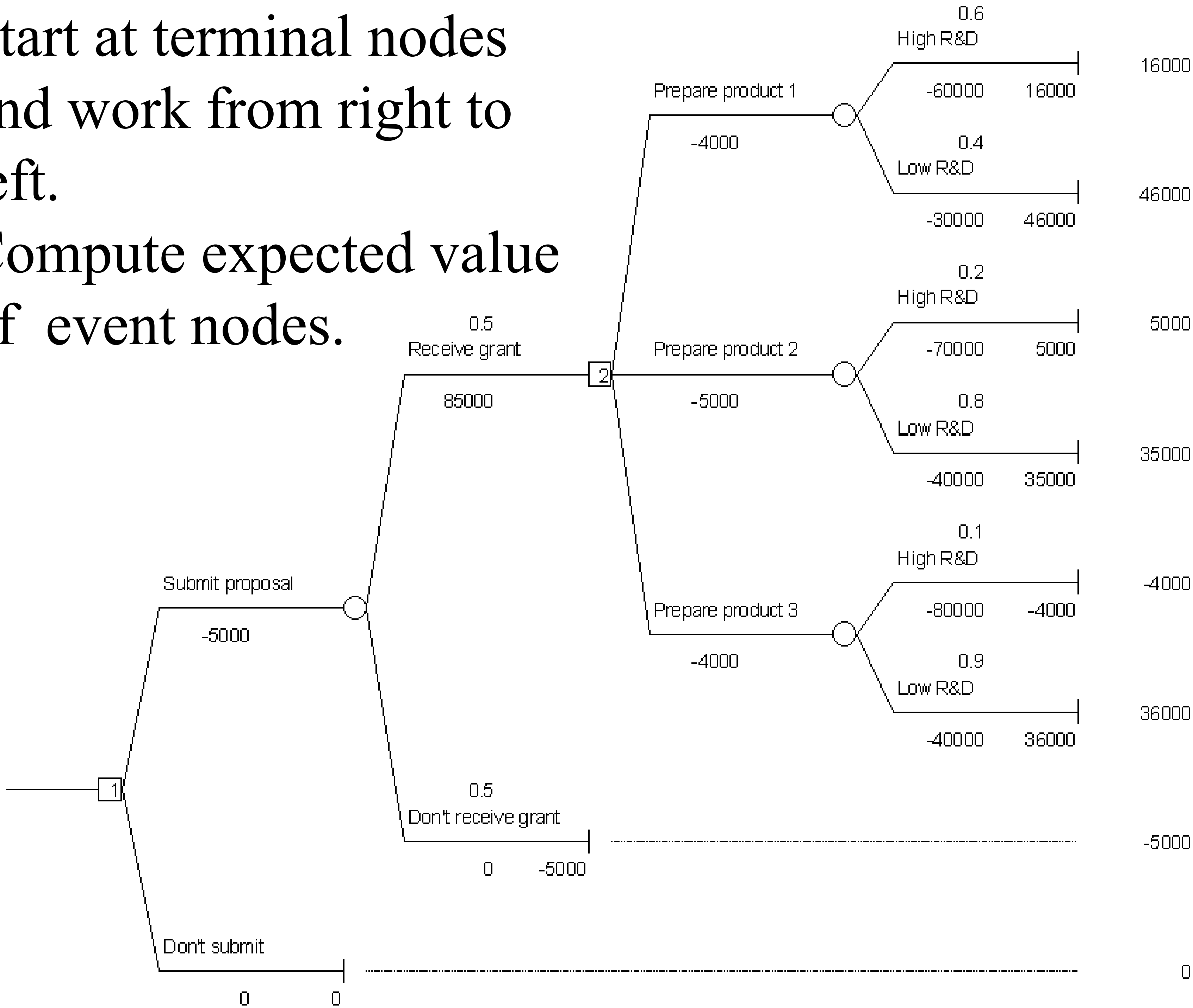


-5000 + 85000 – 4000 -60000
= 16000
0.6
High R&D
-60000
16000
16000
Prepare product 1
-4000
0.4
Low R&D
-30000
46000
46000
0.2
High R&D
-70000
5000
5000
0.5
Receive grant
85000
2
Prepare product 2
-5000
0.8
Low R&D
-40000
35000
35000
0.1
High R&D
-80000
-4000
-4000
Submit proposal
-5000
Prepare product 3
-4000
0.9
Low R&D
-40000
36000
36000
1
0.5
Don't receive grant
0
-5000
-5000
Don't submit
0
0
0

Find best decision by roll back method



Start at terminal nodes and work from right to left.
Compute expected value of event nodes.



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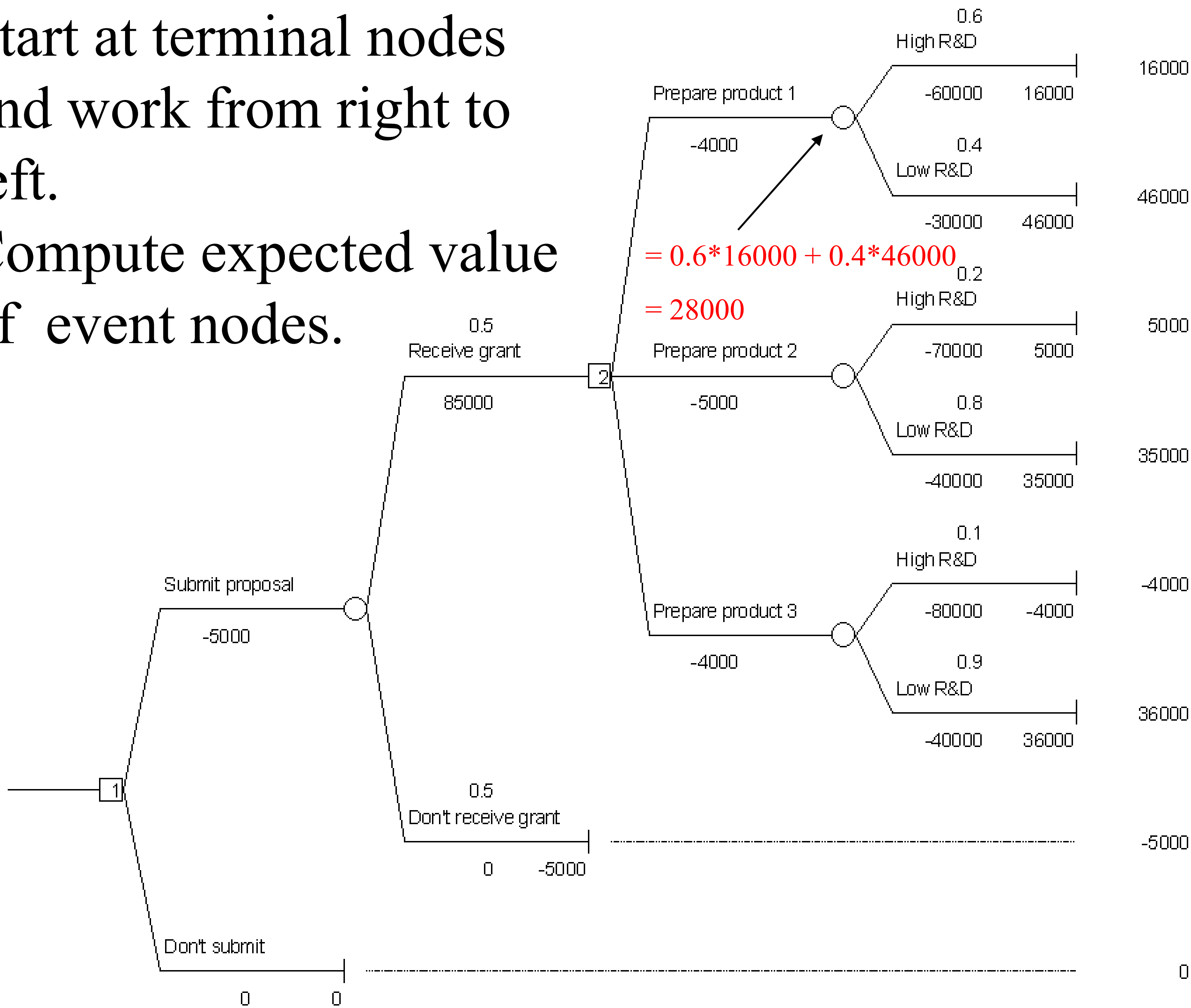

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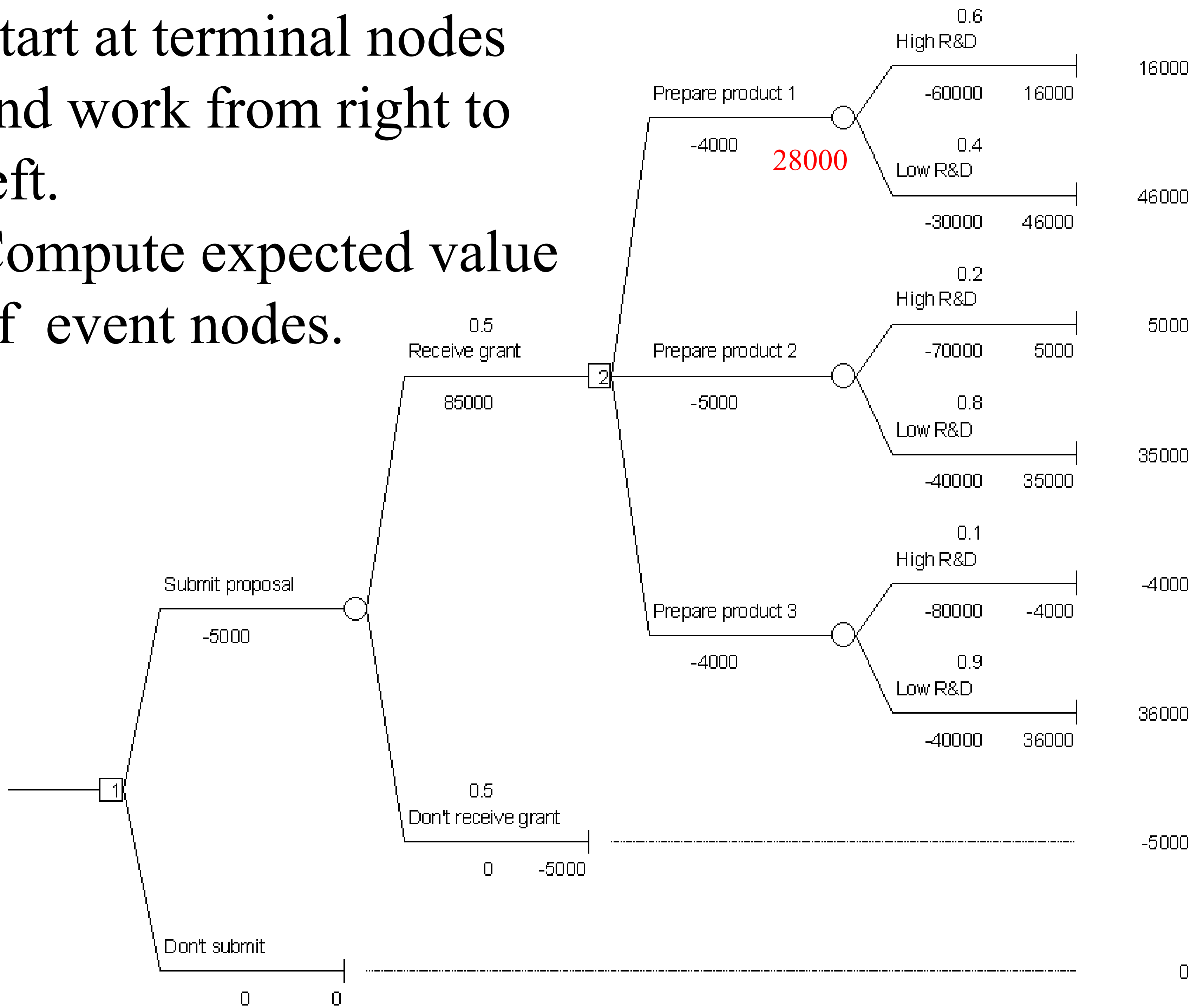
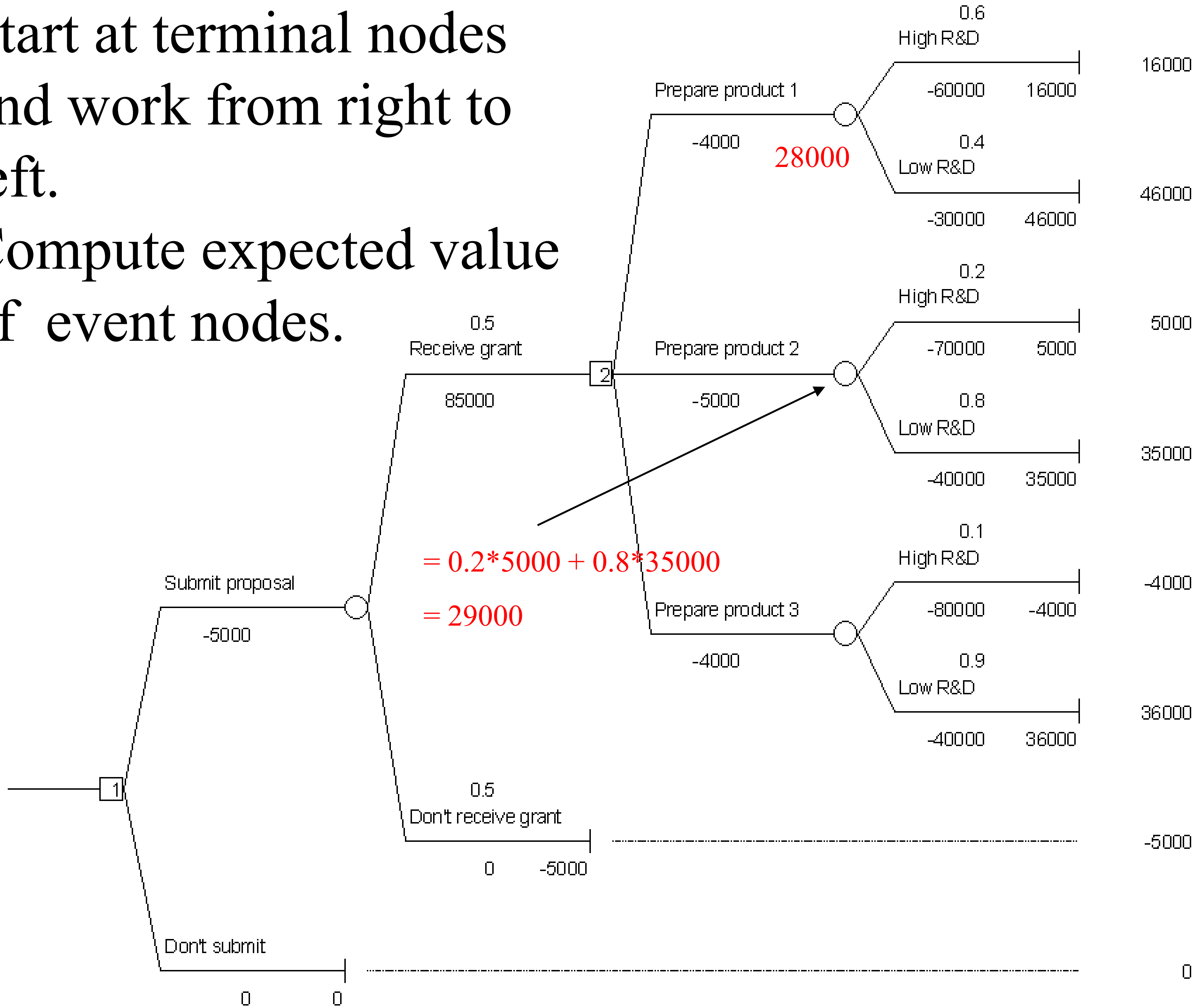


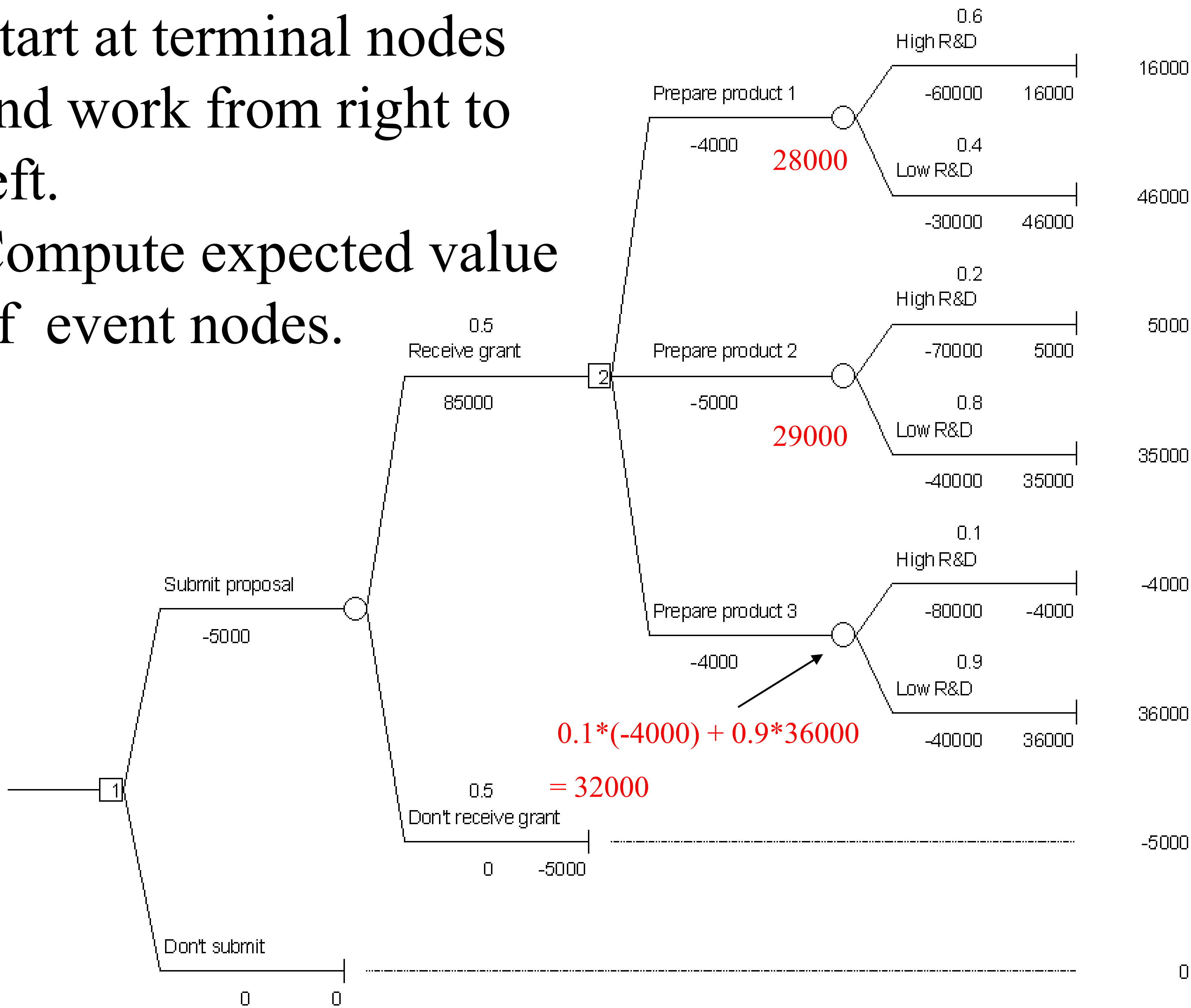
Start at terminal nodes and work from right to left.

Compute expected value of event nodes.

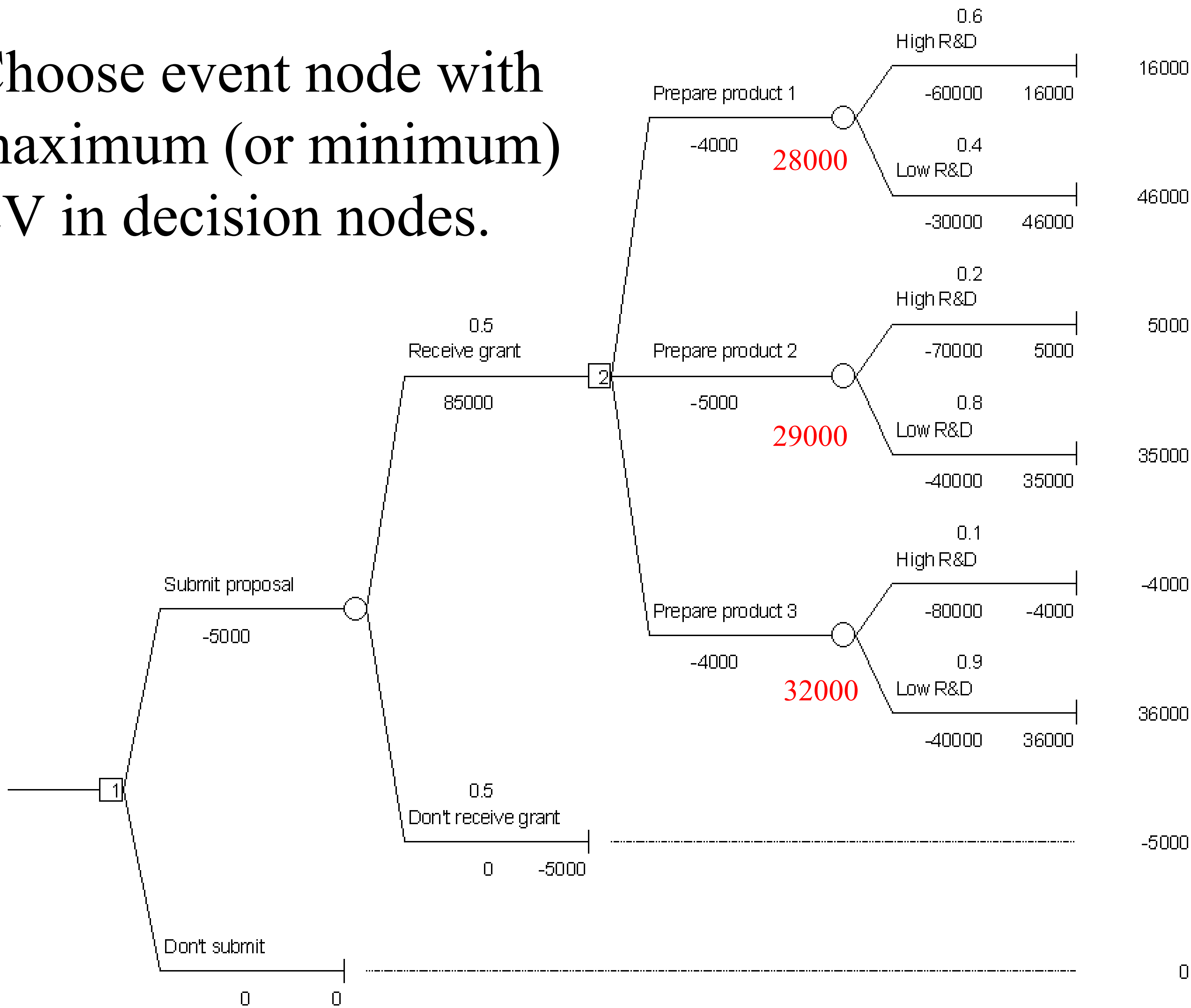


Start at terminal nodes and work from right to left.

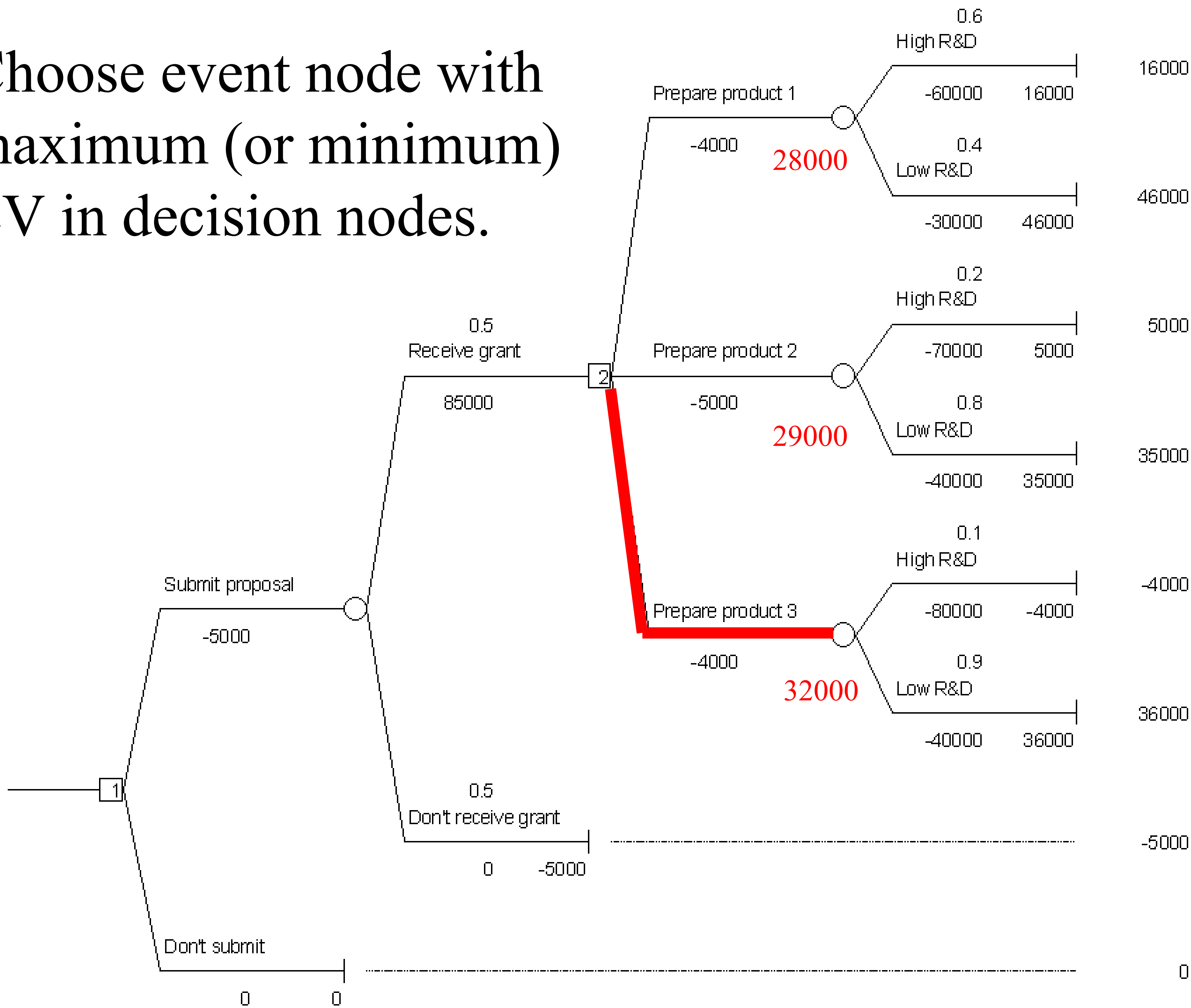
Compute expected value of event nodes.



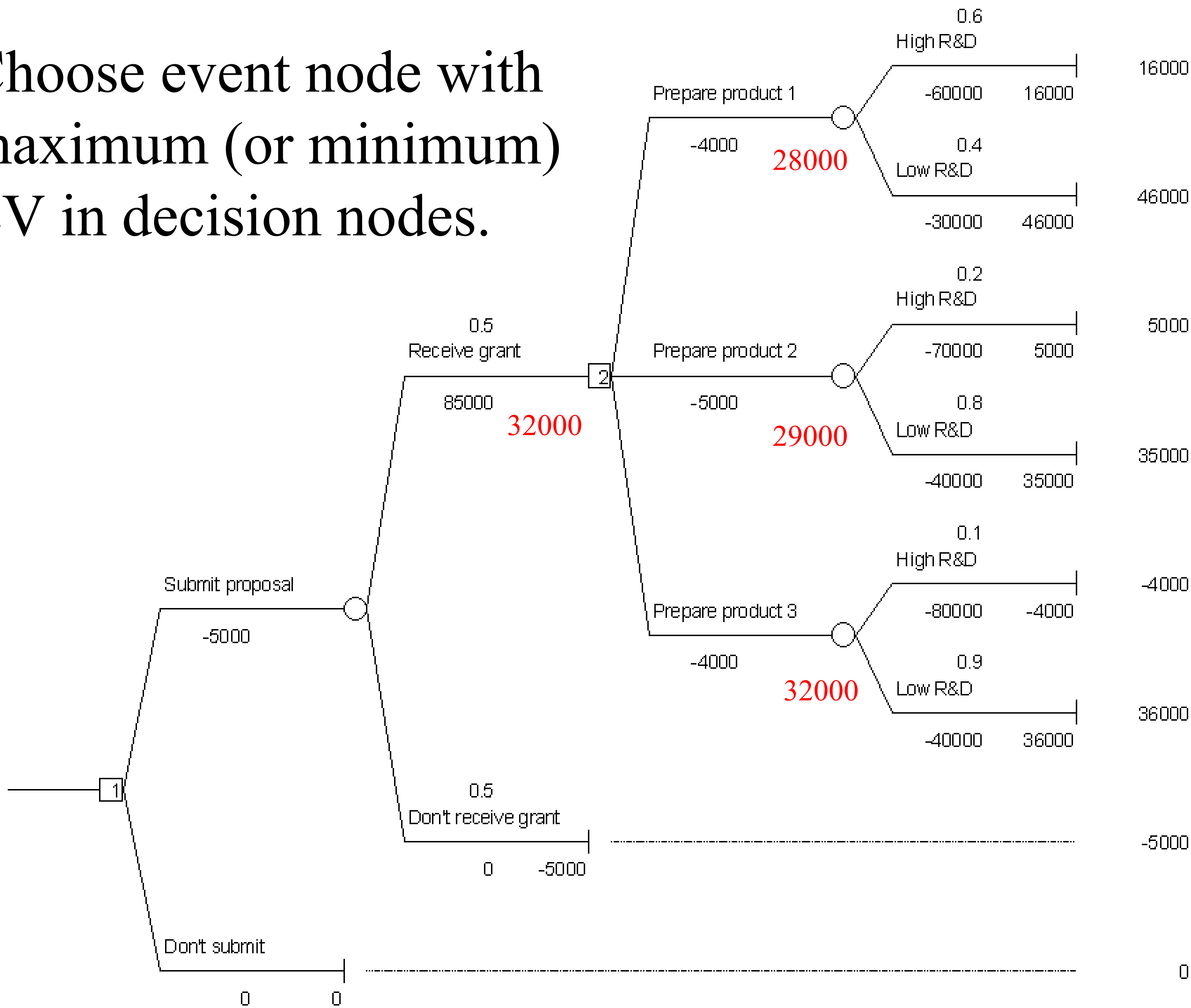
Choose event node with maximum (or minimum) EV in decision nodes.



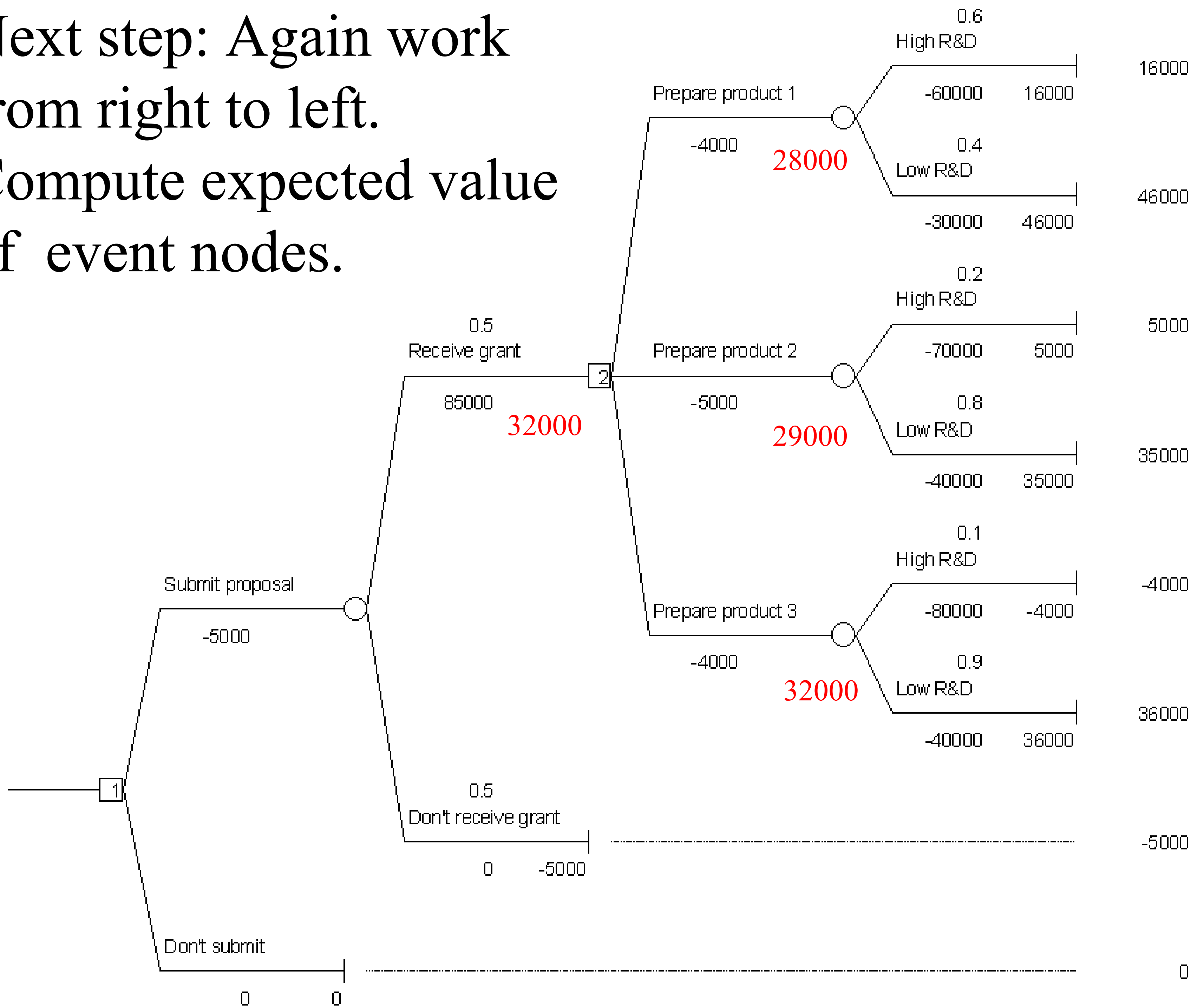
Choose event node with maximum (or minimum) EV in decision nodes.



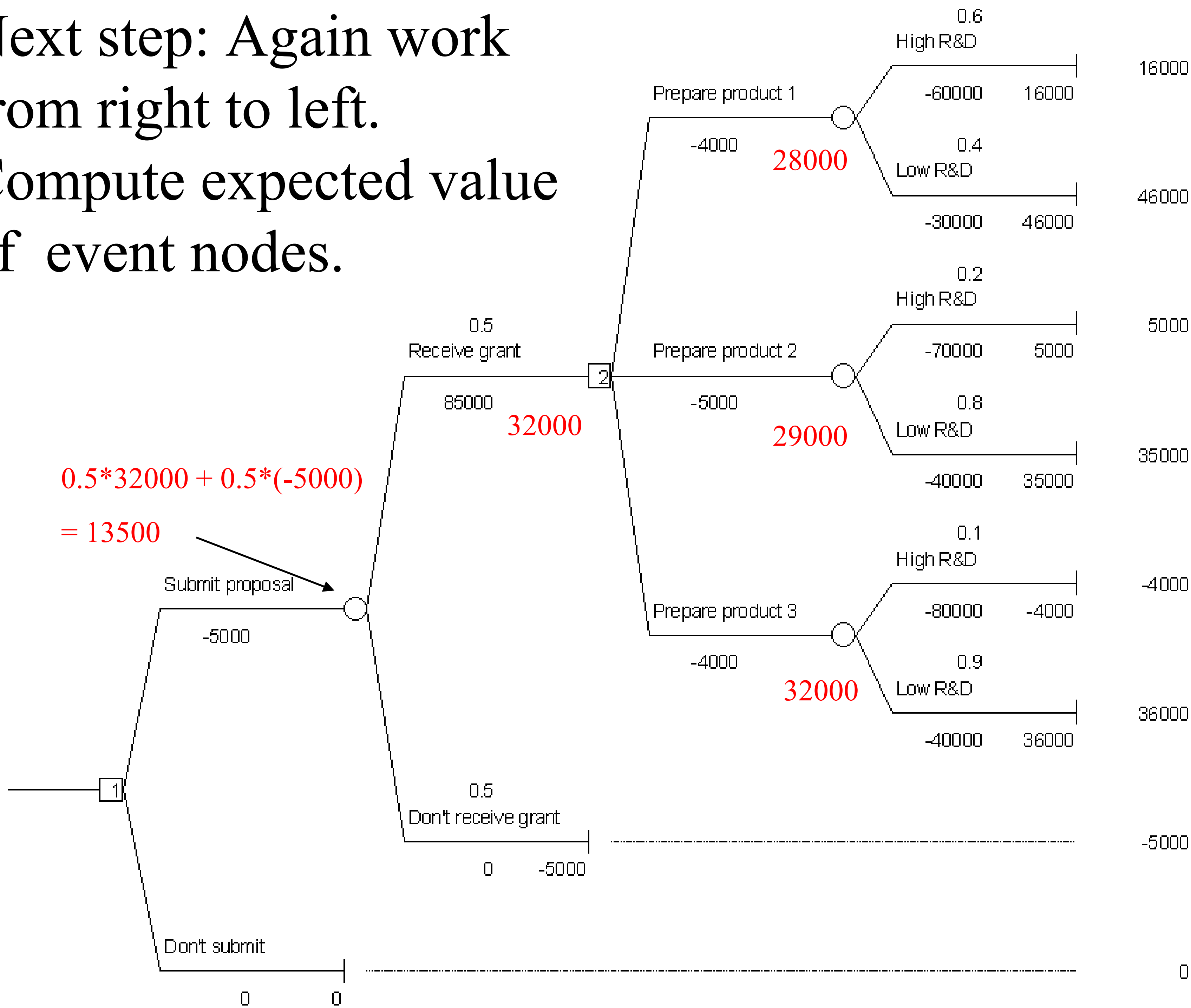
Choose event node with maximum (or minimum) EV in decision nodes.



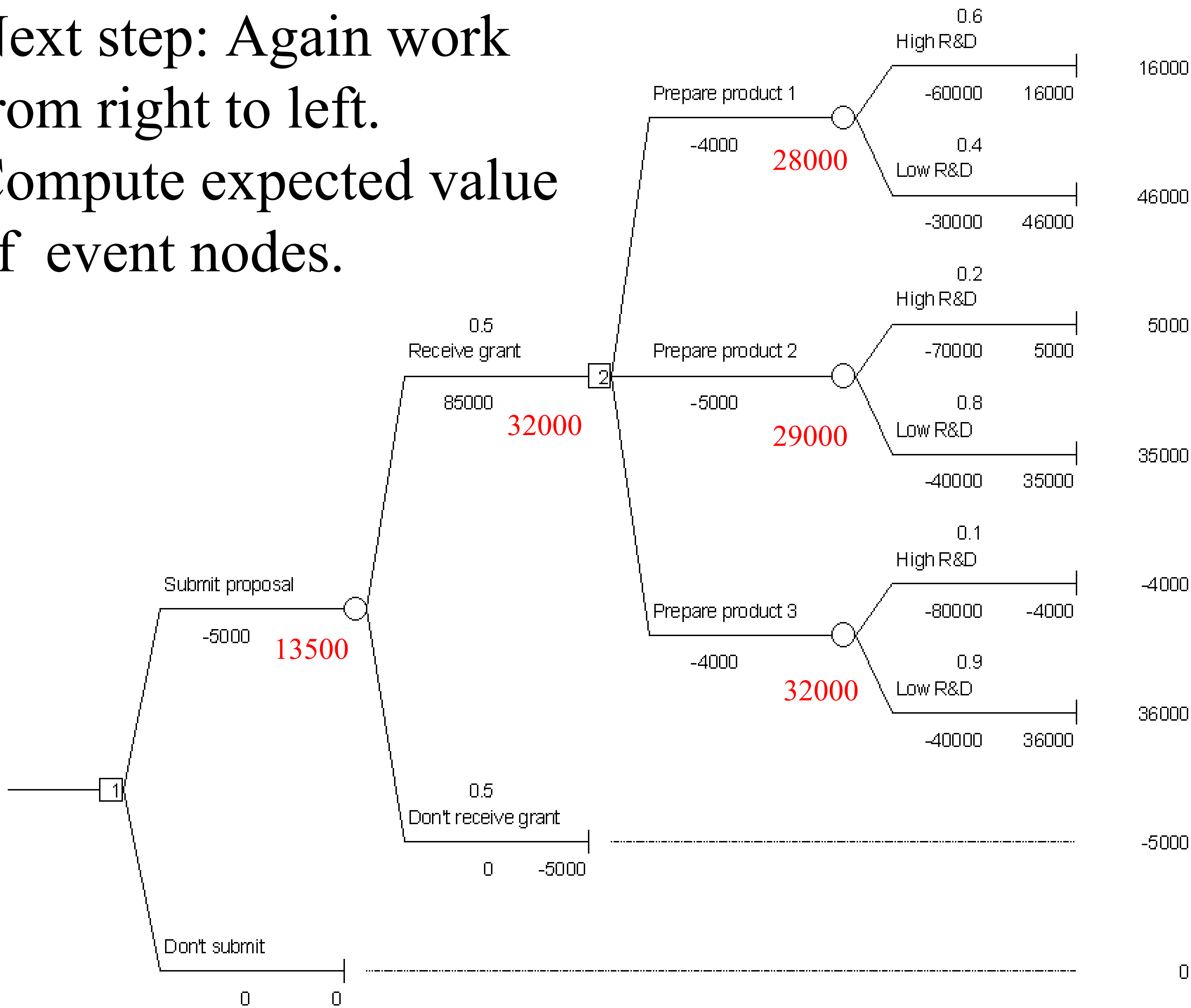
Next step: Again work from right to left. Compute expected value of event nodes.



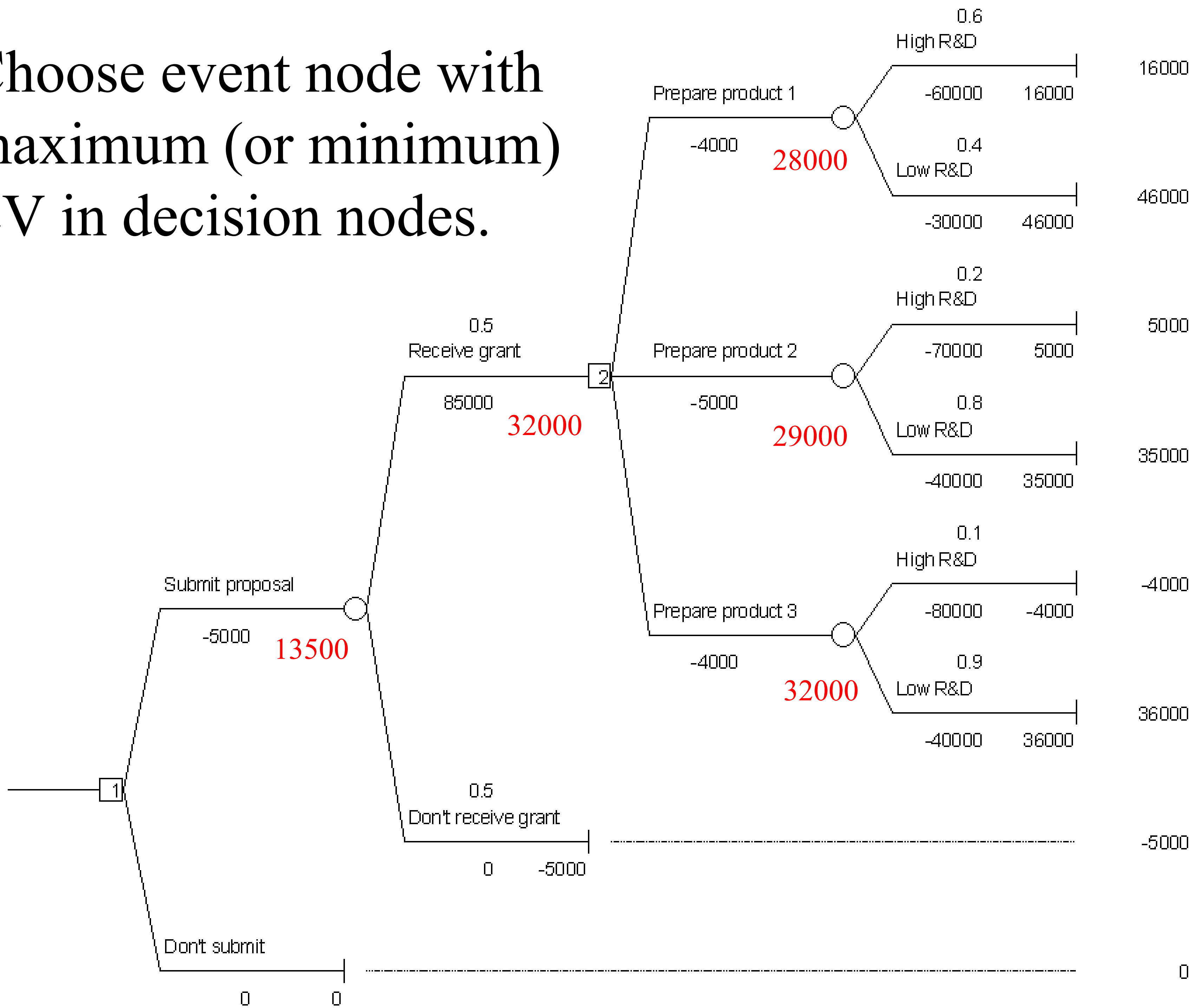
Next step: Again work from right to left. Compute expected value of event nodes.



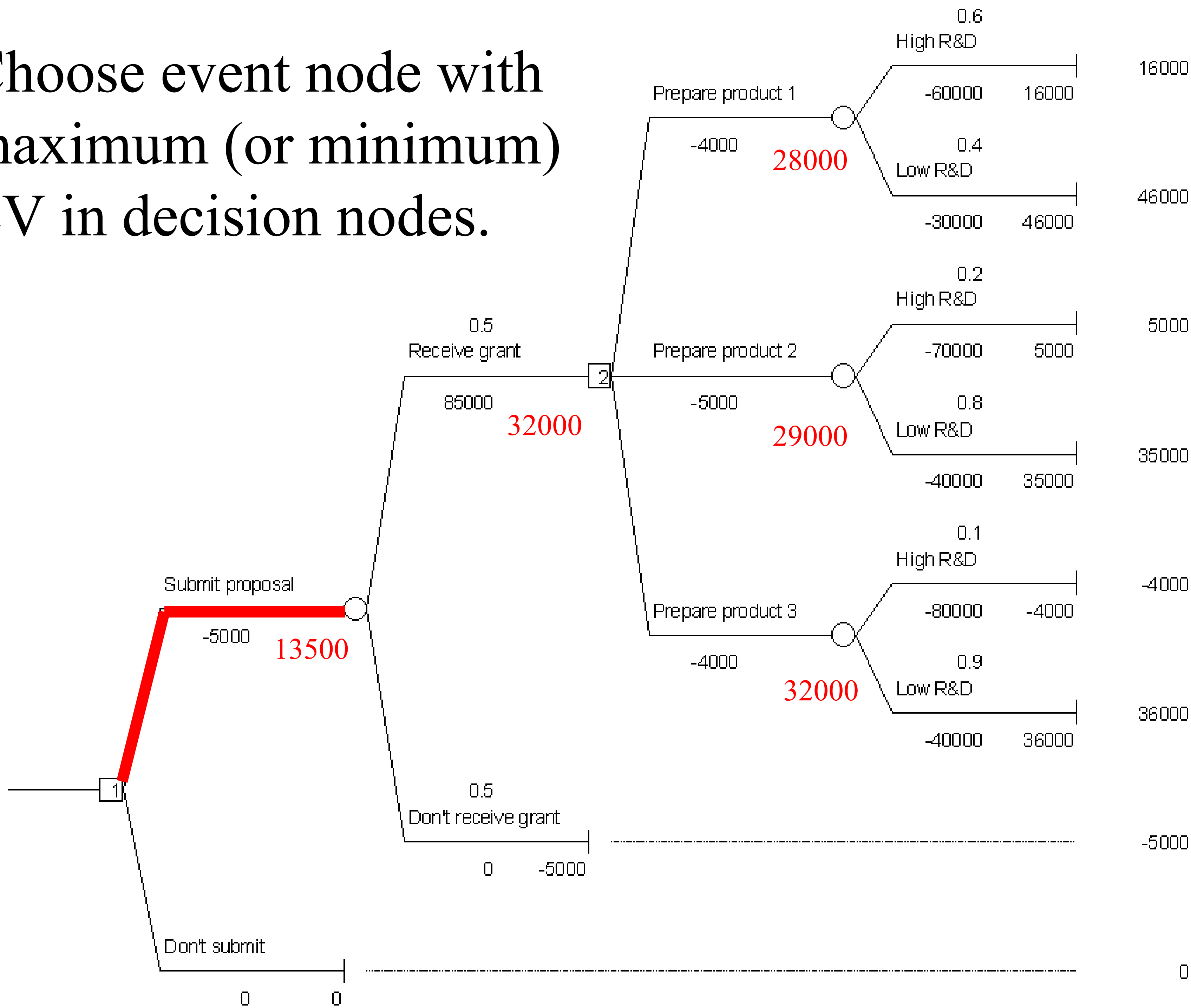
Next step: Again work from right to left. Compute expected value of event nodes.



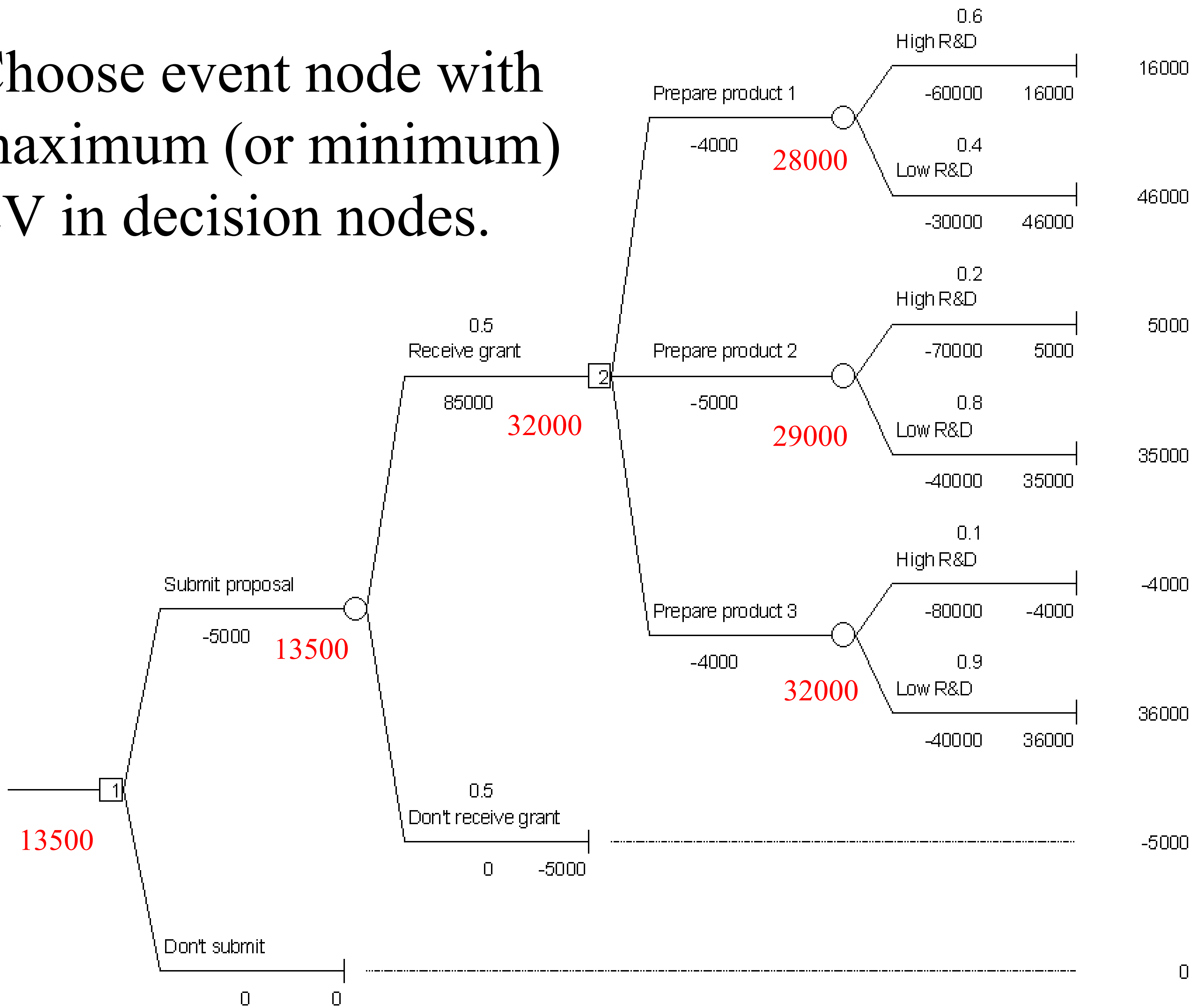
Choose event node with maximum (or minimum) EV in decision nodes.



Choose event node with maximum (or minimum) EV in decision nodes.



Choose event node with maximum (or minimum) EV in decision nodes.



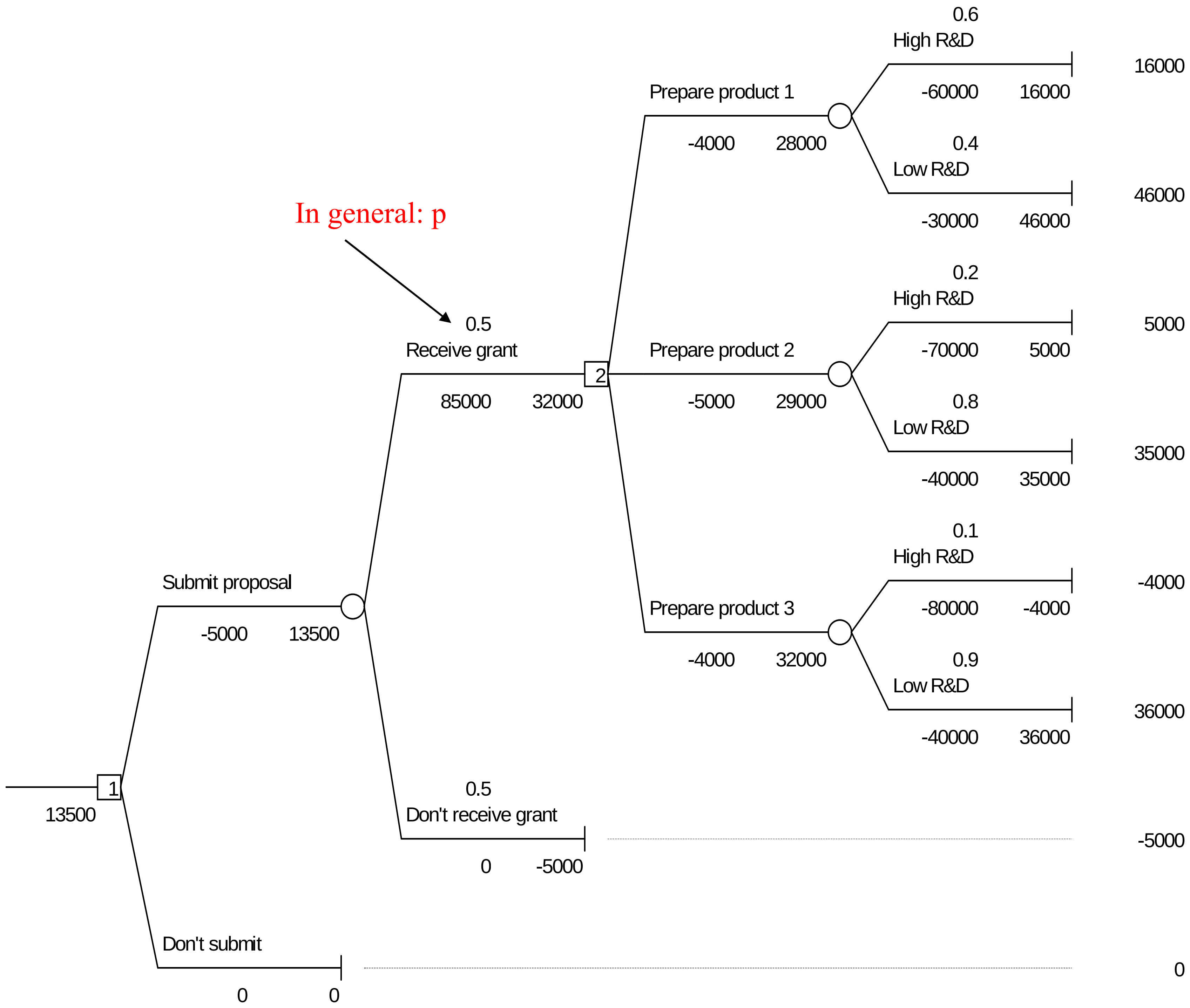
Sensitivity Analysis

- *Sensitivity Analysis*: how sensitive is the result for changes in values of parameters?
- Why is this an interesting question?
- E.g. the probability to get the grant is usually a rough estimation.
- We estimated it to be 0.50.
- Do we get a complete different result if it is 0.49 or 0.51?

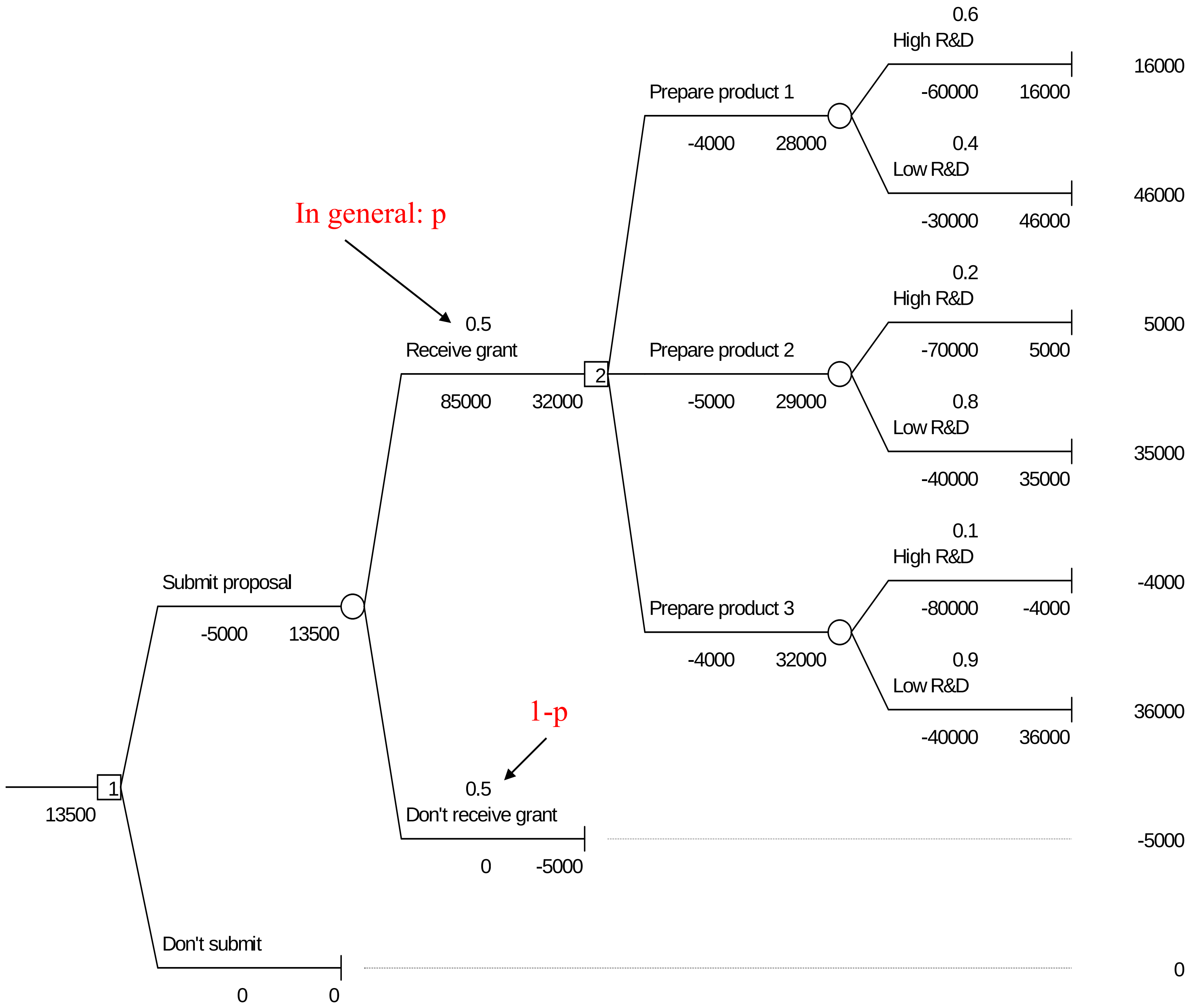
Sensitivity Analysis

Systematic approach of this question:

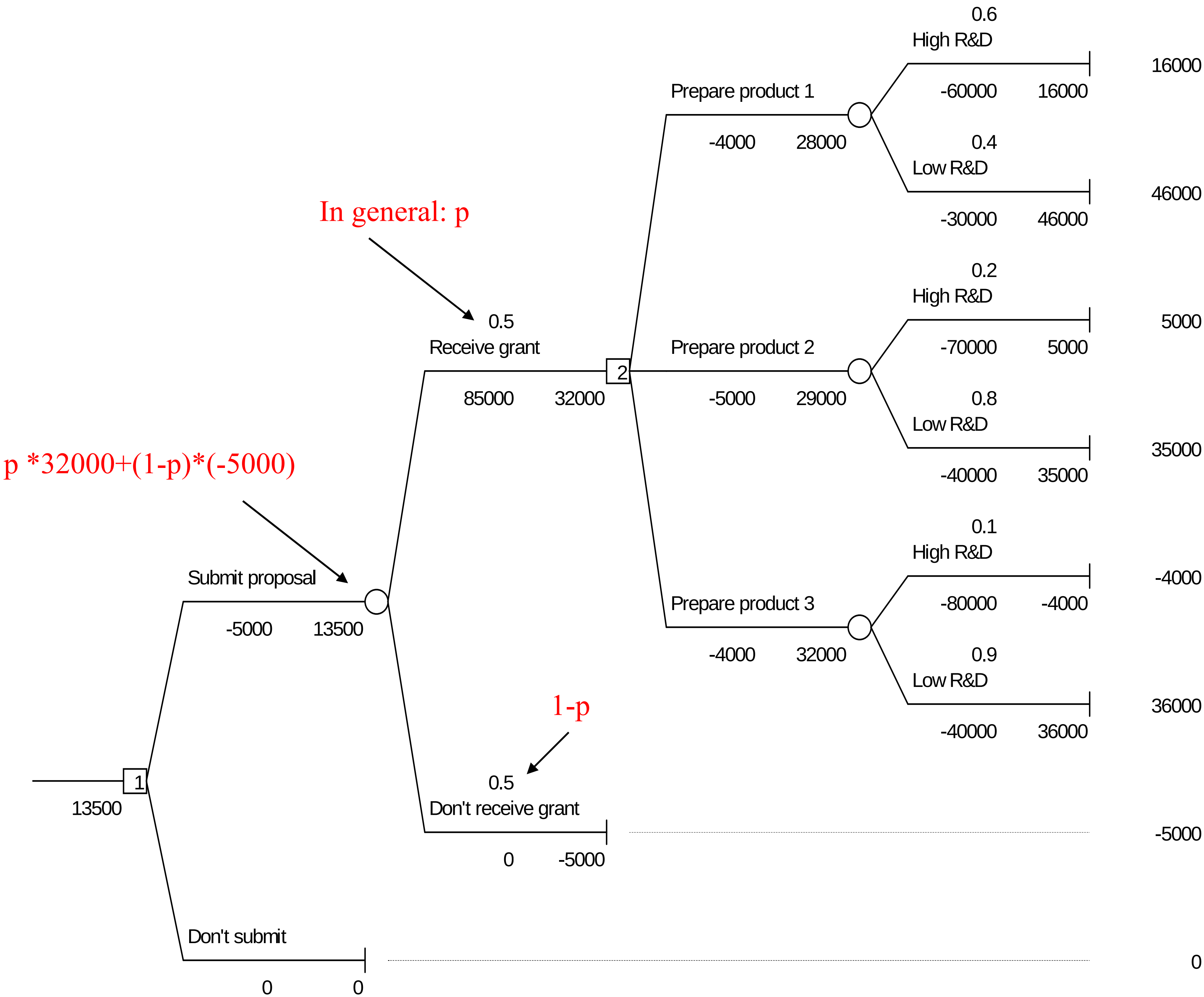
- We came to the conclusion that it makes sense to submit the proposal if chance to get it is 0.5 .
- What is the *minimum probability* for which it still makes sense to submit the proposal (possibly losing €5000)?



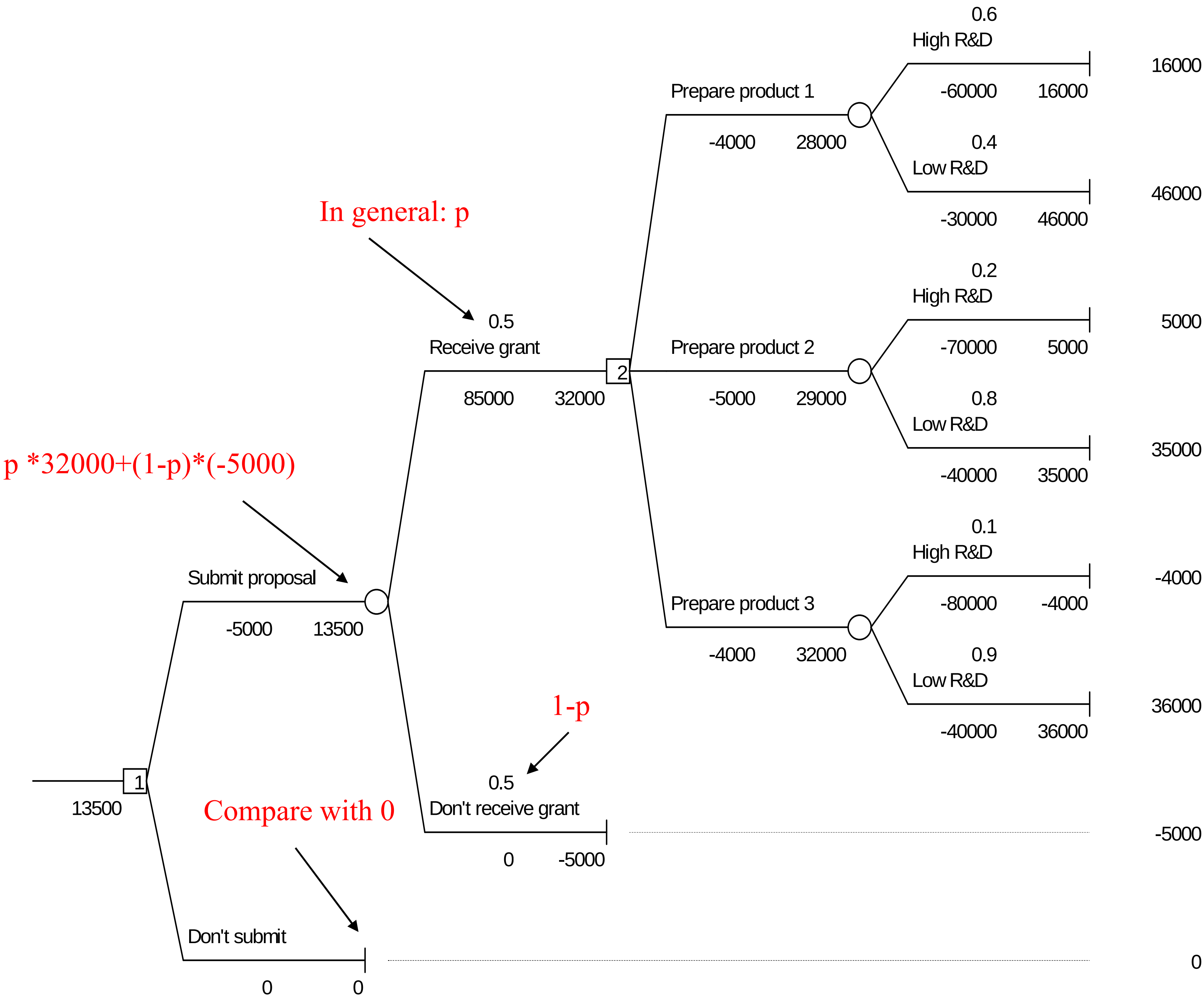
In general: p
0.5
Receive grant
85000
32000
0.5
Don't receive grant
0
-5000
Submit proposal
-5000
13500
Don't submit
0
0
13500
1
2
Prepare product 1
-4000
28000
Prepare product 2
-5000
29000
Prepare product 3
-4000
32000
0.6
High R&D
-60000
16000
0.4
Low R&D
-30000
46000
0.2
High R&D
-70000
5000
0.8
Low R&D
-40000
35000
0.1
High R&D
-80000
-4000
0.9
Low R&D
-40000
36000
16000
46000
5000
35000
-4000
36000
-5000
0



In general: p
1-p
0.5
Receive grant
85000
32000
2
0.5
Don't receive grant
0
-5000
Submit proposal
-5000
13500
1
13500
Don't submit
0
0
Prepare product 1
-4000
28000
Prepare product 2
-5000
29000
Prepare product 3
-4000
32000
0.6
High R&D
-60000
16000
0.4
Low R&D
-30000
46000
0.2
High R&D
-70000
5000
0.8
Low R&D
-40000
35000
0.1
High R&D
-80000
-4000
0.9
Low R&D
-40000
36000
16000
46000
5000
35000
-4000
36000
-5000
0



In general: p
p *32000+(1-p)*(-5000)
1-p
13500
1
Submit proposal
-5000
13500
Don't submit
0
0
0.5
Receive grant
85000
32000
2
0.5
Don't receive grant
0
-5000
Prepare product 1
-4000
28000
Prepare product 2
-5000
29000
Prepare product 3
-4000
32000
0.6
High R&D
-60000
16000
0.4
Low R&D
-30000
46000
0.2
High R&D
-70000
5000
0.8
Low R&D
-40000
35000
0.1
High R&D
-80000
-4000
0.9
Low R&D
-40000
36000
16000
46000
5000
35000
-4000
36000
-5000
0



0.6
High R&D
16000
Prepare product 1
-60000
16000
-4000
28000
0.4
Low R&D
46000
-30000
46000
In general: p
0.2
High R&D
5000
0.5
Receive grant
Prepare product 2
-70000
5000
2
85000
32000
-5000
29000
0.8
Low R&D
35000
p *32000+(1-p)*(-5000)
-40000
35000
0.1
High R&D
-4000
Submit proposal
-80000
-4000
Prepare product 3
-5000
13500
-4000
32000
0.9
Low R&D
36000
1-p
-40000
36000
1
0.5
13500
Compare with 0
Don't receive grant
-5000
0
-5000
Don't submit
0
0
0

Sensitivity Analysis

Solve:

$$32000*p + (1 - p)*(-5000) = 0$$

$$32000*p = 5000*(1 - p)$$

$$32000*p = 5000 - 5000*p$$

Add on both sides 5000*p

$$37000*p = 5000$$

Divide both sides by 37000

$$\Rightarrow p = 5/37 = 0.1351$$

Sensitivity Analysis

Interpretation:

• For $p = 0.1351$, we are indifferent between submitting the proposal or not since it gives us the EMV of zero in both cases.

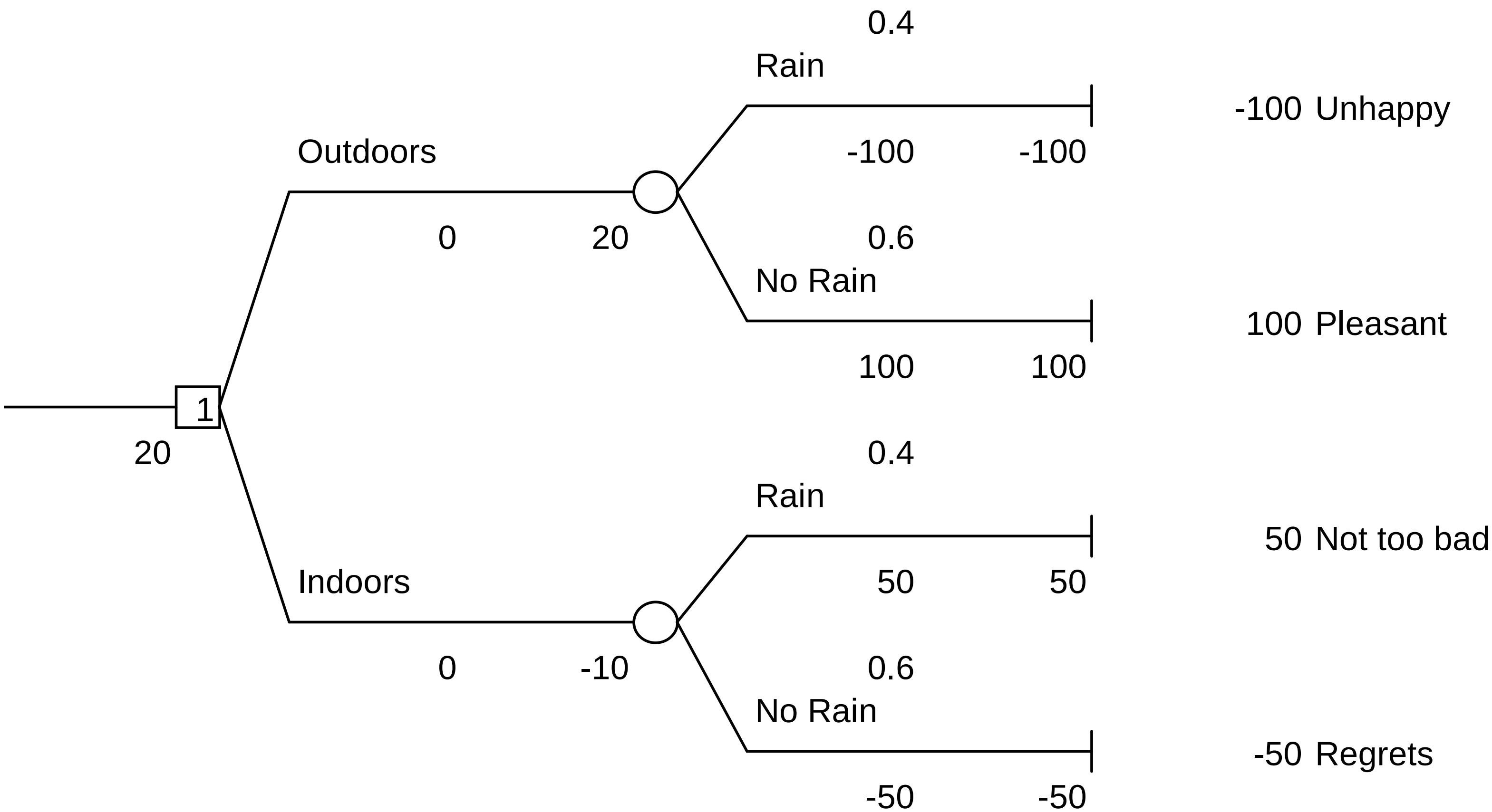
• For any p higher than 0.1351, we should submit the proposal.

Conclusion: the result of the initial assumption $p=0.5$ is <u>not sensitive</u> to small changes around 0.5.

Value of Information

Recall Dilemma: organize party indoors or in garden?

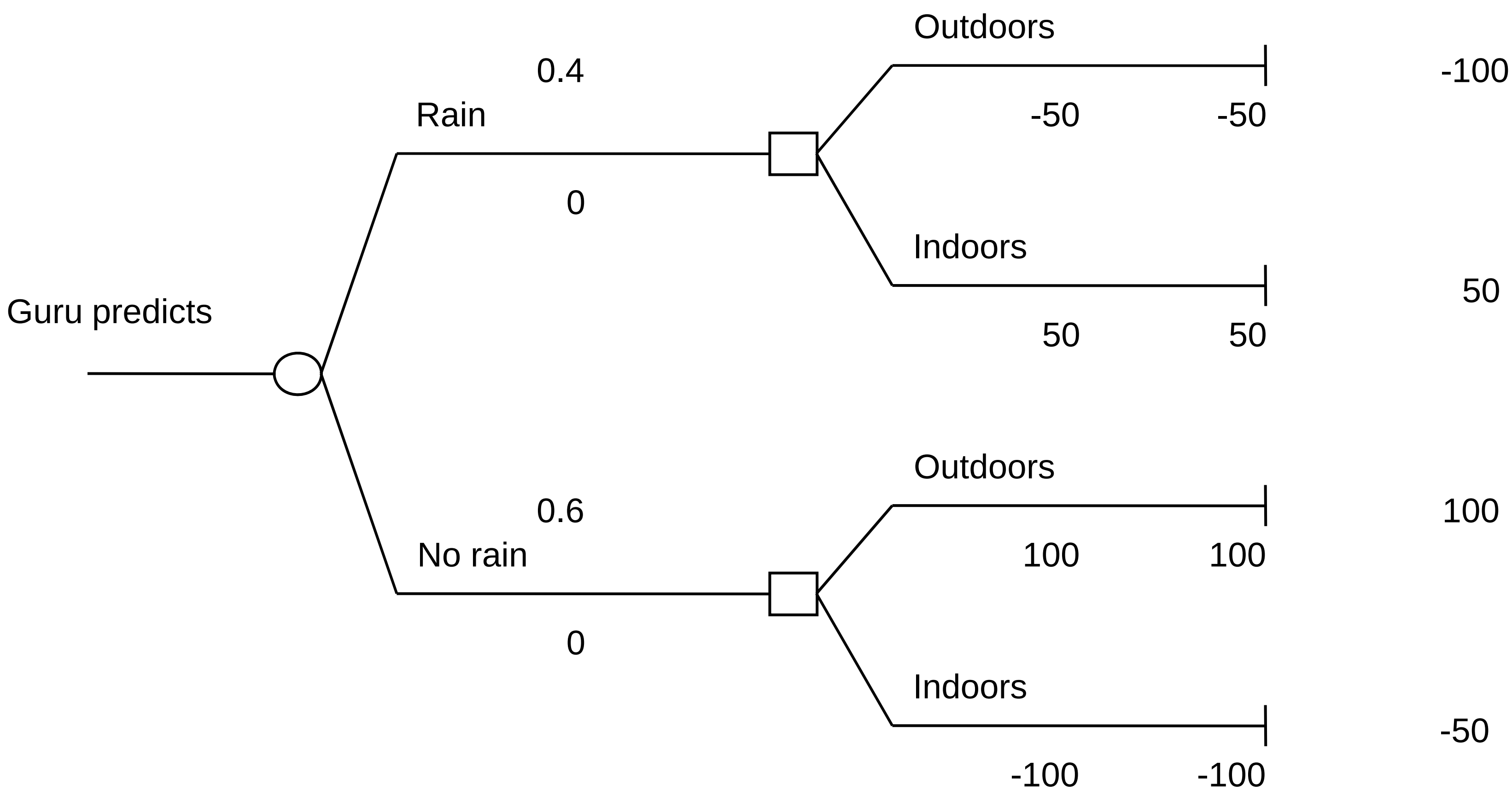
Recall Dilemma: organize party indoors or in garden?



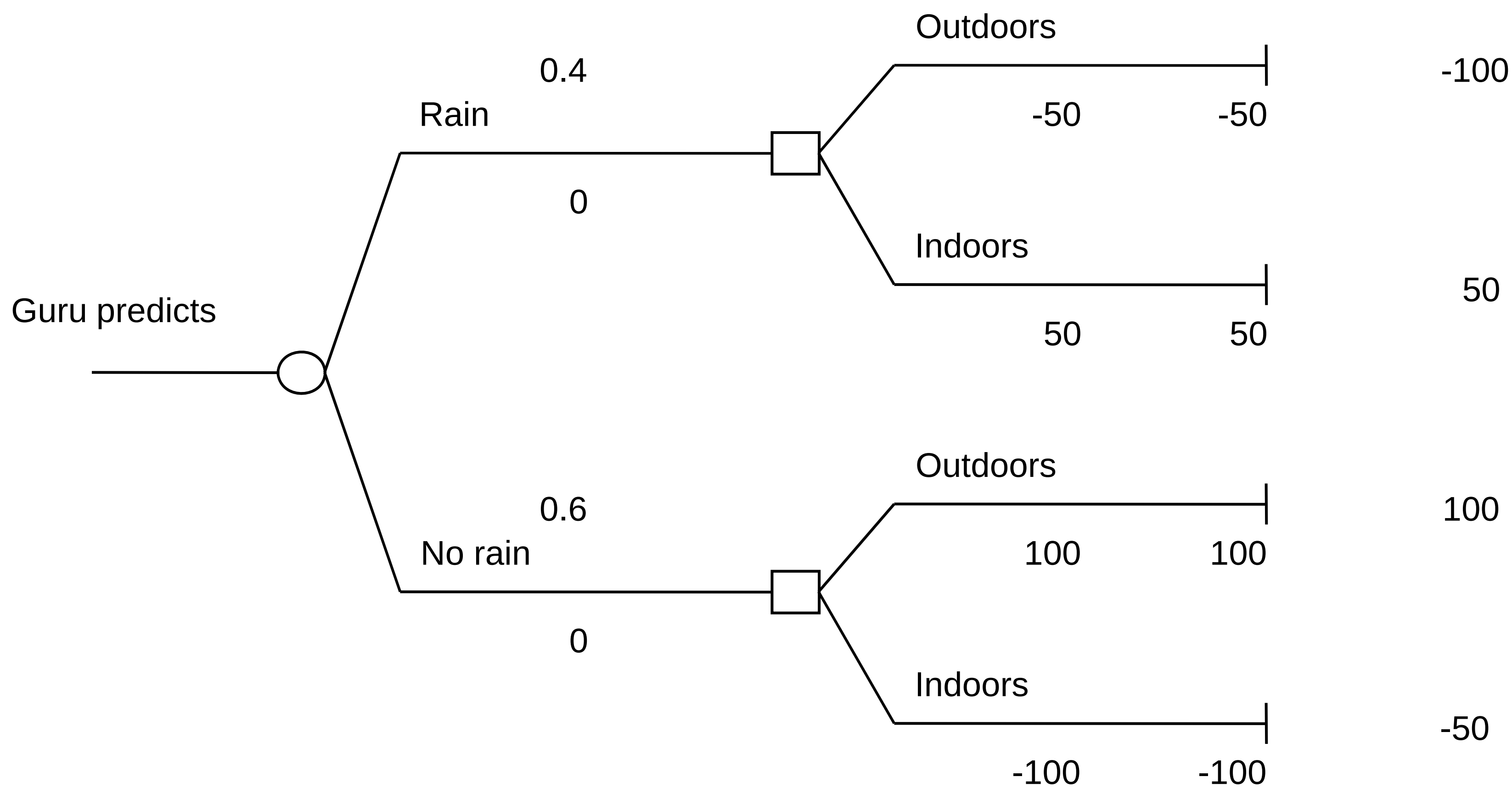
Value of Perfect Information

- 40% of the time rain, 60% no rain.
- Guru is able to perfectly predict weather.
- Guru tells you 40% of the time: “it will rain” => rain.
- Guru tells you 60% of the time: “it will not rain” => no rain.
- Guru is always right. How much would you be prepared to pay for this information?

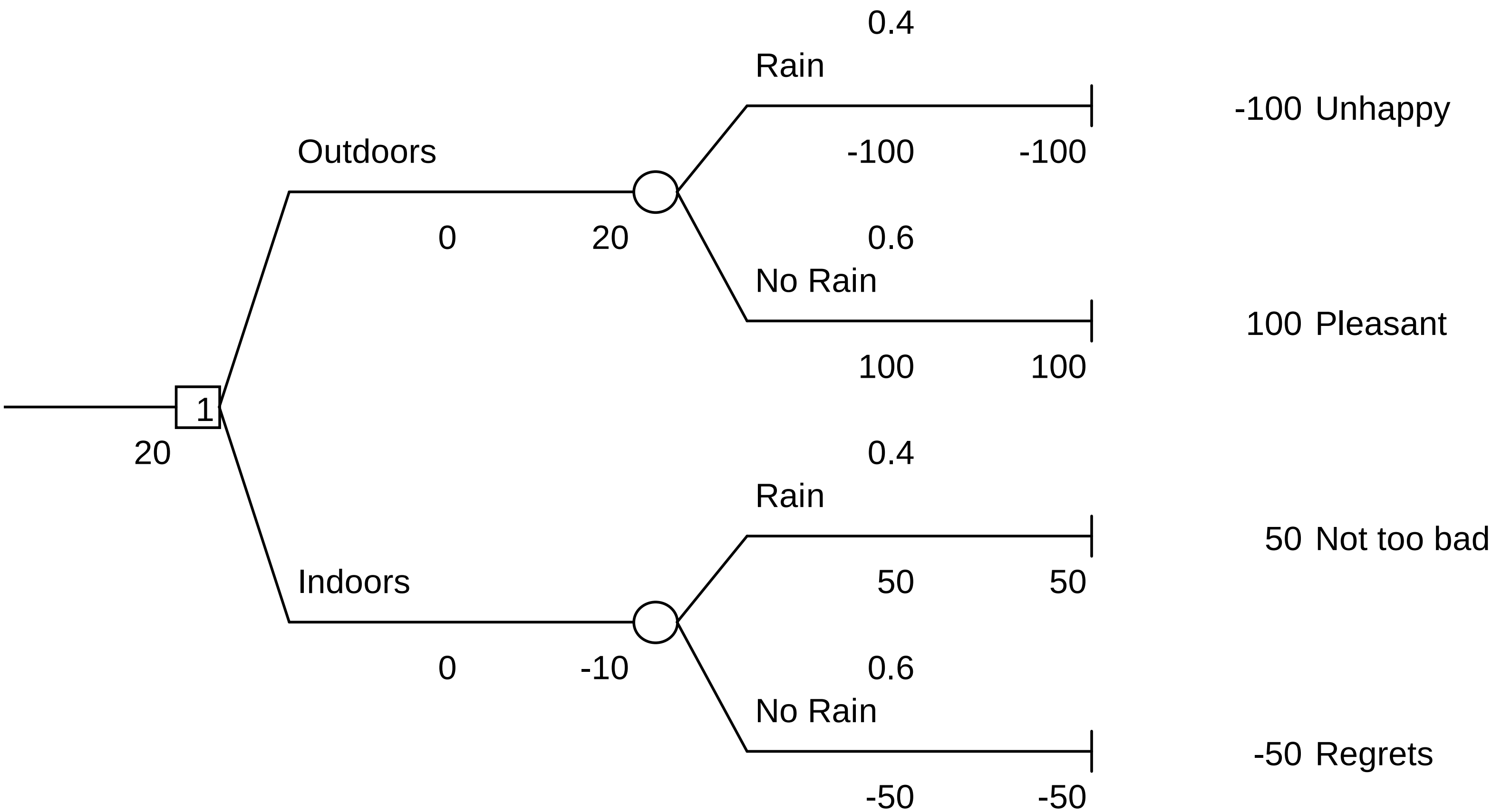
Perfect Information



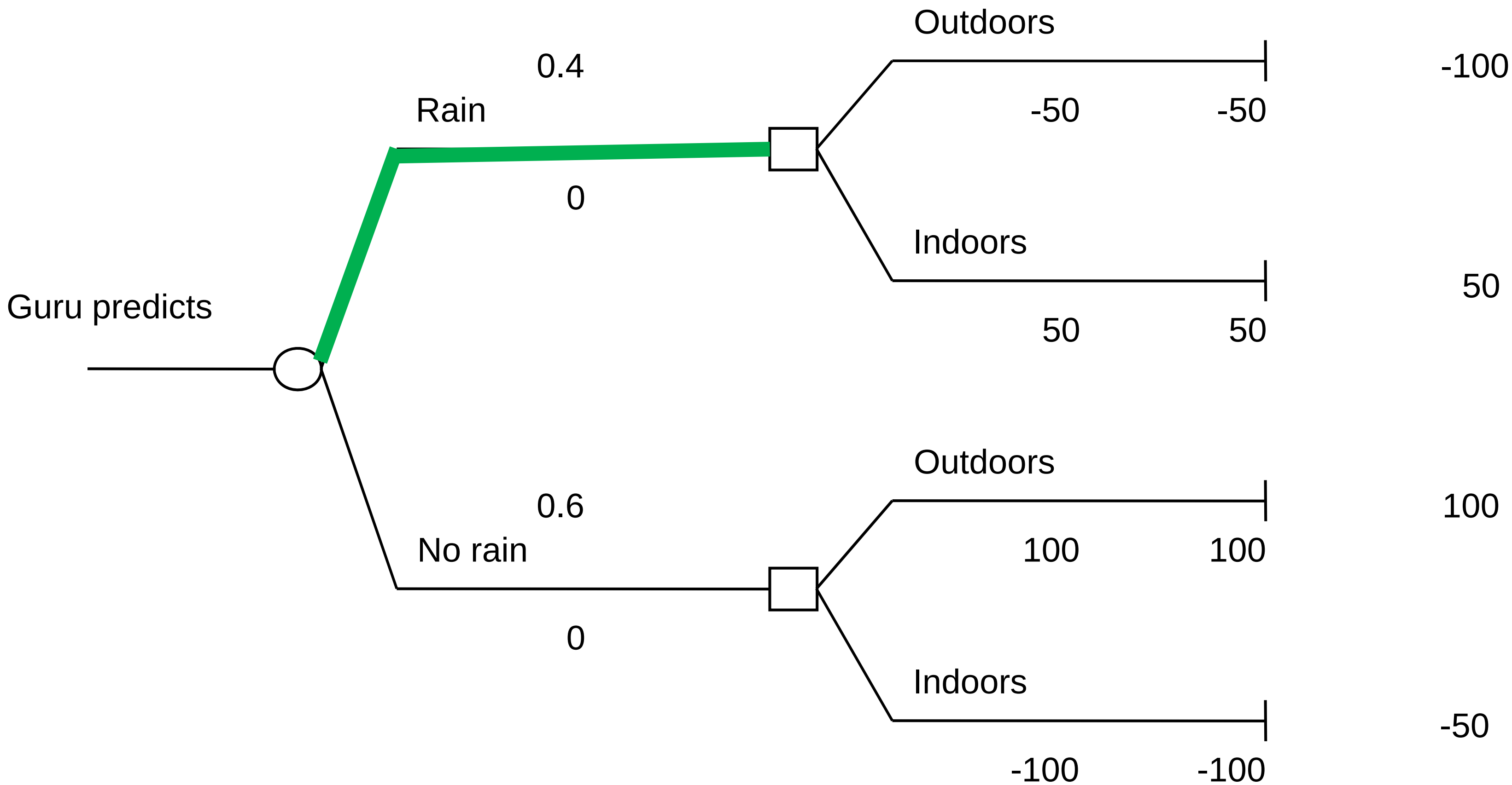
Note that this tree is different from tree without information!



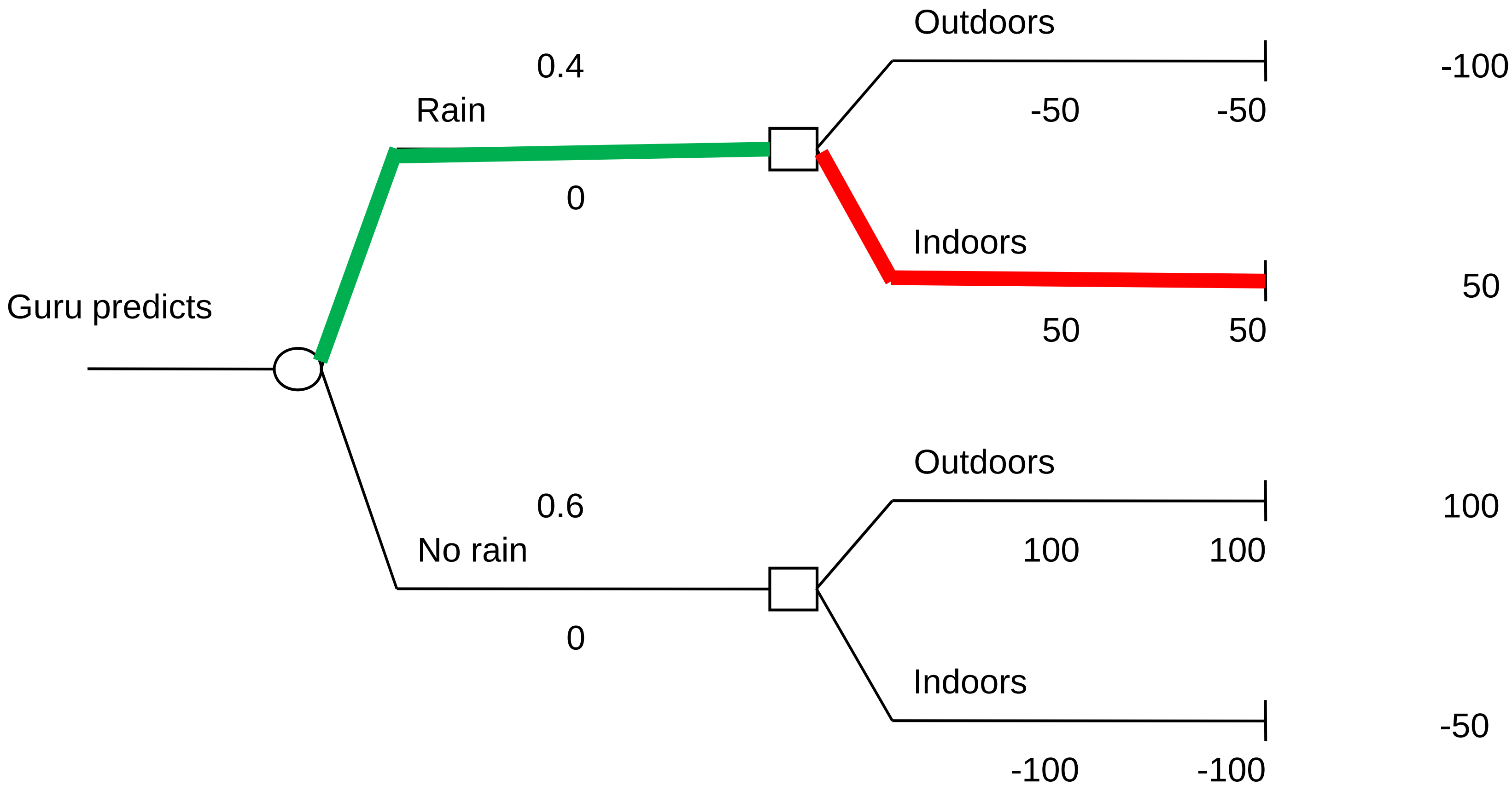
Recall: tree without information



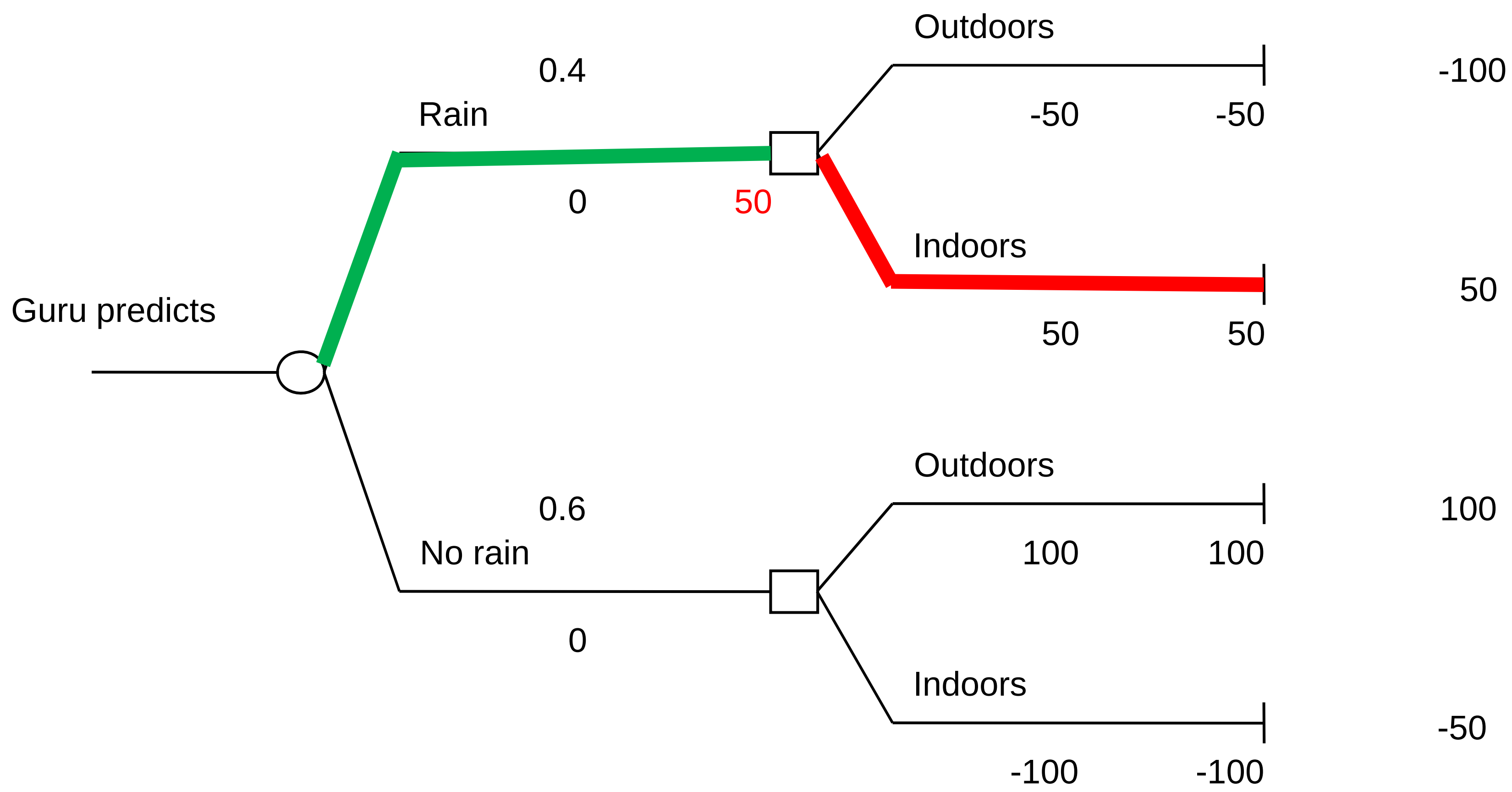
Perfect Information



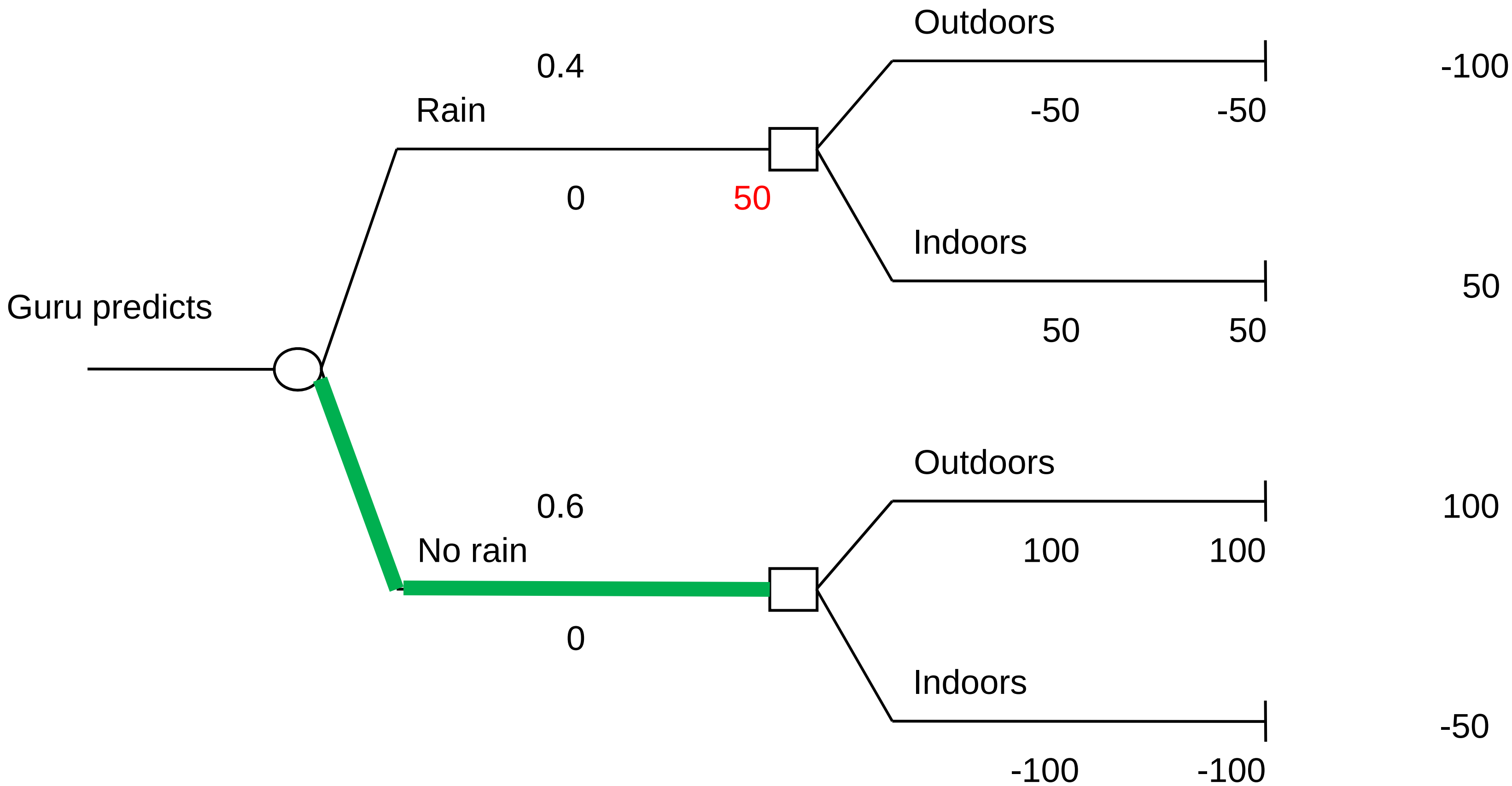
Perfect Information



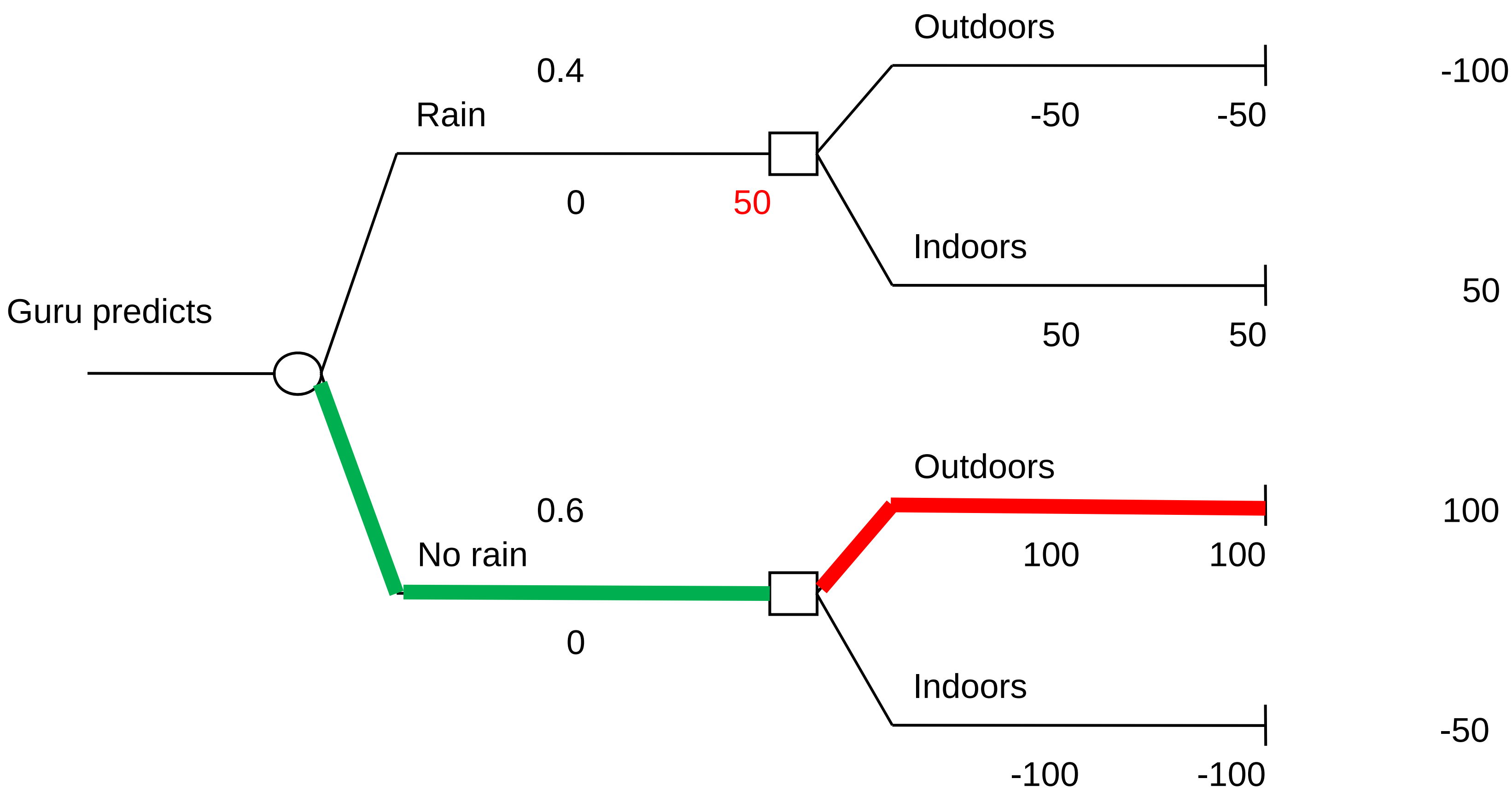
Perfect Information



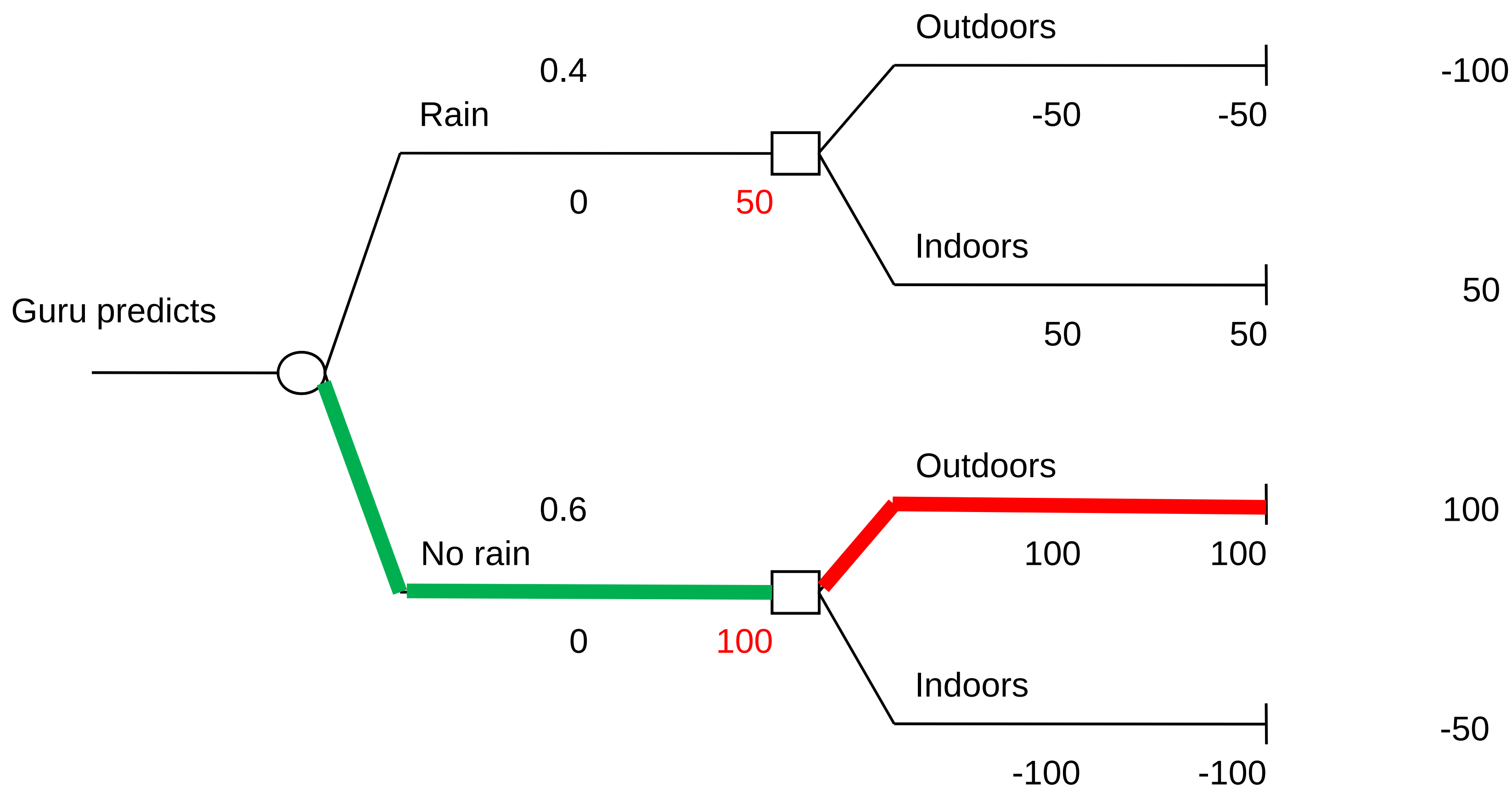
Perfect Information



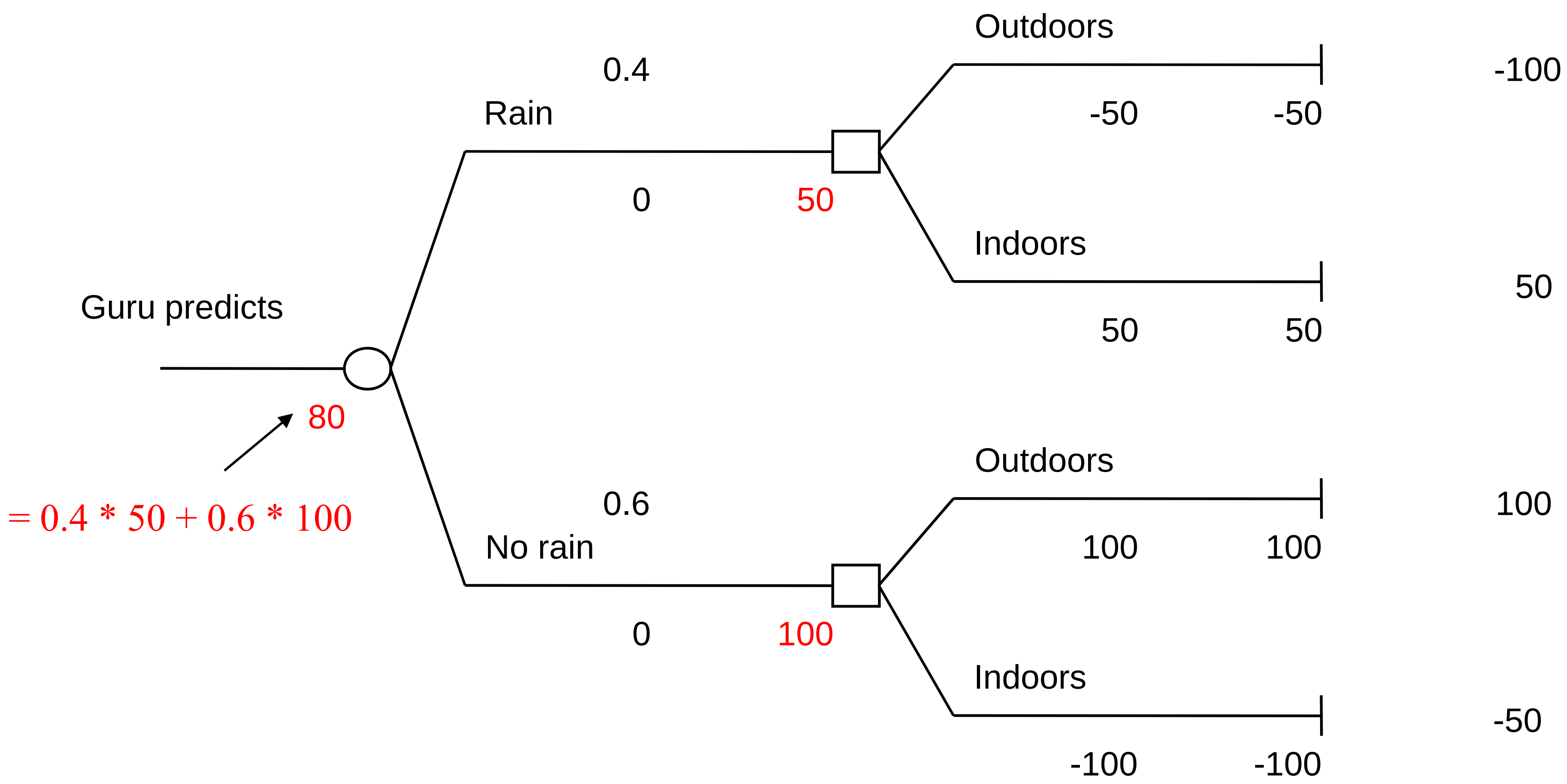
Perfect Information



Perfect Information



Perfect Information



Value of Perfect Information

Value *with* perfect information: 80

Value without information: 20

Value *of* perfect information: 80 – 20 = 60

Value of Perfect Information

Value *with* perfect information: 80

Value without information: 20

Value *of* perfect information: 80 – 20 = 60

What if Guru is not always right and you get imperfect information?

Imperfect Information

- Perfect information: Guru is right 100% of the time.
- Now suppose Guru is *not* always right, but only 65% of the time.
- E.g. after his rain warning it indeed rains 65% of the time.
- How much is his information worth?
- To adress this question we need the notion "conditional probability".

Imperfect Information

Suppose the Guru's rain warning is only right 65% of the time. Let R|W mean "It rains after the Guru has given an warning". P(R|W) = 0.65

Consequence: P(no R|W) = 0.35, i.e. Guru warned for rain but was wrong.

Assume the probability that it will rain is still 40%. Notation: P(R) = 0.4, P(no R) = 0.6

Suppose Guru warns for rain 45% of the time. Notation: P(W) = 0.45 and P(no W) = 0.55

Imperfect Information of Guru

Rain warning	Rain	No Rain
P(W)=45%	P(R|W) = 65%	P(no R|W)=35%

Imperfect Information of Guru

Rain warning	Rain	No Rain
P(W)=45%	P(R|W) = 65%	P(no R|W)=35%

Note:

Warning (information) of Guru is imperfect.

If it *was* perfect: P(R|W)=100%.

But still valuable information since P(R)=0.4, hence P(R|W) is larger.

Conclusion: Rain probability indeed increases when Guru warns.

Imperfect Information of Guru

Rain warning	Rain	No Rain
P(W)=45%	P(R|W) = 65%	P(no R|W)=35%

Conclusion: Information of Guru is worth something – but how much?

In order to answer this we have to complete table.

Imperfect Information of Guru

Rain warning	Rain	No Rain
P(W)=45%	P(R|W) = 65%	P(no R|W)=35%
P(no W)=55%		

Imperfect Information of Guru

Rain warning	Rain	No Rain
$P(W)=45\%$	$P(R\|W) = 65\%$	$P(\text{no } R\|W)=35\%$
$P(\text{no } W)=55\%$	$P(R\|\text{no } W)=?$	$P(\text{no } R\|\text{no } W)=?$

Recall: $P(R) = P(R|W) * P(W) + P(R|\text{no } W) * P(\text{no } W)$

$P(R) = 0.4$ is given by exercise,
rest given by table except $P(R|\text{no } W)$
-> solve equation for $P(R|\text{no } W)$

Imperfect Information of Guru

Rain warning	Rain	No Rain
$P(W)=45\%$	$P(R\|W) = 65\%$	$P(\text{no } R\|W)=35\%$
$P(\text{no } W)=55\%$	$P(R\|\text{no } W)=?$	$P(\text{no } R\|\text{no } W)=?$

Recall: $P(R) = P(R|W) * P(W) + P(R|\text{no } W) * P(\text{no } W)$

Maybe easier:
insert all values we know and *then* solve for $P(R|\text{no } W)$

Imperfect Information of Guru

Rain warning	Rain	No Rain
P(W)=45%	P(R|W) = 65%	P(no R|W)=35%
P(no W)=55%	P(R|no W)=?	P(no R|no W)=?

Recall: $P(R) = P(R|W) * P(W) + P(R|\text{no } W) * P(\text{no } W)$

$$0.4 = 0.65 * 0.45 + P(R|\text{no } W) * 0.55$$
$$0.4 = 0.2925 + P(R|\text{no } W) * 0.55$$
$$0.1075 = P(R|\text{no } W) * 0.55$$
$$0.19545 = P(R|\text{no } W)$$

Imperfect Information of Guru

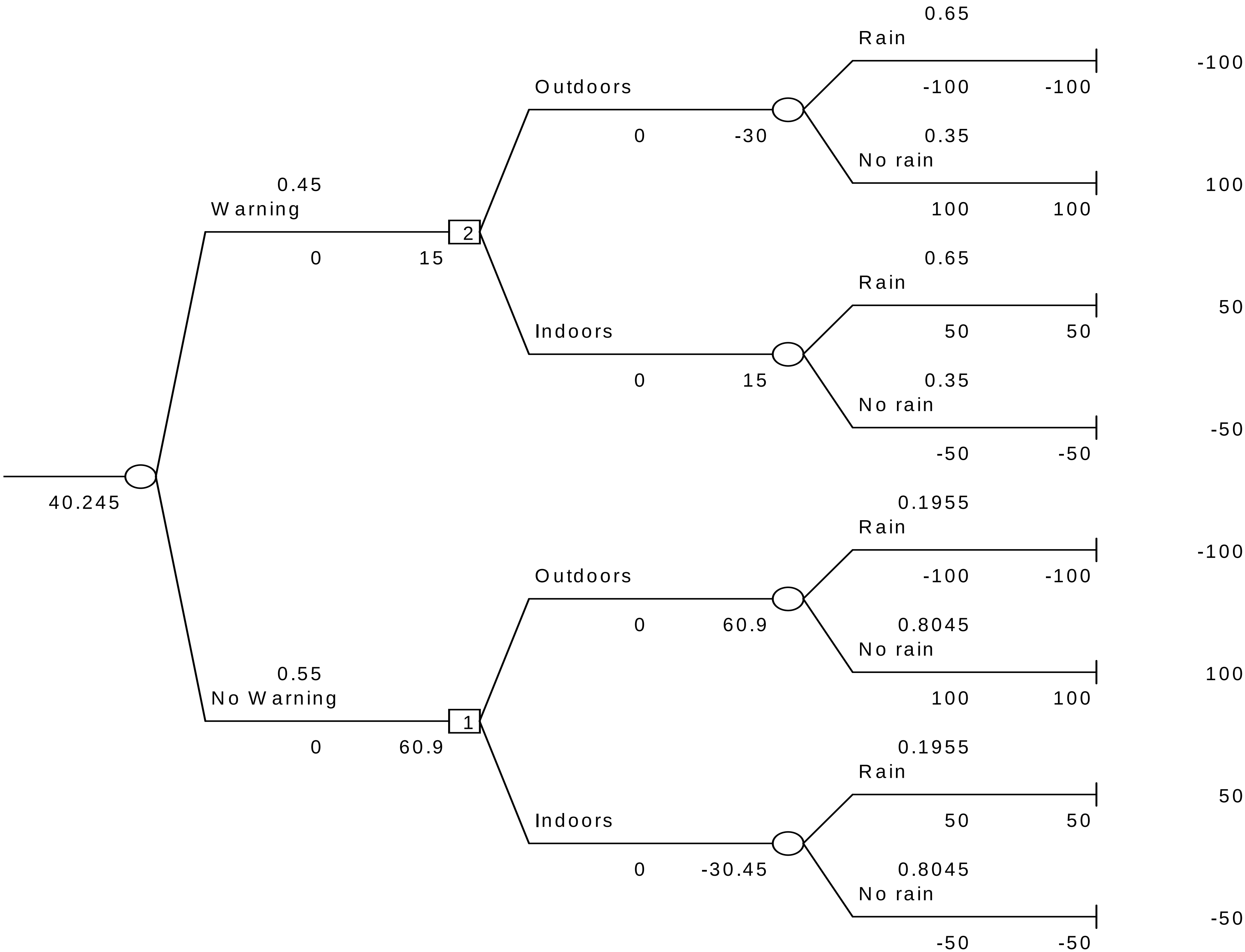
Rain warning	Rain	No Rain
$P(W)=45\%$	$P(R\|W) = 65\%$	$P(\text{no } R\|W)=35\%$
$P(\text{no } W)=55\%$	$P(R\|\text{no } W)=?$	$P(\text{no } R\|\text{no } W)=?$

From $P(R|\text{no } W) = 0.19545$

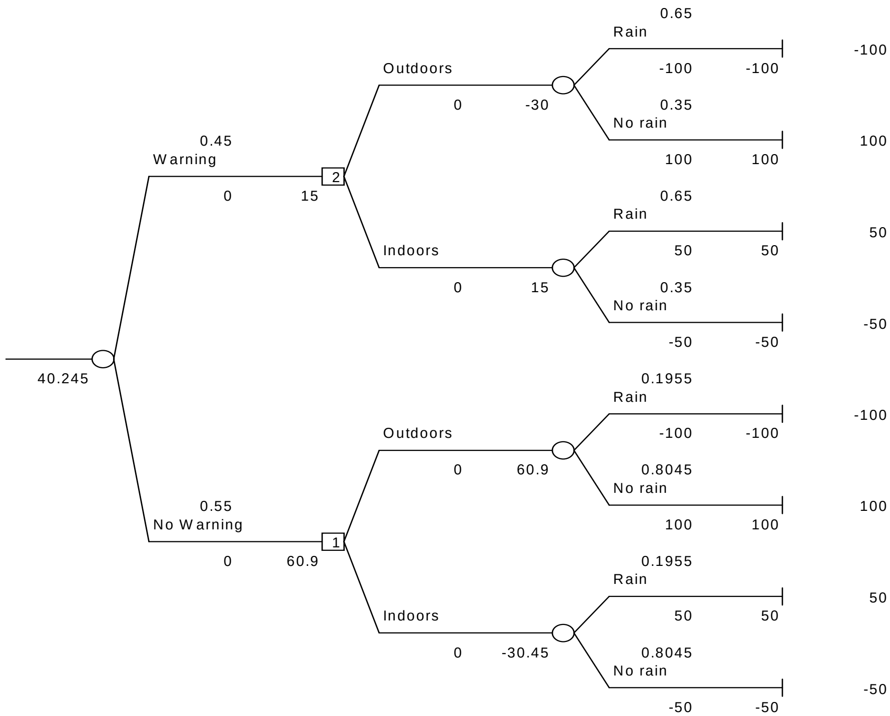
we get $P(\text{no } R|\text{no } W) = 1 - 0.19545 = 0.80455$

Imperfect Information of Guru

Rain warning	Rain	No Rain
P(W)=45%	P(R|W) = 65%	P(no R|W)=35%
P(no W)=55%	P(R|no W) =19.45%	P(no R|no W) =80.455%


0.65
Rain
-100
Outdoors
-100
-100
0
-30
0.35
No rain
100
0.45
Warning
100
100
2
0
15
0.65
Rain
50
Indoors
50
50
0
15
0.35
No rain
-50
-50
-50
40.245
0.1955
Rain
-100
Outdoors
-100
-100
0
60.9
0.8045
No rain
100
0.55
No Warning
100
100
1
0
60.9
0.1955
Rain
50
Indoors
50
50
0
-30.45
0.8045
No rain
-50
-50
-50

Imperfect Information



Expected value without information: EMV=20

Expected value *with* sample (imperfect) information: 40.245

Expected value of sample information: 40.245 – 20 = 20.245

Interpretation: 20.245 = Maximum price we are willing to pay the Guru.

Another example: Should we build a large or small factory?

Problem:

- introduce a new product
- don’t know yet whether demand will be high or low
- if demand is high -> large factory might be optimal
- if demand is low -> small factory might be optimal

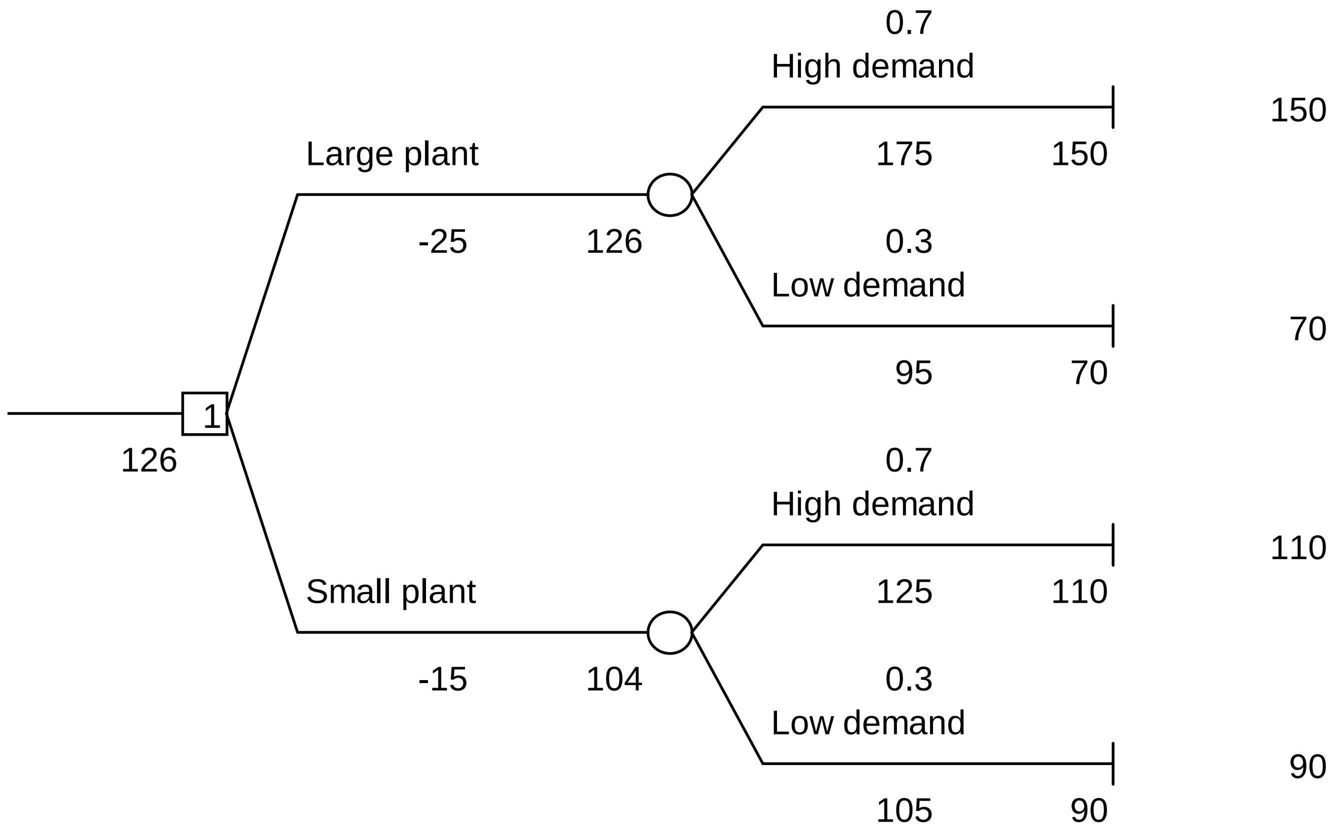
Another example: Should we build a large or small factory?

Construction costs: Large Factory €25 Million
Small Factory €15 Million

Factory Size	**High Demand**	**Low Demand**
Large	175	95
Small	125	105
Probability	70%	30%

Estimated total Revenues (in million €)

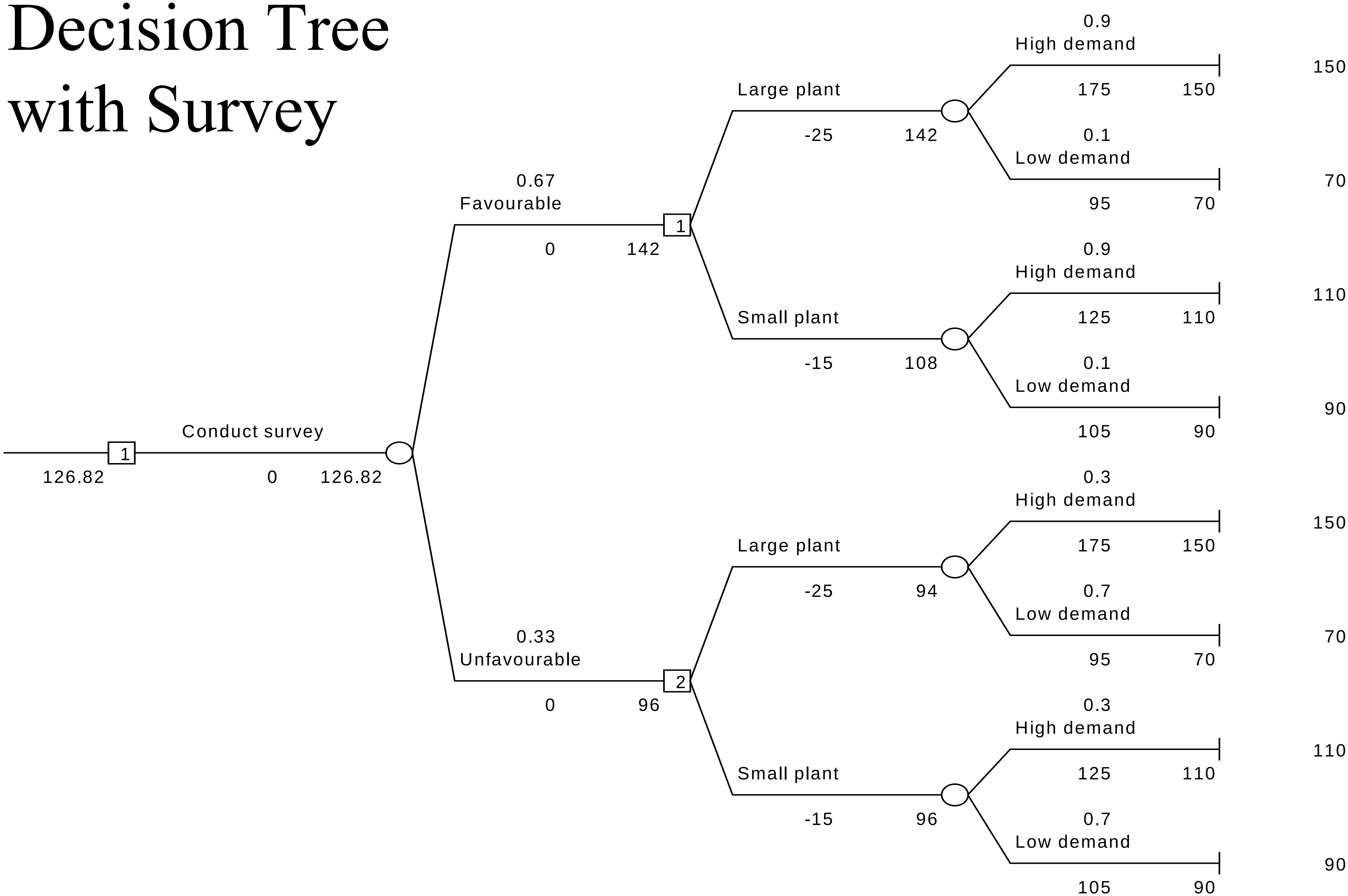
Decision Tree without Survey

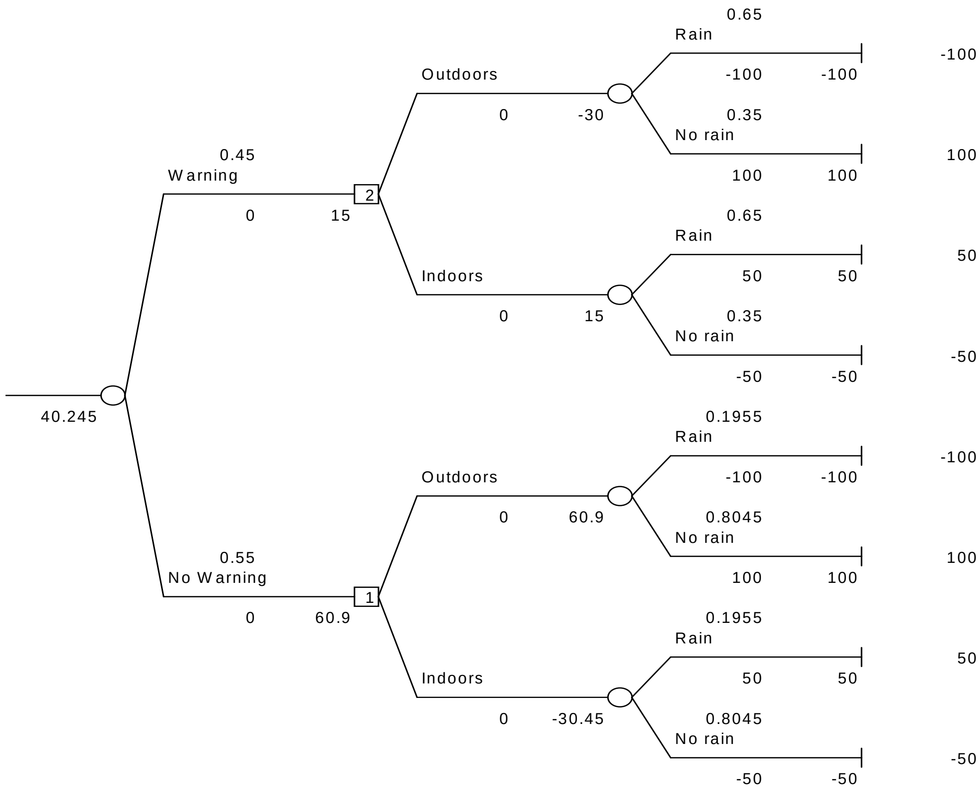


Estimated Results of Survey

Probability	Survey	High Demand	Low Demand
67%	Favourable	90%	10%
33%	Unfavourable	30%	70%

Decision Tree with Survey





Expected value *without* sample information: 126

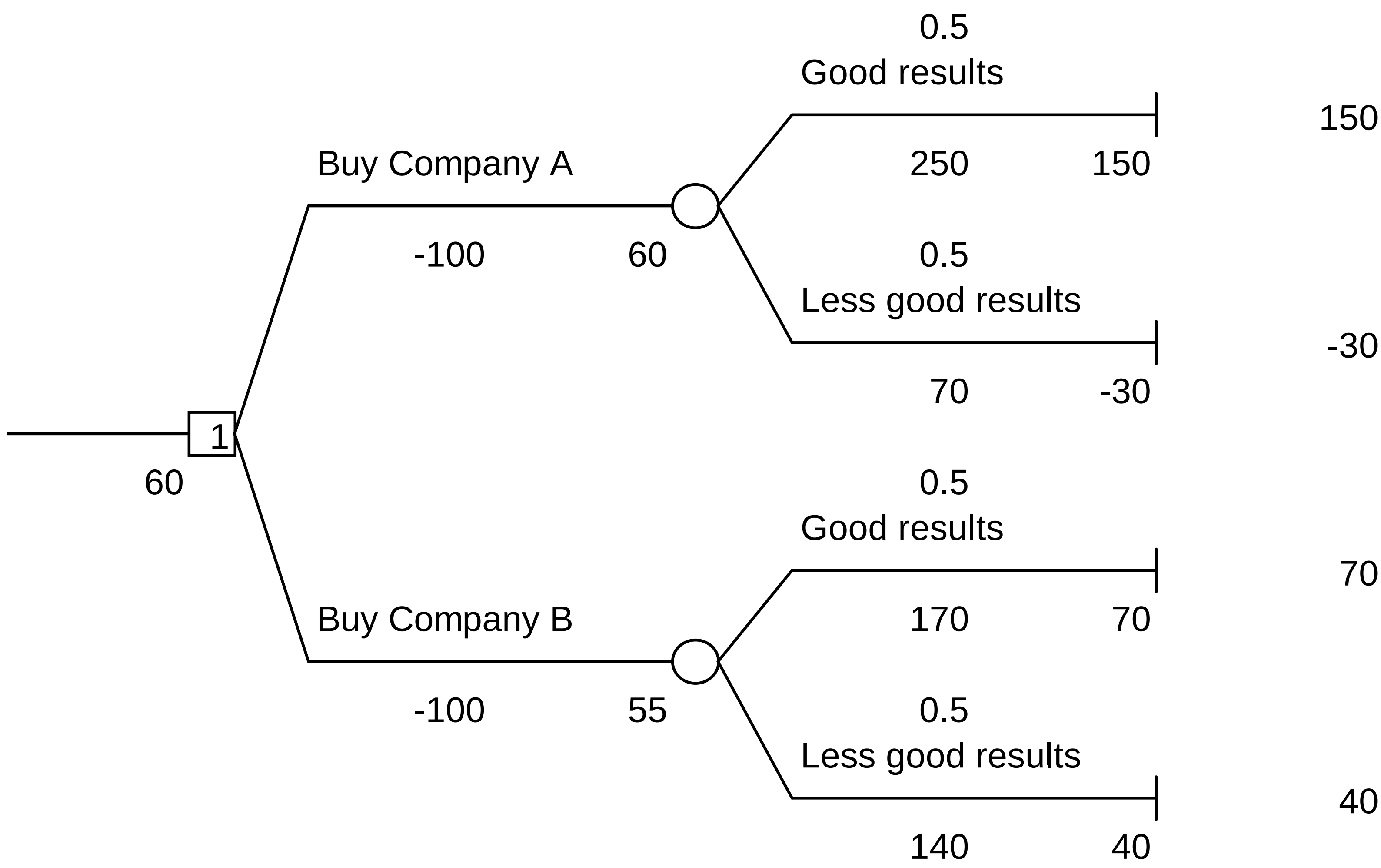
Expected value *with* sample information: 126.82

Expected value *of* sample information: 126.82 – 126 = 0.82

Answer: our maximum willingness to pay for the survey is 820.000 Euros.

Expected Money Value - is this always a good criterion for choice under risk?

Example –
payoff in million Euros



Taking risks

Company	**Payoff**	**Payoff**	**EMV**
A	150	-30	60
B	70	40	55
Probability	0.5	0.5	

- Should we buy company A or B?

Taking risks

Company	Payoff	Payoff	EMV
A	150	-30	60
B	70	40	55
Probability	0.5	0.5	

- Should we buy company A or B?
- Equivalent question: Should we play gamble A or B?

Taking risks

Company	Payoff	Payoff	EMV
A	150	-30	60
B	70	40	55
Probability	0.5	0.5	

- Nobody would hesitate to play gamble B.

Taking risks

Company	Payoff	Payoff	EMV
A	150	-30	60
B	70	40	55
Probability	0.5	0.5	

- Nobody would hesitate to play lottery B.
- Gamble A less attractive due to risk of losing €30 million.

Taking risks

Company	Payoff	Payoff	EMV
A	150	-30	60
B	70	40	55
Probability	0.5	0.5	

- Nobody would hesitate to play gamble B.
- Gamble A less attractive due to risk of losing €30 million.
- Note EMV(A) > EMV(B)

Taking risks

Company	**Payoff**	**Payoff**	**EMV**
A	150	-30	60
B	70	40	55
Probability	0.5	0.5	

Conclusion: EMV doesn't reflect how we feel about risk!

Utility Function

• Fundamental premise: people choose the alternative that has not the highest expected value but the highest **expected utility**

• Assume **utility function** *U* that assigns a numerical measure to the satisfaction associated with different outcomes.

• How does such a utility function look like?

Thought Experiment

Give the following two persons €100 as a gift.

Who’s happiness will increase more?

Thought Experiment

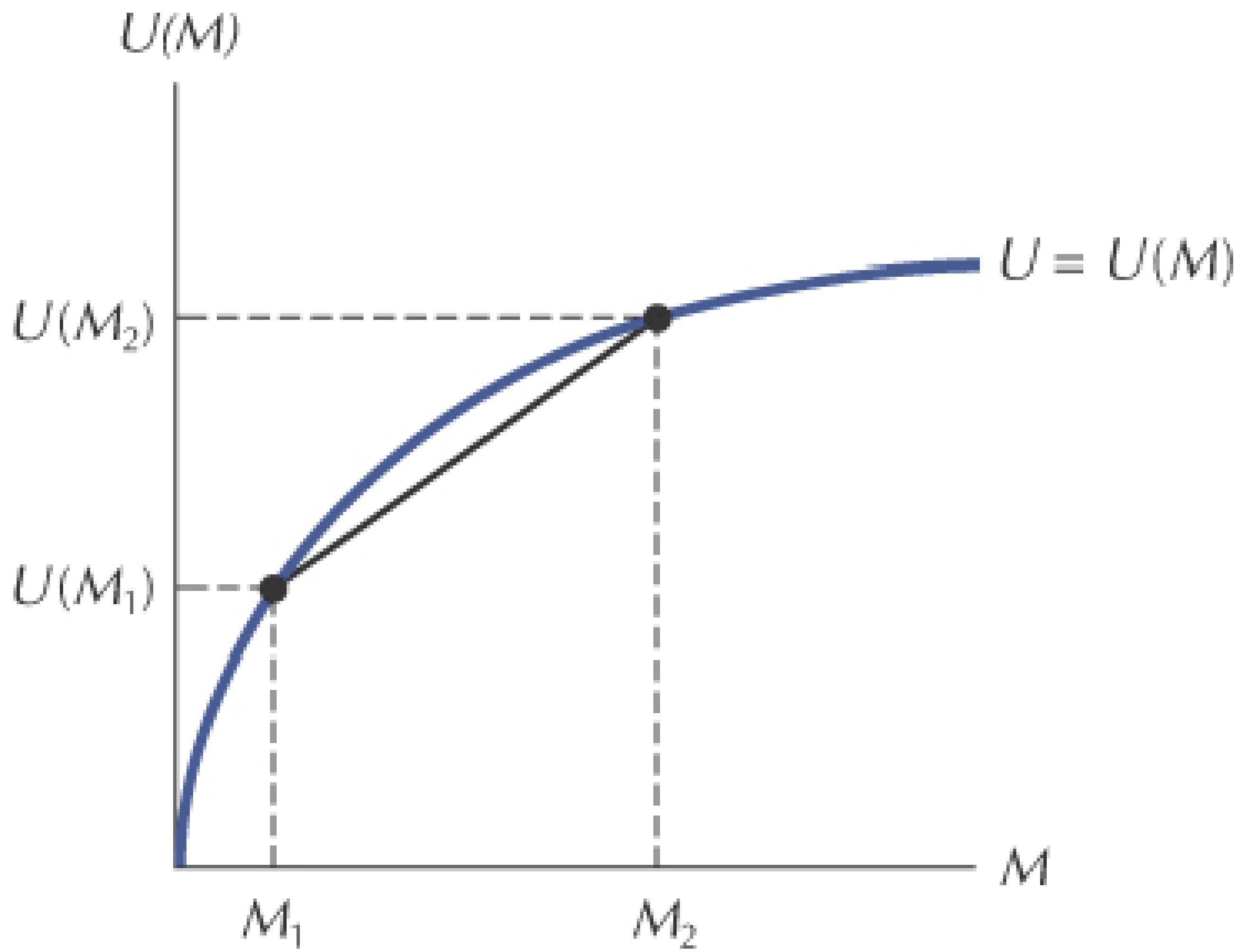
Student

Bill Gates

Thought Experiment

Frequent observation: The more money we have, the less our happiness increases with additional money.

A Concave Utility Function



Thought Experiment

- Warning: utility function is not always concave!
- We will see that it depends on our attitude towards risk, i.e. whether we risk-avert or risk-loving.

Utility Function

- Note: utility is “dimensionless”, i.e. there is no unit such as $, kg, cm.
- This function assigns numbers to outcomes.
- The higher the number, the more we like the outcome.
- Without loss of generality, assume these numbers are between 0 and 1
- Otherwise substract minimum and divide by the maximum

Utility Function

Example:

Assume Anna says her utility level of money in Euros are

$U(100) = 10, \quad U(0) = 5, \quad U(-50) = 1$

Utility Function

$U(100) = 10, \quad U(0) = 5, \quad U(-50) = 1$

Utility Function

$U(100) = 10, \quad U(0) = 5, \quad U(-50) = 1$

If we substract 1 (the minimum) from every value we get

$U(100) = 9, \quad U(0) = 4, \quad U(-50) = 0$

Utility Function

$U(100) = 10$, $U(0) = 5$, $U(-50) = 1$

If we substract 1 (the minimum) from every value we get

$U(100) = 9$, $U(0) = 4$, $U(-50) = 0$

Finally, divide all values by 9 to get

$U(100) = 1$, $U(0) = 4/9=0.44$, $U(-50) = 0$

Utility Function

Note:

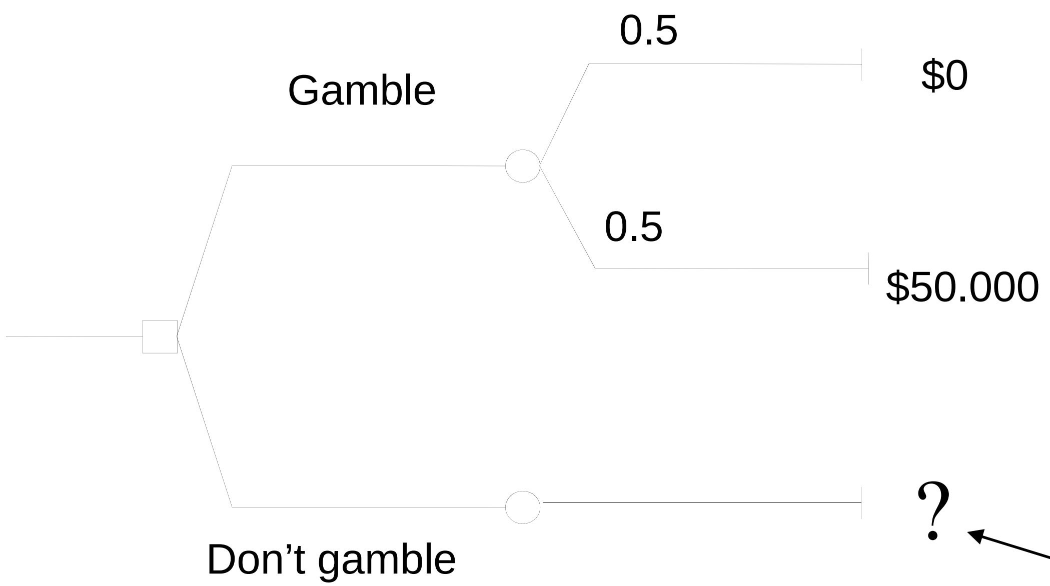
• what we did here was to convert utility into a more convenient form

• comparable as e.g. when you convert prices from dollars to euros

Utility Function

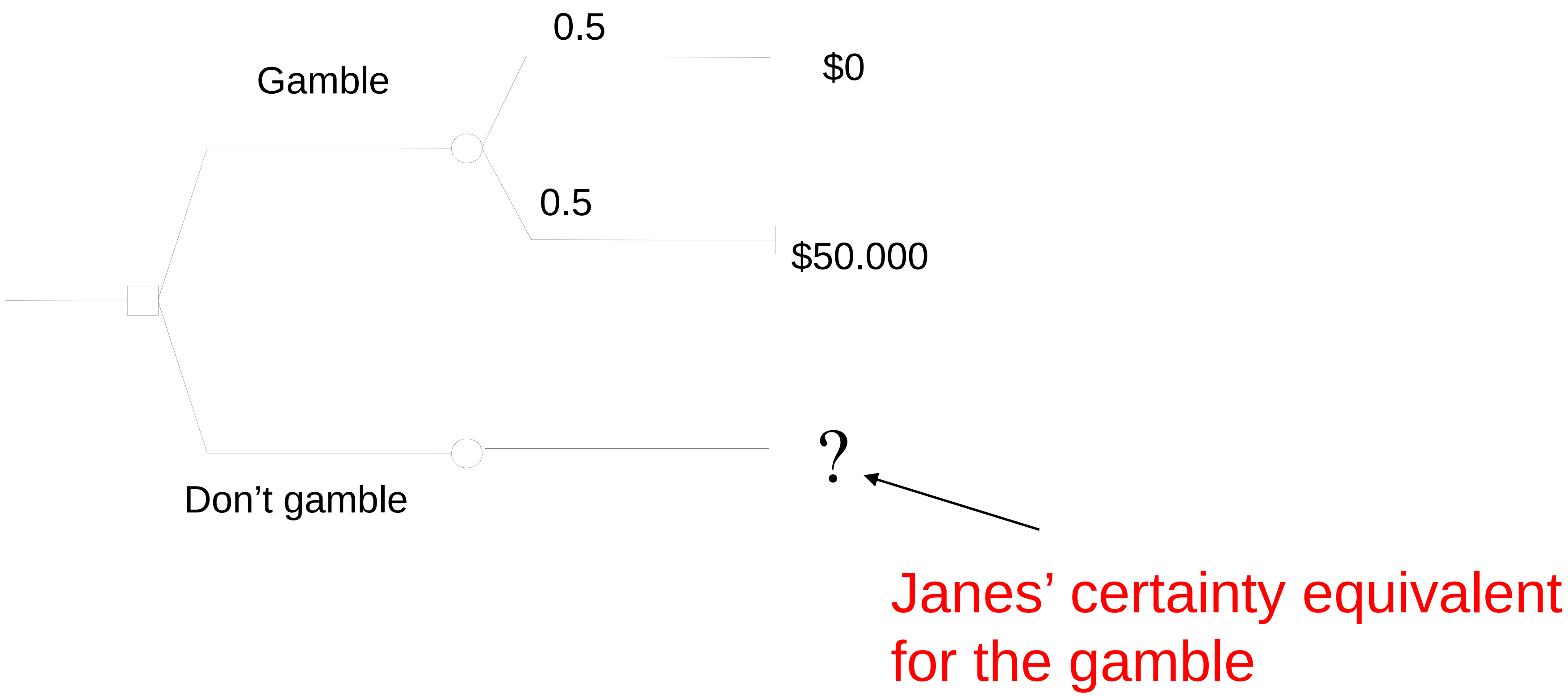
How do we construct a person’s utility function (preference)?

Example: Gambling or not?



What is the minimum amount we have to pay Jane to make her walk away from the gamble?

Example: Gambling or not?



Utility Function

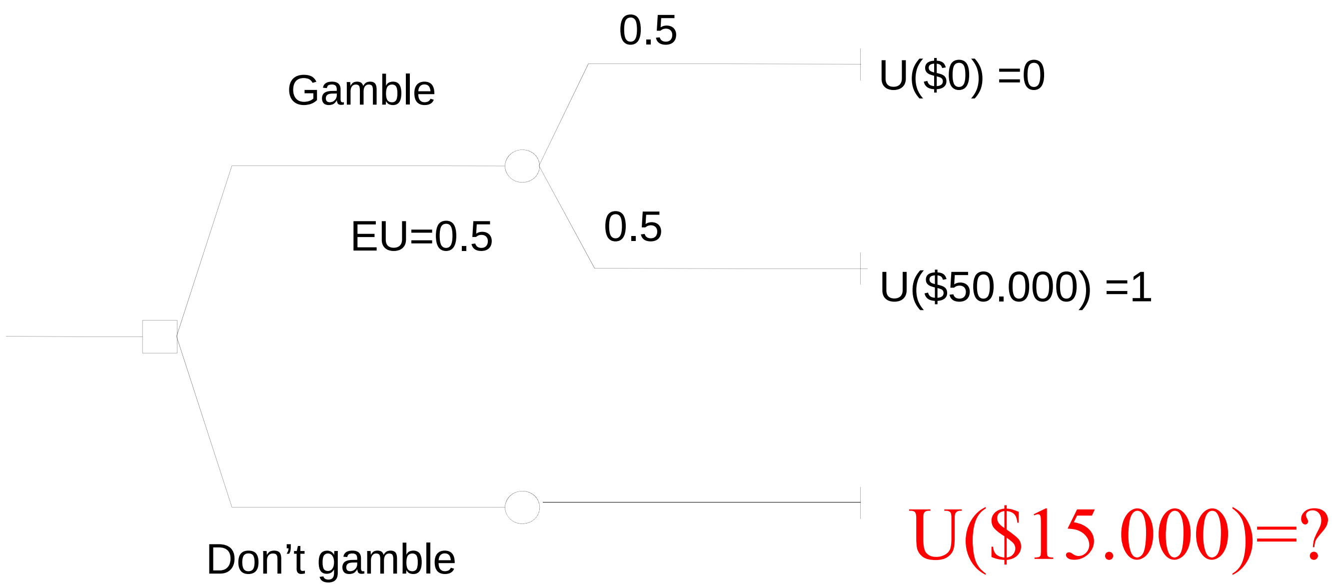
- \$15.000 is Jane's *certainty equivalent of this gamble*.
- For a guaranteed \$15.000 she is willing to give up a potential gain of \$50.000 in order to avoid the risk of winning nothing.
- This varies from person to person.
- Note EMV of the gamble is

$$0.5 * \$0 + 0.5 * \$50.000 = \$ 25.000$$

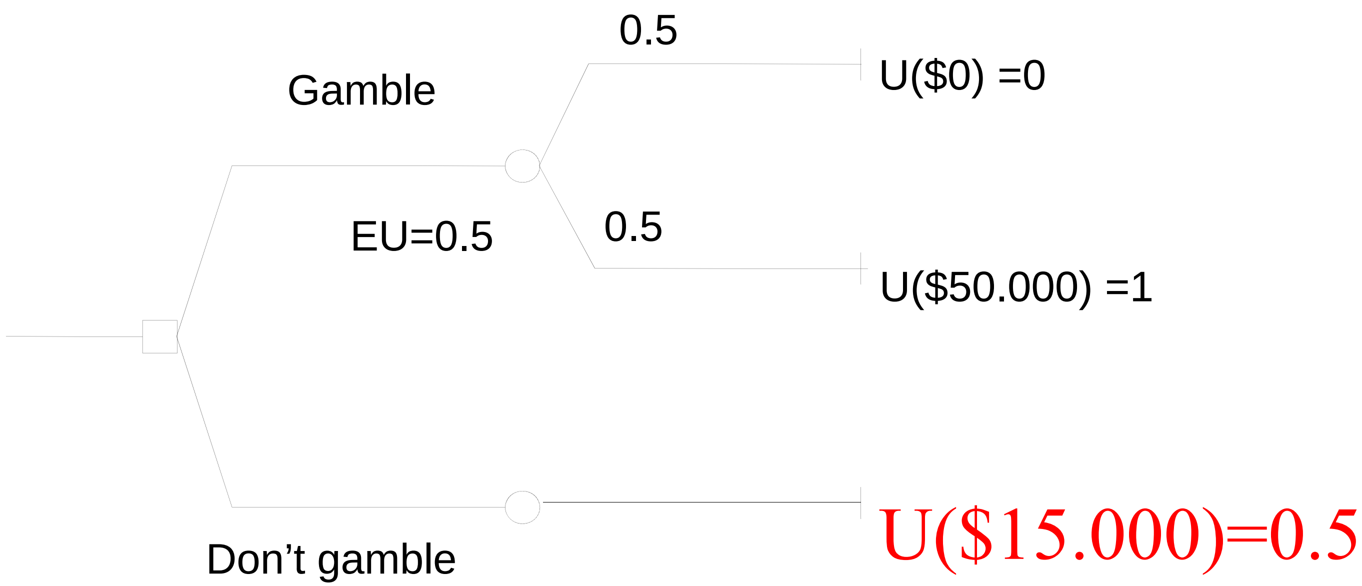
- Certainty equivalent < EMV

=> Jane is risk-averse for this gamble.

Utility Perspective



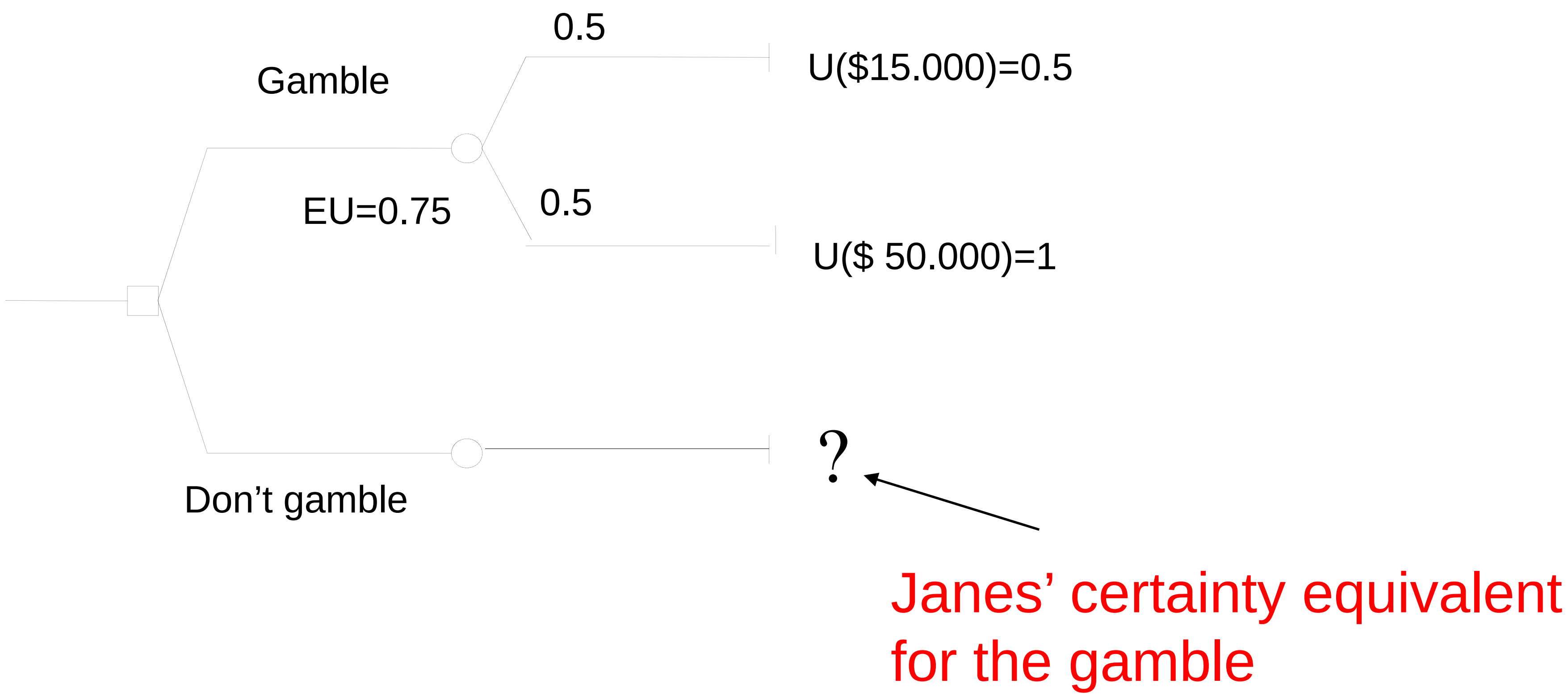
Utility Perspective



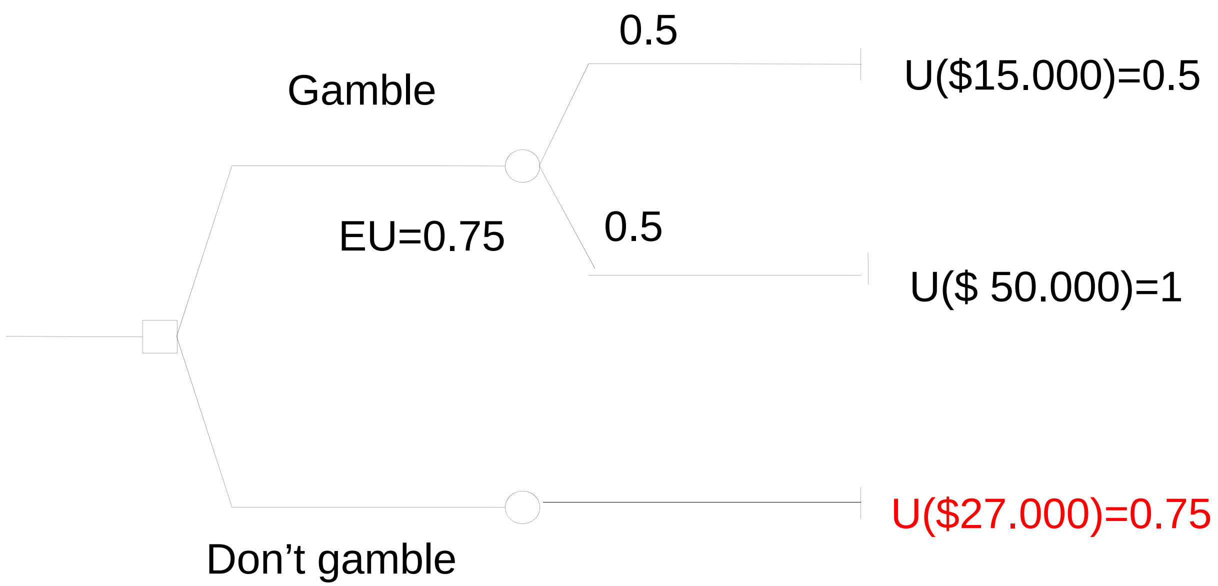
Utility Function

To find more values of U repeat gamble questions.

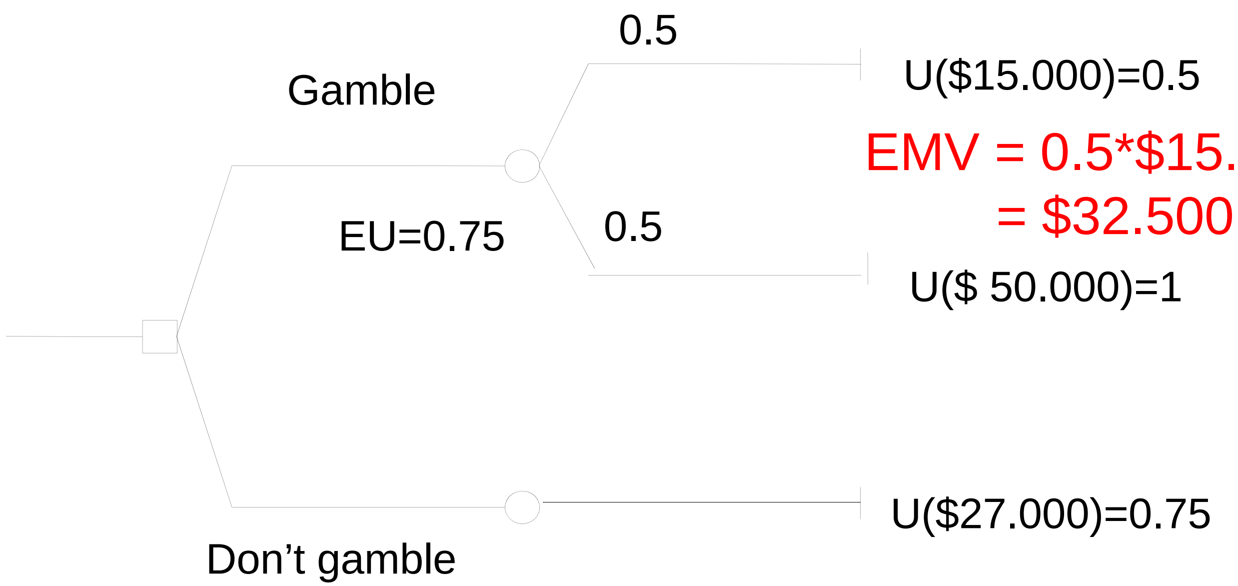
Example: Gambling or not?



Example: Gambling or not?

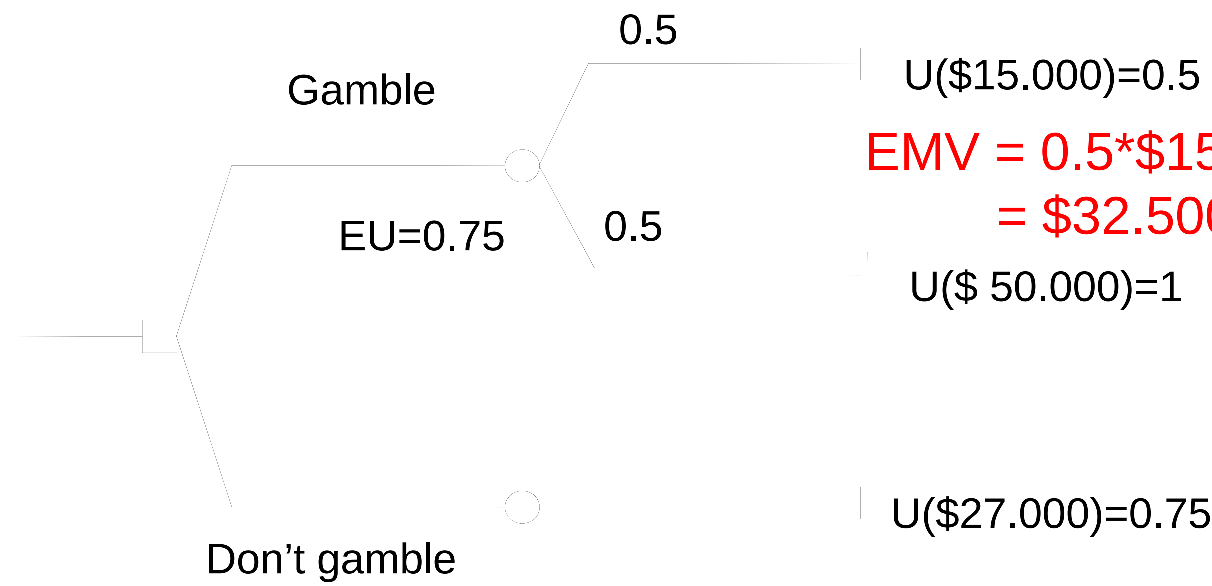


Example: Gambling or not?



EMV = 0.5*$15.000 +0 .5*$50.000
= $32.500

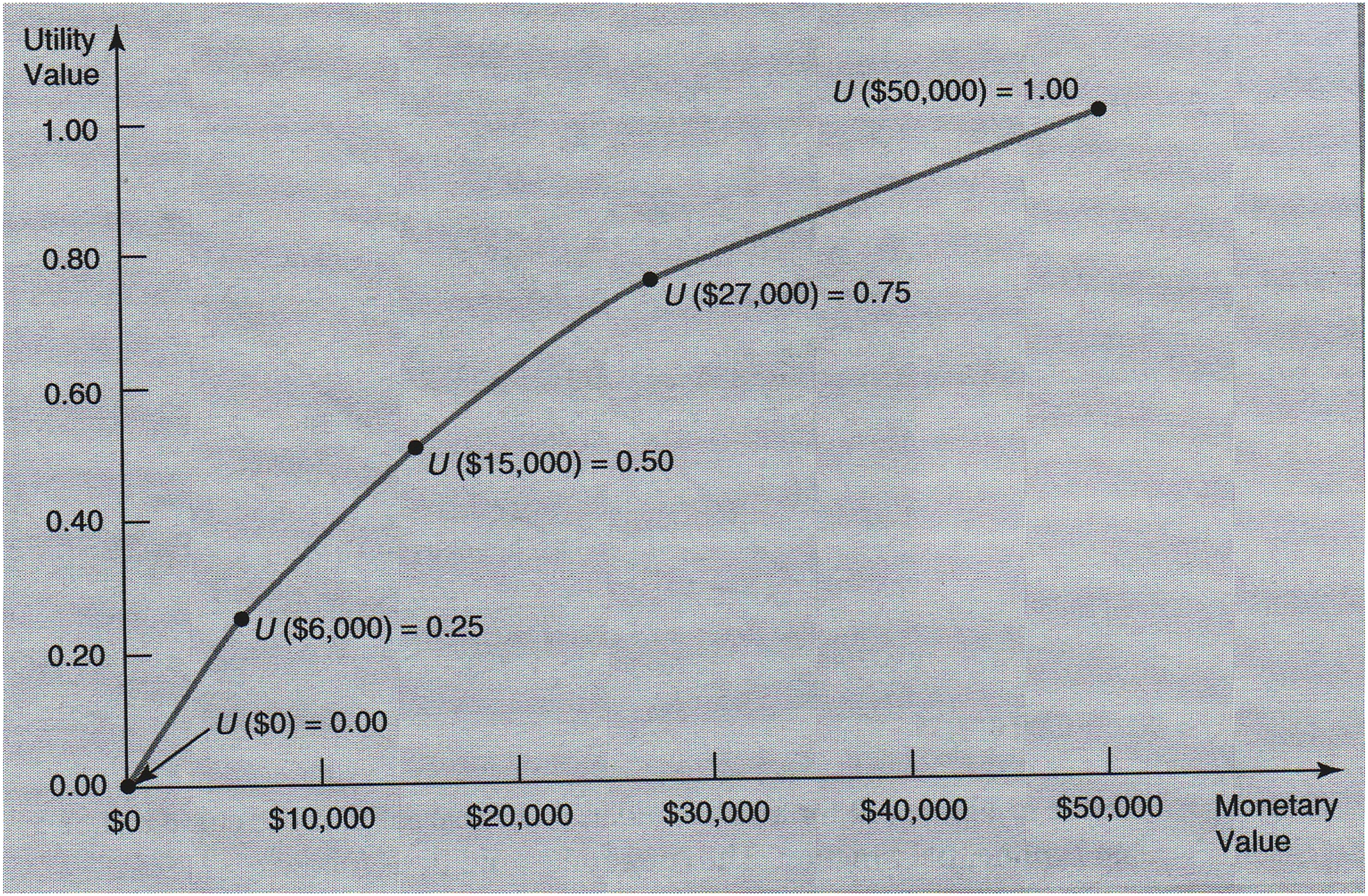
Example: Gambling or not?



EMV = 0.5*$15.000 +0 .5*$50.000
= $32.500

Certainty equivalent < EMV
-> risk averse

Utility Curve for Jane



Risk Aversion

- Jane would be a risk averse person when she is risk averse for any gamble.
- In formulae: gamble with probability p to win amount V_1 and with probability 1-p to win V_2.
- EMV of this gamble = $p^* V_1 + (1-p)^* V_2$
- Certainty equivalent of this gamble is C.
- Then $C < p^* V_1 + (1-p)^* V_2$ means that she is risk averse.

Utility Function

How does the utility function of a risk-*seeker* look like?

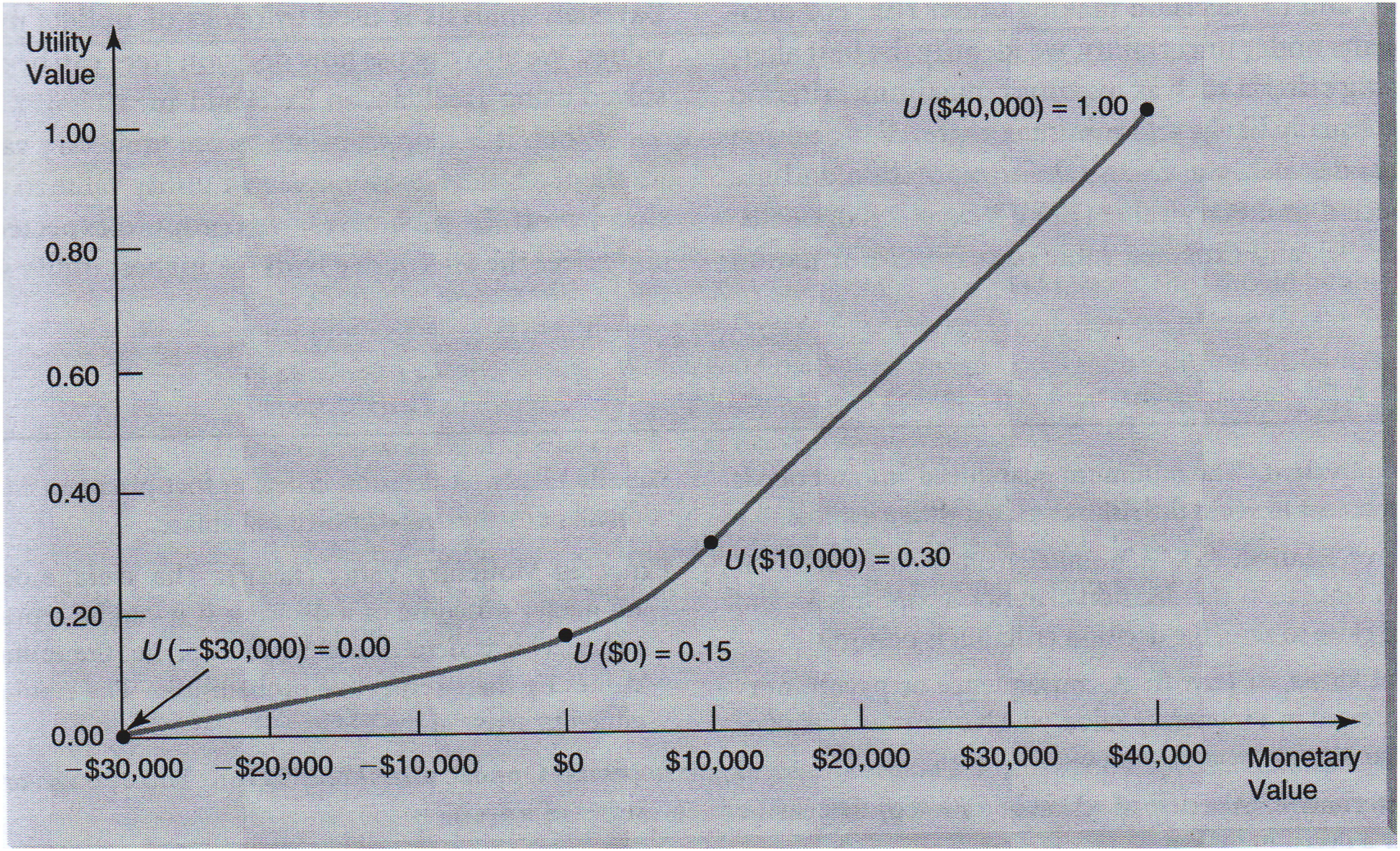
Utility Function

Example:
Mark’s preference between –$30.000 and $40.000

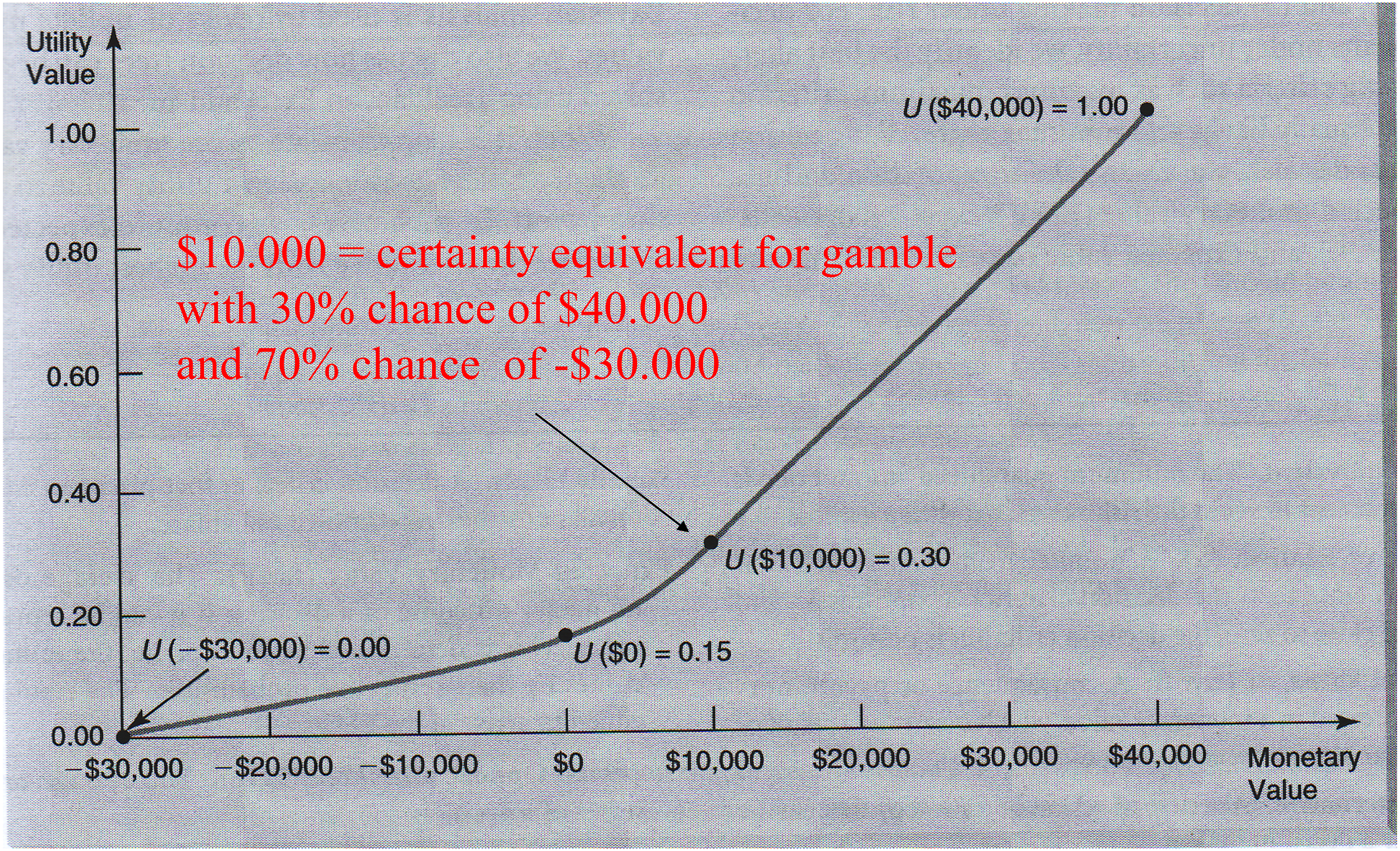
Receipe:

- Assign utility 0 to the lowest (–$30.000)
- Assign utility 1 to the highest ($40.000)
- To find out about in-between values we ask Mark questions about gambles.

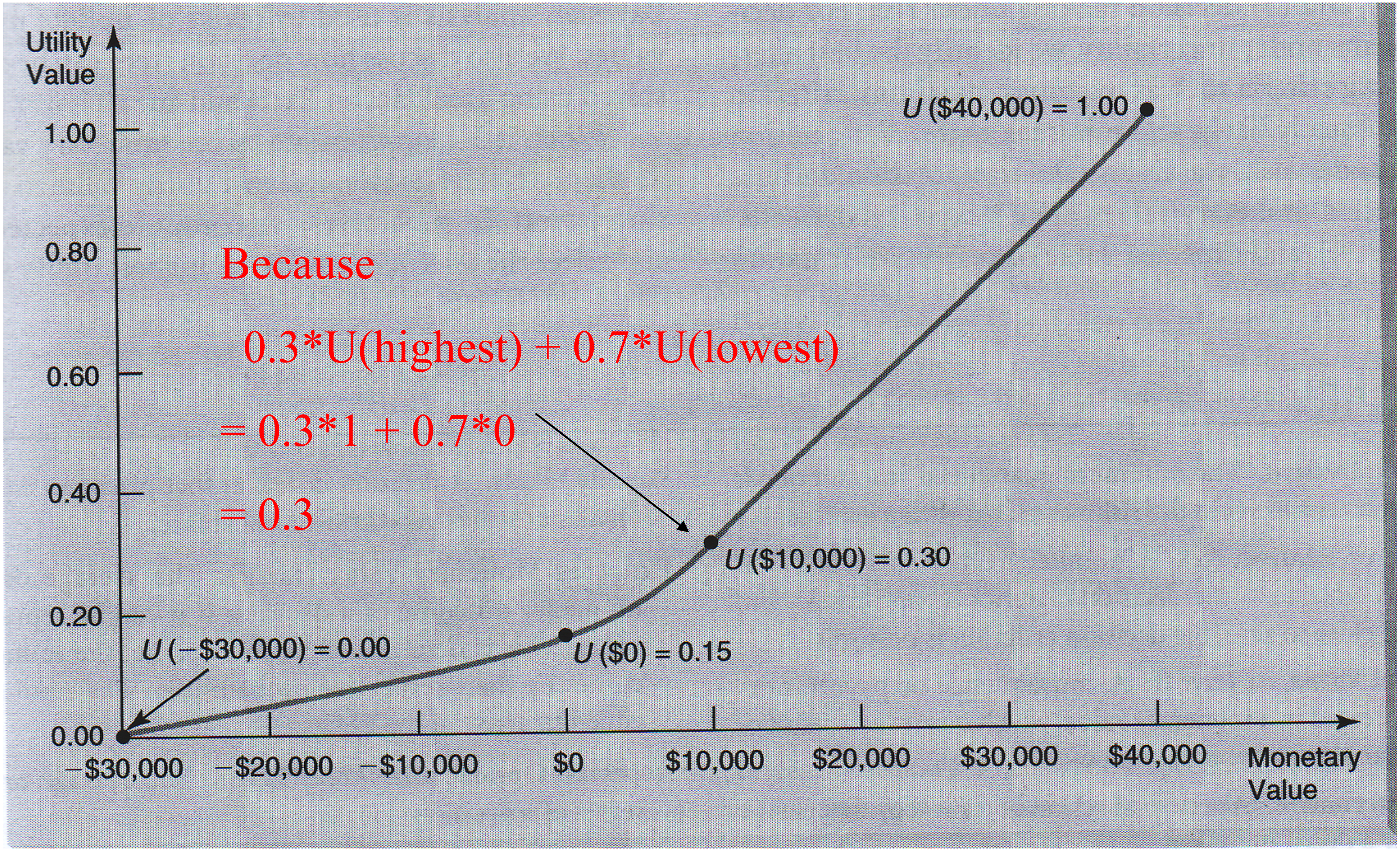
Utility Curve for Mark
Convex => risk-seeker



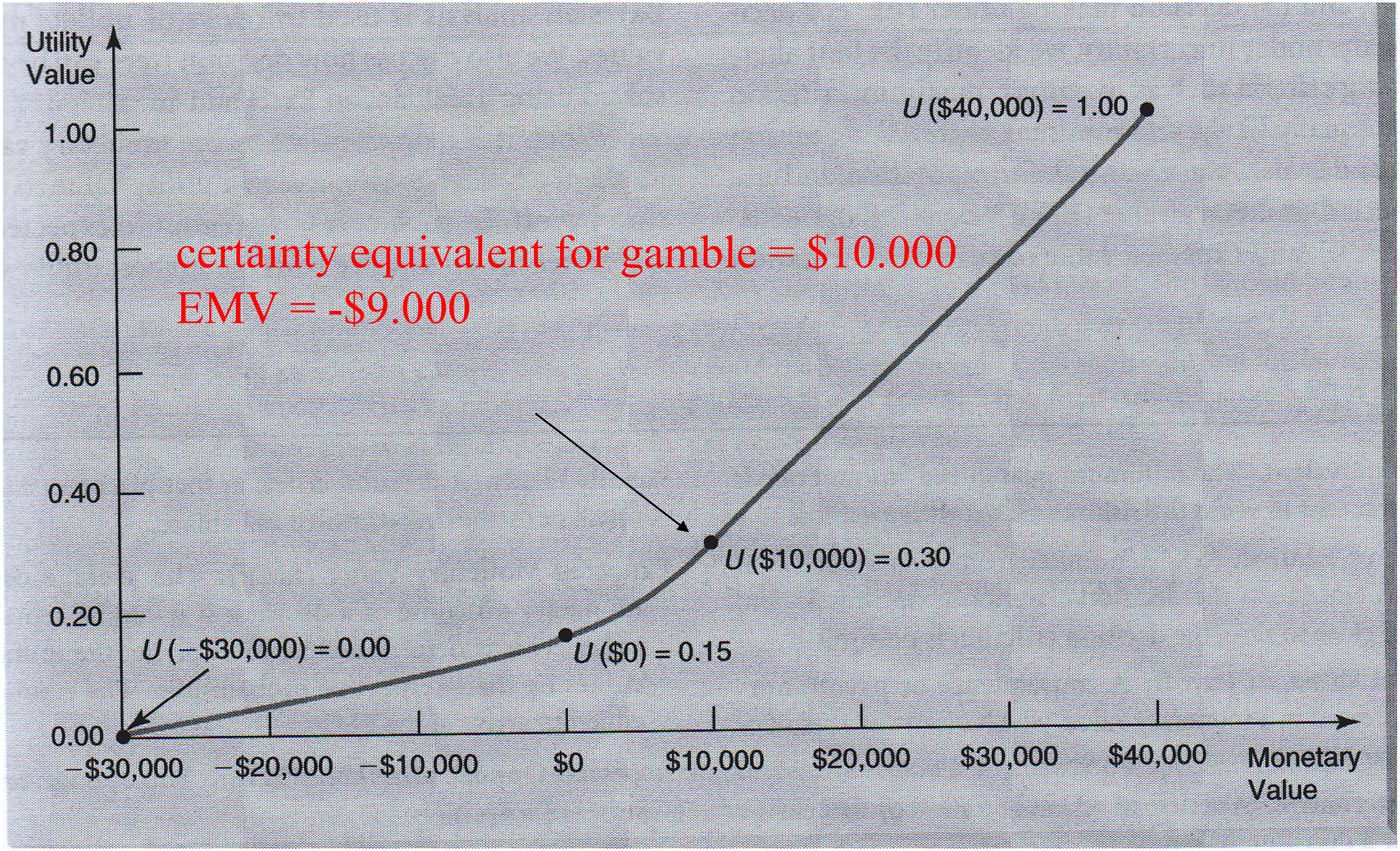
Utility Curve for Mark
Convex => risk-seeker



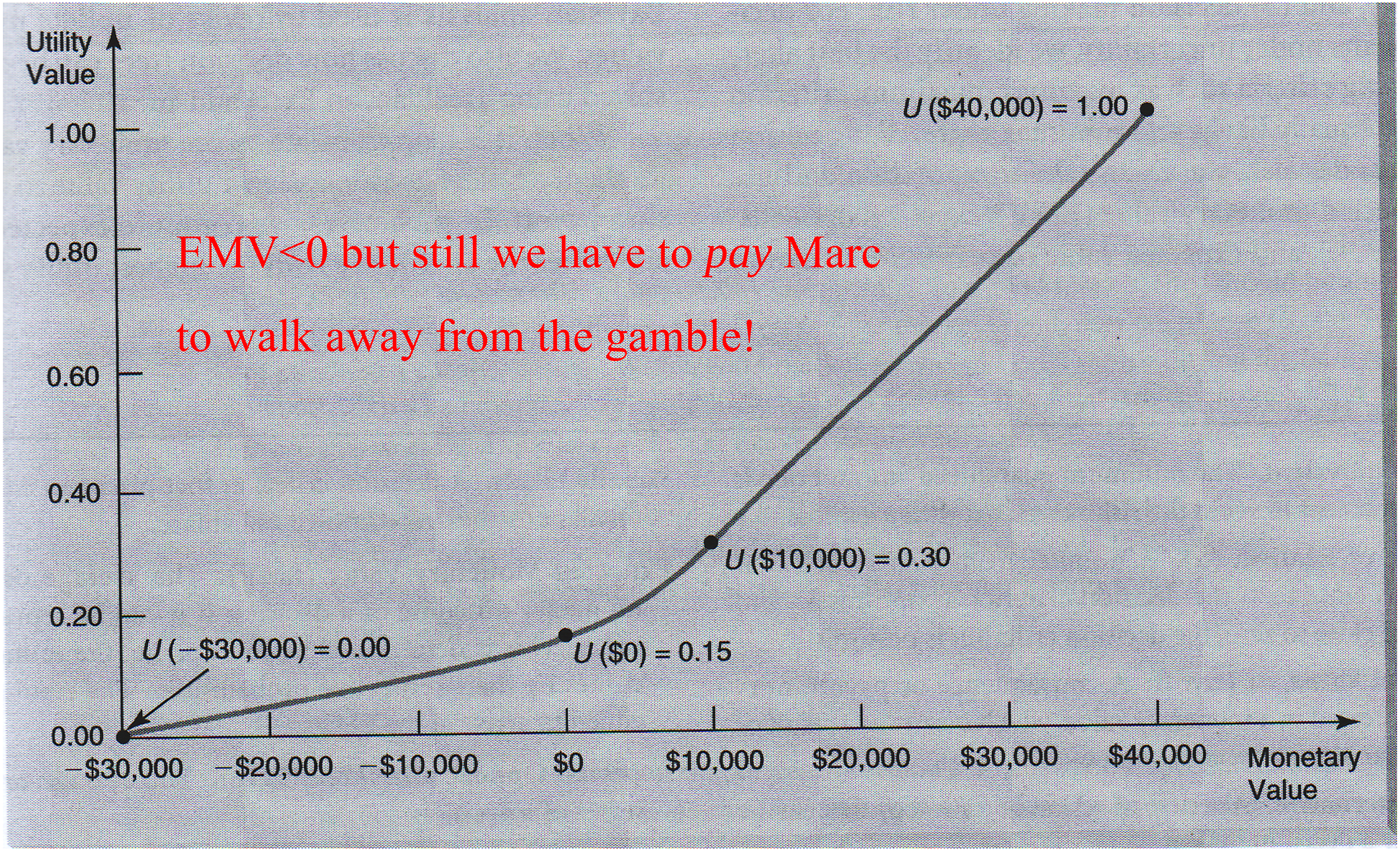
Utility Curve for Mark
Convex => risk-seeker



Utility Curve for Mark
Convex => risk-seeker



Utility Curve for Mark
Convex => risk-seeker



Utility Function

In general, gamble

with chance p to win highest outcome ($40.000)
chance 1-p to win lowest outcome (-$30.000)

⇒ Expected utility:

$$p*U(\text{highest}) + (1-p)*U(\text{lowest})$$

$$= \quad p*1 \qquad\qquad + (1-p)*0$$

$$= \quad p$$

Utility Function

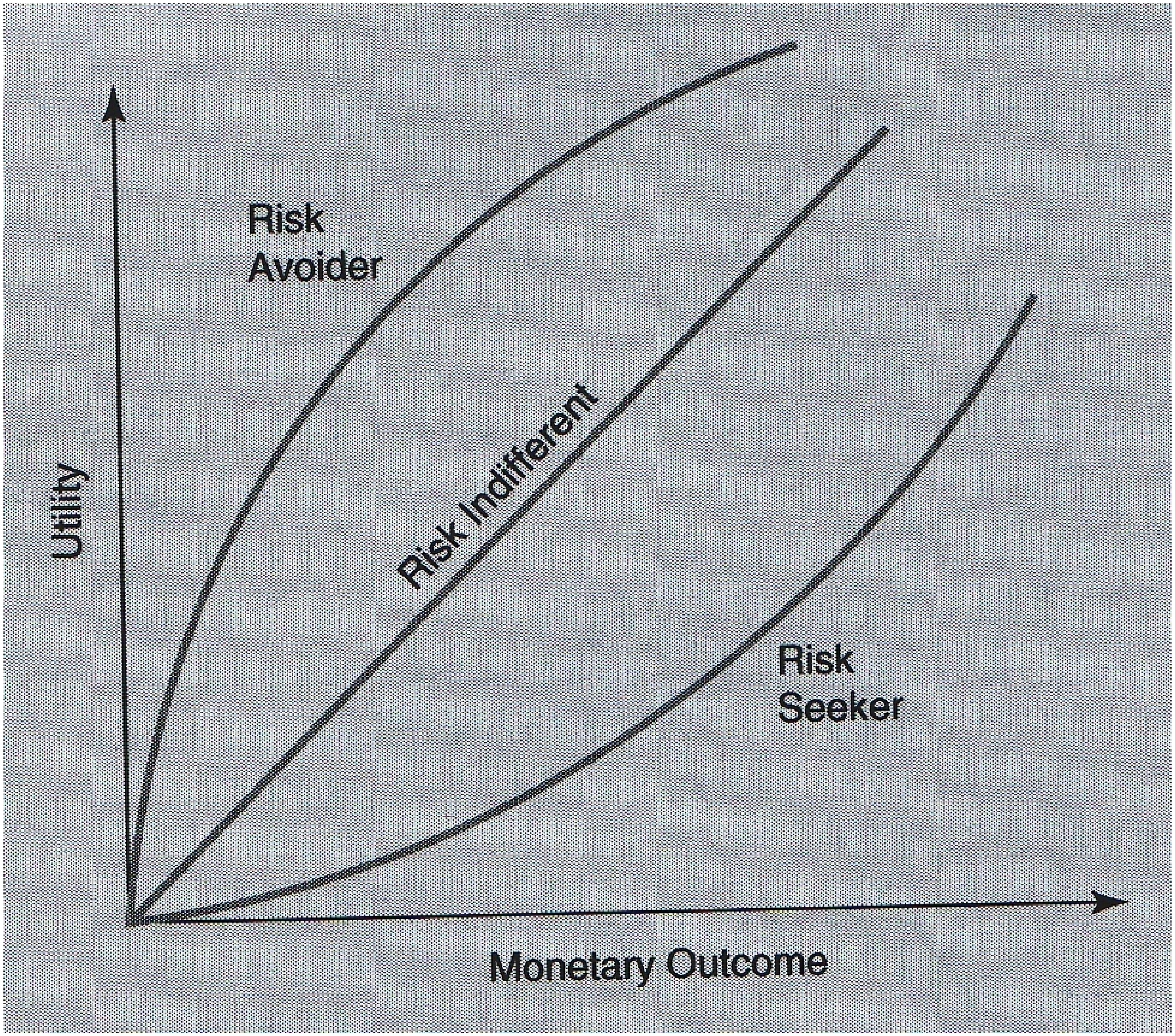
- Expected utility of gamble between lowest and highest outcome = p.

If Mark tells us that U($0)=0.15

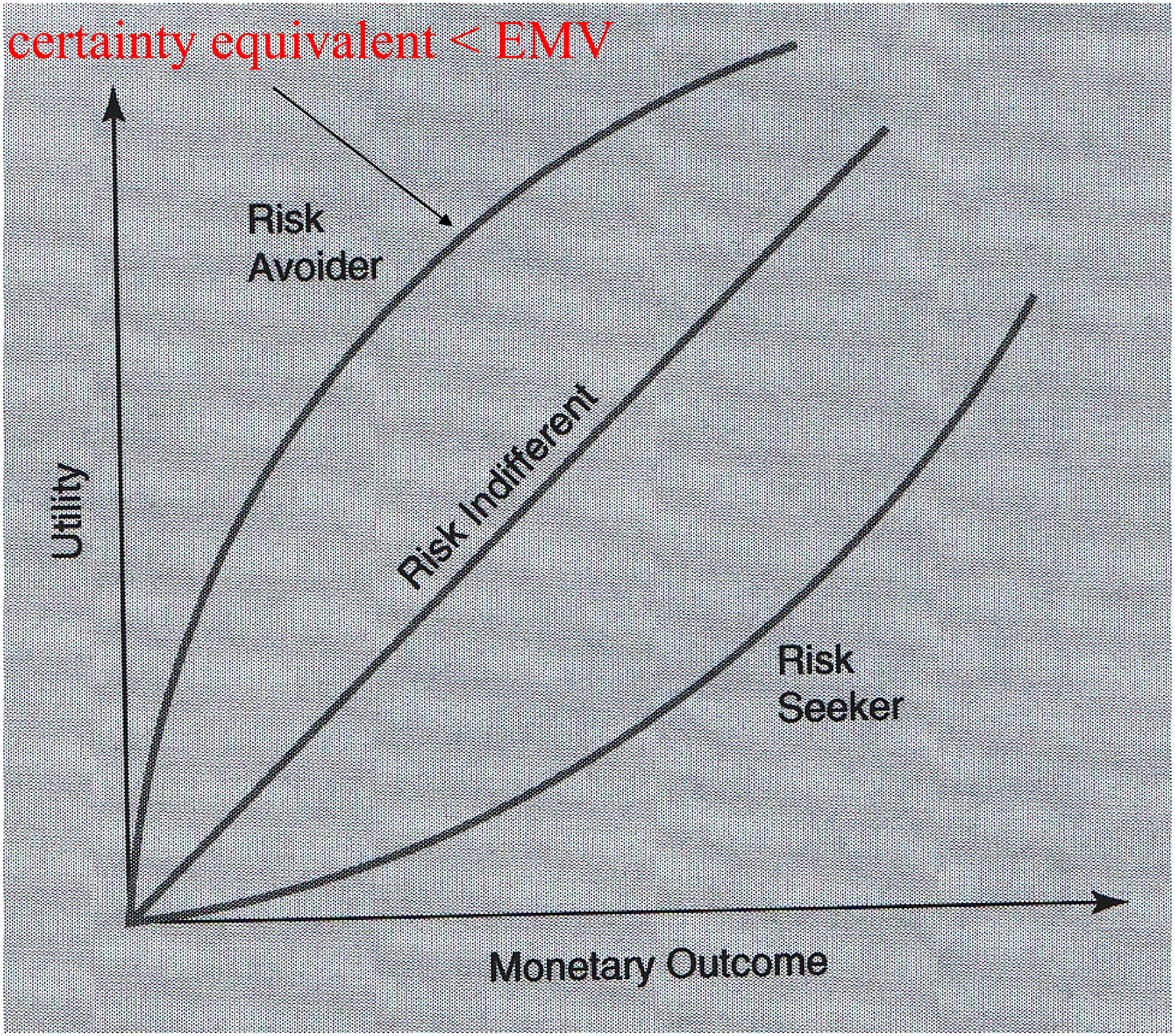
⇒ $0 is certainty equivalent of gamble which wins the highest payoff with 15% chance, the lowest with 85% chance.

⇒ Mark is risk-seeking.

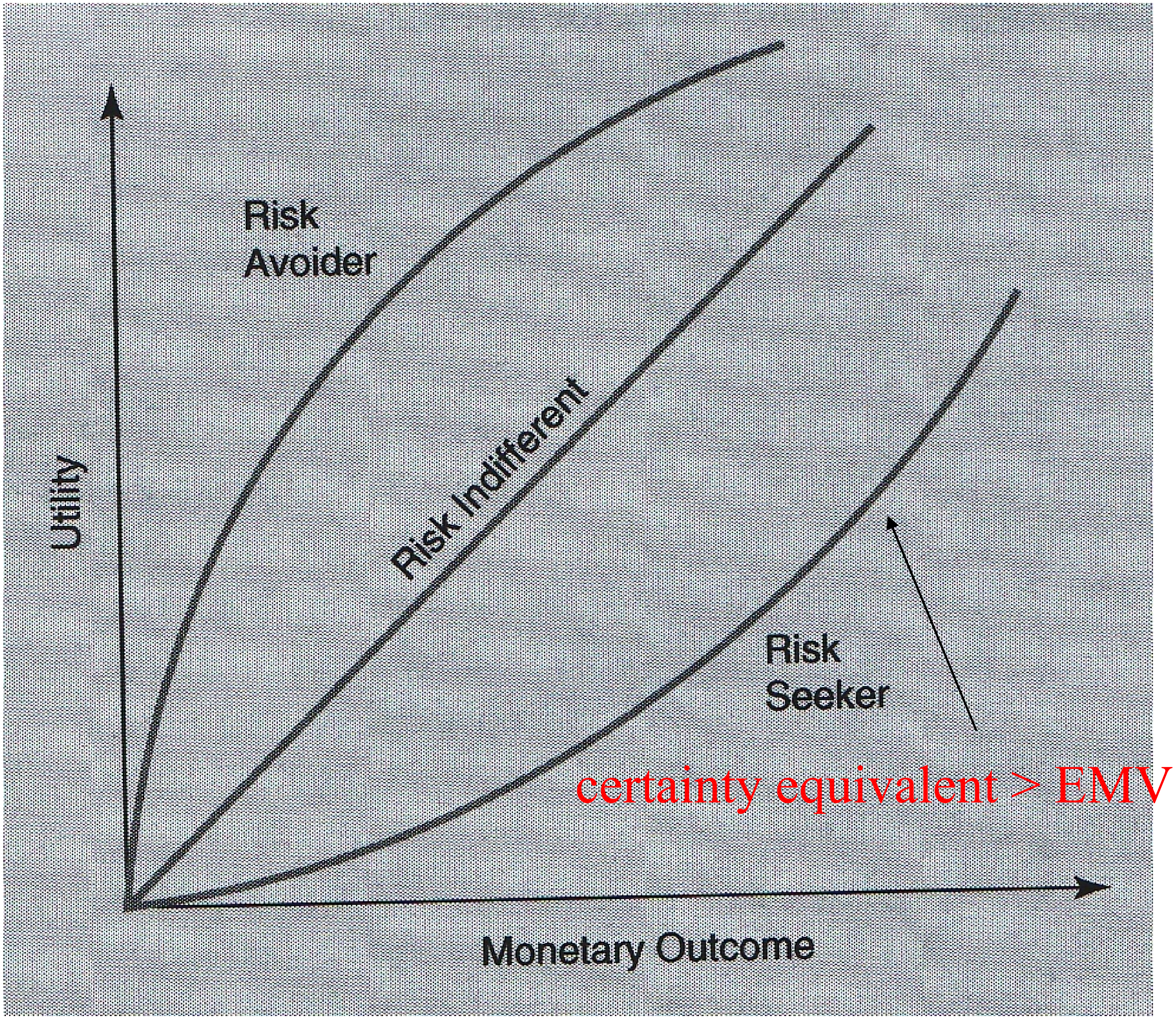
Utility Function



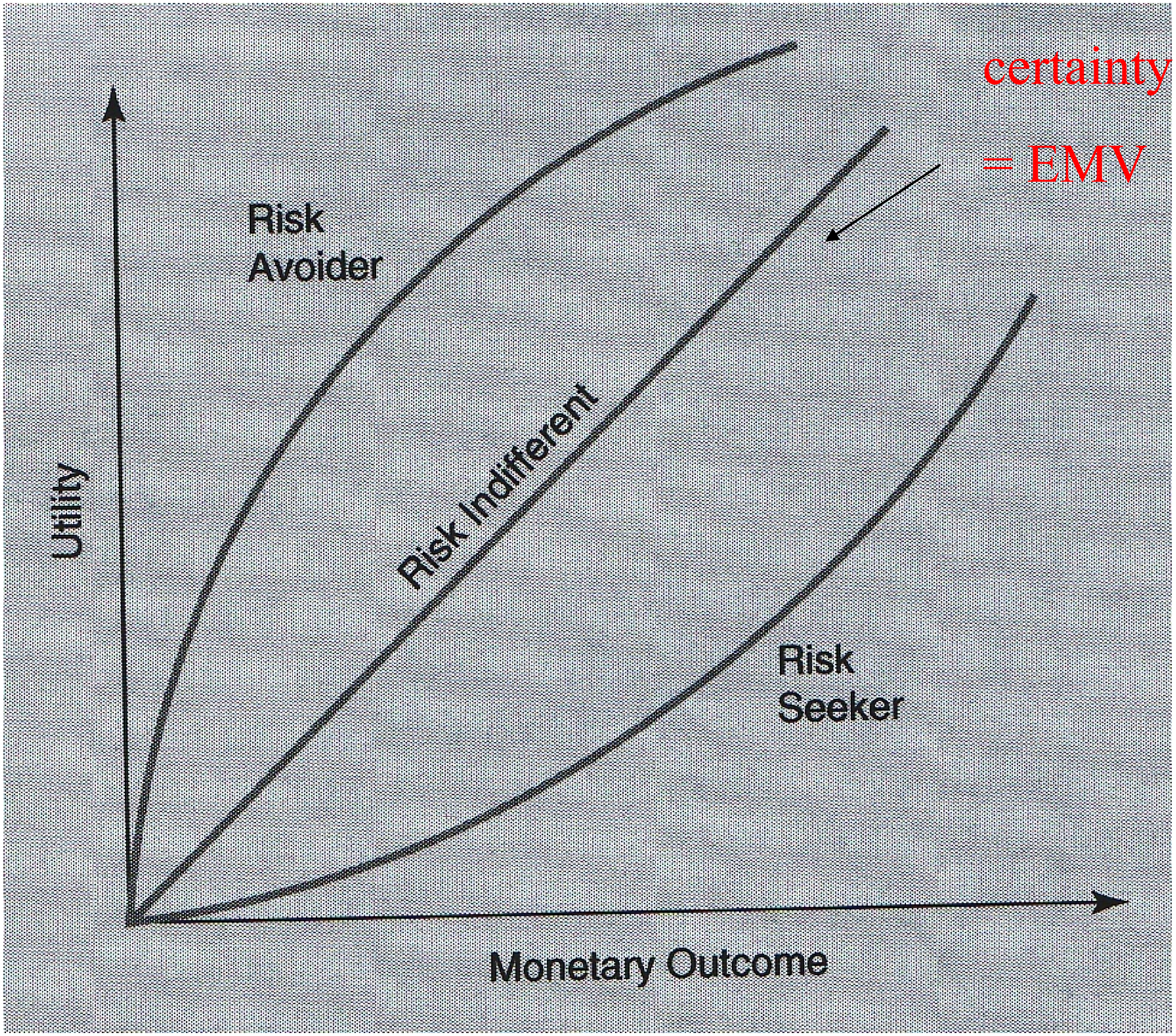
Utility Function



Utility Function



Utility Function



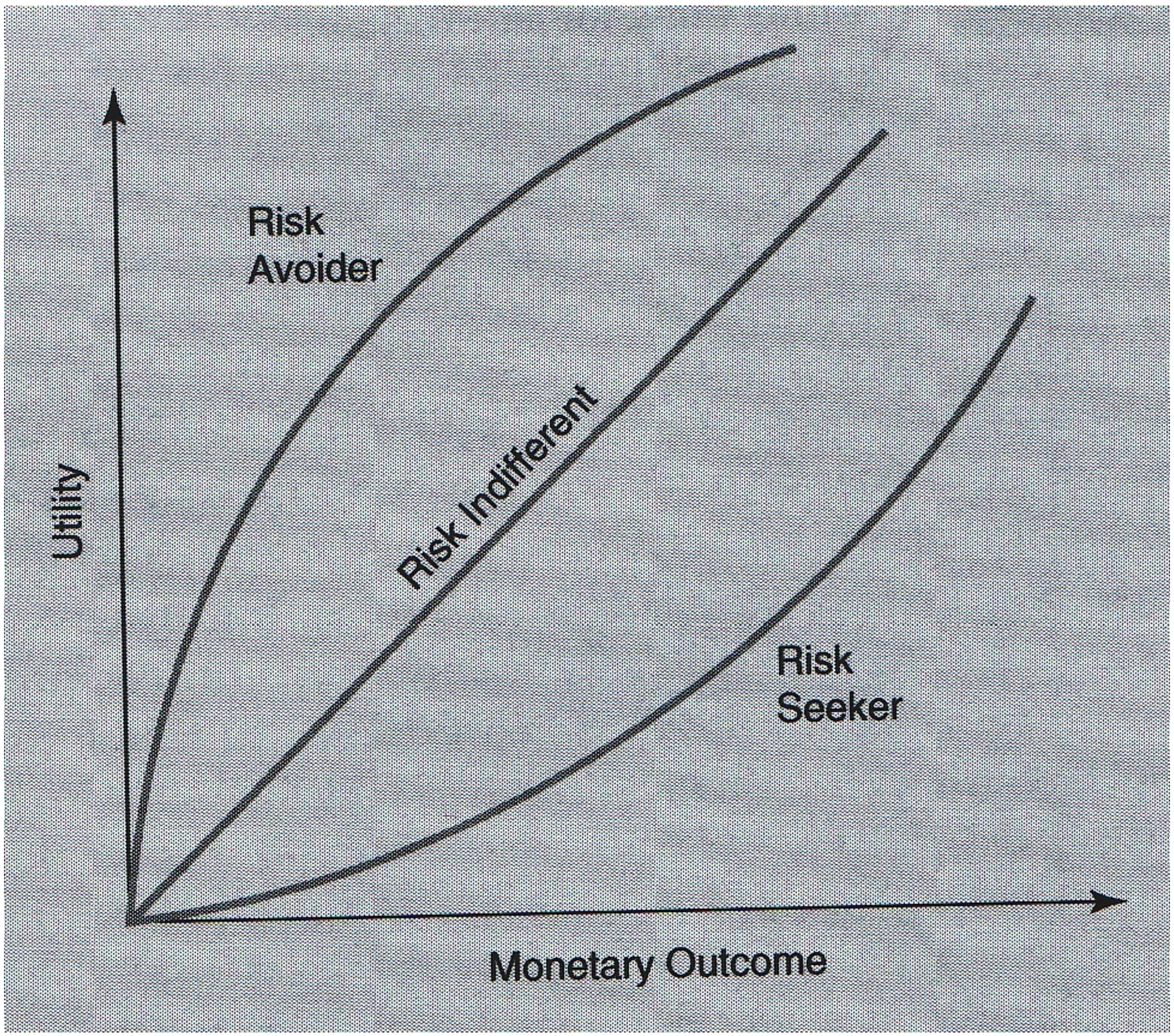
certainty equivalent

= EMV

Risk averse: diminishing marginal utility

Risk neutral: constant marginal utility

Risk seeking: increasing marginal utility



Example: Should Mark Invest in New Business?

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Payoffs in Dollars

Example: Should Mark Invest in New Business?

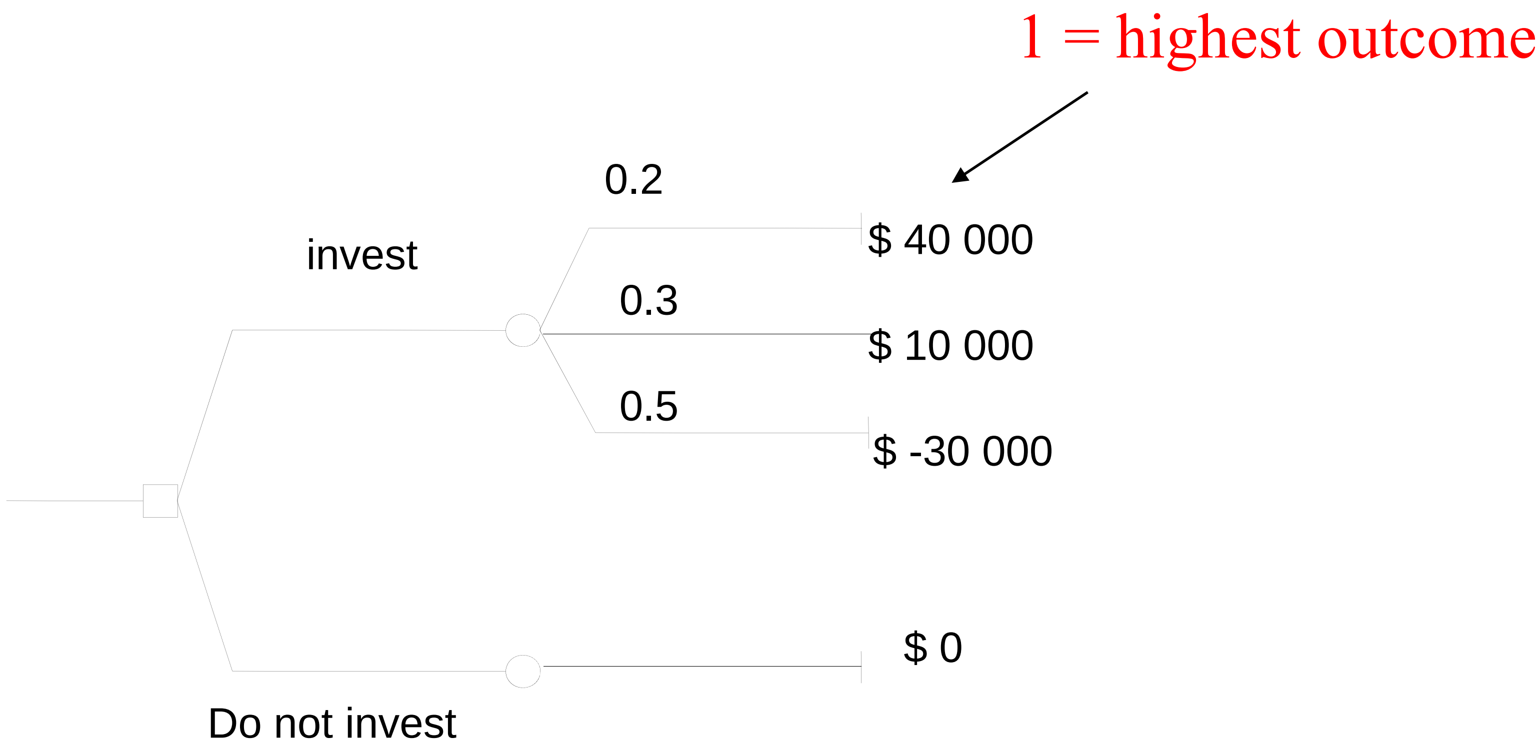
Payoffs in Dollars

Comparing EMV of his options => no investment

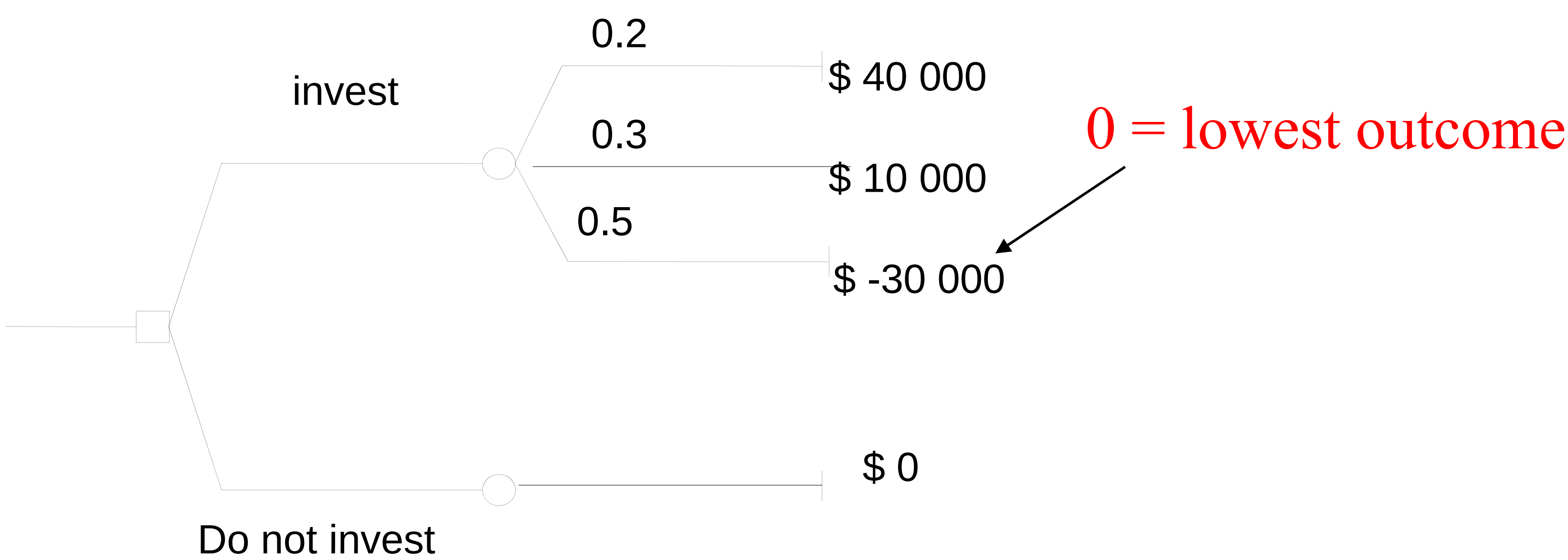
Example: Should Mark Invest in New Business?

Now instead of monetary payoffs consider his utility – which reflects his risk attitude.

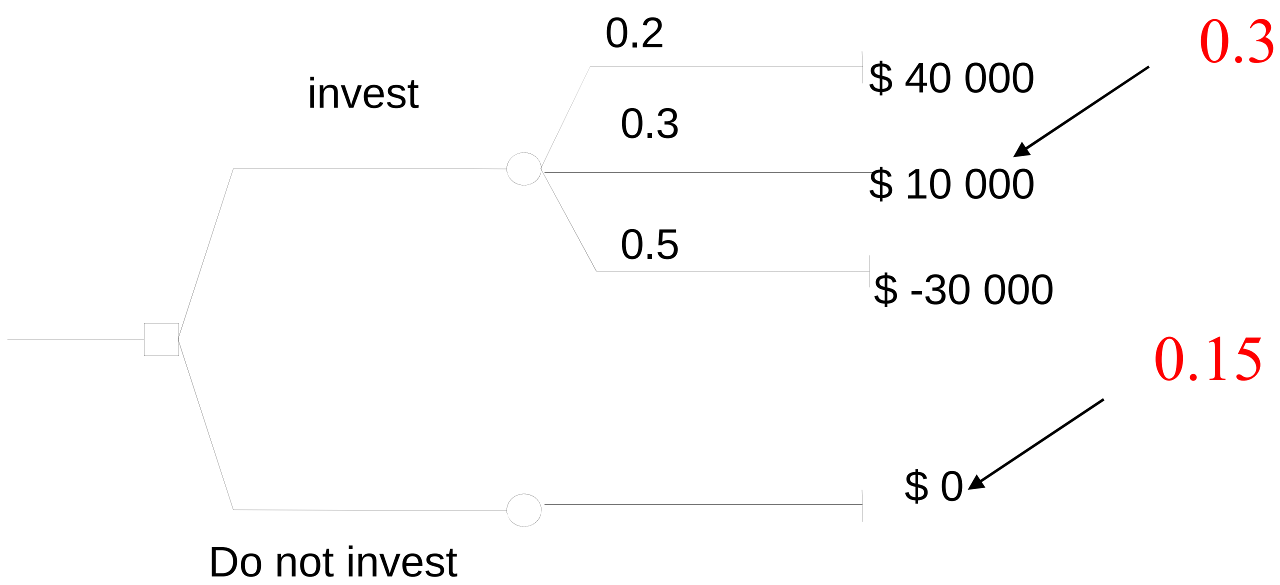
Example: Should Mark Invest in New Business?



Example: Should Mark Invest in New Business?

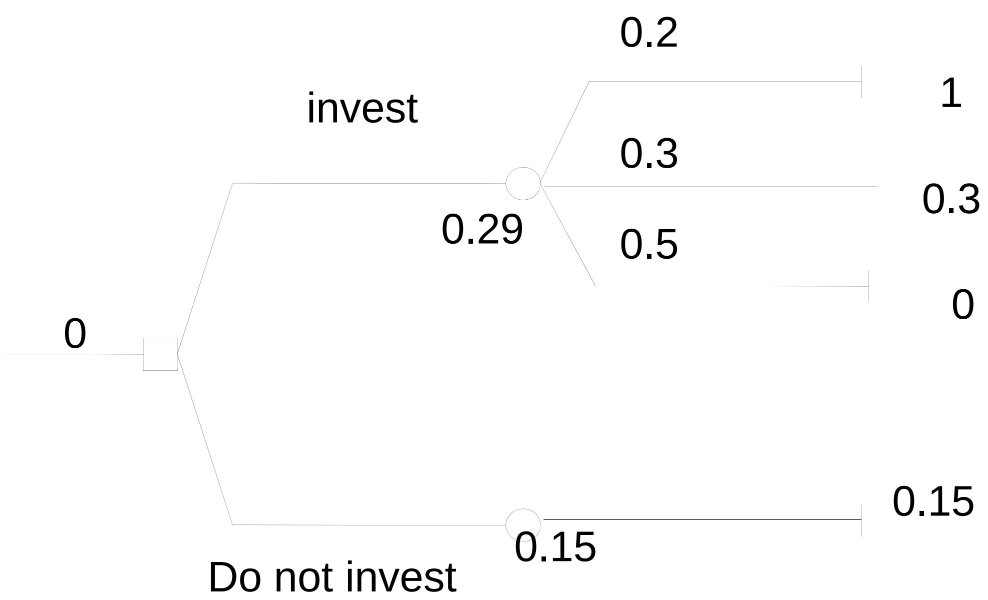


Example: Should Mark Invest in New Business?



Example: Should Mark Invest in New Business?

Marc should invest!



Warnings Using Utility Theory

- Each person’s has his or her own utility function.
- It could change over time, e.g. with getting older or richer.
- A person’s utility function can change with different range.
- E.g. most people are risk seeking when potential losses are small.
- This changes when potential losses get larger!

Warnings Using Utility Theory

=> Consider utility function only over relevant range of monetary values of a specific problem.