

Grade 2 Mathematics Lesson Plan

Teacher: Ken Hayakawa

Students: Class #3, Grade 2

25 students (14 boys, 11 girls)

1 Name of the unit: Addition and subtraction

2 About the unit

(1) About the content

In Grade 1, students learned about two kinds of numbers: cardinal numbers and ordinal numbers. In the study of addition and subtraction, students mostly dealt with cardinal numbers. This is because the main emphasis in Grade 1 is understanding the meanings of addition (combine and increase) and subtraction (take away and compare). In Grade 2, the range of numbers students deal with expands, and they study the addition and subtraction algorithms.

In the unit, Addition and Subtraction, which appears towards the end of Grade 1, students' understanding of operations was deepened by examining addition and subtraction problems that included ordinal numbers. In general, problems involving ordinal numbers are considered to be more difficult for children. One reason for this difficulty is that ordinal numbers themselves are a difficult concept. In addition, in order to solve these kinds of problems, students must convert ordinal numbers to cardinal numbers and also draw diagrams. Students have learned to think about this by examining statements like, "Takeshi is the fifth person from the front of the line." This means that "there are five people from the front of the line to Takeshi (including him)," or "there are four people in front of Takeshi."

In this special unit in Grade 2, we will deal with more advanced problems involving ordinal numbers. For these problems, students cannot just use the numbers in the problem statement in computation to obtain the correct answers. The emphasis is on understanding the structures of the problem situations. In other words, through these problems involving ordinal numbers, students will deepen their understanding of the meanings of the operations of addition and subtraction. Moreover, students will be learning about the importance of representing problem situations with pictures, diagrams, words, math sentences, and manipulatives.

(2) Abilities to be developed through "communication" activities

The problem used in this lesson is, "Yoshiko is the 6th person from the front of the line and the 7th from the back of the line. How many people are in the line altogether?" This problem is complicated for students and often creates confrontation between competing ideas.

The most common answers are either 13 people or 12 people and the math sentences often used to get these answers are:

① Answer: 13 people --- $6 + 7 = 13$

② Answer: 12 people --- $5 + 6 + 1 = 12$, $6 + 6 = 12$, $6 + 7 - 1 = 12$, etc.

Thus, the first point of confrontation is 13 people vs. 12 people.

Answer ① results when the numbers in the problem are used directly. Quite a few children think that writing a math sentence means to just use the numbers in the given problem. Moreover, from the word, “altogether,” they decide that addition is to be used. On the other hand, the answer of 12 people shown in ② is reached by thinking carefully about the meaning of the phrase, “the 6th person from the front of the line,” perhaps using pictures or diagrams. Thus, in the confrontation between ideas ① and ②, we want students to “communicate” by using well grounded reasoning.

The next point of contention is how the answer of 12 people can be obtained. We would like students to represent their ideas mathematically using pictures, diagrams, math sentences, and words. Furthermore, students should pay attention to their classmates’ ideas carefully and try to interpret their representations. Through this “communication” activity, students will be encouraged to connect pictures and diagrams with math sentences.

Therefore, one goal of today’s lesson is for students to understand the structure of the problem by mathematically representing the problem situation using pictures, diagrams, words, and math sentences. Moreover, students will learn to think cohesively by connecting math sentences with pictures and diagrams, based on their understanding that the phrase “Takeshi is the fifth person from the front of the line” means that “there are five people from the front of the line to Takeshi (including him),” or “there are four people in front of Takeshi.” We would like to achieve these goals through “communication” motivated by contentions and complications that arise from students’ ideas. We would also like students to understand that what they have learned about addition and subtraction may be used in problem situations that involve ordinal numbers. This lesson was developed to achieve these goals.

3 Relationship to school-based research (*konaikenshu*)

(1) Concerning the class

Based on the results of the Q-U (school based survey), the students who are satisfied with school life are those who have been given various classroom duties and responsibilities to do collaboratively or other opportunities in which they can express themselves. Positive feedback and public praise are used to help those students who do not appear to be satisfied with their daily lives both within and outside of lessons. That feedback may be directed at such things as how they share their ideas, how they exhibit a positive attitude in listening to others during a lesson, how they talk with other students, and how they willingly fulfill their responsibilities during clean-up time. As a result, these students are beginning to participate in activities with more confidence.

In daily life, we have been focusing on norms such as “walking in the hallway quietly,” and “looking at the person who is talking.” Students have been instructed to encourage each other when someone inadvertently violates any rule or norm. As a result, students are beginning to follow the rules and norms better and to support each other. To develop and deepen friendship among students, we have used singing and games that require students to interact in a group. Students are beginning to feel satisfaction from doing things together with others and share the resulting enjoyment and happiness with each other.

As for thinking together as a group, we continue our efforts from Grade 1. No one is hesitant about speaking in front of the class. Many children respond to the teacher’s questions willingly, and some express their desire to come to the board to explain their ideas. On the other hand, there are some children who are a little hesitant because their voice may be soft or they lack confidence.

While listening to others during whole class discussion, many children listen quietly to others and give their full attention. Some children ask questions and even provide additional ideas. On the other hand, there are children who talk with their neighbors, cannot keep a good posture, start making their own comments before the other person is finished, or do not truly understand what the other person is saying even though they appear to be listening. Thus, the importance of listening to each other is often discussed in various situations.

(2) “Communication” in elementary school mathematics

During mathematics lessons, we have been encouraging students to understand each other by engaging in activities in which students try to read/interpret each other’s ideas and asking questions such as “What was Yoshiko thinking when she did this?” Because there

are some children who are still self-centered, occasionally the classroom atmosphere becomes such that students become indifferent to other students' ideas. Thus, we are currently trying to help students to understand the enjoyment that comes from thinking together as they engage in activities where they can learn something important from other students' comments.

We are also learning about how to express our own ideas by using mathematical representations such as pictures, diagrams, math sentences, and words (particularly mathematical terms such as "ones place" and "tens place"). There is a wide range of individual differences in the way the students represent mathematical ideas. But students are improving their ability to represent mathematically by translating their writing into mathematical terms and imitating the good ways that other students represent their mathematical ideas.

In order to help students develop well-grounded reasoning, we are paying careful attention to some fundamental properties and relationships (such as "you can add two numbers in any order," or "you must add numerals in the ones place only with other numerals in ones place").

4 Goals of the unit

- Students will be able to represent relationships among numbers in quantities in problem situations by using pictures, diagrams, math sentences, and words.

5 Plan of instruction and evaluation (special 2-lesson unit)

Step	Instructional activity and contents	Evaluation criteria	Teacher Support
1 (2)	<p>○ Students will understand that they can use addition and subtraction even when problems include ordinal numbers. They can also use their understanding to solve problems.</p> <p>• Students will solve problems involving ordinal numbers using diagrams. <Extension> Today's lesson: 2 of 2</p>	Can Ss represent and think about relationships among numbers and quantities in problem situations using pictures and diagrams? (Math Thinking)	<ul style="list-style-type: none"> • Provide blocks they can use to represent problem situations.

6 Today's lesson and evaluation

(1) Date and location: July 2 (Mon.), 2007, 13:55 – 14:40

Home Economics Room at Ishida Elementary School

(2) Goals

- To represent relationships among numbers and quantities in problem situations by using pictures, diagrams, math sentences and words.
- To think about problems by associating pictures and diagrams with math sentences.

(3) Strategies to encourage students to “communicate”

- Pose a problem that may produce confrontation and complication among students' ideas. (Understanding the problem)
- Provide opportunities for students to represent the problem situation using pictures, diagrams, math sentences and words in their own notebooks. (Individual problem solving)
- As students communicate with each other, incorporate activities that focus on interpreting other students' ideas. (Comparing and critiquing)

(4) Flow of the lesson (1/2)

Steps	Instructional activities and contents	Points of consideration & evaluation	Notes
Understand	1. Understand the learning task		
	<div>Children are lined up in a straight line. Yoshiko is the 6th person from the front of the line and the 7th person from the back of the line. How many children are there altogether?</div>		
Examine	2. Individual problem solving <ul style="list-style-type: none"> 13 people ---- $6 + 7 = 13$ 12 people <div>----○○○○○●○○○○○○○</div>	<ul style="list-style-type: none"> To develop a sense of ownership of the problem, try to use a familiar situation in students' everyday lives as the setting of the problem. Make sure students can represent their reasoning using pictures, diagrams, math sentences, words, or manipulation of blocks. 	Blocks
Deepen	3. Share the solutions.	<ul style="list-style-type: none"> Make sure that students are sharing the rationale for their answers. Contrast the two answers (13 people vs. 12 people) and examine the pictures, diagrams, and math sentences carefully. 	Magnets (for black board)
Summarize	4. Compare and critique the solutions <ul style="list-style-type: none"> ○○○○○●○○○○○○○ 1 2 3 4 5 6 5 4 3 2 1 There are 5 people in front of Yoshiko, 6 people behind her, and Yoshiko herself. --- $5 + 6 + 1 = 12$. There are 6 people in the front, including Yoshiko. There are 7 people in the back, including Yoshiko. Since Yoshiko is counted twice, we must subtract 1 from the sum. --- $6 + 7 - 1 = 12$ 	<ul style="list-style-type: none"> When the different ways of getting the answer of 12 people becomes the topic of discussion, make sure students can explain their reasoning explicitly, for example, “‘Yoshiko is the 6th person in the line’ means...” Go back and forth between pictures/diagrams and math sentences in order to make the structure of the problem more explicit. <div>Can Ss represent the relationships in the problem situation with pictures, diagrams, math sentences and</div>	
	5. Summarize in math journal entries.		

(5) Evaluation criteria for the lesson

- Were the strategies designed to encourage “communication” that were incorporated in today’s lesson effective in achieving the goals of the lesson?
 - Posing a problem that may produce confrontation and complication among students’ ideas. (Understanding the problem)
 - Providing opportunities for students to represent the problem situation by using pictures, diagrams, math sentences, and words in their own notebooks. (Individual problem solving)
 - Incorporating activities that focus on interpreting other students’ ideas as they communicate with each other. (Comparing and critiquing)

Students' current levels

ID #	Column 1	Column 2	Column 3
1	○		○
2			
3	○	△	○
4		○	
5	○	○	○
6	○	○	
7		○	○
8			
9			○
10	△		
11		△	
12	△	△	
13	△	△	△
14		△	△
15	○		
16	○		○
17	○	○	
18		○	○
19		○	
20			○
21	○		○
22	○		○
23		○	
24		○	
25	○	○	○

Column 1: Speaking

Students can state clearly what happened and how.

Column 2: Listening

Students can listen to each others' ideas completely.

Column 3: Discussing

Students can ask and make comments that are appropriate for the current topic of discussion.

Seating Chart (by ID #)

Teacher's Desk

9	6	24	12	16	5	
17	14	20	13	10	18	7
19	11	15	8	23	4	
25	2	21	3	22	1	

Supplemental material for Ishida Elementary School in-school lesson study

I. About lesson study in mathematics

1. Goals Through lessons that encourage students to communicate their own ideas and reason, foster students' ability to think cohesively.
2. About lessons that facilitate communication to develop the ability to think cohesively

(1) Discovery/inquiry style lesson

The ability to think is not something that can be simply taught directly by telling. We can teach children how to think more effectively by having them actually think about something in context. Therefore, the lessons will be problems-solving style lessons (discovery/inquiry style). Teachers must value each student's ideas and discoveries, and by facilitating students' communication of their ideas with each other, raise the level of students' thinking. Children will develop their own ideas, and through the process of communicating their ideas to others (speaking, listening, acknowledging, contrasting, writing, etc.) they will broaden and deepen their thinking.

← Mastery & Discovery/Inquiry styles

(2) Flow of lessons

① Understanding the problem (Problem development & task)

- Develop problems that are important for children. Provide an opportunity to get started with their reasoning and help students have a clear idea about how to individually approach the task.
- Move from a problem posed by the teacher to students' own problem.
- Intentionally create situations that contradict students' own ideas and common sense. Develop situations where diverse ideas become apparent.
- Devise instructional materials that combine numbers, quantities and shapes. Devise situations where all students can develop their own answers (*kyozaikenkyu*)

② Individual problem solving <Opportunities to think individually>

- Represent their ideas using their own forms of representation.
- Mathematical representations (words, math sentences, pictures and diagrams, manipulation)

③ Critical reflection ("communication") <Opportunity to think collaboratively>

- Develop a broader way of reasoning by thinking collaboratively and reaching solutions that may not have been possible through individual problem solving.

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(Merits of communication)

- Mathematical representation (words, math sentences, pictures and diagrams, manipulation) Devise manipulatives or other forms of representation that will be appropriate for children at their levels.
- Value the process of “create, discover, and use” mathematics.

<Developing opportunities to communicate>

- Develop a classroom atmosphere that invites students to share their ideas freely and also accepts and nurtures everyone’s ideas. Develop the disposition to accept and forgive others’ mistakes, to seek and acknowledge good points in others, and to nurture good ideas collaboratively. Teachers must possess these dispositions themselves. Instead of rejecting not-so-good ideas, always maintain the attitude of acceptance of students’ efforts to express their own reasoning freely.
- The teachers’ role is to coordinate student discussion by making the goals of the lesson clear. Instead of evaluating students’ responses, accept their comments and responses positively. Identify and acknowledge the good points in students’ ideas and provide suggestions that may lead to better solutions. It is also necessary to model different ways of reasoning. Instead of simply accepting students’ ideas as they are, sometimes pose questions to challenge their ideas – through questioning and modeling how to think and reason. It is also important to ask students to explain their rationale (“why?”) and generalize (“Is it always true?”).

④ Summary learning log/journal -- summarize in your own words

(3) About the learning content

Make sure that all students understand fundamental and foundational ideas (those ideas that become the basis for reasoning. This is to be developed throughout a unit and throughout the year). It is also important that students are able to use what they learn.

II. Specific ideas of lesson study

1. Introduction

The goal of the joint (primarily) mathematics and Japanese language lesson study group is “To foster students’ ability to <reason logically (Japanese language)>, <reason cohesively (mathematics)> through lessons in which students communicate their own ideas and reasoning.”

We will present our ideas on what we need to be doing in our everyday lessons to achieve this goal. In particular, assuming that our goal is to teach public lessons in November, what do we need to do to develop “good lessons in which students communicate with each other” as we envision at Ishida Elementary School?

There are many purposes for engaging in lesson study as a whole school. One of them is for teachers to discuss lessons critically so they can improve their vision and philosophy about lessons, teaching and learning through friendly rivalry. Each teacher has his/her own vision and philosophy of lessons and teaching. Students in every classroom have their own vision of learning. By sharing each other’s vision and philosophy about lessons, teaching and learning, and critically discussing our differences, we can begin to share good points and overcome shortcomings. Instead of discussing “communication activities,” <ability to reason logically (Japanese language)>, or <ability to reason cohesively (mathematics)> on paper, we want to discuss these ideas critically through actual lessons. By observing each lesson, we want to pursue a model of “good lessons in which students communicate with each other” at Ishida Elementary School. In this report, we want to propose a system in which we observe each other’s lessons without making any specific reference to the subject areas.

2. To observe each other’s lessons

(1) “Lighten our burden!”

There are many barriers for teachers to observe each other’s lessons. Our busy schedule and the need to prepare the class for the time the teacher is not present are just two examples. Thus, we would like to suggest the following ideas that might make it more feasible for teachers to observe each other’s lessons. (Try either grade-level or group observation once in each term.)

- Observe each other’s lesson within the grade-level group

Arrange to observe lessons within the same grade level. The same lesson will develop very differently when different students are involved. By observing multiple lessons over time, we can observe the way the lesson transforms and

students' grow over time.

- Observe each other in each subject group (Japanese language – grades 1, 3, and 5; mathematics – grades 2, 4 and 6)

Announce the public lesson to the entire school, and those teachers who are able to will observe the lesson. We can discuss what grade-appropriate “communication” activities may be. We will also identify a common theme for each subject area.

- Observe only a part of the lesson

Instead of observing an entire lesson, which might be difficult time-wise, teachers can observe specific sections of lessons, for example, introduction of the task, communication stage, etc.

- Use a simplified lesson plan

Make the focus of the lesson clear. Instead of preparing an elaborate lesson plan, teach and observe everyday lessons. During the planning stage, consult with Kawahara (lesson study coordinator), Maejima, and Hayakawa about the content of the lesson and ideas about “communication.” They will all try to observe as many lessons as possible.

(2) Be self-critical!

In order to make each public lesson fruitful, we should try to do the following.

- Set aside a time to discuss the lesson among the teacher and the observers.

Before the lesson: Make the goals clear.

After the lesson: Self-reflection and criticism.

Post-lesson discussion: Observers will share their ideas openly. Discuss any struggle.

Discuss the way the lesson was developed, methods of communication, views on the instructional materials, specific <ability to reason logically (Japanese language), ability to reason cohesively (mathematics)> in today's lesson, effective use of students' records. Moreover, try to learn about own idiosyncrasy in posing questions and choices of words, attitudes, blackboard writing etc. Also try to learn things you did not notice or could not observe about children's manner of learning.

- Video- and/or audio-record the lesson, and take digital photographs of blackboard writing. Use these records for self-reflection.

Although other people's opinions and suggestions are useful, to change lessons,

teachers must examine their own teaching carefully. Teachers may ask how the lesson was deepened through communication or how clearly the lesson's flow can be understood from the end-of-class blackboard writing. Teachers must develop a disposition to view self-reflection and self-criticism on specific focus issues as something useful to improve their own teaching.

- Share about the lesson, post-lesson discussion, and future research questions for the entire school.

In order to develop shared understanding, we must summarize (concisely) our discussion and further research questions on “communication activities” and <ability to think logically (Japanese language), ability to reason cohesively (mathematics)> and share the summaries with the entire school.

III. Strategies for “communication” activities in mathematics lessons

1. Strategies for the discussion/critique stage of the lesson

(1) When sharing one’s own ideas:

Use language that makes the communication easier to understand

- “first,” “then” --- to present the idea in a clear, sequential manner
- “first,” “second,” ... --- to describe the steps of a procedure
- “because,” “the reason is...” --- to share the rationale for the idea
- “for example” --- to share some specific examples
- “but,” “what about --- to respond to someone else’s idea, to express disagreement
- “what if,” “if” --- to present a conjecture or prediction, to present a counter example
- “therefore” --- to present a conclusion

Also incorporate styles of discourse such as, “I think... because ...”.

(2) When “interpreting” or “reading” each other’s ideas:

Since communication in a mathematics lesson uses mathematical representations and terms, we will include activities that involve reading and interpreting math sentences, pictures, diagrams, words, and manipulations to facilitate mutual communication.

Students must make sense of what others are saying through their math sentences, pictures, diagrams, words, and manipulations. For example, when someone uses the math sentence, “ $7 \times 6 + 1$,” others have to be able to interpret the sentence. They need to interpret the person’s intention by figuring out what s/he meant by “7,” “6” “+1,” etc.. This is why it is important to clearly explaining one’s idea with supporting reasons. When there is misunderstanding between the speaker and the listener, communication is further enhanced because there is a need to explain their differences.

(3) When discussing with others

Students can deepen their own thinking by communicating with each other.

However, there are students who hesitate to participate in whole class discussion.

Therefore, we will incorporate small group discussion (with neighbor, within pre-assigned groups) so students can express their ideas more easily. By sharing their own ideas and listening to other’s ideas in small groups, students can engage in whole class discussion more comfortably. Moreover, they may learn that others have similar ideas, or that their ideas may be well regarded by others. Such experiences will allow

them to participate in the whole class discussion with more confidence.

(4) When comparing and critiquing shared ideas:

It is during the time in the lesson when students communicate with each other (comparing and critiquing each other's ideas and solutions) they deepen their own thinking and raise their level of mathematical reasoning. Therefore, we would like to facilitate this time by showing more explicitly how such comparing and critiquing should proceed.

- “Let’s think about the reason.” --- ask for the rationale
- “Let’s find out their similarities.” “How are they different?” --- identify similarities and differences
- “Can we always say that?” --- seek generalization
- “What is good about this idea?” --- look for merits
- “Which one is simpler?” “Which is easier to understand?” --- examine clarity and simplicity

2. Strategies for the comparing/critiquing stage of the lesson:

(1) Devise learning tasks (understanding the problem)

In order to have lively discussion and communication, it is important to think carefully about the learning task for the lesson. We must devise tasks that promote diverse ideas, cause confrontation of competing ideas, and examine the conditions (too much or too little) of the problem. We also need to consider that these types of problems reveal common misconceptions and errors. We need to examine the numbers used in the problem by anticipating students' responses.

(2) Express their own ideas (individual problem solving) (summarizing --- learning journals)

Express students' own ideas using mathematical representations.

Pictures and diagrams --- clarify the relationship among quantities in the problem so it is helpful in determining the operation to be used.

Math sentences --- represent the problem situation concisely by using numbers and symbols. Compound math sentences may clarify the problem situation.

Language --- express their own ideas using mathematical terms.

Manipulation --- deepen one's thinking through the use of actual objects or manipulatives such as blocks and counters.

Learning journals --- reflection on the lesson. Examine the reasoning process.

Capture the progress of students' thinking and increase in their level of mathematical reasoning.