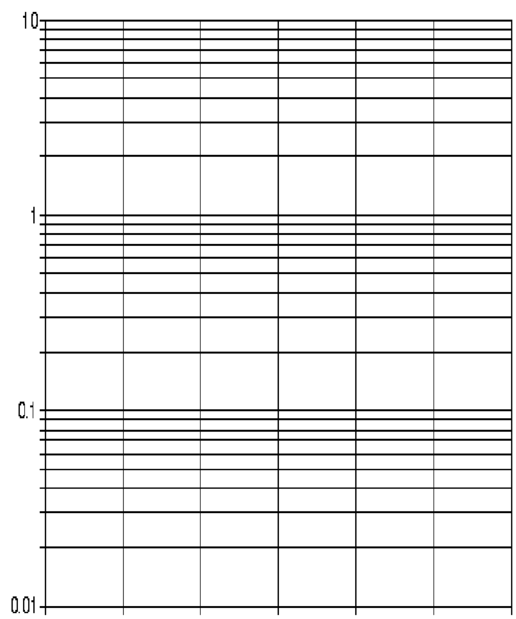
How to Use Semilog Graph Paper

In semilog paper the y-axis is marked off so that the height represents the log of the value to be graphed. If you look at a semilog graph you'll notice that the horizontal lines are not drawn at even intervals, they start out sort of spread apart then get closer and closer until, *bing!*, they jump and are spread out again, and the same process repeats. In this way the graph is broken up into *cycles*. Each cycle has 10 lines (counting the lines it shares with the cycles immediately above and below.)



Semilog Graph
(3-cycles)

The bottom line is the *base line* for the first cycle, each line corresponding to the bottom line is the base line for the cycle above it. The base lines are those where the spaces just below are small and those immediately above are big. Each base line always represents a power of 10, that is 10^n for some whole number n. (The power, n, can be negative or zero.) For example the first line stands for 1 or 10 or 100 or 1000, ... or one of 1, 0.1, 0.01, 0.001, ... (20 is *not* a power of 10). The next base line is labeled 10 *times* the bottom line, and each base line is always 10 times the preceding base line.

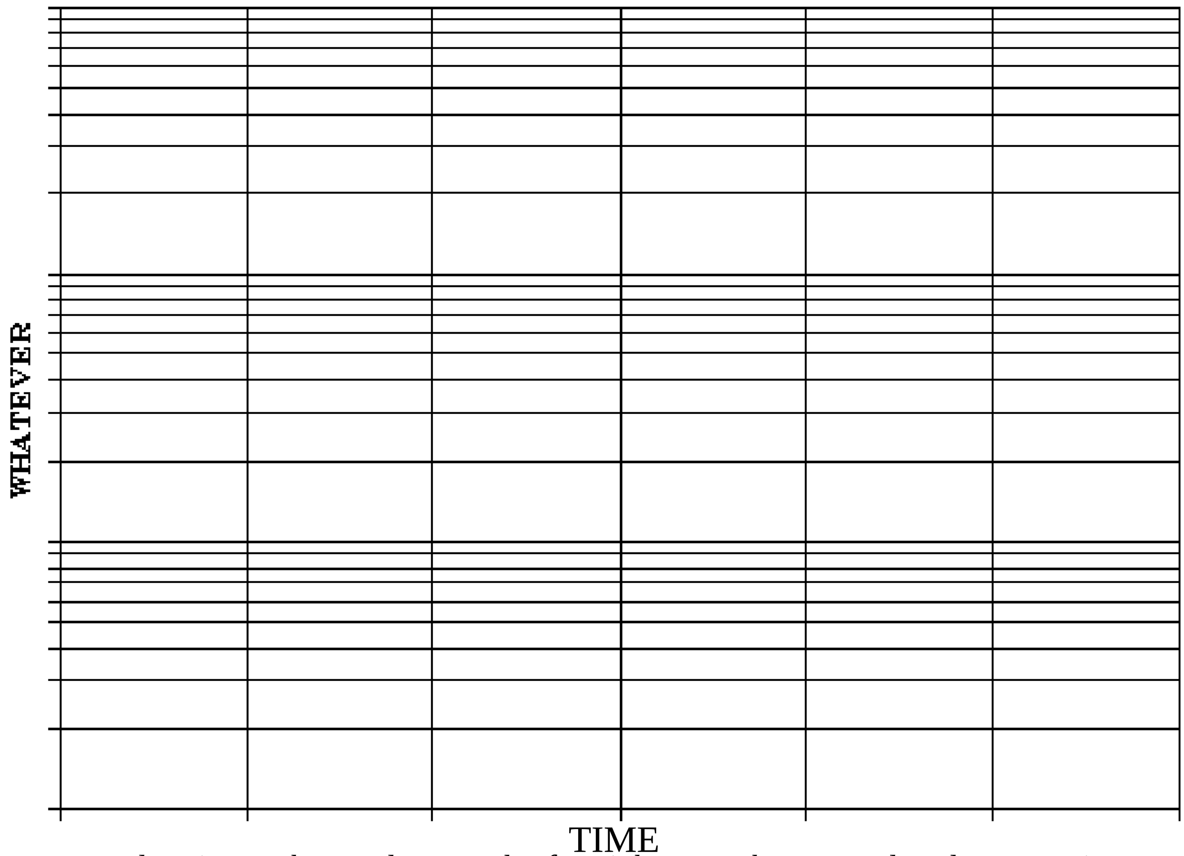
In each cycle, the line immediately above the base line represents **2 times** the base line, the line above that **3 times** the base line, etc. In a cycle all the lines correspond to multiples of the base line. So, for example, if the bottom line is labeled 10 the next line would be 20, then 30, 40, 50, until 100 ($= 10 \times 10$), which is the base line for next cycle; then continuing the second cycle, the values of the lines would go 200, 300, ... up to 1000 ($= 10 \times 100$) which is the base line for the third cycle, and the third cycle would continue 2000, 3000, ... 10000.

You can download a program to draw semilog and all sorts of graph paper at

http://www.graphtablet.com/

Or go to Google and type "Graph Tablet".

Traditionally we represent the *in*dependent variable along the horizontal (x-axis) and the *de*pendent variable along the vertical (y-axis). The independent variable is the one which is not affected by the other. This is not the place to get into a long (or even a short) discussion of cause and effect. Suffice it to say that *time* (waits for no man, heals all wounds....) is not affected by such things as concentration of drugs or populations of rabbits or most of the things that we are likely to run up against. So TIME (seconds, minutes, hours, days, weeks, fortnights, months, years, centuries) is usually the Independent Variable and is measured along the bottom of the graph.



seconds, minutes, hours, days, weeks, fortnights, months, years, decades, centuries etc.

0-order and 1^{st}-order

Now that you know what semilog graphs are you may be wondering what use they could be. If you are wondering (even if you're not) I'll tell you. Semilog graphs help us determine the "**order**" of the processes. Of course this isn't very useful unless you know what "order" means. Let's let $y(t)$ be the outcome of some process (chemical reaction, population growth, radioactive decay, osmosis .....) then we say $y(t)$ is n^{th}-order if $\frac{dy}{dt} = ky^n$.

We'll only need to worry about 0- and first-order, that is the two cases $n = 0$ and $n = 1$.

$$\boxed{y \text{ is 0-order means that } y = mt + b}$$

Presumably you already know that this is a linear equation whose graph (on ordinary graph paper) is a straight line.

> You should also know how to get m and b from the graph: b is the y-intercept and m is calculated from two points on the line (t_1, y_1) and (t_2, y_2) by the formula $m = \frac{y_1 - y_2}{t_1 - t_2}$.

$$\boxed{y \text{ is } 1^{st}\text{-order means } y = Ce^{kt}}$$

Taking the natural log of both sides of this equation gives us:

$$\ln y = \ln C + kt$$

which you will recognize as a straight line with slope k and intercept $\ln C$.
So in a *semilog* graph we plot, not y, but $\ln y$ as a function of t the graph is a straight line and you can find the constants C and k.

★C is gotten from the y-intercept, and

★$k = \dfrac{\ln y_1 - \ln y_2}{t_1 - t_2}$ where (t_1, y_1) and (t_2, y_2) are two points *on the line*

(it is important to take **<u>points on the line</u>** and not use the data itself to compute the slope.)

A 1^{st}-order y on a normal plot is concave up, and a 0-order y on a semilog plot is concave down. You should realize that not every concave up curve in ordinary coordinates is an exponential.

To tell if your data is 0- or 1^{st}-order graph it first on normal graph paper. If the data is on (or close to) a straight line it is 0-order, $y = mt + b$, and you can compute m and b. If the data is concave up it *may* be 1^{st}-order. Plot it on semilog paper to check.

On semilog paper 1^{st}-order data ($y = Ce^{kt}$) will fall on (or near) a straight line. If the data on a semilog plot is concave down it *may* be 0-order, plot it on ordinary graph paper to see.

Form of Graphs
of Different Type of Data
According to Type of Plot

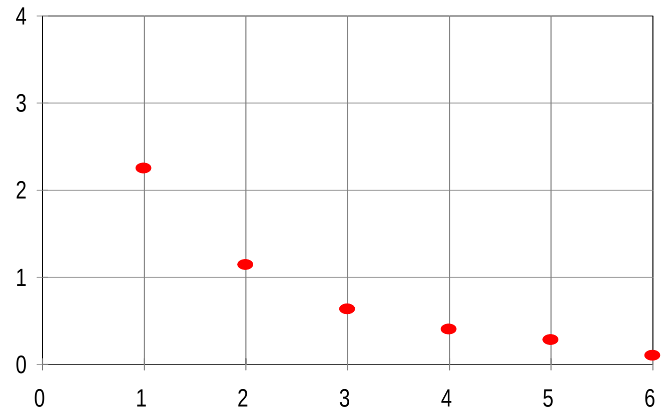
	↓Type of Data↓	
Plot ⇓	0-order	1^{st}order
ordinary	*straight line*	*concave up*
semilog	*concave down*	*straight line*

Example

a. Graph the given data. (label each axis clearly indicating the values)
b. Give the equation that best describes the data.

Blood concentration of a drug in the process of being metabolized.

Time (*hours*)	*Conc.* (*mg/cc*)
1	*2.25*
2	*1.143*
3	*0.63*
4	*0.396*
5	*0.279*
6	*0.09*



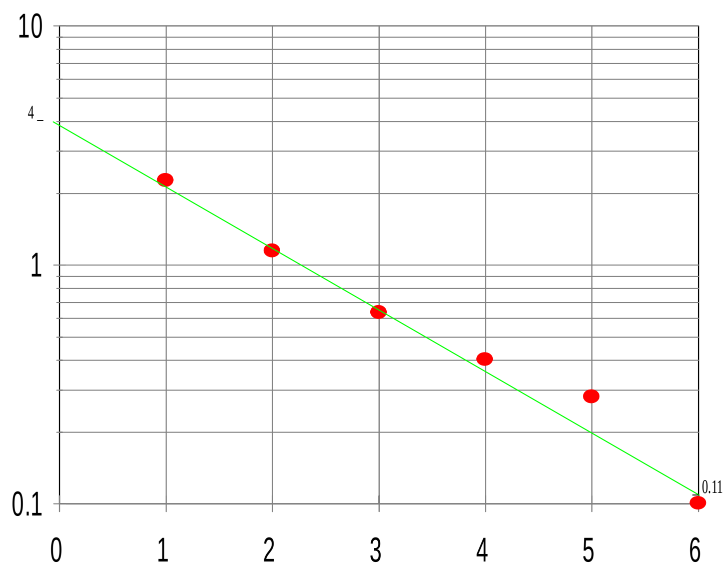
Data plotted on normal graph

From the normal graph above, the data is *not* 0-order. It is however concave up so we plot it on a semilog graph and note that the data seems to fall close to a straight line (at least it seems to me).
This indicates that it is 1^{st}-order ($y = Ce^{kt}$). From the y-intercept we see that $C = 4$ and from the line we calculate

$$k = \frac{\ln 4 - \ln 0.11}{0 - 6} = -0.6$$

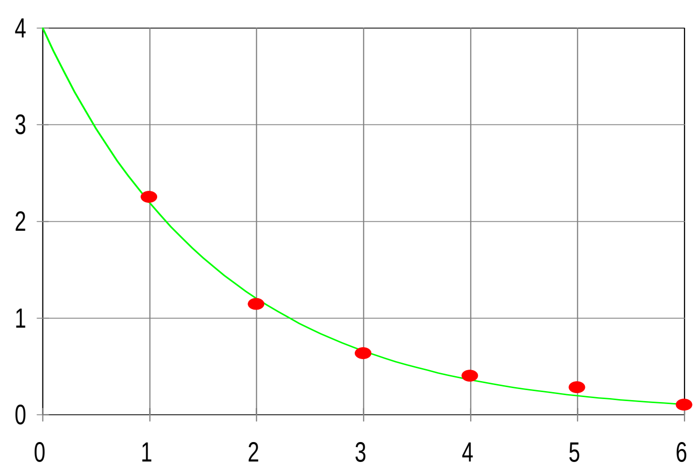
And so we get

$$\boxed{y = 4e^{-0.6t}}$$



Data plotted on semilog graph

Here's a plot of this exponential along with the data:



Data plotted along with its exponential

Homework

Graph the following sets of data on regular and on semilog graph paper. Determine the equation that best describes the data.

#1

Concentration of Drug During Absorption

Time hr	Conc x 100 M
1	7.4
2	6.05
3	5
4	4.08
5	3.34
6	2.75
7	2.25
8	1.85
10	1.25
24	0.0001

#2

Percentage of Product Relative to the Final Concentration

Time hr	Conc 100%-A%
1	96.6
5	84
8	75.7
12	65.9
17	55.3
22	46.5
30	35.2
39	25.7
54	15.3
86	5
120	2

#3

Amount of Drug Remaining as a Function of Time

time hr	theophylline mg
1	278
2	246
4	205
6	146
8	105
10	50
12	6