Algebraic Fractions (Addition and Subtraction).

Solving Equations with Algebraic Fractions. Higher KS4 (KS3)

A. Simplify (write them as a single fraction in its simplest):

1. $\frac{3}{5x}-\frac{1}{10x}$ 2. $\frac{8}{5x}-\frac{4}{15x}$ 3. $\frac{1}{4x}+\frac{1}{5x}$

4. $\frac{2}{x}+\frac{3}{2x}$ 5. $\frac{1}{x^2}+\frac{1}{x}$ 6. $\frac{1}{xy}+\frac{1}{x}$ 7. $\frac{y}{x}-\frac{x}{y}$

8. $\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}$ 9. $\frac{1}{x}+\frac{1}{(x+1)}$ 10. $\frac{1}{x}-\frac{1}{(x+1)}$

11. $\frac{1}{x}+\frac{1}{(x-1)}$ 12. $\frac{1}{(x+1)}+\frac{1}{(x+2)}$ 13. $\frac{1}{(x+1)}-\frac{1}{(x+2)}$

14. $\frac{3}{(x+3)}-\frac{2}{(x+2)}$ 15. $\frac{2}{(x+1)}+\frac{1}{(x+1)^2}$ 16. $\frac{1}{(x+1)}+\frac{1}{(x-1)}$

17. $\frac{1}{(x+1)}-\frac{1}{(x-1)}$ 18. $\frac{1}{(x-1)}-\frac{1}{(x+1)}$ 19. $\frac{1}{(x+1)}-\frac{1}{x^2-1}$

20. $\frac{1}{x-x^2}-\frac{1}{x+x^2}$ 21. $\frac{x+1}{x+2}+\frac{x-3}{3x+6}$ 22. $\frac{x+2}{2x-1}-\frac{x-2}{2x+1}$

23. $\frac{1}{x^2-4}-\frac{1}{x^2-x-6}$ 24. $\frac{x+2}{x+3}-\frac{x-1}{x}$ * 25. $\frac{y}{x-y}+\frac{x}{y-x}$

B. Solve the following equations:

1. $\frac{x}{3} = 5$ 2. $\frac{x}{3} - 2 = 3$ 3. $\frac{10}{x} = 4$ 4. $\frac{12}{x} + 1 = 7$

5. $\frac{2x-3}{7} = 5$ 6. $\frac{15}{2x-1} = 3$ 7. $\frac{2x-3}{7} - 1 = 4$

8. $\frac{1}{x-1} = \frac{2}{5x}$ 9. $\frac{2}{2x-5} = \frac{3}{x-3}$ 10. $\frac{1}{x} - \frac{10}{3x} = \frac{1}{9}$

11. $\frac{1}{6}(x+2) - \frac{1}{9}(2x-3) = \frac{1}{12}(1-3x)$

12. $\frac{3}{4}(2x-5) + \frac{1}{6}(x+7) = \frac{2}{3}(x-2)$ 13. $\frac{x}{4} = \frac{9}{x}$

14. $x + 2 = \frac{25}{x+2}$ 15. $\frac{x+5}{x-3} = x$ 16. $\frac{x+4}{x-2} = x$ 17. $\frac{2x+1}{3} = \frac{1}{x-2}$

18. $\frac{1}{x+1} - \frac{1}{x+4} = \frac{1}{6}$ 19. $\frac{3}{x+3} + \frac{2}{x-2} = \frac{-10}{x+7}$ 20. $\frac{1}{2x+3} + \frac{3}{x-1} = \frac{2}{x-3}$

21. $\frac{4}{3x+1} + \frac{1}{2x-1} = \frac{9}{5x}$ 22. $\frac{x+3}{x} - \frac{2x+1}{x+3} = \frac{5}{6}$ 23. $\frac{1}{2p+3} + \frac{3}{p-1} = \frac{2}{p-3}$

24. $\frac{3}{a-2} + \frac{5}{a-4} - \frac{4}{a-5} = 0$ 25. $\frac{2}{m+1} + \frac{2}{m+8} + \frac{3}{4m+2} = 0$

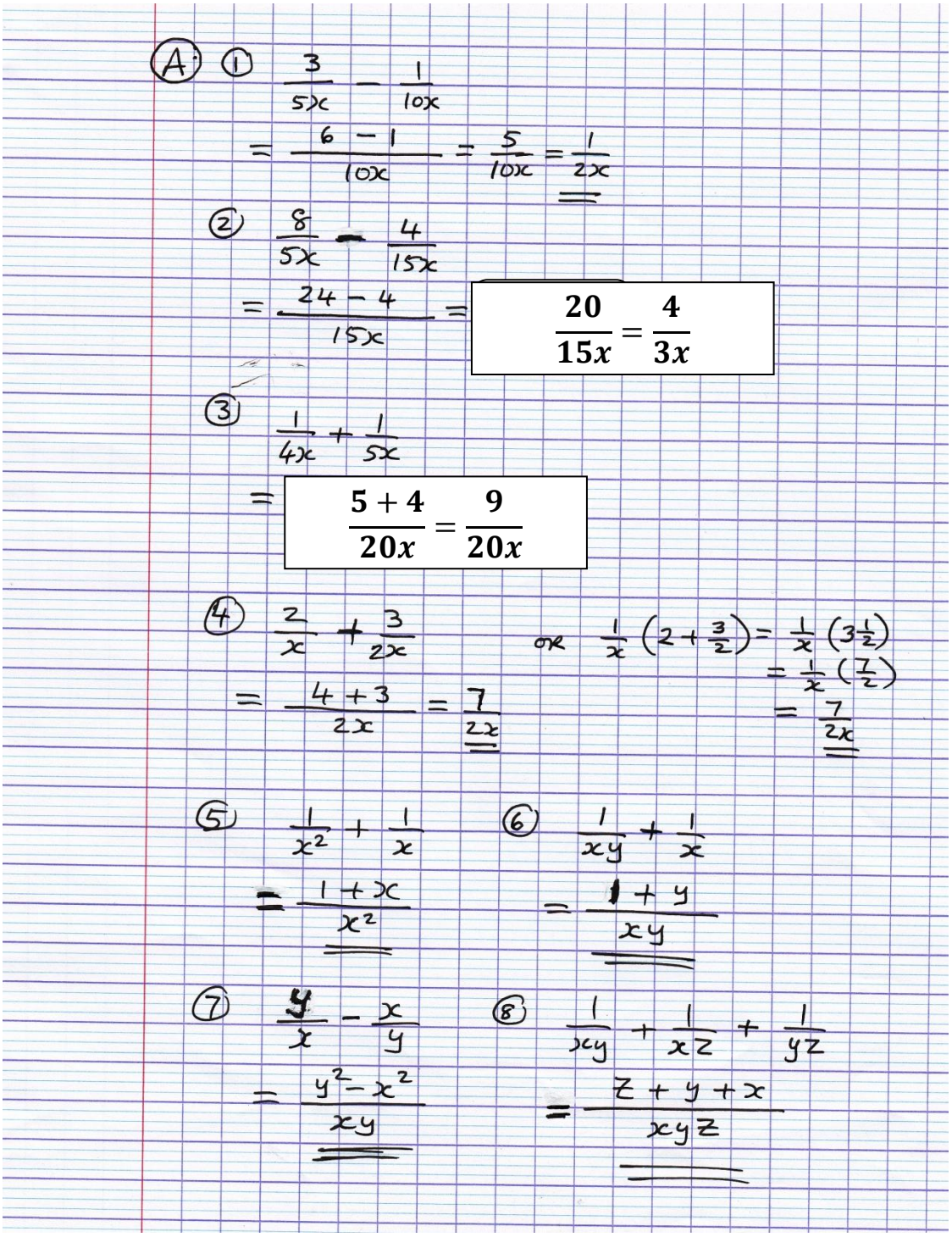
26. Find the exact solutions of: (a) $x - \frac{8}{x} = 4$ (b) $\frac{4}{x} - x = 8$

27. $\frac{3(4x+1)^2}{6x+5} = 8x - 3$ 28. $\frac{3(2x+1)^2}{3x-1} = 4x + 5$

29. $\frac{4(3x-2)^2}{6x-1} = 6x - 5$ 30. $\frac{9x}{x+7} \times 3\frac{1}{2} = x + 7$

Apologies for the spacing of the questions. I was trying to get them all on two pages.

Answers/Solutions (solutions not unique):



A ① $\frac{3}{5x} - \frac{1}{10x}$

$= \frac{6-1}{10x} = \frac{5}{10x} = \frac{1}{2x}$

② $\frac{8}{5x} - \frac{4}{15x}$

$= \frac{24-4}{15x} = \frac{20}{15x} = \frac{4}{3x}$

③ $\frac{1}{4x} + \frac{1}{5x}$

$= \frac{5+4}{20x} = \frac{9}{20x}$

④ $\frac{2}{x} + \frac{3}{2x}$

$= \frac{4+3}{2x} = \frac{7}{2x}$

or $\frac{1}{x}\left(2+\frac{3}{2}\right) = \frac{1}{x}\left(3\frac{1}{2}\right)$

$= \frac{1}{x}\left(\frac{7}{2}\right)$

$= \frac{7}{2x}$

⑤ $\frac{1}{x^2} + \frac{1}{x}$

$= \frac{1+x}{x^2}$

⑥ $\frac{1}{xy} + \frac{1}{x}$

$= \frac{1+y}{xy}$

⑦ $\frac{y}{x} - \frac{x}{y}$

$= \frac{y^2 - x^2}{xy}$

⑧ $\frac{1}{xy} + \frac{1}{xz} + \frac{1}{yz}$

$= \frac{z+y+x}{xyz}$

(9) $\frac{1}{x} + \frac{1}{x+1}$

$= \frac{x+1+x}{x(x+1)} = \frac{2x+1}{x(x+1)}$

(10) $\frac{1}{x} - \frac{1}{x+1}$

$= \frac{x+1-x}{x(x+1)} = \frac{1}{x(x+1)}$

(11) $\frac{1}{x} + \frac{1}{x-1}$

$= \frac{x-1+x}{x(x-1)} = \frac{2x-1}{x(x-1)}$

(12) $\frac{1}{x+1} + \frac{1}{x+2}$

$= \frac{x+2+x+1}{(x+1)(x+2)}$

$= \frac{2x+3}{(x+1)(x+2)}$

(13) $\frac{1}{x+1} - \frac{1}{x+2}$

$= \frac{x+2-(x+1)}{(x+1)(x+2)}$

$= \frac{x+2-x-1}{(x+1)(x+2)} = \frac{1}{(x+1)(x+2)}$

(14) $\frac{3}{(x+3)} - \frac{2}{(x+2)}$

$= \frac{3(x+2)-2(x+3)}{(x+3)(x+2)} = \frac{6x+6-2x-6}{(x+3)(x+2)} = \frac{4x}{(x+3)(x+2)}$

(15) $\frac{2}{x+1} + \frac{1}{(x+1)^2}$

$= \frac{2(x+1)+1}{(x+1)^2} = \frac{2x+2+1}{(x+1)^2} = \frac{2x+3}{(x+1)^2}$

(16) $\frac{1}{x+1} + \frac{1}{x-1}$

$= \frac{x-1+x+1}{(x+1)(x-1)} = \frac{2x}{(x+1)(x-1)}$

(17) $\frac{1}{x+1} - \frac{1}{x-1}$

$= \frac{x-1-(x+1)}{(x+1)(x-1)} = \frac{x-1-x-1}{(x+1)(x-1)} = \frac{-2}{(x+1)(x-1)}$ OR $\frac{2}{(x+1)(1-x)}$

(18) $\frac{1}{x-1} - \frac{1}{x+1}$

$= \frac{x+1-(x-1)}{(x-1)(x+1)} = \frac{x+1-x+1}{(x-1)(x+1)} = \frac{2}{(x-1)(x+1)}$

(19) $\frac{1}{x+1} - \frac{1}{x^2-1}$

$= \frac{1}{(x+1)} - \frac{1}{(x+1)(x-1)}$

$= \frac{x-1-1}{(x+1)(x-1)} = \frac{x-2}{(x+1)(x-1)}$

(20) $\frac{1}{x-x^2} - \frac{1}{x+x^2} = \frac{1}{x(1-x)} - \frac{1}{x(1+x)}$

$= \frac{1+x-(1-x)}{x(1+x)(1-x)} = \frac{1+x-1+x}{x(1+x)(1-x)} = \frac{2x}{x(1+x)(1-x)}$

$= \boxed{\frac{2}{(1+x)(1-x)}}$

(21) $\frac{x+1}{x+2} + \frac{x-3}{3x+6} = \frac{x+1}{(x+2)} + \frac{x-3}{3(x+2)}$

$= \frac{3(x+1)+x-3}{3(x+2)} = \frac{3x+3+x-3}{3(x+2)}$

$= \frac{4x}{3(x+2)}$

(22) $\frac{x+2}{2x-1} - \frac{x-2}{2x+1}$

$= \frac{(2x+1)(x+2) - (2x-1)(x-2)}{(2x-1)(2x+1)} = \frac{2x^2+5x+2-(2x^2-5x+2)}{(2x-1)(2x+1)}$

$= \frac{2x^2+5x+2-2x^2+5x-2}{(2x-1)(2x+1)} = \frac{10x}{(2x-1)(2x+1)}$

(23) $\frac{1}{x^2-4} - \frac{1}{x^2-x-6} = \frac{1}{(x+2)(x-2)} - \frac{1}{(x+2)(x-3)}$

$= \frac{x-3-(x-2)}{(x+2)(x-2)(x-3)} = \frac{x-3-x+2}{(x+2)(x-2)(x-3)}$

$= \frac{-1}{(x+2)(x-2)(x-3)}$ OR $\frac{1}{(x+2)(x-2)(3-x)}$

(24) $\frac{x+2}{x+3} - \frac{x-1}{x} = \frac{x(x+2)-(x+3)(x-1)}{x(x+3)}$

$= \frac{x^2+2x-(x^2+2x-3)}{x(x+3)} = \frac{x^2+2x-x^2-2x+3}{x(x+3)}$

$= \frac{3}{x(x+3)}$

*(25) This question is easy to do if you realise that $y-x = -(-y+x) = -(x-y)$

$\frac{y}{x-y} + \frac{x}{y-x} = \frac{y}{(x-y)} + \frac{x}{-(x-y)}$

$= \frac{y}{(x-y)} - \frac{x}{(x-y)} = \frac{y-x}{(x-y)} = \frac{-(x-y)}{(x-y)}$

$= -1$ (unusual but interesting!)

(B) (1) $\frac{x}{3} = 5 \Rightarrow x = 3 \times 5 = \underline{\underline{15}}$

(2) $\frac{x}{3} - 2 = 3 \Rightarrow \frac{x}{3} = 5 \Rightarrow \underline{\underline{x = 15}}$

(3) $\frac{10}{x} = 4$

$4x = 10$

$x = \frac{10}{4} \left(= 2{\cdot}5\right)$

(4) $\frac{12}{x} + 1 = 7$

$\frac{12}{x} = 6$

$6x = 12$

$x = \underline{\underline{2}}$

(5) $\frac{2x-3}{7} = 5$

$2x - 3 = 7 \times 5$

$2x - 3 = 35$

$2x = 38$

$x = \underline{\underline{19}}$

(6) $\frac{15}{2x-1} = 3$

$3(2x-1) = 15$ OR $2x - 1 = 5$

$6x - 3 = 15$ $\quad 2x = 6$

$6x = 18$ $\quad x = \underline{\underline{3}}$

$x = \underline{\underline{3}}$

(7) $\frac{2x-3}{7} - 1 = 4$

$\frac{2x-3}{7} = 5 \Rightarrow x = \underline{\underline{19}}$ (see Q5)

(8) $\frac{1}{x-1} = \frac{2}{5x}$

$5x = 2(x-1)$

$5x = 2x - 2$

$3x = -2$

$x = \underline{\underline{-\frac{2}{3}}}$

(9) $\frac{2}{2x-5} = \frac{3}{x-3}$

$3(2x-5) = 2(x-3)$

$6x - 15 = 2x - 6$

$6x - 2x = -6 + 15$

$4x = 9$

$x = \frac{9}{4}$ $(= 2\frac{1}{4})$

(10) $\frac{1}{x} - \frac{10}{3x} = \frac{1}{9}$

$\frac{3 - 10}{3x} = \frac{1}{9}$

$\frac{-7}{3x} = \frac{1}{9}$

$3x = -63$

$x = -21$

OR

$\frac{1}{x}\left(1 - \frac{10}{3}\right) = \frac{1}{9}$

$\frac{1}{x}\left(-\frac{7}{3}\right) = \frac{1}{9}$

$\frac{1}{x} = \frac{1}{9} \div \left(-\frac{7}{3}\right)$

$\frac{1}{x} = \frac{1}{9_3} \times -\frac{3^1}{7} = -\frac{1}{21}$

$x = -21$

(11) $\frac{1}{6}(x+2) - \frac{1}{9}(2x-3) = \frac{1}{12}(1-3x)$

$\times 36 \Rightarrow \quad 6(x+2) - 4(2x-3) = 3(1-3x)$

$6x + 12 - 8x + 12 = 3 - 9x$

$6x - 8x + 9x = 3 - 12 - 12$

$7x = -21$

$x = -3$

(12) $\frac{3}{4}(2x-5) + \frac{1}{6}(x+7) = \frac{2}{3}(x-2)$

$\times 12 \Rightarrow \quad 9(2x-5) + 2(x+7) = 8(x-2)$

$18x - 45 + 2x + 14 = 8x - 16$

$18x + 2x - 8x = -16 + 45 - 14$

$12x = 15$

$x = \frac{15}{12}$ $(= \frac{5}{4} = 1\frac{1}{4})$

(13) $\frac{x}{4} = \frac{9}{x}$

$x^2 = 36$

$x = \pm 6$

(14) $(x+2) = \frac{25}{x+2}$

$(x+2)^2 = 25$

$x+2 = \pm 5$

$x = 3$ OR $x = -7$

(15) $\frac{x+5}{x-3} = x$

$x(x-3) = x+5$

$x^2 - 3x = x+5$

$x^2 - 4x + 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ OR $x = -1$

(16) $\frac{x+4}{x-2} = x$

$x(x-2) = x+4$

$x^2 - 2x = x+4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ OR $x = -1$

(17) $\frac{2x+1}{3} = \frac{1}{x-2}$

$(2x+1)(x-2) = 3$

$2x^2 - 3x - 2 = 3$

$2x^2 - 3x - 5 = 0$

$(2x-5)(x+1) = 0$

$x = \frac{5}{2}$ OR $x = -1$

(18) $\frac{1}{x+1} - \frac{1}{x+4} = \frac{1}{6}$

$\frac{x+4-(x+1)}{(x+1)(x+4)} = \frac{1}{6} \Rightarrow \frac{x+4-x-1}{(x+1)(x+4)} = \frac{1}{6}$

$\frac{3}{(x+1)(x+4)} = \frac{1}{6} \Rightarrow (x+1)(x+4) = 18$

$x^2 + 5x + 4 = 18$

$x^2 + 5x - 14 = 0$

$(x+7)(x-2) = 0$

$x = -7$ OR $x = 2$

(19) $\frac{3}{x+3} + \frac{2}{x-2} = \frac{-10}{x+7}$

$\frac{3(x-2)+2(x+3)}{(x+3)(x-2)} = \frac{-10}{x+7}$

$\frac{3x-6+2x+6}{(x+3)(x-2)} = \frac{-10}{x+7}$

$\frac{5x}{(x+3)(x-2)} = \frac{-10}{x+7}$

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(19) Continued:

$5x(x+7) = -10(x+3)(x-2)$

$5x^2+35x = -10(x^2+x-6)$

$5x^2+35x = -10x^2-10x+60$

$15x^2+45x-60=0$

$\div 15 \Rightarrow \quad x^2+3x-4=0 \Rightarrow (x+4)(x-1)=0$

$x=-4$ OR $x=1$

(20) $\frac{1}{(2x+3)}+\frac{3}{(x-1)}=\frac{2}{x-3}$

$\frac{x-1+3(2x+3)}{(2x+3)(x-1)}=\frac{2}{x-3} \Rightarrow \frac{x-1+6x+9}{(2x+3)(x-1)}=\frac{2}{x-3}$

$\frac{7x+8}{(2x+3)(x-1)}=\frac{2}{x-3} \Rightarrow (7x+8)(x-3)=2(2x+3)(x-1)$

$7x^2-13x-24=2(2x^2+x-3)$

$7x^2-13x-24=4x^2+2x-6$

$3x^2-15x-18=0$

$\div 3 \Rightarrow \quad x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6$ OR $x=-1$

(21) $\frac{4}{3x+1}+\frac{1}{2x-1}=\frac{9}{5x}$

$\frac{4(2x-1)+(3x+1)}{(3x+1)(2x-1)}=\frac{9}{5x}$

$\frac{8x-4+3x+1}{(3x+1)(2x-1)}=\frac{9}{5x} \Rightarrow \frac{11x-3}{(3x+1)(2x-1)}=\frac{9}{5x}$

$5x(11x-3)=9(3x+1)(2x-1)$

$55x^2-15x=9(6x^2-x-1)$

$55x^2-15x=54x^2-9x-9$

$x^2-6x+9=0 \Rightarrow (x-3)(x-3)=0$

$x=3$

(22) $\frac{x+3}{x} - \frac{2x+1}{x+3} = \frac{5}{6}$

$\frac{(x+3)(x+3) - x(2x+1)}{x(x+3)} = \frac{5}{6}$

$\frac{x^2+6x+9-2x^2-x}{x(x+3)} = \frac{5}{6} \Rightarrow \frac{-x^2+5x+9}{x(x+3)} = \frac{5}{6}$

$6(x^2+5x+9) = 5x(x+3)$

$6x^2+30x+54 = 5x^2+15x$

$x^2+15x+54 = 0$

$(x+6)(x+9) = 0$

$x = -6$ or $x = -9$

(23) $\frac{1}{2p+3} + \frac{3}{p-1} = \frac{2}{p-3}$

$\frac{p-1+3(2p+3)}{(2p+3)(p-1)} = \frac{2}{p-3}$

$\frac{p-1+6p+9}{(2p+3)(p-1)} = \frac{2}{p-3}$

$\frac{7p+8}{(2p+3)(p-1)} = \frac{2}{p-3}$

$(7p+8)(p-3) = 2(2p+3)(p-1)$

$7p^2-13p+24 = 2(2p^2+p-3)$

$= 4p^2+2p-6$

$3p^2-15p+30 = 0$

$\div 3 \Rightarrow p^2-5p+6 = 0$

$(p-3)(p-2) = 0$

$p = 3$ or $p = 2$

(24) $\frac{3}{a-2}+\frac{5}{a-4}-\frac{4}{a-5}=0 \Rightarrow \frac{3}{a-2}+\frac{5}{a-4}=\frac{4}{a-5}$

$$\frac{3(a-4)+5(a-2)}{(a-2)(a-4)}=\frac{4}{a-5}$$

$$\frac{3a-12+5a-10}{(a-2)(a-4)}=\frac{4}{a-5} \Rightarrow \frac{8a-22}{(a-2)(a-4)}=\frac{4}{a-5}$$

$$(a-5)(8a-22)=4(a-2)(a-4)$$
$$8a^2-62a+110=4(a^2-6a+8)$$
$$8a^2-62a+110=4a^2-24a+32$$
$$4a^2-38a+78=0$$

$\div 2 \Rightarrow$ $2a^2-19a+39=0$

$$(2a-13)(a-3)=0$$
$$a=\frac{13}{2} \text{ OR } a=3$$

(25) rearrange to: $\frac{2}{m+1}+\frac{2}{m+8}=\frac{-3}{4m+2}$

$$\Rightarrow \frac{2(m+8)+2(m+1)}{(m+1)(m+8)}=\frac{-3}{4m+2}$$

$$\frac{2m+16+2m+2}{(m+1)(m+8)}=\frac{-3}{4m+2}$$

$$\frac{4m+18}{(m+1)(m+8)}=\frac{-3}{4m+2}$$

$$(4m+2)(4m+18)=-3(m+1)(m+8)$$
$$16m^2+80m+36=-3(m^2+9m+8)$$
$$=-3m^2-27m-24$$

$$\Rightarrow 19m^2+107m+60=0$$
$$(19m+12)(m+5)=0$$
$$m=-\frac{12}{19} \text{ OR } m=-5$$

(26) (a) $x - \frac{8}{x} = 4$

$\times x \Rightarrow x^2 - 8 = 4x$

$x^2 - 4x - 8 = 0$

Complete the square or used the formula

$(x-2)^2 - 4 - 8 = 0$

$(x-2)^2 = 12$

$x - 2 = \pm\sqrt{12}$

$x = 2 \pm \sqrt{12}$

$x = 2 \pm 2\sqrt{3}$

OR

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{16 - 4(1)(-8)}}{2}$

$x = \frac{4 \pm \sqrt{16 + 32}}{2}$

$x = \frac{4 \pm \sqrt{48}}{2}$

$x = \frac{4 \pm 4\sqrt{3}}{2}$

$x = 2 \pm 2\sqrt{3}$

(b) $\frac{4}{x} - x = 8$

$4 - x^2 = 8x$

$0 = x^2 + 8x - 4$

$(x+4)^2 - 16 - 4 = 0$

$(x+4)^2 = 20$

$x + 4 = \pm\sqrt{20}$

$x = -4 \pm \sqrt{20}$

$x = -4 \pm 2\sqrt{5}$

Try using the formula and see if you get the same answers.

(27) $\frac{3(4x+1)^2}{6x+5} = 8x - 3$

$3(4x+1)^2 = (6x+5)(8x-3)$

$3(16x^2 + 8x + 1) = 48x^2 + 22x - 15$

$48x^2 + 24x + 3 = 48x^2 + 22x - 15$

$2x = -18$

$x = -9$

(28) $\frac{3(2x+1)^2}{3x-1} = 4x+5$

$3(2x+1)^2 = (3x-1)(4x+5)$

$3(4x^2+4x+1) = 12x^2-11x-5$

$12x^2+12x+3 = 12x^2-11x-5$

$x = -8$

(29) $\frac{4(3x-1)^2}{6x+1} = 6x-5$

$4(3x-1)^2 = (6x+1)(6x-5)$

$4(9x^2-6x+1) = 36x^2-36x+5$

$36x^2-24x+4 = 36x^2-36x+5$

$12x = -1$

$x = -\frac{1}{12}$

(30) $\frac{9x}{x+7} \times \frac{7}{2} = x+7$

$\frac{63x}{2(x+7)} = x+7$

$2(x+7)(x+7) = 63x$

$2(x^2+14x+49) = 63x$

$2x^2+28x+98 = 63x$

$2x^2-35x+98 = 0$

$(2x-7)(x-14) = 0$

$x = \frac{7}{2}$ or $x = 14$

I hope you find this useful and challenging for the most able students.

Please check answers and let me know if you find any errors so that I may make the necessary changes. Thank you in advance.