



Topic H

Multiplication of Two-Digit by Two-Digit Numbers

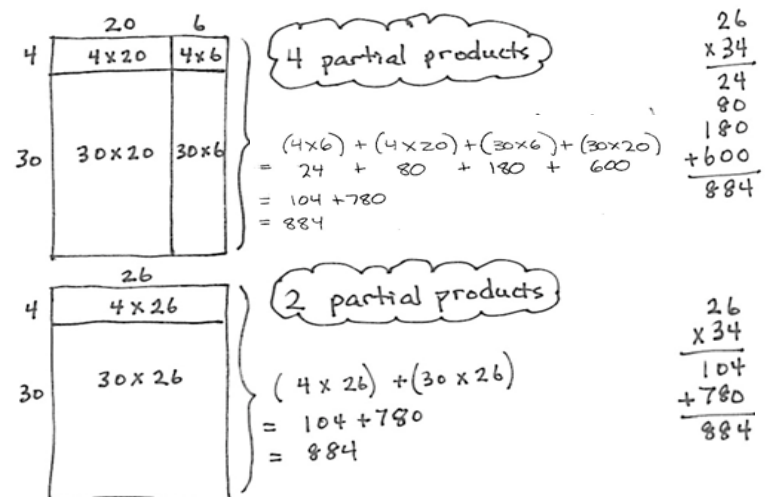
4.NBT.5, 4.OA.3, 4.MD.3

Focus Standard:	4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Instructional Days:	5	
Coherence -Links from:	G3–M1	Properties of Multiplication and Division and Problem Solving with Units of 2–5 and 10
	G3–M3	Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10
-Links to:	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations

The module closes with Topic H as students multiply two-digit by two-digit numbers.

Lesson 34 begins this topic by having students use the area model to represent and solve the multiplication of two-digit multiples of 10 by two-digit numbers using a place value chart. Practice with this model helps to prepare students for two-digit by two-digit multiplication and builds the understanding of multiplying units of 10. In Lesson 35, students extend their learning to represent and solve the same types of problems using area models and partial products.

In Lesson 36, students make connections to the distributive property and use both the area model and four partial products to solve problems. Lesson 37 deepens students' understanding of multi-digit multiplication by transitioning from four partial products with representation of the area model to two partial products with representation of the area model and then to two partial products without representation of the area model.



Topic H culminates at the most abstract level with Lesson 38 as students are introduced to the multiplication algorithm for two-digit by two-digit numbers. Knowledge from Lessons 34–37 provides a firm foundation for understanding the process of the algorithm as students make connections from the area model to partial products to the standard algorithm (**4.NBT.5**). Students see that partial products written vertically are the same as those obtained via the distributive property: $4 \text{ twenty-sixes} + 30 \text{ twenty-sixes} = 104 + 780 = 884$.

A Teaching Sequence Towards Mastery of Multiplication of Two-Digit by Two-Digit Numbers

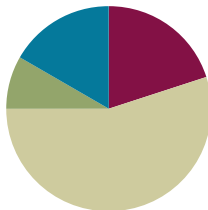
- Objective 1: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.**
(Lesson 34)
- Objective 2: Multiply two-digit multiples of 10 by two-digit numbers using the area model.**
(Lesson 35)
- Objective 3: Multiply two-digit by two-digit numbers using four partial products.**
(Lesson 36)
- Objective 4: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.**
(Lessons 37–38)

Lesson 34

Objective: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Draw a Unit Fraction **3.G.2** (4 minutes)
- Divide Three Different Ways **4.NBT.6** (4 minutes)
- Multiply Units **4.NBT.1** (4 minutes)

Draw a Unit Fraction (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews Grade 3 geometry and fraction concepts in anticipation of G4–Modules 4 and 5. Accept reasonable drawings. Using rulers and protractors is not necessary to review the concept and will take too long.

T: Draw a quadrilateral with 4 equal sides and 4 right angles.

S: (Draw.)

T: What's the name of a quadrilateral with 4 equal sides and 4 right angles?

S: Square.

T: Partition the square into 3 equal parts.

S: (Draw.)

T: Shade in 1 part of 3.

S: (Draw.)

T: Write the fraction of the square that's shaded.

S: Write $\frac{1}{3}$.

Repeat process partitioning a rhombus into fourths, a rectangle into fifths, and a rectangle into eighths.

Divide Three Different Ways (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews content from G4–M3–Lessons 31–33.

T: (Write $732 \div 6$.) Solve this problem by drawing number disks.

S: (Solve.)

T: Solve $732 \div 6$ using the area model.

S: (Solve.)

T: Solve $732 \div 6$ using the standard algorithm.

S: (Solve.)

Continue with this possible suggestion: $970 \div 8$.

Multiply Units (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M3–Lesson 4.

T: (Write $4 \text{ ones} \times 3$.) Say the multiplication sentence in unit form.

S: $4 \text{ ones} \times 3 = 12 \text{ ones}$.

T: Write the equation in standard form.

S: (Write $4 \times 3 = 12$.)

T: (Write $4 \text{ tens} \times 3$.) Write the equation in standard form.

S: $40 \times 3 = 120$.

T: (Write $4 \text{ tens} \times 3 \text{ tens}$.) Write the equation in standard form.

S: $40 \times 30 = 1200$.

T: (Write 3×2 .) Say the multiplication sentence.

S: $3 \times 2 = 6$.

T: Write the equation in unit form.

S: (Write $3 \text{ ones} \times 2 = 6 \text{ ones}$.)

T: (Write 30×2 .) Write the equation in unit form.

S: (Write $3 \text{ tens} \times 2 = 6 \text{ tens}$.)

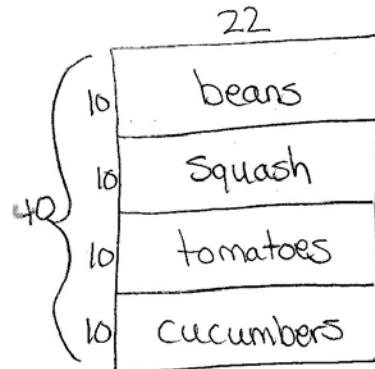
T: (Write 30×20 .) Write the equation in unit form.

S: (Write $3 \text{ tens} \times 2 \text{ tens} = 6 \text{ hundreds}$.)

Continue with the following possible sequence: 30×5 , 30×50 , $3 \text{ tens} \times 6$, $3 \text{ tens} \times 6 \text{ tens}$, 50×4 , $5 \text{ tens} \times 8$ tens, and 60×50 .

Application Problem (5 minutes)

Mr. Goggins planted 10 rows of beans, 10 rows of squash, 10 rows of tomatoes, and 10 rows of cucumbers in his garden. He put 22 plants in each row. Draw an area model, label each part, and then write an expression that represents the total number of plants in the garden.



$$(4 \times 10) \times 22$$

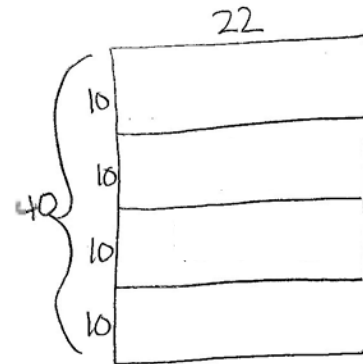
Note: This Application Problem builds on Topic B where students learned to multiply by multiples of 10 and Topic C where students learned to multiply two-digit by one-digit numbers using an area model. The Application Problem helps bridge to today's lesson as students learn to multiply multiples of 10 by two-digit numbers.

Concept Development (33 minutes)

Materials: (S) Personal white board

Problem 1: Discover that $4 \times 10 \times 22$ and 40×22 represent the same amount.

- T: Look at the area model that we drew for the Application Problem. (Erase the names of the plants.)
- T: How many 10×22 rectangles are there in the model?
- S: 4.
- T: Write an expression to show the entire length of the rectangle. (Indicate the vertical side length).
- S: 4×10 .
- T: What is the area formula?
- S: Length times width.
- T: Discuss with your partner an expression to show the total area of the largest rectangle.
- S: 40 times 22. $\rightarrow (4 \times 10) \times 22$.
- T: What can we say about the expressions $(4 \times 10) \times 22$ and 40×22 ?
- S: They represent the same amount.



- T: How can you tell? Turn and talk to your partner.
- S: The total length is 4 tens. → We can write 4 tens as 4 times 10 or 40. → You can either find the area of the smallest rectangle and multiply it by four or just multiply 40 times 22.
- T: We can say that $40 \times 22 = 4 \times 10 \times 22$ or $10 \times 4 \times 22$.
- S: Yes.
- T: That's good because we can solve 4 times 10 times 22 but 40 times 22 is new territory.

Problem 2: Multiply 40×22 using a place value chart.



Write:

$$40 \times 22 = (4 \times 10) \times 22$$

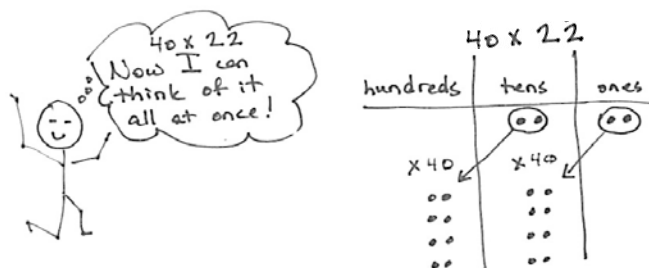
$$40 \times 22 = 4 \times (10 \times 22)$$

$$40 \times 22 = 10 \times (4 \times 22)$$

- T: Show 22 on your place value chart.
- T: Show 10 times as many. 10×22 is?
- S: (Draw disks to show 22 and then draw arrows to show 10 times that amount.)
- T: How many hundreds? How many tens?
- S: 2 hundreds and 2 tens.
- T: Show 4 times as many.
- S: (Draw disks to show 3 more groups of 2 hundreds 2 tens.)
- T: Tell how many you have now.
- S: 8 hundreds and 8 tens.
- T: What number does that represent? Say the number sentence.
- S: $4 \times (10 \times 22) = 880$. → $40 \times 22 = 880$.

Repeat the process, this time beginning by multiplying 22 by 4 as in the model above and to the right. (Write $40 \times 22 = (10 \times 4) \times 22 = 10 \times (4 \times 22)$.)

Next have students see that they can conceive of the problem as 40 times 22 as pictured below right, without breaking the process into the two steps of multiplying by 4 and 10 in whatever order.



Problem 3: Multiply 50×31 using a place value chart.

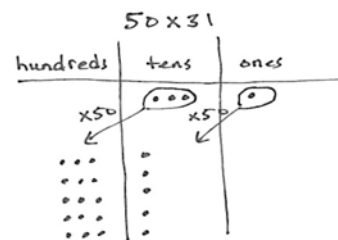
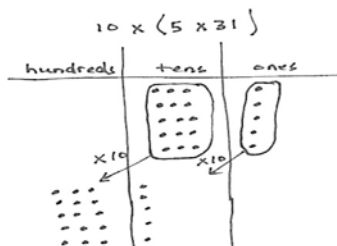
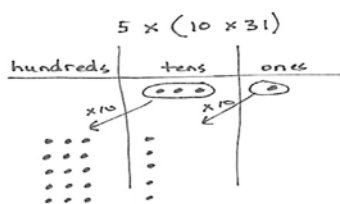
- T: What is another way to express 50×31 ?
- S: $10 \times (5 \times 31)$. $\rightarrow 5 \times (10 \times 31)$.
- T: Tell your partner why you chose to represent 50 using the numbers 5 and 10.
- S: $50 = 5 \times 10$. I know how to multiply by 10 and by 5, but I don't know how to multiply by 50.
- T: Yes, you are using the **associative property**, which allows us to use the factors of a number to help us multiply. What factors did we use for 50?
- S: 5 and 10.
- T: Show 31 on your place value chart.
- T: Show 10 times as many.
- S: (Draw disks to show 31 and then draw arrows to show 10 times that amount.)
- T: How many hundreds? How many tens?
- S: 3 hundreds and 1 ten.
- T: Show 5 times as many.
- S: (Draw disks to show 4 more groups of 3 hundreds 1 ten.)
- T: Tell how many you have now.
- S: 15 hundreds and 5 tens. I change 10 hundreds for one thousand. I have 1 thousand, 5 hundreds, and 5 tens.
- T: What number does that represent?
- S: 1,550.



**NOTES ON
MULTIPLE MEANS OF
REPRESENTATION:**

You may present the following scaffolds to assist students to organize and keep track of arrays of number disks:

- Students may alternate colors as they draw 5 rows of 3 hundreds.
- Give extra time to model.
- Couple disks with words, e.g., 3 hundreds \times 5.
- Use manipulatives at first, then transition to drawings.



Problem 4: Multiply 50×31 without using a place value chart.

- T: 50×31 is the same $50 \times (30 + 1)$. Discuss with your partner why this is true.
- S: We drew 31 as 3 tens and 1 one on the place value chart and multiplied each unit by 50. \rightarrow 50 groups of 31 is the same as 50 groups of 30 and 50 groups of 1. \rightarrow It's the break apart and distribute property! \rightarrow It's the distributive property!
- T: $50 \times (30 + 1) = 50 \times 30 + 50 \times 1$. Can you see that on our place value chart? (Show that on the place value chart.) At first we broke apart 50 into 10×5 or 5×10 but in the end we just thought of it as 50×31 .
- T: Let's say $50 \times 30 + 50 \times 1$ in unit language.
- S: 5 tens \times 3 tens + 5 tens \times 1 one.
- T: 5 tens times 3 tens is?
- S: 15 hundreds.
- T: 5 tens times 1 one is?
- S: 5 tens.
- T: 15 hundreds + 5 tens is?
- S: 1,550.
- T: 50×31 is?
- S: 1,550.
- T: Did we get the same product just by multiplying that we did when we used the chart?
- S: Yes.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students working above grade level to do mental computations in their head. For example, students can use the distributive property to think of 6×560 as $(6 \times 500) + (6 \times 60)$ and then solve using what they know about multiplying multiples of 10.

Students who are not ready for the mental computations may continue to draw the area model used in previous lessons to confirm their final product.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 34 Problem Set 4•3

Name Jack Date _____

1. Use the associative property to rewrite each expression. Solve using disks and then complete the number sentences.

a. 30×24

$= (3 \times 10) \times 24$
 $= 3 \times (10 \times 24)$
 $= 720$

hundreds	tens	ones
7	2	0

b. 40×43

$= (4 \times 10) \times 43$
 $= 4 \times (10 \times 43)$
 $= 1,720$

thousands	hundreds	tens	ones
1	7	2	0

c. 30×37

$= (3 \times 10) \times 37$
 $= 3 \times (10 \times 37)$
 $= 1,110$

thousands	hundreds	tens	ones
1	1	1	0

COMMON CORE Lesson 34: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart. Date: 8/28/13

engage^{ny} 3.H.10

© 2013 Common Core, Inc. Some rights reserved. commoncore.org

Student Debrief (10 minutes)

Lesson Objective: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- In Problem 1(a), is it best to model 30 or 24 on the chart initially? Why?
- Tell your partner how you used the **associative property** in Problem 3(a). Is there an order you find easier for multiplying the three factors like when multiplying using the place value chart?
- Why was it helpful to break the multiple of 10 into two factors before solving?
- How did distributing the second factor in Problem 4 of the Concept Development make it easier to solve?
- Compare Problems 3(a) and 4(a). Why did you get the same answer by using two different methods? What does this tell you about the associative and distributive property? Compare their processes. How are they different?
- How did representing the multiplication with disks help you solve and understand the multiplication?
- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 34 Problem Set 4•3

2. Use the associative property and number disks to solve.

a. 20×27
 $= (2 \times 10) \times 27$
 $= 2 \times (10 \times 27)$
 $= 540$

b. 40×31
 $= (4 \times 10) \times 31$
 $= 4 \times (10 \times 31)$
 $= 1,240$

3. Use the associative property without number disks to solve.

a. 40×34
 $= (4 \times 10) \times 34$
 $= 4 \times (10 \times 34)$
 $= 1,360$

b. 50×43
 $= (5 \times 10) \times 43$
 $= 5 \times (10 \times 43)$
 $= 2,150$

4. Use the distributive property to solve the following problems. Distribute the second factor.

a. 40×34
 $= (40 \times 30) + (40 \times 4)$
 $= 1,200 + 160$
 $= 1,360$

b. 60×25
 $= (60 \times 20) + (60 \times 5)$
 $= 1,200 + 300$
 $= 1,500$

COMMON CORE Lesson 34: Multiply two-digit multiples of 10 by two-digit numbers using a place value chart. 8/28/13

engage^{ny} 3.H.11

© 2013 Common Core, Inc. Some rights reserved. commoncore.org

Name _____

Date _____

1. Use the associative property to rewrite each expression. Solve using disks and then complete the number sentences.

a. 30×24

$$= (\quad \times 10) \times 24$$

$$= \quad \times (10 \times 24)$$

$$= \quad$$

hundreds	tens	ones

b. 40×43

$$= (4 \times 10) \times \quad$$

$$= 4 \times (10 \times \quad)$$

$$= \quad$$

thousands	hundreds	tens	ones

c. 30×37

$$= (3 \times \quad) \times \quad$$

$$= 3 \times (10 \times \quad)$$

$$= \quad$$

thousands	hundreds	tens	ones

2. Use the associative property and number disks to solve.

a. 20×27

b. 40×31

3. Use the associative property without number disks to solve.

a. 40×34

b. 50×43

4. Use the distributive property to solve the following problems. Distribute the second factor.

a. 40×34

b. 60×25

Name _____

Date _____

1. Use the associative property to rewrite each expression. Solve using disks and then complete the number sentences.

a. $20 \times 41 =$

_____ \times _____ \times _____ $=$

hundreds	tens	ones

2. Distribute 32 as $30 + 2$ and solve.

60×32

Name _____

Date _____

1. Use the associative property to rewrite each expression. Solve using disks and then complete the number sentences.

a. 20×34

$$= (\underline{\quad} \times 10) \times 34$$

$$= \underline{\quad} \times (10 \times 34)$$

$$= \underline{\quad}$$

hundreds	tens	ones

b. 30×34

$$= (3 \times 10) \times \underline{\quad}$$

$$= 3 \times (10 \times \underline{\quad})$$

$$= \underline{\quad}$$

thousands	hundreds	tens	ones

c. 30×42

$$= (3 \times \underline{\quad}) \times \underline{\quad}$$

$$= 3 \times (10 \times \underline{\quad})$$

$$= \underline{\quad}$$

thousands	hundreds	tens	ones

2. Use the associative property and number disks to solve.

a. 20×16

b. 40×32

3. Use the associative property without number disks to solve.

a. 30×21

b. 60×42

4. Use the distributive property to solve the following. Distribute the second factor.

a. 40×43

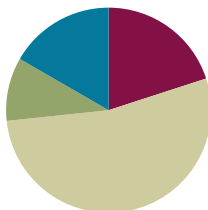
b. 70×23

Lesson 35

Objective: Multiply two-digit multiples of 10 by two-digit numbers using the area model.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(32 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Draw and Label Unit Fractions **3.G.2** (4 minutes)
- Divide Three Different Ways **4.NBT.6** (4 minutes)
- Multiply by Multiples of 10 **4.NBT.1** (4 minutes)

Draw and Label Unit Fractions (4 minutes)

Materials: (S) Personal white boards

Notes: This fluency reviews Grade 3 geometry and fraction concepts in anticipation of G4–Modules 4 and 5. Accept reasonable drawings. Using rulers and protractors is not necessary to review the concept and will take too long.

T: On your boards, write down the name for any four-sided figure.

S: (Write *quadrilateral*.)

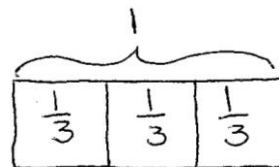
T: Draw a quadrilateral that has 4 right angles but not 4 equal sides.

S: (Draw a rectangle that is not a square.)

T: Partition the rectangle into 3 equal parts.

S: (Do so.)

T: Label the whole rectangle as 1. Write the unit fraction into each part.



Continue partitioning and labeling with the following possible sequence: a square as 4 fourths, a rhombus as 2 halves, a square as 5 fifths, and a rectangle as 6 sixths.

Divide Three Different Ways (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews content from G4–M3–Lessons 31–33.

T: (Write $348 \div 6$.) Find the quotient using number disks.

S: (Do so.)

T: Find the quotient using the area model.

S: (Do so.)

T: Find the quotient using the standard algorithm.

S: (Do so.)

Continue for $2,816 \div 8$.

Multiply by Multiples of 10 (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M3–Lesson 34's content.

T: (Write $40 \times 22 = 22 \times 10 \times \underline{\hspace{1cm}}$.) On your boards, fill in the missing factor to create a true number sentence.

S: (Write $40 \times 22 = 22 \times 10 \times 4$.)

T: What's 22×10 ?

S: $22 \times 10 = 220$.

T: (Write $220 \times 4 = \underline{\hspace{1cm}}$.) On your boards, write the answer.

S: (Write $220 \times 4 = 880$.)

$$40 \times 22 = 22 \times 10 \times 4$$

$$40 \times 22 = 220 \times 4$$

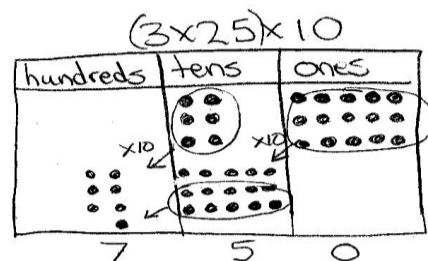
$$40 \times 22 = 880$$

Continue with the following possible sequence: 30×21 , 30×43 , and 50×39 .

Application Problem (6 minutes)

For 30 days out of one month, Katie exercised for 25 minutes a day. What is the total number of minutes that Katie exercised? Solve using a place value chart.

Note: This Application Problem builds on the content of Lesson 34 by using a place value chart to represent and then multiply a multiple of 10 by a two-digit number. Although some students may easily solve this problem using mental math, encourage them to see that the model verifies their mental math skills. Students can use their mental math and place value chart solution to verify their answer in Problem 1 of the Concept Development.



Katie exercised for 750 minutes.

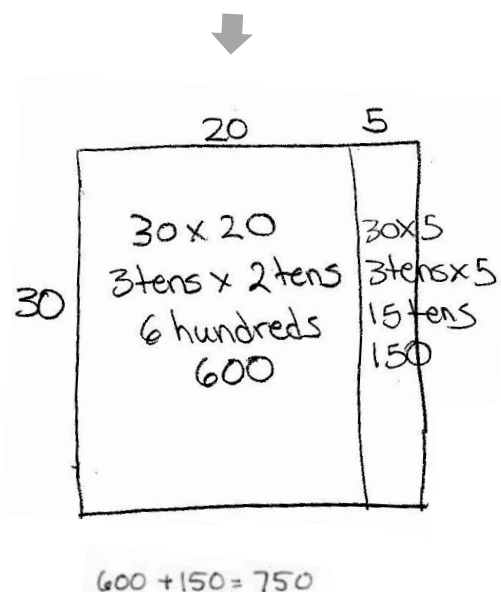
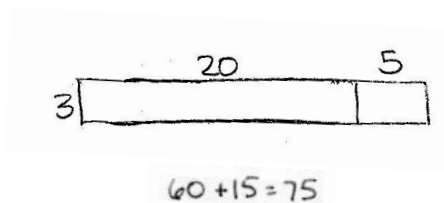
Concept Development (32 minutes)

Materials: (S) Personal white boards

Problem 1

Find the product of 30 and 25 using an area model to solve.

- T: Aside from the place value chart, what is another way that we have represented multiplication?
- S: Arrays. → Equal groups. → The area model.
- T: We will use an area model to show 30×25 . Since $30 \times 25 = 10 \times (3 \times 25)$, let's represent 3×25 first since we already know how to draw area models for one-digit by two-digit multiplication. (Draw an area model to represent 3×25 .) We've decomposed 3×25 into what two products? Give me an expression for each in unit form.
- S: 3×2 tens and 3×5 ones.
- T: 3×2 tens is?
- S: 6 tens.
- T: And 3×5 ones?
- S: 15 ones.
- T: So 3×25 is?
- S: 75.
- T: What unit does this 3 have right now?
- S: Ones.
- T: Let's change that unit. Let's make it tens. (Draw the new area model.) What new multiplication problem is represented?
- S: 30×25 .
- T: Let's find the total area by finding partial products again. (Point to the 30 by 5 rectangle.) In unit form, give me a multiplication sentence to find the area of this portion.
- S: $3 \text{ tens} \times 5 = 15 \text{ tens}$.
- T: Do we need to put a unit on the 5?
- S: It would be ones. → We don't always have to say the unit when it's just ones.
- T: (Record as shown. Then point to the 30 by 20 rectangle.) In unit form, give me a multiplication sentence to find the area of this rectangle.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

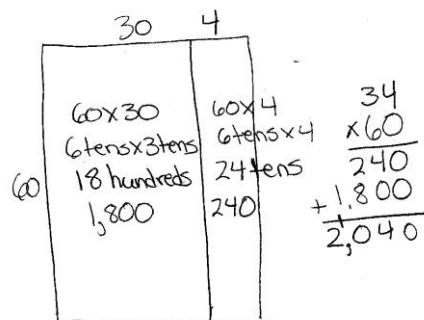
Help students understand that multiplying tens, unlike adding, will result in a larger unit. Here, 3 tens times 2 tens is 6 hundreds, not 6 tens. To clarify, refer back to the magnifying arrows on the place value chart, the number form, or place value blocks (cubes, longs, and flats).

- S: $3 \text{ tens} \times 2 \text{ tens} = 6 \text{ hundreds}$.
- T: I noticed this time you gave me the units on both factors, why?
- S: They were both tens. \rightarrow This way I can just think of 3×2 , and all I have to do is figure out what the new unit will be. \rightarrow Tens times tens gives me hundreds.
- T: Find the product for 30×25 and discuss with your partner how the two products, 3×25 and 30×25 , are related.
- S: One was 75 and the other was 750. That's 10 times as much. \rightarrow The first was 6 tens plus 15 ones. The other was 6 hundreds plus 15 tens. \rightarrow For the first one, we did 3×5 and 3×20 . On the second, we just multiplied the 3 by 10 and got 30×5 and 30×20 . That's $150 + 600$, or 750. \rightarrow The only difference was the unit on the 3. Ones became tens.

Problem 2

Find the product of 60 and 34 using an area model. Record the partial products to solve.

- T: Draw an area model to represent 60×34 and then write the expressions that solve for the area of each rectangle.
- S: (Draw area model and write expressions.)
- T: Write 60×34 vertically next to the area model and then record the partial products beginning with the area of the smaller rectangle.
- S: (Record partial products as 240 and 1,800.)
- T: What does the partial product of 240 represent?
- S: The area of the small rectangle. \rightarrow 6 tens times 4.
- T: What does the partial product of 1,800 represent?
- S: The area of the larger part. \rightarrow 6 tens times 3 tens.
- T: How do we find the product for 60×34 ?
- S: We need to add the partial products. $240 + 1,800 = 2,040$. \rightarrow $60 \times 34 = 2,040$.



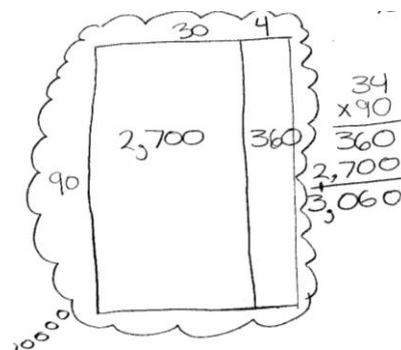
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some learners may benefit from graph paper or lines outlining the place values to assist their accurate recording of the partial products.

Problem 3

Find the product of 90 and 34 without using an area model. Record the partial products to solve.

- T: Write 90×34 vertically. If we were to create an area model to solve 90×34 , what would it look like?
- S: It would be 90 units by 34 units. The 34 would be split into two parts: 30 and 4.
- T: Imagine the area model and use it to record the two partial products using the vertical written method. Then use unit language to explain to your partner how you solved the problem.



MP.8

Circulate and listen for phrases such as 9 tens times 4 and 9 tens \times 3 tens. Ensure students are accurately lining up digits in the appropriate place value columns.

Repeat with 30×34 .

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Multiply two-digit multiples of 10 by two-digit numbers using the area model.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- How is Problem 1 of the Problem Set less complex than the others?
- How do Problems 3–7 lend themselves to the use of the area model?
- Can you explain why Problems 6 and 7 have the same product?
- What can you say about area models for Problems 8 and 9?
- When we record partial products, do we have to start with the one with the smallest place value? Will we get a different result if we start with the tens?

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 35 Problem Set 4•3

Name Jack Date _____

Use an area model to represent the following expressions. Then record the partial products and solve.

1. 20×22

20	2
20×20 4 hundreds 400	20×2 4 tens 40

22
 $\times 20$
40
+ 400
440

2. 50×41

40	1
50×40 5 tens \times 4 tens 20 hundreds 2,000	50×1 5 tens 50

41
 $\times 50$
50
+ 2,000
2,050

3. 60×73

70	3
60×70 6 tens \times 7 tens 42 hundreds 4,200	60×3 6 tens \times 3 18 tens 180

73
 $\times 60$
180
+ 4,200
4,380

COMMON CORE Lesson 35: Multiply two-digit multiples of 10 by two-digit numbers using the area model. 8/22/13

engage^{ny} 3.H.21

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 35 Problem Set 4•3

Draw an area model to represent the following expressions. Then record the partial products vertically and solve.

4. 80×32

30	2
80×30 8 tens \times 3 tens 24 hundreds 2,400	80×2 8 tens \times 2 16 tens 160

32
 $\times 80$
160
+ 2,400
2,560

5. 70×54

50	4
70×50 7 tens \times 5 tens 35 hundreds 3,500	70×4 7 tens \times 4 28 tens 280

54
 $\times 70$
280
+ 3,500
3,780

Visualize the area model and solve the following products numerically.

6. 30×68

68
 $\times 30$
240
+ 1,800
2,040

7. 60×34

34
 $\times 60$
240
+ 1,800
2,040

8. 40×55

55
 $\times 40$
200
+ 2,000
2,200

9. 80×55

55
 $\times 80$
400
+ 4,000
4,400

COMMON CORE Lesson 35: Multiply two-digit multiples of 10 by two-digit numbers using the area model. 8/22/13

engage^{ny} 3.H.22

- When we multiply by a multiple of 10, why is there always a 0 in the ones place?
- What significant math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?

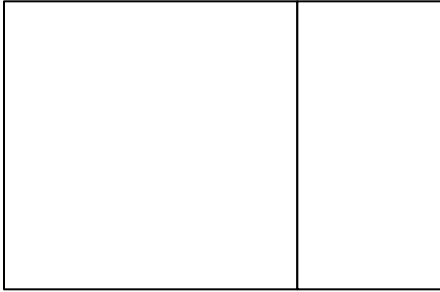
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____ Date _____

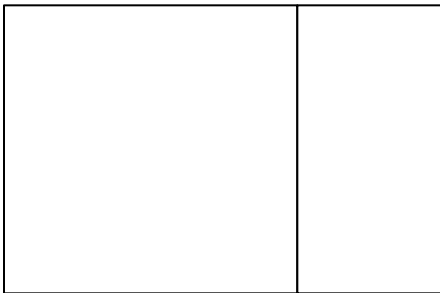
Use an area model to represent the following expressions. Then record the partial products and solve.

1. 20×22



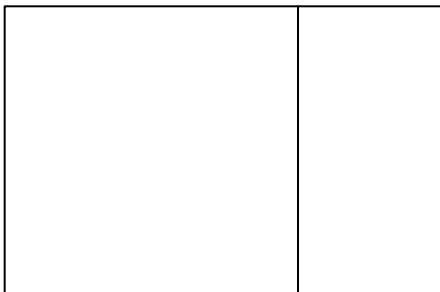
$$\begin{array}{r} 22 \\ \times 20 \\ \hline \\ + \\ \hline \end{array}$$

2. 50×41



$$\begin{array}{r} 41 \\ \times 50 \\ \hline \\ + \\ \hline \end{array}$$

3. 60×73



$$\begin{array}{r} 73 \\ \times 60 \\ \hline \\ + \\ \hline \end{array}$$

Draw an area model to represent the following expressions. Then record the partial products vertically and solve.

4. 80×32

5. 70×54

Visualize the area model and solve the following products numerically.

6. 30×68

7. 60×34

8. 40×55

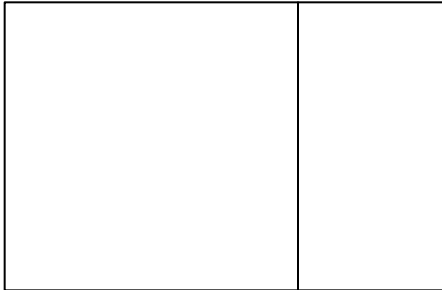
9. 80×55

Name _____

Date _____

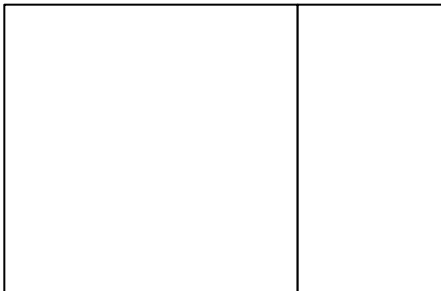
Use an area model to represent the following expressions. Then record the partial products and solve.

1. 30×93



$$\begin{array}{r} 93 \\ \times 30 \\ \hline \\ + \\ \hline \end{array}$$

2. 40×76

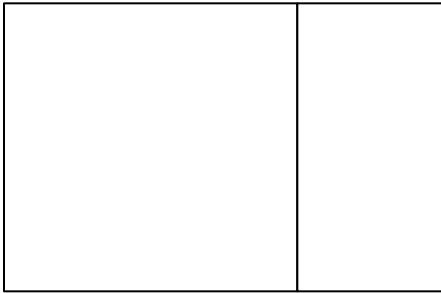


$$\begin{array}{r} 76 \\ \times 40 \\ \hline \\ + \\ \hline \end{array}$$

Name _____ Date _____

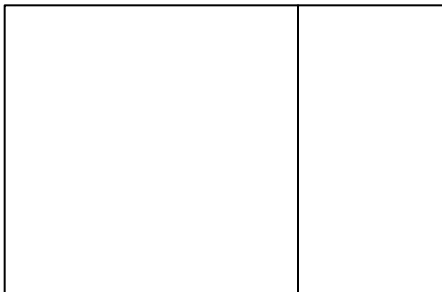
Use an area model to represent the following expressions. Then record the partial products and solve.

1. 30×17



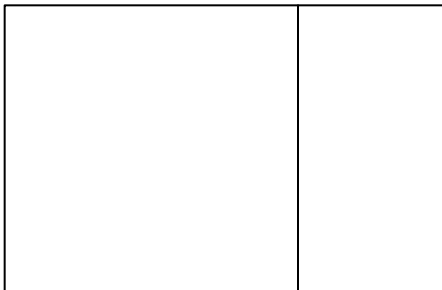
$$\begin{array}{r} 17 \\ \times 30 \\ \hline \\ + \\ \hline \end{array}$$

2. 40×58



$$\begin{array}{r} 58 \\ \times 40 \\ \hline \\ + \\ \hline \end{array}$$

3. 50×38



$$\begin{array}{r} 38 \\ \times 50 \\ \hline \\ + \\ \hline \end{array}$$

Draw an area model to represent the following expressions. Then record the partial products vertically and solve.

4. 60×19

5. 20×44

Visualize the area model and solve the following products numerically.

6. 20×88

7. 30×88

8. 70×47

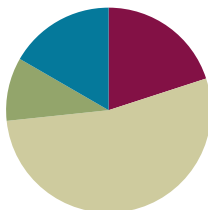
9. 80×65

Lesson 36

Objective: Multiply two-digit by two-digit numbers using four partial products.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(32 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Draw a Unit Fraction **3.G.2** (4 minutes)
- Divide Three Different Ways **4.NBT.6** (4 minutes)
- Multiply by Multiples of 10 Written Vertically **4.NBT.5** (4 minutes)

Draw a Unit Fraction (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews Grade 3 geometry and fraction concepts in anticipation of G4–Modules 4 and 5. Accept reasonable drawings. Using rulers is not necessary to review the concept and will take too long.

T: On your boards, write down the name for any four-sided figure.

S: (Write *quadrilateral*.)

T: Draw a quadrilateral that has 4 right angles and 4 equal sides.

S: (Draw a square.)

T: Partition the square into 4 equal parts.

S: (Do so.)

T: Shade in 1 of the parts.

S: (Do so.)

T: Write the fraction of the square that you shaded.

S: (Write $\frac{1}{4}$.)

Continue with the following possible sequence: Partition a rectangle into 5 equal parts, shading $\frac{1}{5}$; partition a

rhombus into 2 equal parts, shading $\frac{1}{2}$; partition a square into 12 equal parts shading $\frac{1}{12}$; and partition a rectangle into 8 equal parts, shading $\frac{1}{8}$.

Divide Three Different Ways (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M3–Lessons 32 and 33

T: (Write $406 \div 7$.) Find the quotient using number disks.

T: Find the quotient using the area model.

T: Find the quotient using the standard algorithm.

Repeat using $3,168 \div 9$.

Multiply by Multiples of 10 Written Vertically (4 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M3–Lesson 35's content.

T: (Write 30×23 vertically.) When I write 30×23 , you say 3 tens times 3 ones plus 3 tens times 2 tens.

S: (Point to the corresponding expressions as students speak.) 3 tens times 3 ones + 3 tens times 2 tens.

T: Write and solve the entire equation vertically.

T: What is 30 times 23?

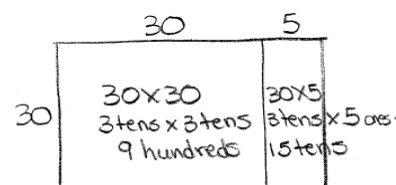
S: 690.

Continue with the following possible sequence: 30×29 , 40×34 , and 50×45 .

Application Problem (6 minutes)

Mr. Goggins set up 30 rows of chairs in the gymnasium. If each row had 35 chairs, how many chairs did Mr. Goggins set up? Draw an area model to represent and to help solve this problem. Discuss with a partner how the area model can help you to solve 30×35 .

Note: This Application Problem builds on prior learning from Lesson 35 where students used an area model and partial products to multiply a two-digit multiple of 10 by a two-digit number using an area model. The Application Problem helps bridge to today's lesson in that students will apply prior knowledge of the area model and partial products to represent and solve two-digit by two-digit multiplication.



$$\begin{array}{r} 35 \\ \times 30 \\ \hline 150 \\ + 900 \\ \hline 1,050 \end{array}$$

Mr. Goggins set up 1,050 chairs.

Concept Development (32 minutes)

Problem 1

Use the distributive property to represent and solve two-digit by two-digit multiplication.

T: (Use the context of the Application Problem to continue with today's lesson.) Mr. Goggins set up an additional 4 rows of chairs with 35 chairs in each row. Let's change our area model to represent the additional rows. (Revise the area model.)

T: What is the length of this entire side? (Show the vertical length.)

S: 34.

T: And the length of this side? (Show the horizontal length.)

S: 35.

T: Use the area formula. What expression is shown by the area model now?

S: 34×35 .

T: We can use the area model to help us represent two-digit times two-digit multiplication. Write the expressions that represent the areas of the two smaller rectangles that we just created.

S: 4×5 and 4×30 .

T: Let's say the expressions in unit form to help us understand their value. Using the units for each factor, say 4×5 and 4×30 in unit form.

S: 4 ones \times 5 ones and 4 ones \times 3 tens.

T: Write those unit expressions in each rectangle. How can we use these expressions and the expressions of the other two rectangles to find the area of the whole rectangle?

S: We can find the sum of all of the smaller areas.

T: Let's represent this using the distributive property. We are going to move from top to bottom, right to left to represent the areas of the smaller rectangles. You tell me the numerical expressions as I point to each of the smaller rectangles. I will write down what you say. 34×35 equals?

S: $34 \times 35 = (4 \times 5) + (4 \times 30) + (30 \times 5) + (30 \times 30)$.

T: Now express this same number sentence in unit form (without rewriting).

S: $34 \times 35 = (4 \text{ ones} \times 5 \text{ ones}) + (4 \text{ ones} \times 3 \text{ tens}) + (3 \text{ tens} \times 5 \text{ ones}) + (3 \text{ tens} \times 3 \text{ tens})$.

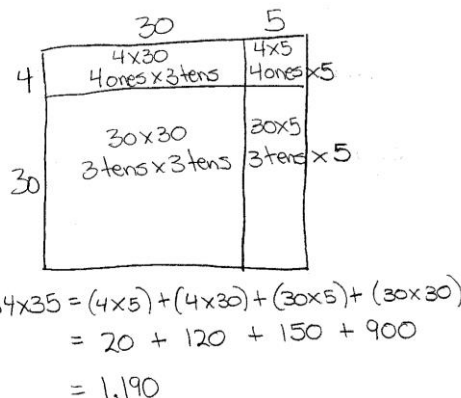
T: Now we are ready to solve! First, let's find each of the four partial products. Then add the four partial products to find 34×35 .

S: $20 \text{ ones} + 12 \text{ tens} + 15 \text{ tens} + 9 \text{ hundreds} = 20 + 120 + 150 + 900 = 1,190$.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Lead a discussion with students in order to deepen their understanding of representing expressions in numerical form and unit form. Be sure that students understand that there are different ways to express numbers in both written and oral form.

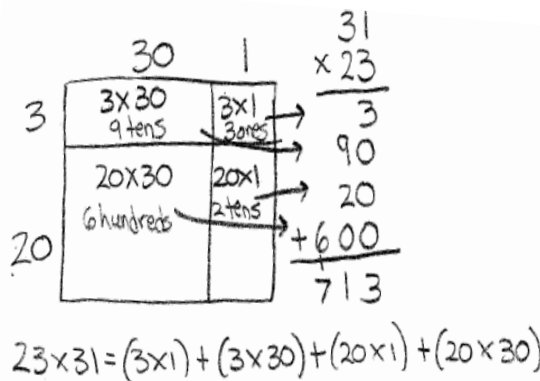


MP.4

Problem 2

Find the product of 23 and 31 using an area model and partial products to solve.

- T: Let's solve 23×31 using area to model the product.
- T: (Draw a rectangle.) Break down the length and width according to place value units.
- S: 2 tens 3 ones and 3 tens 1 one. \rightarrow 20 and 3. 30 and 1.
- T: (Draw one vertical and one horizontal line subdividing the rectangle.) Turn and tell your partner the length and width of each of the 4 smaller rectangles we just created.
- S: 3 and 1, 3 and 30, 20 and 1, and 20 and 30.
- T: Using the area model that you just drew, write an expression that represents the product of 23 and 31 as the sum of those four areas.
- S: $23 \times 31 = (3 \times 1) + (3 \times 30) + (20 \times 1) + (20 \times 30)$.
- T: Now we are ready to solve!
- T: Let's look at a way to record the partial products. (Write 23×31 vertically.) Recall that when we multiplied a one-digit number by a two-, three-, or four-digit number, we recorded the partial products. We also recorded partial products when we multiplied a two-digit number by a multiple of 10. Let's put it all together and do precisely the same thing here.
- T: (Point to the area model and the expression showing the distributive property.) What is the product of 3 ones and 1 one?
- S: 3 ones.
- T: Record the product below. Draw an arrow connecting the rectangle with the corresponding partial product. How about 3 ones times 3 tens?
- S: 9 tens.
- T: Record the product below the first partial product. Draw an arrow connecting the rectangle with the corresponding partial product. What is 2 tens times 1 one?
- S: 2 tens or 20.
- T: As before, record the partial product below the other two and do the same with 2 tens times 3 tens.
- T: Draw arrows to connect the new partial products with the corresponding rectangles. Now, let's add the partial products together. What is the sum?
- S: The sum is 713. That means that $23 \times 31 = 713$.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students working below grade level may benefit from continuing to write out the expressions used to find each of the partial products. Students may write the expressions in numerical form or in unit form.

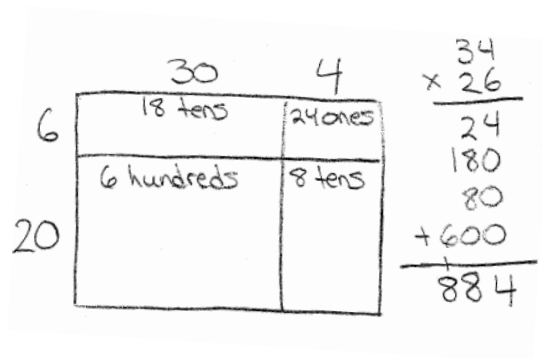
To help solidify place value, it might also be helpful to have students shade, in different colors, the rectangles that represent the ones, tens, and hundreds.

Students working above grade level may be ready to use the four partial product algorithm and can be encouraged to do so.

Problem 3

Find the product of 26 and 34 using partial products. Verify partial products using the area model.

- T: Draw an area model to represent 26×34 .
 T: How do I find the area of the smallest rectangle?
 S: Multiply 6 ones times 4 ones.
 T: Point to 6 ones times 4 ones in the algorithm. What is 6 ones times 4 ones?
 S: 24 ones.
 T: Record 24 beneath the expression and in the corresponding area.
 T: Point to the next area to solve for. Tell me the number sentence.
 S: 6 ones times 3 tens.
 T: Locate those numbers in the algorithm. Solve for 6 ones times 3 tens.
 S: 18 tens.
 T: Record 18 tens under the expression.
 S: We can also record 18 tens in this rectangle.



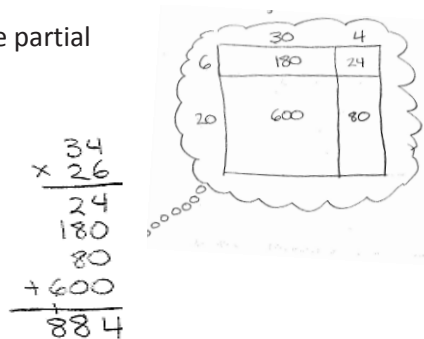
Continue connecting the width and length of each rectangle in the model to the location of those units in the algorithm. Record the partial products first under the expression and then inside the area.

- T: What is the last step?
 S: Add together all of the partial products. $24 + 180 + 80 + 600 = 884$. $\rightarrow 26 \times 34 = 884$.

Problem 4

Find the product of 26 and 34 without using an area model. Record the partial products to solve.

- T: Take a mental picture of your area model before you erase it, the partial products, and the final product.
 T: When we multiplied these numbers before, with what did we start?
 S: 6 ones \times 4 ones.
 T: Do you see 6 ones \times 4 ones?
 S: Yes.



Students point to 6 ones \times 4 ones. You might model on the board as students also record.

- T: What is 6 ones \times 4 ones?
 S: 24 ones.
 T: Record 24 ones as a partial product.
 T: What did we multiply next?
 S: 6 ones \times 3 tens. That's 18 tens or 180.

- T: Where do we record 180?
- S: Below the 24.
- T: Now what?
- S: We multiply the tens. $2 \text{ tens} \times 4 \text{ ones}$ and then $2 \text{ tens} \times 3 \text{ tens}$.
- T: What are $2 \text{ tens} \times 4 \text{ ones}$ and $2 \text{ tens} \times 3 \text{ tens}$?
- S: 8 tens and 6 hundreds. Let's record these as partial products.
- T: Notice that we have four partial products. Let's again identify from where they came. (Point to each part of the algorithm as students chorally read the expressions used to solve the two-digit by two-digit multiplication.)
- S: $6 \text{ ones} \times 4 \text{ ones} = 24 \text{ ones}$. $6 \text{ ones} \times 3 \text{ tens} = 18 \text{ tens}$. $2 \text{ tens} \times 4 \text{ ones} = 8 \text{ tens}$. $2 \text{ tens} \times 3 \text{ tens} = 6 \text{ hundreds}$.
- T: What is their sum?
- S: $24 + 180 + 80 + 600 = 884$. $\rightarrow 26 \times 34 = 884$.
- T: Visualize to relate this back to the area model that we drew earlier.

Repeat for 38×43 . You might first draw the area model (without multiplying out the partial products) and then erase it so that students again visualize the connection.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Multiply two-digit by two-digit numbers using four partial products.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 36 Problem Set 4•3

Name Jack Date _____

1. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.

4 $4 \text{ ones} \times 1 \text{ ten}$ $4 \text{ ones} \times 2 \text{ ones}$
 $10 \text{ ten} \times 1 \text{ ten}$ $1 \text{ ten} \times 2 \text{ ones}$

$4 \times 10 = 40$ $4 \times 2 = 8$
 $10 \times 10 = 100$ $10 \times 2 = 20$

b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form and then read in unit form.

$34 \times 12 = (4 \times \underline{2}) + (4 \times \underline{10}) + (10 \times \underline{2}) + (10 \times \underline{10})$

2. Use an area model to represent the following expressions. Record the partial products and solve.

a. 14×22

4 80 8
 10 200 20

22
 $\times 14$
 \hline
 8
 80
 20
 200
 \hline
 308

COMMON CORE Lesson 36: Multiply two-digit by two-digit numbers using four partial products. 8/22/13 engage^{ny} 3.H.3.3

© 2013 Common Core, Inc. Some rights reserved. commoncore.org This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

You may choose to use any combination of the questions below to lead the discussion.

- How does Problem 1(a) support your understanding of the distributive property and partial products?
- How do Problems 1 and 2 help to prepare you to solve Problems 3, 4, 5, and 6?
- How did our previous work with area models and partial products help us to be ready to solve two-digit by two-digit multiplication problems using partial products?
- How is it helpful to think about the areas of each rectangle in terms of *units*?
- How could you explain to someone that *ones* \times *tens* equals *tens* but *tens* \times *tens* equals *hundreds*?
- What significant math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 36 Problem Set 4•3

Draw an area model to represent the following expressions. Record the partial products vertically and solve.

3. 25×32

	30	2
5	150	10
20	600	40

$$\begin{array}{r} 32 \\ \times 25 \\ \hline 160 \\ 640 \\ \hline 800 \end{array}$$

4. 35×42

	40	2
5	200	10
30	1,200	60

$$\begin{array}{r} 42 \\ \times 35 \\ \hline 210 \\ 1,260 \\ \hline 1,470 \end{array}$$

Visualize the area model and solve the following numerically using four partial products. (You may sketch an area model if it helps.)

5. 42×11

$$\begin{array}{r} 11 \\ \times 42 \\ \hline 22 \\ 400 \\ \hline 462 \end{array}$$

6. 46×11

$$\begin{array}{r} 11 \\ \times 46 \\ \hline 66 \\ 460 \\ \hline 506 \end{array}$$

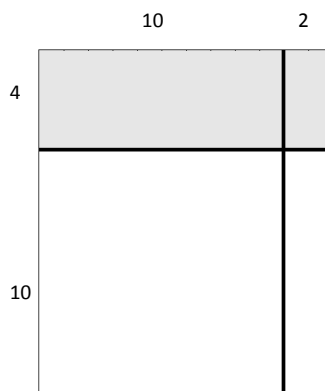
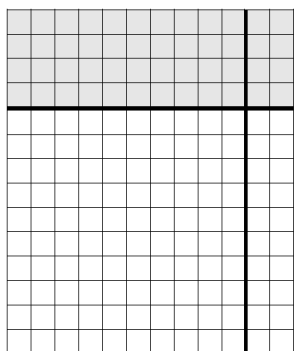
COMMON CORE Lesson 36: Multiply two-digit by two-digit numbers using four partial products. Date: 8/28/13 engage^{ny} 3.H.32

© 2013 Common Core, Inc. Some rights reserved. commoncore.org This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

Name _____

Date _____

- 1.
- a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.

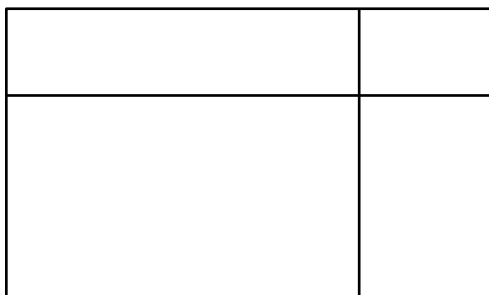


- b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form and then read in unit form.

$$14 \times 12 = (4 \times \underline{\quad}) + (4 \times \underline{\quad}) + (10 \times \underline{\quad}) + (10 \times \underline{\quad})$$

2. Use an area model to represent the following expressions. Record the partial products and solve.

- a. 14×22



$$\begin{array}{r} 22 \\ \times 14 \\ \hline \\ \hline \\ \hline \\ \hline \\ + \hline \end{array}$$

Draw an area model to represent the following expressions. Record the partial products vertically and solve.

3. 25×32

4. 35×42

Visualize the area model and solve the following numerically using four partial products. (You may sketch an area model if it helps.)

5. 42×11

6. 46×11

Name _____

Date _____

Record the partial products to solve.

Draw an area model first to support your work, or draw the area model last to check your work.

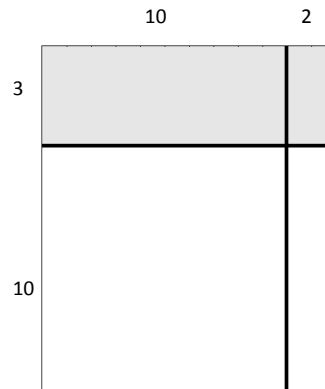
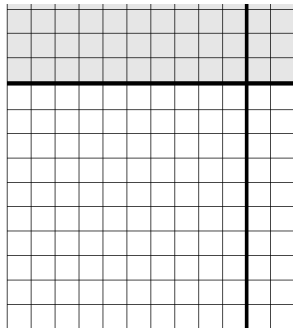
1. 26×43

2. 17×55

Name _____

Date _____

- 1.
- a. In each of the two models pictured below, write the expressions that determine the area of each of the four smaller rectangles.

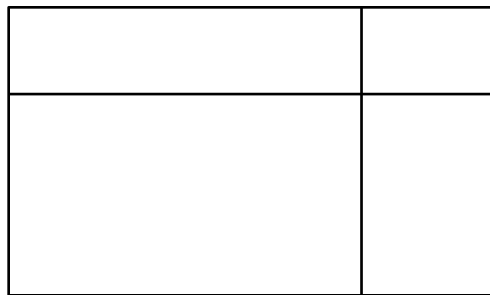


- b. Using the distributive property, rewrite the area of the large rectangle as the sum of the areas of the four smaller rectangles. Express first in number form and then read in unit form.

$$13 \times 12 = (3 \times \underline{\quad}) + (3 \times \underline{\quad}) + (10 \times \underline{\quad}) + (10 \times \underline{\quad})$$

Use an area model to represent the following expressions. Record the partial products and solve.

2. 17×34



$$\begin{array}{r} 34 \\ \times 17 \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$$

Draw an area model to represent the following expressions. Record the partial products vertically and solve.

3. 45×18

4. 45×19

Visualize the area model and solve the following numerically using four partial products. (You may sketch an area model if it helps.)

5. 12×47

6. 23×93

7. 23×11

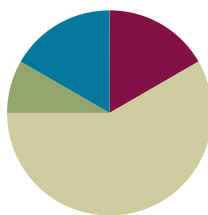
8. 23×22

Lesson 37

Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Decompose 90 and 180 **4.MD.7** (4 minutes)
- Multiply by Multiples of 10 Written Vertically **4.NBT.5** (6 minutes)

Decompose 90 and 180 (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for finding complementary and supplementary angles in G4–Module 4.

T: (Project a number bond with a whole of 90 and a part of 10.) On your boards, fill in the missing part in the number bond.

S: (Fill in 80.)

T: (Write $90 - 10 = \underline{\quad}$.) Say the subtraction sentence.

S: $90 - 10 = 80$.

Continue decomposing 90, taking away the following possible suggested parts: 20, 30, 85, 40, 45, 25, 35, and 15.

Using the same process, take away the following possible suggested parts from 180: 10, 100, 90, 70, 150, 60, 5, 15, 75, 65, and 45.

Multiply by Multiples of 10 Written Vertically (6 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M3–Lesson 35’s content.

T: Solve 30×23 vertically as you say the unit form: 3 tens times 3 ones plus 3 tens times 2 tens. You

have one minute. If you finish early, go on to 40×23 .

T: (Allow students a minute to work.) 3 tens times 3 ones is?

S: 9 tens. (Write 90.)

T: 3 tens times 2 tens is?

S: 6 hundreds. (Write 600.)

T: The sum of 90 and 600 is?

S: 690.

T: 30 groups of 23 is?

S: 690.

Continue with the following possible sequence: 40×23 , 40×34 , 50×45 , and 60×39 .

Application Problem (5 minutes)

Sylvie's teacher challenged the class to draw an area model to represent the expression 24×56 and then to solve using partial products. Sylvie solved the expression as seen to the right. Is her answer correct? Why or why not?

Note: This Application Problem builds on the content of Lessons 34, 35, and 36. Students now have a solid foundation upon which to build understanding of two-digit by two-digit multiplication. They move from pictorial representations to abstract representations. This Application Problem guides such movement and builds to the content of today's lesson where students will see how all of the work that they have done fits together and prepares them to solve using the standard algorithm (Lesson 38).

$$\begin{array}{r} 56 \\ \times 24 \\ \hline 24 \\ 200 \\ 12 \\ + 100 \\ \hline 1344 \end{array}$$

	50	6
4	200	24
20	1000	120

Concept Development (35 minutes)

Materials: (S) Paper and pencil

Problem 1

Solve 26×35 using four partial products and two partial products.

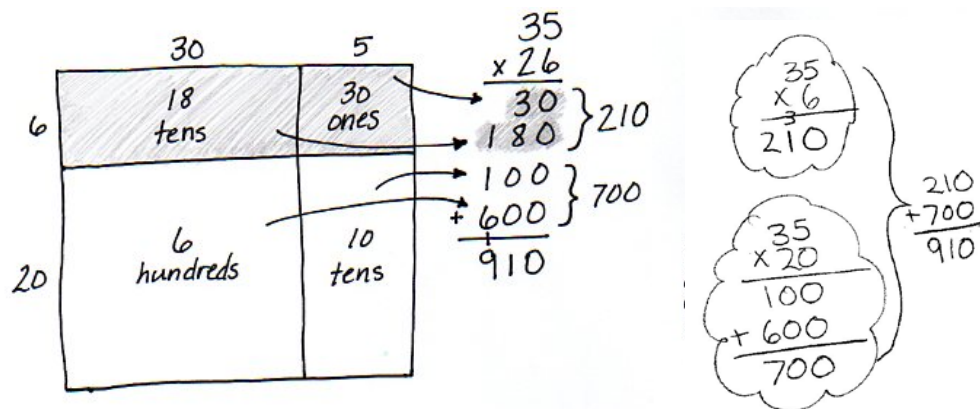
T: Work with a partner:

1. Draw an area model for 26×35 .
2. Record the partial products within each of four smaller rectangles.
3. Write the expression 26×35 vertically.
4. Write the four partial products under the expression.
5. Find their sum.

6. Connect the rectangles in the area model to the partial products using arrows.

S: (Draw area model and solve.) $26 \times 35 = 910$.

T: Shade the top half of the area model with the side of your pencil. Shade the corresponding partial products as well.



T: Use mental math to add the two partial products that you just shaded.

S: $30 + 180 = 210$.

T: What multiplication expression can be used to represent the entire shaded area?

S: 6×35 .

T: Find the total for 6 thirty-fives. (Students solve.)

S: $6 \times 35 = 210$. Hey, that's the same as when we added the two partial products that are shaded.

T: Explain why they are the same.

S: The two smaller rectangles in the shaded portion take up the same amount of space as the larger rectangle in the shaded portion.

T: Use mental math to add the two partial products that are not shaded.

S: $100 + 600 = 700$.

T: What expression can be used to represent the area of the larger rectangle that is not shaded?

S: 20×35 .

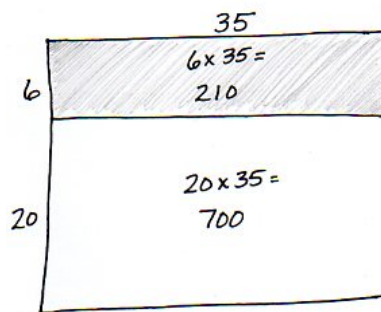
T: Solve for 20 thirty-fives. (Students solve.)

S: 700. It's the same!

T: (Draw an area model to show two partial products.) Say an addition sentence for the sum of the two parts.

S: $210 + 700 = 910$. That's the same answer as when we added the four partial products.

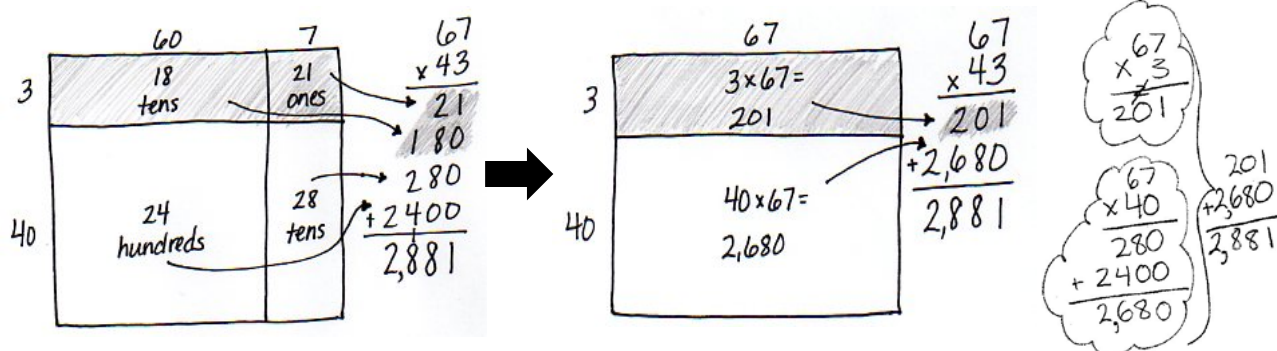
T: We can solve by finding two partial products instead of four!



Problem 2

Solve 43×67 using four partial products and two partial products.

- T: Work with a partner to draw and label an area model for 43×67 and solve.
- T: Draw arrows to show how the parts of the area model relate to the partial products.
- T: Draw and label another area model, as we did in Problem 1, which shows how we can combine the rectangles in the top portion and the rectangles in the bottom portion. (Guide students as they draw and label.) What expressions do the rectangles represent? Write the expressions in the rectangles. Solve for each expression.



MP.4

- S: (3×67) and (40×67) . 3 sixty-sevens is 201. 40 sixty-sevens is 2,680.
- T: Write the two partial products within their corresponding rectangles. Write 43×67 vertically and then write the partial products. Draw arrows to show how the parts of the area model relate to the partial products. (Guide students as they make the connections.)
- T: What is the sum of the two partial products?
- S: $201 + 2,680 = 2,881$. Again, it's the same as when we solved using four partial products.
- T: We found the value of 3 sixty-sevens and then added the value of 40 sixty-sevens.



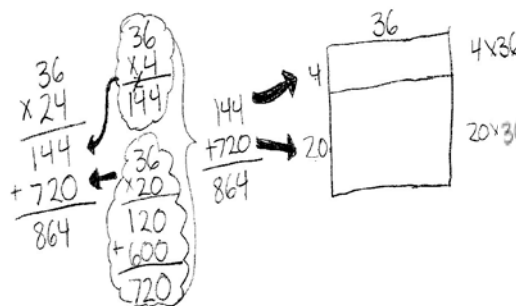
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If students are not ready to complete the transition away from the area model, encourage them to quickly sketch an area model so that they can visually see the partial products.

Problem 3

Solve 24×36 using two partial products and an area model.

- T: Write 24×36 vertically and then represent by drawing an area model. Discuss with a partner how to solve with two partial products.
- S: One rectangle represents 4 thirty-sixes and the other represents 20 thirty-sixes. We can find the

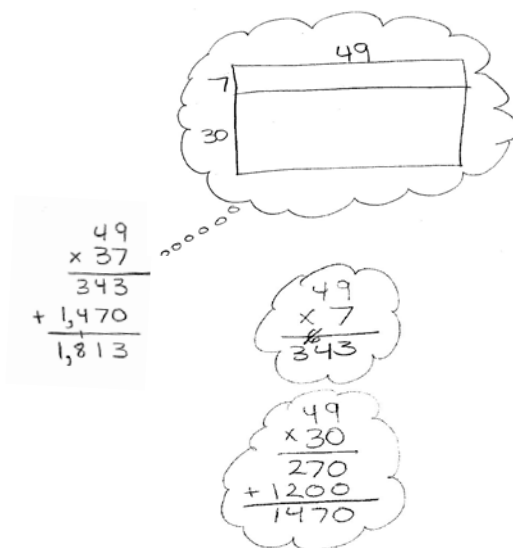


area of each rectangle and then find their sum.

T: (Draw arrows next to where the partial products will be written.) Let's write the expressions for each partial product. (Write expressions next to where the partial products will be written as students do the same.) Find each partial product and then determine their sum.

S: $144 + 720 = 864$.

T: Look at the area model that you drew. Connect the partial products in the area model to the partial products in the vertical expression. Close your eyes and create a picture in your mind of how the partial products relate to the area model.



Problem 4

Solve 37×49 using two partial products without an area model.

- T: Write 37×49 vertically on your board. How can we solve?
- S: I can think about what the area model would look like. I know that the expressions for the partial products are 7×49 and 30×49 .
- T: Find each partial product. (Allow time for students to calculate answers.)
- S: 7 forty-nines is 343; 30 forty-nines is 1,470.
- T: How did you solve for the partial products?
- S: I multiplied the numbers off to the side because I know how to multiply one-digit numbers by two-digit numbers.
- T: Write the partial products beneath the expression. What is their sum?
- S: $343 + 1,470 = 1,813$. $\rightarrow 37 \times 49 = 1,813$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Lead students through steps that will help them to create a picture in their mind of the area model. Ask questions such as the following to support the visualization.

- Can you see the rectangle?
- How many parts does it have?
- What are the two lengths of the vertical side?
- What is the length of the horizontal side?
- What expressions give us the area of each smaller rectangle?
- Look at the vertical expression that we have written. Do you see the expressions for the area of each smaller rectangle?

Student Debrief (10 minutes)

Lesson Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Did you record the 15 or 57 as the width of the rectangle in Problem 3? Does it matter the order? Which number as the width is easiest for you to solve the rest of the problem? Explain.
- Imagine the area models for Problems 4(c) and 4(d). Notice how the rectangle in (d) is half as wide and double the length of (c). What might the areas look like? Why does that result in the same product?
- How does the shading on the area models help you understand the movement from four partial products to two partial products?
- Why would we want to represent the area model using two partial products instead of four?
- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 37 Problem Set 4•3

Name Jack Date _____

1. Solve 14×12 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.

$$\begin{array}{r} 10 \quad 2 \\ 4 \text{ ones} \times 1 \text{ ten} = 40 \\ 1 \text{ ten} \times 1 \text{ ten} = 100 \\ 2 \text{ ones} \times 1 \text{ ten} = 20 \\ 2 \text{ ones} \times 2 \text{ ones} = 4 \\ \hline 168 \end{array}$$

$$\begin{array}{r} 12 \\ 4 \times 12 = 48 \\ 10 \times 12 = 120 \\ \hline 168 \end{array}$$

2. Solve 32×43 using 4 partial products and 2 partial products. Match each partial product to its area on the models. Remember to think in terms of units as you solve.

$$\begin{array}{r} 40 \quad 2 \\ 2 \text{ ones} \times 4 \text{ tens} = 80 \\ 30 \text{ tens} \times 4 \text{ tens} = 1200 \\ 2 \text{ ones} \times 3 \text{ ones} = 6 \\ 30 \text{ tens} \times 3 \text{ ones} = 90 \\ \hline 1376 \end{array}$$

$$\begin{array}{r} 43 \\ 2 \times 43 = 86 \\ 30 \times 43 = 1290 \\ \hline 1376 \end{array}$$

COMMON CORE Lesson 37: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication. 8/22/13

engage^{ny} 3.H.4.4

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 37 Problem Set 4•3

3. Solve 57×15 using 2 partial products. Match each partial product to its rectangle on the area model.

$$\begin{array}{r} 15 \\ 7 \times 15 = 105 \\ 50 \times 15 = 750 \\ \hline 855 \end{array}$$

$$\begin{array}{r} 15 \\ 57 \times 15 = 855 \end{array}$$

4. Solve the following using 2 partial products. Visualize the area model to help you.

a. 25×6

$$\begin{array}{r} 25 \\ \times 6 \\ \hline 150 \end{array}$$

b. 18×62

$$\begin{array}{r} 18 \\ \times 62 \\ \hline 1116 \end{array}$$

c. 39×46

$$\begin{array}{r} 39 \\ \times 46 \\ \hline 1794 \end{array}$$

d. 78×23

$$\begin{array}{r} 78 \\ \times 23 \\ \hline 1794 \end{array}$$

COMMON CORE Lesson 37: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication. 8/22/13

engage^{ny} 3.H.4.5

Name _____ Date _____

1. Solve 14×12 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.

10 2

$$\begin{array}{r} 12 \\ \times 14 \\ \hline \end{array}$$

4 ones \times 2 ones

4 ones \times 1 ten

1 ten \times 2 ones

1 ten \times 1 ten

12

$$\begin{array}{r} 12 \\ \times 14 \\ \hline \end{array}$$

4 ones \times 12 ones

1 ten \times 12 ones

2. Solve 32×43 using 4 partial products and 2 partial products. Match each partial product to its area on the models. Remember to think in terms of units as you solve.

40 3

$$\begin{array}{r} 43 \\ \times 32 \\ \hline \end{array}$$

2 ones \times 3 ones

2 ones \times 4 tens

3 tens \times 3 ones

3 tens \times 4 tens

43

$$\begin{array}{r} 43 \\ \times 32 \\ \hline \end{array}$$

2 ones \times 43 ones

3 tens \times 43 ones

3. Solve 57×15 using 2 partial products. Match each partial product to its rectangle on the area model.

4. Solve the following using 2 partial products. Visualize the area model to help you.

a.
$$\begin{array}{r} 25 \\ \times 46 \\ \hline \end{array}$$

$$\begin{array}{r} \\ \times 46 \\ \hline \end{array}$$

$$ \times $$

$$ $$

$$ \times $$

$$ $$

b.
$$\begin{array}{r} 18 \\ \times 62 \\ \hline \end{array}$$

$$\begin{array}{r} \\ \times 62 \\ \hline \end{array}$$

$$ \times $$

$$ $$

$$ \times $$

$$ $$

c.
$$\begin{array}{r} 39 \\ \times 46 \\ \hline \end{array}$$

$$\begin{array}{r} \\ \times 46 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 78 \\ \times 23 \\ \hline \end{array}$$

$$\begin{array}{r} \\ \times 23 \\ \hline \end{array}$$

Name _____

Date _____

1. Solve 43×22 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.

	20	2	
3			$\begin{array}{r} 22 \\ \times 43 \\ \hline \end{array}$
40			<p>_____ <i>3 ones \times 2 ones</i></p> <p>_____ <i>3 ones \times 2 tens</i></p> <p>_____ <i>4 tens \times 2 ones</i></p> <p>_____ <i>4 tens \times 2 tens</i></p>

	22	
3		$\begin{array}{r} 22 \\ \times 43 \\ \hline \end{array}$
40		<p>_____ <i>3 ones \times 22 ones</i></p> <p>_____ <i>4 tens \times 22 ones</i></p>

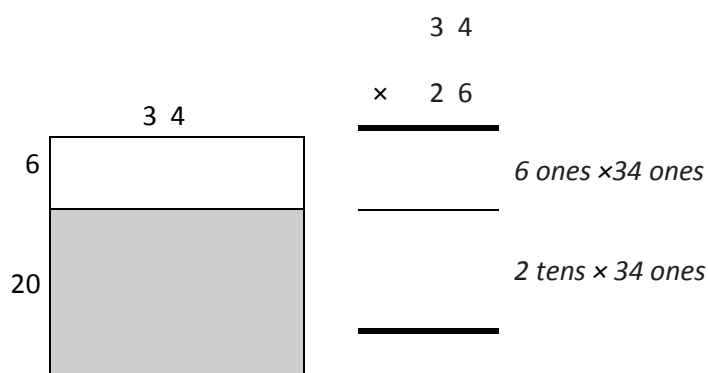
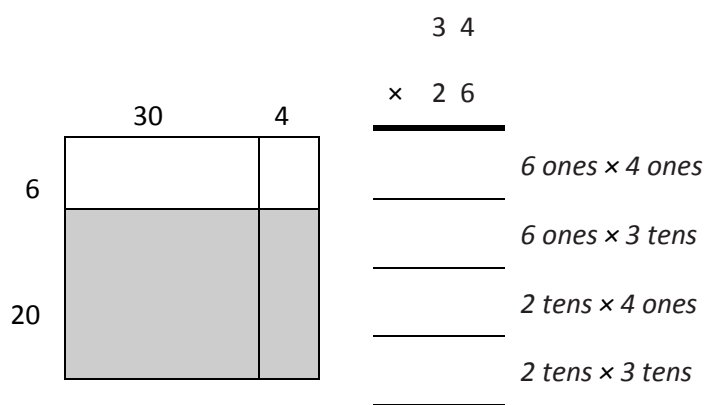
2. Solve the following using 2 partial products.

	64	
	$\begin{array}{r} 64 \\ \times 15 \\ \hline \end{array}$	<p>_____ <i>5 ones \times 64 ones</i></p> <p>_____ <i>1 ten \times 64 ones</i></p>

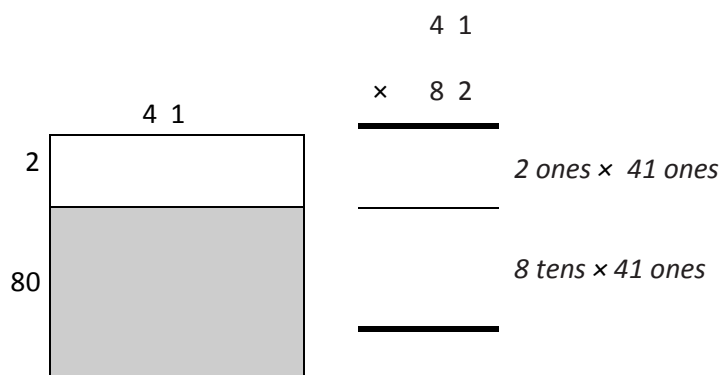
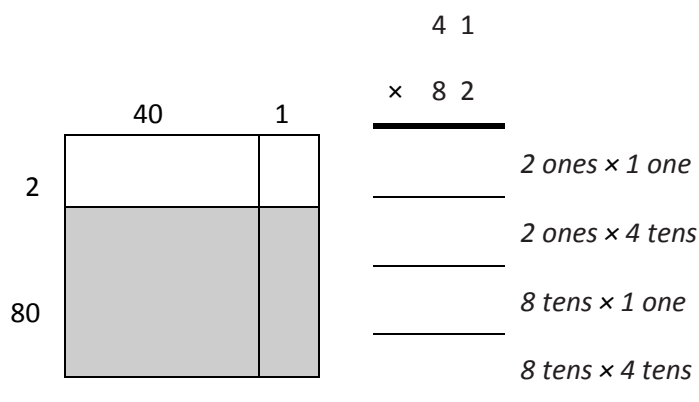
Name _____

Date _____

1. Solve 26×34 using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.



2. Solve using 4 partial products and 2 partial products. Remember to think in terms of units as you solve. Write an expression to find the area of each smaller rectangle in the area model.



3. Solve 52×26 using 2 partial products and an area model. Match each partial product to its area on the model.

4. Solve the following using 2 partial products. Visualize the area model to help you.

a. 68

$$\begin{array}{r} \times 23 \\ \hline \end{array}$$

$$\underline{\quad} \times \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad}$$

b. 49

$$\begin{array}{r} \times 33 \\ \hline \end{array}$$

$$\underline{\quad} \times \underline{\quad}$$

$$\underline{\quad} \times \underline{\quad}$$

c. 16

$$\begin{array}{r} \times 25 \\ \hline \end{array}$$

d. 54

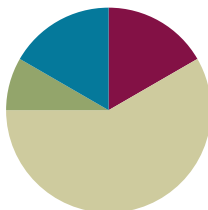
$$\begin{array}{r} \times 71 \\ \hline \end{array}$$

Lesson 38

Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

Suggested Lesson Structure

■ Fluency Practice	(10 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (10 minutes)

- Decompose 90 and 180 **4.MD.7** (4 minutes)
- Multiply by Multiples of 10 Written Vertically **4.NBT.5** (6 minutes)

Decompose 90 and 180 (4 minutes)

Materials: (S) Personal white boards

Note: This fluency prepares students for finding complementary and supplementary angles in G4–Module 4.

T: (Project a number bond with a whole of 90 and a part of 10.) On your boards, fill in the missing part in the number bond.

S: (Fill in 80.)

T: (Write $90 - 10 = \underline{\quad}$.) Say the subtraction sentence.

S: $90 - 10 = 80$.

Continue decomposing 90 taking away the following possible suggested parts: 20, 30, 85, 40, 45, 25, 35, and 15.

Repeat the process taking away the following possible suggested parts from 180: 10, 100, 90, 70, 150, 60, 5, 15, 75, 65, and 45.

Multiply by Multiples of 10 Written Vertically (6 minutes)

Materials: (S) Personal white boards

Note: This fluency reviews G4–M3–Lesson 35's content.

T: Solve 20×67 vertically as you say the unit form: 2 tens times 7 ones plus 2 tens times 6 tens. You

have one minute. If you finish early, go on to 20×78 .

T: (Allow students a minute to work.) 2 tens times 7 ones is?

S: 14 tens. (Write 140.)

T: 2 tens times 6 tens is?

S: 12 hundreds. (Write 1,200.)

T: The sum of 140 and 1,200 is?

S: 1,340.

T: 20 groups of 67 is?

S: 1,340.

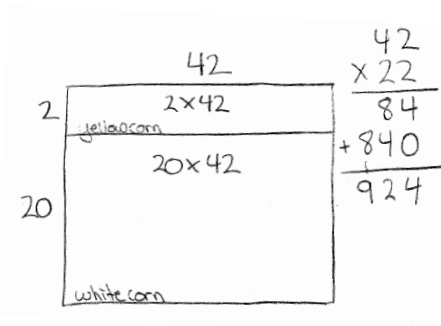
Continue with the following possible sequence: 20×78 , 30×45 , 30×67 , and 40×75 .

Application Problem (5 minutes)

Sandy's garden has 42 plants in each row. She has 2 rows of yellow corn and 20 rows of white corn.

Draw an area model (representing two partial products) to show how much yellow corn and white corn has been planted in the garden.

Note: This problem revisits the area model that focuses on two partial products in preparation for work with the standard algorithm. The area model used in the Application Problem will be used in Problem 1 of the Concept Development.



Concept Development (35 minutes)

Materials: (S) Personal white boards

Problem 1

Represent 22×42 with the distributive property and connect the two partial products to the standard algorithm.

T: Look at the model you drew in the Application Problem. We found the total for 22 rows of 42, or 22 forty-twos. What multiplication expression is that?

S: 22×42 .

T: Write 22×42 vertically. 22 units of 42.

T: Which expression represents the first of the two partial products that we recorded?

S: 2×42 .



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

When multiplying two-digit by two-digit numbers, use Hide Zero cards to represent the factors. The cards provide concrete representation of the place value of each digit within the factors and are another way to promote understanding of the multiplication algorithm.

T: 2 ones times 2 ones equals?

S: 4 ones.

T: Let's record the 4 ones in the ones place.

T: 2 ones times 4 tens equals?

S: 8 tens.

T: Let's record the 8 tens in the tens place. What's the first partial product?

S: 84.

T: Draw an arrow to the area model to show where the partial product is represented.

T: In unit form, which expression represents the second of the two partial products that we recorded?

S: 2 tens \times 4 tens 2 ones.

T: Let's solve.

T: What is 2 tens times 2 ones?

S: 4 tens.

T: Let's record 4 tens as 40 to start the second partial product. (Model.)

T: 2 tens times 4 tens equals how many hundreds?

S: 8 hundreds.

T: Record 8 hundreds in the hundreds place. Draw an arrow to the area model to show where the partial product is represented.

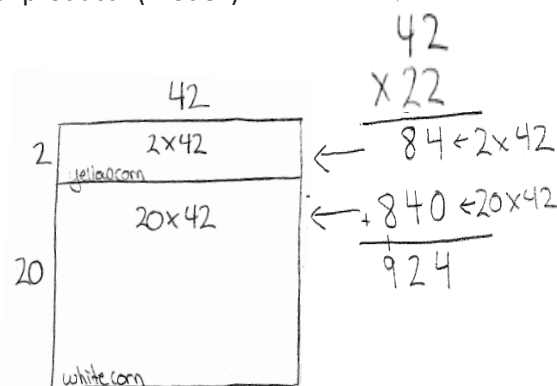
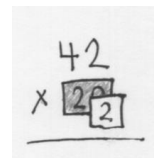
T: What's the second partial product?

S: 840.

T: Find the sum of the two partial products.

T: What is 22×42 ? Say the equation.

S: $22 \times 42 = 924$.



Problem 2

Represent 29×62 involving a regrouping in the first partial product.

T: We want to find the value of 29 sixty-twos using the algorithm.

T: What multiplication expression will I use?

S: 29×62 .

T: First let's find the value of 9 sixty-twos.

T: 9 ones times 2 ones is?

S: 18 ones.

T: Let's record the new groups below just as we have done in the past. (Write the 1 on the line under the tens place first and the 8 in the ones place second.)

T: 9 ones times 6 tens is?

S: 54 tens.

$$\begin{array}{r} 62 \\ \times 29 \\ \hline 558 \end{array} \leftarrow 9 \times 62$$

- T: (Point to the regrouped ten.) 54 tens plus 1 ten is?
- S: 55 tens. Now we need to cross off the 1 ten that we regrouped.
- T: What is 9×62 ?
- S: 558.
- T: Now let's find the value of the second partial product, 20 sixty-twos.
- T: 2 tens times 2 ones is?
- S: 4 tens.
- T: Record the 4 tens as 40 ones. 2 tens times 6 tens is?
- S: 12 hundreds.
- T: Record 12 hundreds in the second partial product. What is our second partial product?
- S: 1,240.
- T: What is the sum of our partial products?
- S: 1,798.
- T: What is 29×62 ? Say the complete equation.
- S: $29 \times 62 = 1,798$.
- T: Yes, 9 sixty-twos plus 20 sixty-twos is 29 sixty-twos. The product is 1,798.

$$\begin{array}{r}
 62 \\
 \times 29 \\
 \hline
 558 \leftarrow 9 \times 62 \\
 + 1240 \leftarrow 20 \times 62 \\
 \hline
 1,798
 \end{array}$$



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Use graph paper or a template that allows for wide rows to show how the regrouping is within the same partial product and how it relates to the value of that row. Students can then see the regrouped number is intentionally placed in the next column within the same partial product. Relate back to the number disk model of representation as needed.

Problem 3

Solve 46×63 involving a regrouping in the second partial product.

- T: Let's find the value of 46 sixty-threes. Write the multiplication expression.
- S: (Write 46×63 .)
- T: Which partial product do we find first?
- S: 6×63 .
- T: 6 ones times 3 ones is?
- S: 18 ones.
- T: Let's record. (Write the 1 on the line under the tens place first, and the 8 in the ones place second.)
- T: What do we multiply next?
- S: 6 ones times 6 tens. That's 36 tens. When I add the 1 ten I get 37 tens.
- T: Record 37 tens. The value of 6 sixty-threes is?
- S: 378.

$$\begin{array}{r}
 63 \\
 \times 46 \\
 \hline
 378 \leftarrow 6 \times 63 \\
 + 2520 \leftarrow 40 \times 63 \\
 \hline
 2,898
 \end{array}$$

MP.8

MP.8

- T: Now let's find the value of 40 sixty threes. What do we do first?
- S: 4 tens times 3 ones equals 12 tens.
- T: 12 tens is 1 hundreds 2 tens. Record the 1 hundred in the hundreds column of the second partial product. Record 2 tens as 20.
- T: What do we multiply next?
- S: 4 tens times 6 tens. That's 24 hundreds.
- T: The total number of hundreds is?
- S: We had 24 hundreds, plus one more hundred is 25 hundreds.
- T: Cross out the 1 hundred and record 25 hundreds.
- T: What is the second partial product?
- S: 2,520.
- T: Turn and tell your partner what the next step is.
- S: We add the partial products.
- T: What is 46 sixty-threes?
- S: $46 \times 63 = 2,898$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 38 Problem Set 4•3

Name Jack Date _____

1. Express 23×54 as two partial products using the distributive property. Solve.

$$\begin{array}{r} 54 \\ 3 \times 54 \\ 20 \times 54 \\ \hline \end{array}$$

$$23 \times 54 = (3 \text{ fifty-fours}) + (20 \text{ fifty-fours})$$

$$\begin{array}{r} 54 \\ \times 23 \\ \hline 162 \\ + 1080 \\ \hline 1242 \end{array}$$

2. Express 46×54 as two partial products using the distributive property. Solve.

$$\begin{array}{r} 54 \\ 6 \times 54 \\ 40 \times 54 \\ \hline \end{array}$$

$$46 \times 54 = (6 \text{ fifty-fours}) + (40 \text{ fifty-fours})$$

$$\begin{array}{r} 54 \\ \times 46 \\ \hline 324 \\ + 2160 \\ \hline 2484 \end{array}$$

3. Express 55×47 using two partial products using the distributive property. Solve.

$$\begin{array}{r} 47 \\ 5 \times 47 \\ 50 \times 47 \\ \hline \end{array}$$

$$55 \times 47 = (5 \times 47) + (50 \times 47)$$

$$\begin{array}{r} 47 \\ \times 55 \\ \hline 235 \\ + 2350 \\ \hline 2585 \end{array}$$

COMMON CORE Lesson 38: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication. 8/22/13

engage^{ny} 3.H.5.5

© 2013 Common Core, Inc. Some rights reserved. commoncore.org

You may choose to use any combination of the questions below to lead the discussion.

- What is the relationship between the product for Problem 1 and Problem 2 of the Problem Set?
- How does the structure of determining the answers to Problems 1 and 2 help you to solve Problem 3?
- How is recording multiplication using the multiplication algorithm the same as when we solved using two partial products? How is it different?
- How did your understanding of two partial products help you to learn the multiplication algorithm?
- How is the multiplication algorithm similar to the algorithm for addition? How is it different?
- What might be an advantage of using the multiplication algorithm to multiply?
- Explain to your partner how to multiply using the multiplication algorithm.
- What new (or significant) math vocabulary did we use today to communicate precisely?
- How did the Application Problem connect to today's lesson?

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 38 Problem Set 4•3

4. Solve the following using 2 partial products.

$$\begin{array}{r} 58 \\ \times 45 \\ \hline 290 \\ + 2320 \\ \hline 2610 \end{array}$$

$$\begin{array}{r} 58 \\ \times 5 \\ \hline 290 \end{array}$$

$$\begin{array}{r} 58 \\ \times 40 \\ \hline 2320 \\ + 290 \\ \hline 2610 \end{array}$$

5. Solve using the multiplication algorithm.

$$\begin{array}{r} 82 \\ \times 55 \\ \hline 410 \\ + 4100 \\ \hline 4510 \end{array}$$

6. 53×63

$$\begin{array}{r} 63 \\ \times 53 \\ \hline 189 \\ + 3150 \\ \hline 3339 \end{array}$$

7. 84×73

$$\begin{array}{r} 73 \\ \times 84 \\ \hline 292 \\ + 5840 \\ \hline 6132 \end{array}$$

COMMON CORE Lesson 38: Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication. Date: 8/22/13

engage^{ny} 3.H.54

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.

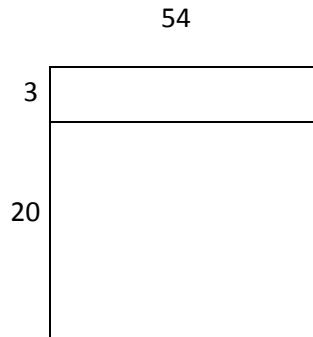
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____

Date _____

1. Express 23×54 as two partial products using the distributive property. Solve.

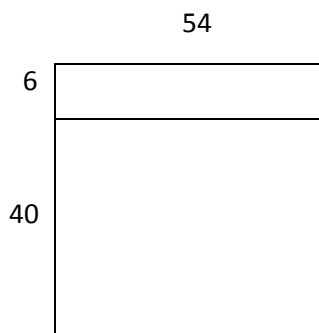


$$23 \times 54 = (\text{___ fifty-fours}) + (\text{___ fifty-fours})$$

$$\begin{array}{r} 54 \\ \times 23 \\ \hline \end{array}$$

3 × _____
20 × _____

2. Express 46×54 as two partial products using the distributive property. Solve.



$$46 \times 54 = (\text{___ fifty-fours}) + (\text{___ fifty-fours})$$

$$\begin{array}{r} 54 \\ \times 46 \\ \hline \end{array}$$

_____ × _____
_____ × _____

3. Express 55×47 using two partial products using the distributive property. Solve.



$$55 \times 47 = (\text{___} \times \text{___}) + (\text{___} \times \text{___})$$

$$\begin{array}{r} 47 \\ \times 55 \\ \hline \end{array}$$

_____ × _____
_____ × _____

4. Solve the following using 2 partial products.

$$\begin{array}{r} 58 \\ \times 45 \\ \hline \end{array}$$

_____ \times _____

_____ \times _____

5. Solve using the multiplication algorithm.

$$\begin{array}{r} 82 \\ \times 55 \\ \hline \end{array}$$

_____ \times _____

_____ \times _____

6. 53×63

7. 84×73

Name _____

Date _____

Solve using the multiplication algorithm.

1.

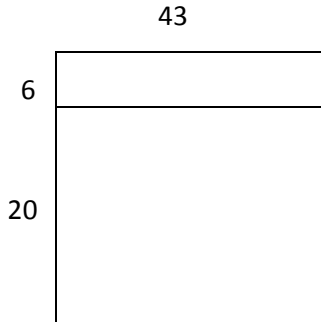
$$\begin{array}{r} 72 \\ \times 43 \\ \hline \end{array}$$
$$\begin{array}{r} ______ \\ \times ______ \\ \hline \end{array}$$
$$\begin{array}{r} ______ \\ \times ______ \\ \hline \end{array}$$

2. 35×53

Name _____

Date _____

1. Express 26×43 as two partial products using the distributive property. Solve.

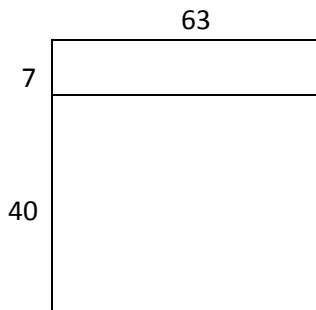


$$26 \times 43 = (\text{ } \text{forty-threes}) + (\text{ } \text{forty-threes})$$

$$\begin{array}{r} 43 \\ \times 26 \\ \hline \end{array}$$

$6 \times \text{ } = \text{ }$
 $20 \times \text{ } = \text{ }$

2. Express 47×63 as two partial products using the distributive property. Solve.

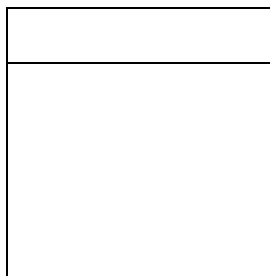


$$47 \times 63 = (\text{ } \text{sixty-threes}) + (\text{ } \text{sixty-threes})$$

$$\begin{array}{r} 63 \\ \times 47 \\ \hline \end{array}$$

$\text{ } \times \text{ } = \text{ }$
 $\text{ } \times \text{ } = \text{ }$

3. Express 54×67 as two partial products using the distributive property. Solve.



$$54 \times 67 = (\text{ } \times \text{ }) + (\text{ } \times \text{ })$$

$$\begin{array}{r} 67 \\ \times 54 \\ \hline \end{array}$$

$\text{ } \times \text{ } = \text{ }$
 $\text{ } \times \text{ } = \text{ }$

Solve the following using 2 partial products.

4.

$$\begin{array}{r} 52 \\ \times 34 \\ \hline \end{array}$$

$$\begin{array}{r} \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \end{array}$$

Solve using the multiplication algorithm.

5.

$$\begin{array}{r} 86 \\ \times 56 \\ \hline \end{array}$$

$$\begin{array}{r} \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \end{array}$$

6. 54×52

7. 44×76

8. 63×63

9. 68×79

Name _____

Date _____

1. Draw an area model to solve the following. Find the value of the following expressions.

a. 30×60

b. 3×269

2. Use any place value strategy to multiply.

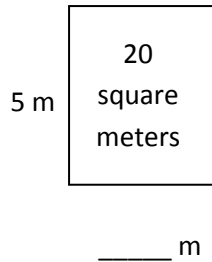
a. 3×68

b. 4×371

c. $7 \times 1,305$

d. $6,034 \times 5$

5. Last year Mr. Petersen's rectangular garden had a width of 5 meters and an area of 20 square meters. This year he wants to make the garden three times as long and two times as wide.
- a. Solve for the length of last year's garden using the area formula. Then, draw and label the measurements of this year's garden.

Last Year**This Year**

- b. How much area for planting will Mr. Petersen have in the new garden?

- c. Last year, Mr. Petersen had a fence all the way around his garden. He can reuse all of the fence he had around the garden last year, but he needs to buy more fencing to go around this year's garden. How many more meters of fencing is needed for this year's garden than last year's?
- d. Last year Mr. Petersen was able to plant 4 rows of carrots with 13 plants in each row. This year he plans to plant twice as many rows with twice as many carrot plants in each. How many carrot plants will he plant this year? Write a multiplication equation to solve. Assess the reasonableness of your answer.

Mid-Module Assessment Task Standards Addressed

Topics A–D

Use the four operations with whole numbers to solve problems.

- 4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- 4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
- 4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

- 4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

- 4.MD.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student CAN do now, and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 4.NBT.5	The student is unable to complete both area models with correct calculations.	The student correctly solves one part using the area model.	The student correctly solves both parts but has an error in one of the area models; or the student represents both area models correctly but miscalculates for one part.	The student correctly completes all components—draws area model for each problem with correct answer: a. 1,800 b. 807
2 4.NBT.5	The student is unable to solve more than one problem correctly.	The student correctly solves at least two of the four problems with evidence of some place value knowledge.	The student correctly solves at least three of the problems showing reasoning through a place value strategy, or the student correctly answers four problems, only showing solid reasoning for three problems.	Student correctly answers all parts showing all work using area models, partial products, or the general method: a. 204 b. 1,484 c. 9,135 d. 30,170
3 4.NBT.5 4.OA.1 4.OA.2 4.OA.3	The student answers incorrectly with little attempt at solving the problem.	The student attempts to use an equation or model, resulting in an incorrect answer.	The student solves using an equation or model but results with an incorrect answer or answers correctly showing only some reasoning.	The student correctly answers 648 seats in an answer statement and uses an equation or model correctly to solve.

A Progression Toward Mastery

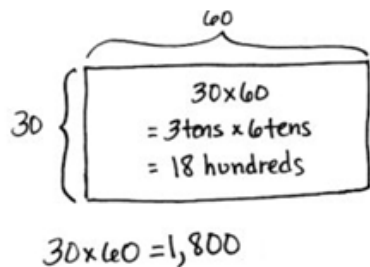
4 4.NBT.5 4.OA.1 4.OA.3	<p>The student answers incorrectly and provides little or no evidence of reasoning through estimation.</p>	<p>The student answers incorrectly but shows some evidence in reasoning through estimation.</p>	<p>The student correctly answers 1,092 boxes using a model or equation accurately, but is unable to clearly reason using estimation, or the student provides clear reasoning and attempt at solving but provides an incorrect answer.</p>	<p>The student correctly answers 1,092 boxes in an answer statement, uses an area model or equation to solve, and validates his answer is reasonable through estimation.</p>
5 4.NBT.5 4.OA.1 4.OA.2 4.OA.3 4.MD.3	<p>The student shows little to no reasoning and answers more than two parts incorrectly.</p>	<p>The student correctly answers two of four parts and shows little reasoning in Part (c) and little evidence of place value understanding.</p>	<p>The student answers three of the four parts correctly, answers all four parts correctly with unclear reasoning in Part (c), or doesn't show solid evidence of place value understanding in all solutions.</p>	<p>The student correctly answers:</p> <ol style="list-style-type: none"> $5 \text{ m} \times 4 \text{ m} = 20$ square meters; draws a rectangle; labels the width as 10 meters and length as 12 meters. 120 square meters; 26 meters; 208 plants; shows a written equation and reasons correctly through estimation.

Name Jack

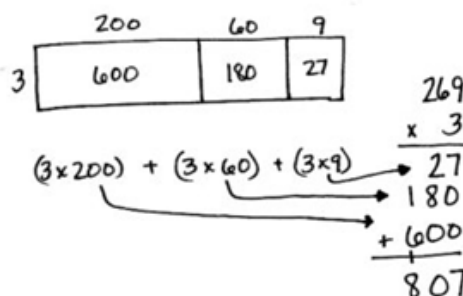
Date _____

1. Draw an area model to solve the following. Find the value of the following expressions.

a. 30×60



b. 3×269



2. Use any place value strategy to multiply.

a. 3×68

$$\begin{array}{r} 68 \\ \times 3 \\ \hline 24 \\ + 180 \\ \hline 204 \end{array}$$

b. 4×371

$$\begin{array}{r} 371 \\ \times 4 \\ \hline 4 \\ 28 \\ + 120 \\ \hline 152 \end{array}$$

c. $7 \times 1,305$

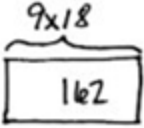
$$\begin{array}{r} 1305 \\ \times 7 \\ \hline 35 \\ 2100 \\ + 7000 \\ \hline 9,135 \end{array}$$

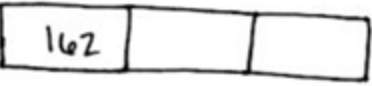
d. $6,034 \times 5$

$$\begin{array}{r} 6034 \\ \times 5 \\ \hline 20 \\ 150 \\ + 30,000 \\ \hline 30,170 \end{array}$$

Directions: Solve using a model or equation. Show your work and write your answer as a statement.

3. A movie theater has two rooms. Room A has 9 rows of seats with 18 seats in each row. Room B has three times as many seats as Room A. How many seats are there in both rooms?

A 

B 

} S

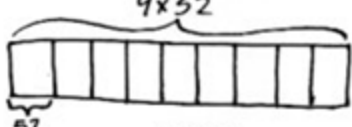
$$\begin{array}{r} 18 \\ \times 9 \\ \hline 162 \end{array}$$


$$\begin{array}{r} 162 \\ \times 3 \\ \hline 486 \end{array}$$

$$\begin{array}{r} 486 \\ + 162 \\ \hline 648 \end{array}$$

There are 648 seats in both rooms.

4. The high school art teacher has 9 cases of crayons with 52 boxes in each case. The elementary school art teacher has 6 cases of crayons with 104 boxes in each case. How many total boxes of crayons do both teachers have? Is your answer reasonable? Explain.

HS 

ES 

} B

$$10 \times 52 = 520$$

$$6 \times 100 = 600$$

$$600 + 520 = 1120$$

$$1,120 \text{ boxes is my estimate.}$$

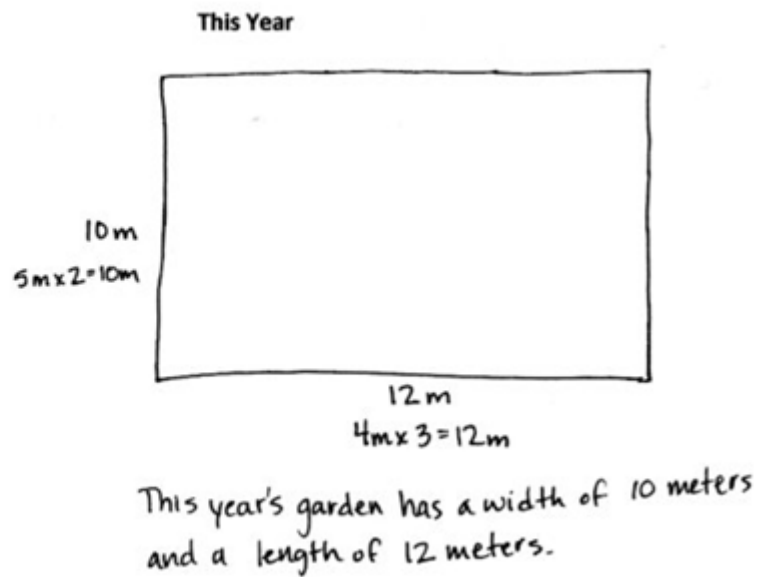
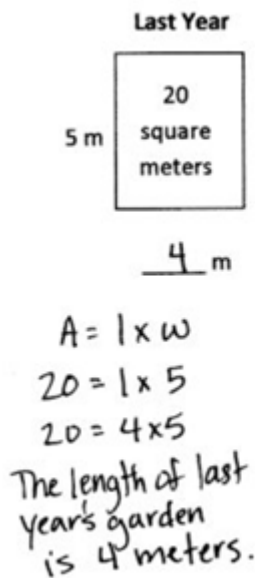
$$\begin{array}{r} 52 \\ \times 9 \\ \hline 468 \end{array}$$

$$\begin{array}{r} 104 \\ \times 6 \\ \hline 624 \end{array}$$

$$\begin{array}{r} 624 \\ + 468 \\ \hline 1,092 \end{array}$$

Both teachers have 1,092 boxes of crayons. My answer is reasonable because it is very close to my estimate of 1,120 boxes.

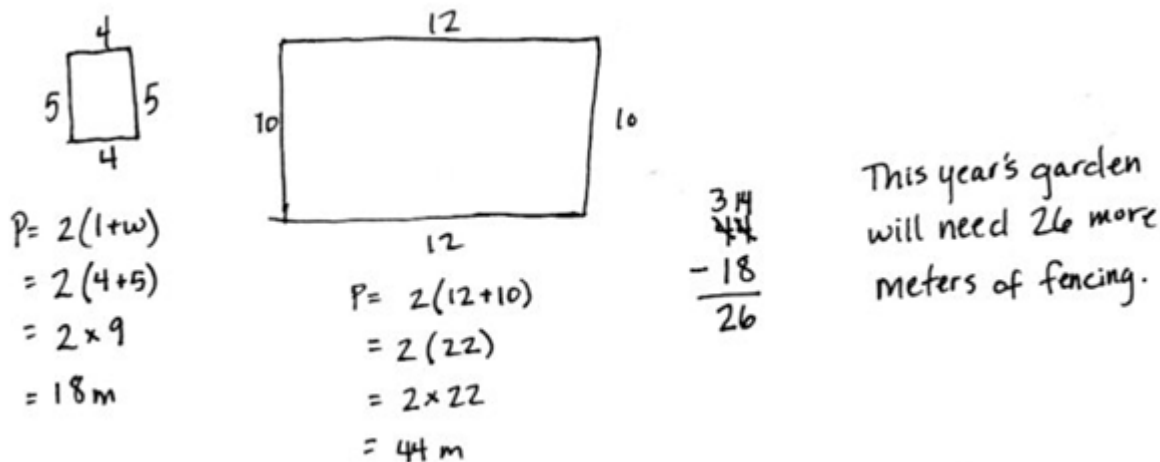
5. Last year Mr. Petersen's rectangular garden had a width of 5 meters and an area of 20 square meters. This year he wants to make the garden three times as long and two times as wide.
- a. Solve for the length of last year's garden using the area formula. Then, draw and label the measurements of this year's garden.



- b. How much area for planting will Mr. Petersen have in the new garden?

$A = l \times w$
 $= 12 \times 10$
 $= 120$
 Mr. Petersen's new garden has an area of 120 square meters.

- c. Last year, Mr. Petersen had a fence all the way around his garden. He can reuse all of the fence he had around the garden last year, but he needs to buy more fencing to go around this year's garden. How many more meters of fencing is needed for this year's garden than last year's?



$$P = 2(l + w)$$

$$= 2(4 + 5)$$

$$= 2 \times 9$$

$$= 18 \text{ m}$$

$$P = 2(12 + 10)$$

$$= 2(22)$$

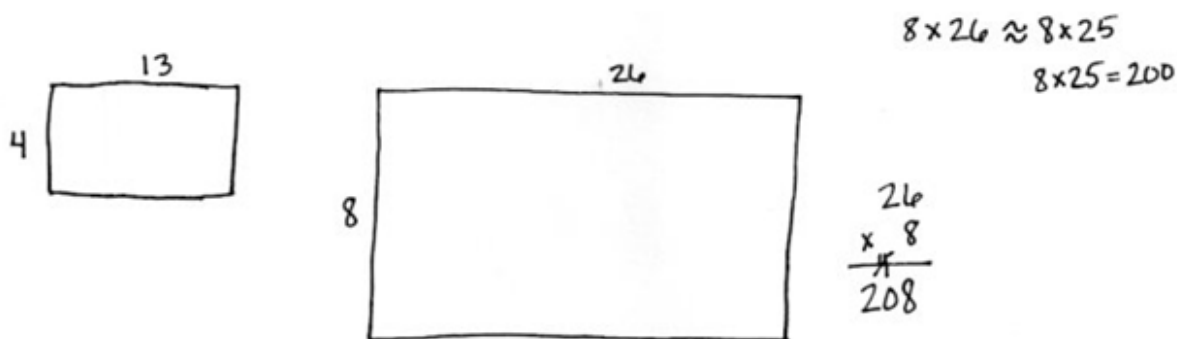
$$= 2 \times 22$$

$$= 44 \text{ m}$$

$$\begin{array}{r} 34 \\ 44 \\ - 18 \\ \hline 26 \end{array}$$

This year's garden will need 26 more meters of fencing.

- d. Last year Mr. Petersen was able to plant 4 rows of carrots with 13 plants in each row. This year he plans to plant twice as many rows with twice as many carrot plants in each. How many carrot plants will he plant this year? Write a multiplication equation to solve. Assess the reasonableness of your answer.



$$8 \times 26 \approx 8 \times 25$$

$$8 \times 25 = 200$$

$$\begin{array}{r} 26 \\ \times 8 \\ \hline 208 \end{array}$$

He will plant 208 carrot plants this year.
My answer is reasonable because it is very close to my estimate of 200 plants.

Name _____

Date _____

1. What is the greatest multiple of 7 that is less than 60?

2. Identify each number as prime or composite. Then list all of its factors.

a. 3 _____

b. 6 _____

c. 15 _____

d. 24 _____

e. 29 _____

3. Use any place value strategy to divide.

a. $3,600 \div 9$

b. 96 pencils come in a box. If 4 teachers share 3 boxes equally, how many pencils does each teacher receive?

4. $427 \div 3$

a. Solve by drawing number disks:

b. Solve numerically:

5. Use any place value strategy to multiply or divide.

a. $5,316 \div 3$

b. $3,809 \div 5$

c. 29×56

d. 17×43

Solve using a model or equation. Show your work and write your answer as a statement.

6. A new grocery store is opening next week.
- The store's rectangular floor is 42 meters long and 39 meters wide. How many square meters of flooring do they need? Use estimation to assess the reasonableness of your answer.
 - The store ordered small posters and large posters to promote their opening. Twelve times as many small posters were ordered as large posters. If there were 48 large posters, how many more small posters were ordered than large posters?

- c. Uniforms are sold in packages of 8. The store's 127 employees will each be given 3 uniforms. How many packages will the store need to order?
- d. There are 3 numbers for the combination to the store's safe. The first number is 17. The other 2 numbers can be multiplied together to give a product of 28. What are all of the possibilities? Write your answers as a multiplication equation.

End-of-Module Assessment Task Standards Addressed

Topics A–H

Use the four operations with whole numbers to solve problems.

- 4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- 4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
- 4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

- 4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1 – 100 is prime or composite.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

- 4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- 4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

- 4.MD.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student **CAN** do now, and what they need to work on next.

A Progression Toward Mastery				
Assessment Task Item	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 4.OA.4	The student answers incorrectly with a number that is not a multiple of 7.	The student answers incorrectly with a number that is a multiple of 7 but greater than 60.	The student answers with a multiple of 7 that is less than 60 but not 56.	The student correctly answers: The greatest multiple of 7 that is less than 60 is 56.
2 4.OA.4	The student is unable to complete the majority of Parts(a–e).	The student correctly answers prime or composite for at least three parts and misses more than three factors overall.	The student correctly answers prime or composite for four of the five parts and misses three or fewer factors overall.	The student correctly answers: <ul style="list-style-type: none"> a. prime; 1, 3 b. composite; 1, 2, 3, 6 c. composite; 1, 3, 5, 15 d. composite; 1, 2, 3, 4, 6, 8, 12, 24 e. prime; 1, 29

A Progression Toward Mastery

3 4.OA.3 4.NBT.5 4.NBT.6	The student incorrectly answers both parts and shows no reasoning.	The student correctly answers one part and shows little reasoning.	The student answers one part correctly but shows solid reasoning in both problems, or the student shows some reasoning with correct answers for both parts.	The student correctly answers using any place value strategy: a. 400 b. Each teacher received 72 pencils.
4 4.NBT.6	The student incorrectly represents division using number disks and incorrectly solves numerically.	The student incorrectly solves the numeric equation but shows some understanding of the place value chart and use of the algorithm.	The student decomposes incorrectly in one place value or does not interpret the remainder.	The student correctly decomposes and divides using the number disks and provides a numerical answer of 142 with a remainder of 1.
5 4.NBT.6	The student answers fewer than two parts correctly showing little to no evidence of place value strategies.	The student correctly solves two parts showing little evidence of place value strategies.	The student correctly solves three parts with understanding of place value strategies, or the student correctly solves all four parts but does not show solid evidence of place value understanding.	The student solves all parts correctly using any place value strategy: a. 1,772 b. 761 with a remainder of 4 c. 1,624 d. 731
6 4.MD.3 4.OA.1 4.OA.2 4.OA.3 4.NBT.5 4.NBT.6	The student incorrectly answers two or more of the four parts, showing little to no reasoning.	The student correctly answers two of four parts, showing some reasoning.	The student answers all four parts correctly but shows little reasoning in Part (a), or the student answers three of four parts correctly showing solid reasoning and understanding mathematically.	The student correctly answers all four parts showing solid evidence of place value understanding: a. 1,638 square meters of flooring (estimate $40 \times 40 = 1,600$ square m). It is a reasonable because the answer and estimate have a difference of only 38 square

A Progression Toward Mastery

- meters.
- b. 528 more small posters than large posters.
 - c. 48 packages.
 - d. The possibilities are
 - $1 \times 28 = 28$
 - $28 \times 1 = 28$
 - $2 \times 14 = 28$
 - $14 \times 2 = 28$
 - $4 \times 7 = 28$
 - $7 \times 4 = 28$

Name Jack

Date _____

1. What is the greatest multiple of 7 that is less than 60?

7, 14, 21, 28, 35, 42, 49, 56, 63

56 is the greatest multiple of 7 that is less than 60.

2. Identify each number as prime or composite. Then list all of its factors.

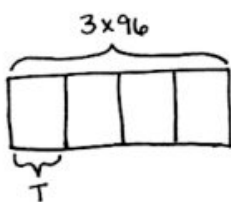
- a. 3 prime 1, 3
- b. 6 composite 1, 2, 3, 6
- c. 15 composite 1, 3, 5, 15
- d. 24 composite 1, 2, 3, 4, 6, 8, 12, 24
- e. 29 prime 1, 29

3. Use any place value strategy to divide.

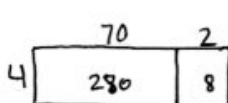
- a.
- $3,600 \div 9$

$$36 \text{ hundreds} \div 9 = 4 \text{ hundreds} \\ = 400$$

- b. 96 pencils come in a box. If 4 teachers share 3 boxes equally, how many pencils does each teacher receive?



$$\begin{array}{r} 96 \\ \times 3 \\ \hline 288 \end{array}$$

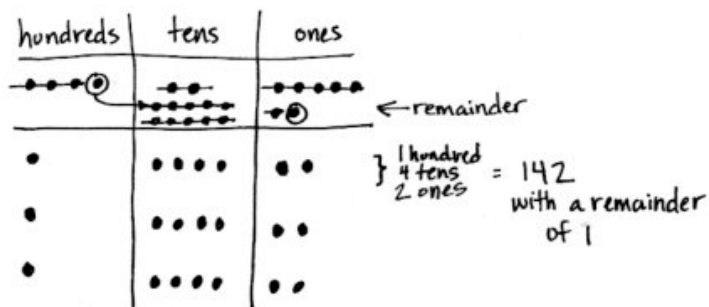


$$\begin{array}{r} 70 \\ 4 \overline{) 288} \\ \underline{-280} \\ 8 \\ \underline{-8} \\ 0 \end{array}$$

$70 + 2 = 72$
Each teacher
receives 72
pencils.

4. $427 \div 3$

a. Solve by drawing number disks:



b. Solve numerically:

$$\begin{array}{r} 142 \text{ R}1 \\ 3 \overline{)427} \\ \underline{-3} \\ 12 \\ \underline{-12} \\ 07 \\ \underline{-6} \\ 1 \end{array}$$

$$\begin{array}{r} \checkmark 142 \\ \times 3 \\ \hline 426 \\ 426 + 1 = 427 \end{array}$$

5. Use any place value strategy to multiply or divide.

a. $5316 \div 3$

$$\begin{array}{r} 1772 \\ 3 \overline{)5316} \\ \underline{-3} \\ 23 \\ \underline{-21} \\ 21 \\ \underline{-21} \\ 06 \\ \underline{-6} \\ 0 \end{array}$$

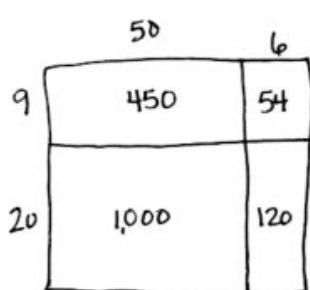
$$\begin{array}{r} \checkmark 1,772 \\ \times 3 \\ \hline 5,316 \end{array}$$

b. $3,809 \div 5$

$$\begin{array}{r} 761 \text{ R}4 \\ 5 \overline{)3809} \\ \underline{-35} \\ 30 \\ \underline{-30} \\ 09 \\ \underline{-5} \\ 4 \end{array}$$

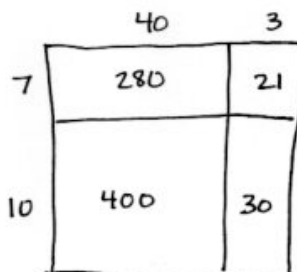
$$\begin{array}{r} \checkmark 761 \\ \times 5 \\ \hline 3,805 \\ 3,805 + 4 = 3,809 \end{array}$$

c. 29×56



$$\begin{array}{r} 56 \\ \times 29 \\ \hline 504 \\ 1000 \\ \hline 1,624 \end{array}$$

d. 17×43

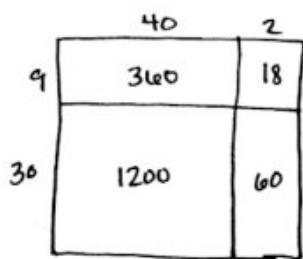


$$\begin{array}{r} 43 \\ \times 17 \\ \hline 21 \\ 280 \\ \hline 731 \end{array}$$

Directions: Solve using a model or equation. Show your work and write your answer as a statement.

6. A new grocery store is opening next week.

- a. The store's rectangular floor is 42 meters long and 39 meters wide. How many square meters of flooring do they need? Use estimation to assess the reasonableness of your answer.



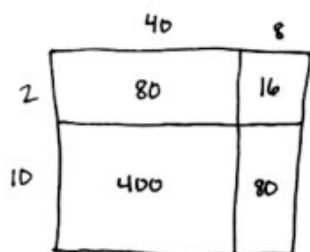
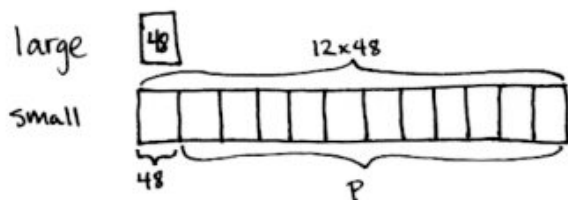
$$\begin{array}{r} 42 \\ \times 39 \\ \hline 18 \\ 360 \\ 60 \\ + 1200 \\ \hline 1,638 \end{array}$$

They need 1,638 square meters of flooring. My answer is reasonable because it is close to my estimate of 1,600 square meters.

$$42 \times 39 \approx 40 \times 40$$

$$40 \times 40 = 1,600$$

- b. The store ordered small posters and large posters to promote their opening. 12 times as many small posters were ordered as large posters. If there were 48 large posters, how many more small posters were ordered than large posters?

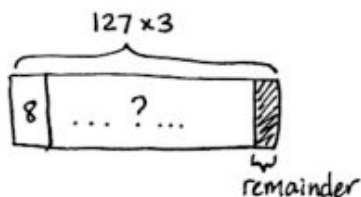


$$\begin{array}{r} 48 \\ \times 12 \\ \hline 16 \\ 80 \\ 80 \\ + 400 \\ \hline 576 \end{array}$$

$$\begin{array}{r} 576 \\ - 48 \\ \hline 528 \\ P = 528 \end{array}$$

528 more small posters were ordered than large posters.

- c. Uniforms are sold in packages of 8. The store's 127 employees will each be given 3 uniforms. How many packages will the store need to order?



$$\begin{array}{r} 127 \\ \times 3 \\ \hline 381 \end{array}$$

$$\begin{array}{r} 47 \text{ R } 5 \\ 8 \overline{) 381} \\ \underline{-32} \\ 61 \\ \underline{-56} \\ 5 \end{array}$$

$$\begin{array}{r} \checkmark 47 \\ \times 8 \\ \hline 376 \end{array}$$

$$376 + 5 = 381$$

The store needs to order 48 packages. If they order 47 packages, only 376 uniforms will come and they will need 5 more uniforms.

- d. There are 3 numbers for the combination to the store's safe. The first number is 17. The other 2 numbers can be multiplied together to give a product of 28. What are all of the possibilities? Write your answers as a multiplication equation.

$$\begin{aligned} 28 &= 1 \times 28 \\ 28 &= 28 \times 1 \\ 28 &= 2 \times 14 \\ 28 &= 14 \times 2 \\ 28 &= 4 \times 7 \\ 28 &= 7 \times 4 \end{aligned}$$

The combination possibilities are:

$$\begin{aligned} &17, 1, 28 \\ &17, 28, 1 \\ &17, 2, 14 \\ &17, 14, 2 \\ &17, 4, 7 \\ &17, 7, 4 \end{aligned}$$