8.2 Arithmetic Sequences

An **arithmetic sequence** is a sequence in which each term differs by a constant amount. This difference between two terms that are right next to each other is called the **common difference**, d. This is found by subtracting any two consecutive terms.

EXAMPLE: Find the common difference, d: -8, -3, 2, 7, 12, …

To find the difference, subtract any two terms that are next to each other. d = 12 – 7 = 5. So the common difference is 5. Notice that it does not matter which two terms you pick; the common difference is still 5.

EXAMPLE: Write the first 6 terms of the arithmetic sequence in which $a_1 = 100$ and $d = 12$.

$a_1 = 100$	To find the second term, a_2, add d to the first term.
$a_2 = 100 + 12 = 112$	To find the third term, a_3, add d to the second term.
$a_3 = 112 + 12 = 124$	To find the fourth term, a_4, add d to the third term.
$a_4 = 124 + 12 = 136$	To find the fifth term, a_5, add d to the fourth term.
$a_5 = 136 + 12 = 148$	To find the sixth term, a_6, add d to the fifth term.
$a_6 = 148 + 12 = 160$	

EXAMPLE: Write the first 6 terms of the arithmetic sequence in which $a_1 = 8$ and $d = -3$.

$a_1 = 8$	To find the second term, a_2, add d to the first term.
$a_2 = 8 + (-3) = 5$	To find the third term, a_3, add d to the second term.
$a_3 = 5 + (-3) = 2$	To find the fourth term, a_4, add d to the third term.
$a_4 = 2 + (-3) = -1$	To find the fifth term, a_5, add d to the fourth term.
$a_5 = -1 + (-3) = -4$	To find the sixth term, a_6, add d to the fifth term.
$a_6 = -4 + (-3) = -7$	

EXAMPLE: Write the first 6 terms of the arithmetic sequence in which $a_n = a_{n-1} + 6$ and $a_1 = 7$.

$a_1 = 7$

Now let's find a_2. Put in a 2 for n in the recursion formula: $a_2 = a_{2-1} + 6$. This tells us $a_2 = a_1 + 6$. Since we are given $a_1 = 7$, plug this into our formula: $a_2 = 7 + 6$, so $a_2 = 13$.
Now let's find a_3. Put in a 3 for n in the recursion formula: $a_3 = a_{3-1} + 6$. This tells us $a_3 = a_2 + 6$. Since we found $a_2 = 13$, plug this into our formula: $a_3 = 13 + 6$, so $a_3 = 19$.
Now let's find a_4. Put in a 4 for n in the recursion formula: $a_4 = a_{4-1} + 6$. This tells us $a_4 = a_3 + 6$. Since we found $a_3 = 19$, plug this into our formula: $a_4 = 19 + 6$, so $a_4 = 25$.
Now let's find a_5. Put in a 5 for n in the recursion formula: $a_5 = a_{5-1} + 6$. This tells us $a_5 = a_4 + 6$. Since we found $a_4 = 25$, plug this into our formula: $a_5 = 25 + 6$, so $a_5 = 31$.
Now let's find a_6. Put in a 6 for n in the recursion formula: $a_6 = a_{6-1} + 6$. This tells us $a_6 = a_5 + 6$. Since we found $a_5 = 31$, plug this into our formula: $a_6 = 31 + 6$, so $a_6 = 37$.

General Term of an Arithmetic Sequence

Suppose you wanted to find the 100th term of an arithmetic sequence and you don't want to write out all the terms. You can find the nth term of a sequence using the below formula. Here a_1 is the first term and d is the common difference:

$$a_n = a_1 + (n-1)d$$

EXAMPLE: Use the formula for the general term (the nth term) of an arithmetic sequence to find the sixth term of the sequence with $a_1 = 3$, and $d = 8$.

To find the sixth term, a_6, we know that n = 6. We are also given $a_1 = 3$, and $d = 8$. Plug all of these into the formula: $a_n = a_1 + (n-1)d$. You will get: $a_6 = 3 + (6-1)(8)$. So $a_6 = 3 + (5)(8) = 43$.

EXAMPLE: Use the formula for the general term (the nth term) of an arithmetic sequence to find the sixth term of the sequence with $a_1 = 11$, and $d = -3$.

To find the sixth term, a_6, we know that n = 6. We are also given $a_1 = 11$, and $d = -3$. Plug all of these into the formula: $a_n = a_1 + (n-1)d$. You will get: $a_6 = 11 + (6-1)(-3)$. So $a_6 = 11 + (5)(-3) = -4$.

EXAMPLE: Write a formula for the general term (the nth term) of the given arithmetic sequence. Then use the formula for a_n to find a_{20}, the 20th term of the sequence: 6, 3, 0, -3, …

From this sequence, we can see the first term is 6, so we know $a_1 = 6$. If you subtract any two consecutive terms, you will find that $d = -3$. To find the twentieth term, a_{20}, we know that n = 20. Plug all of these into the formula: $a_n = a_1 + (n-1)d$. You will get: $a_{20} = 6 + (20-1)(-3)$. So $a_{20} = 6 + 19(-3) = -51$.

EXAMPLE: Write a formula for the general term (the nth term) of an arithmetic sequence in which $a_1 = 17$ and $d = 3$. Do not use a recursion formula. Then use the formula for a_n to find a_{20}, the 20th term of the sequence.

We are still going to start with the general formula for arithmetic sequences: $a_n = a_1 + (n-1)d$. We will plug in 17 for a_1 and 3 for d. You will get: $a_n = 17 + (n-1)(3)$. Now that we have the formula, we need to find the 20th term by putting in a 20 for n: $a_{20} = 17 + (20-1)(3)$. This gives $a_{20} = 17 + 19(3) = 74$.

Sum of the First n Terms of an Arithmetic Sequence

$S_n = \frac{n}{2}(a_1 + a_n)$ This give the sum of the first n terms where a_1 is the first term and a_n is the last term.

EXAMPLE: Find the sum of the first 20 terms of the sequence: 2, 12, 22, 32,…

We know that $a_1 = 2$. We need to find the last term. In order to do this, we need to use $a_n = a_1 + (n-1)d$. From our sequence, we see that d = 10 since that is the difference between and two consecutive terms. For the twentieth term, n = 20. Now plug all of these into the formula: $a_{20} = 2 + (20-1)(10)$. This gives $a_{20} = 2 + 19(10) = 192$.

So now since we know $a_1 = 2$, n = 20, and $a_{20} = 192$, we can put these into the arithmetic sum formula: $S_n = \frac{n}{2}(a_1 + a_n)$. You will get: $S_{20} = \frac{20}{2}(2 + 192)$. This simplifies into: $S_{20} = 10(194) = 1940$.

EXAMPLE: Write out the first three terms and the last term. Then use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum: $\sum_{i=1}^{16} 2i - 5$

When i = 1 our expression is $2(1) - 5 = -3$. So $a_1 = -3$.
When i = 2, our expression is $2(2) - 5 = -1$. So $a_2 = -1$.
When i = 3, our expression is $2(3) - 5 = 1$. So $a_3 = 1$.

When i = 16, our expression is $2(16) - 5 = 27$. So $a_{16} = 27$.

So now since we know $a_1 = -3$, n = 16, and $a_{16} = 27$, we can put these into the arithmetic sum formula: $S_n = \frac{n}{2}(a_1 + a_n)$. You will get: $S_{16} = \frac{16}{2}(-3 + 27)$. This simplifies into: $S_{20} = 8(24) = 192$.