APPENDIX A: ENGINEERING REPORT

ABSTRACT: The objective of this study is to gain a structural understanding of the Burr arch-truss, specifically as found in the Pine Grove Bridge. The scope of the study involves first-order linear elastic analysis of the truss, but does not include analysis of specific connections. From our research we found that the loading of the arch can be as much as three times greater than the truss, as the arch is more efficient in carrying dead load, and the truss provides necessary bending rigidity during concentrated live loads. Maximum stresses are found to occur at the springing of the arch, and no elements are overstressed by current design standards. Based on this we conclude that in the Pine Grove Bridge the arch is structurally dominant, and the truss provides necessary reinforcement under large concentrated live loads.

AUTHORS: Dylan Lamar, HAER Engineering Technician, Summer 2002, and Benjamin W. Schafer, Ph.D., Assistant Professor of Civil Engineering, Johns Hopkins University.

INTRODUCTION

The Pine Grove Bridge, which crosses Octoraro Creek on the Chester and Lancaster county line in southeastern Pennsylvania, is an excellent example of timber engineering. Constructed in 1884 by Civil War veteran, Captain Elias McMellen, the bridge consists of two spans of approximately 90 feet each, both framed with the Burr arch-truss, as shown below in Figure 1. Although the wooden Burr arch-truss was arguably outdated in terms of contemporary 1880s bridge technology for spans of this length, it was a beautiful example of one of the most popular wooden bridge forms. The Burr arch-truss certainly deserved its popularity, as many Burr arch-trusses of the 19th century are still carrying vehicular traffic today.

This report focuses on the significant engineering aspects of the Pine Grove Bridge. The bridge's historical context, in terms of engineering technology, will first be explored, followed by a brief discussion of design and construction methods of the period and an overall structural analysis of the Burr arch-truss form as found in the Pine Grove Bridge. Of particular interest in this case is the dual nature of the Burr arch-truss as both a truss and an arch system. The analysis was structured to determine which system, if any, is dominant and to what extent the two systems enhance one another. The final section will examine at the bridge's camber and the later addition of steel ties to the bridge.

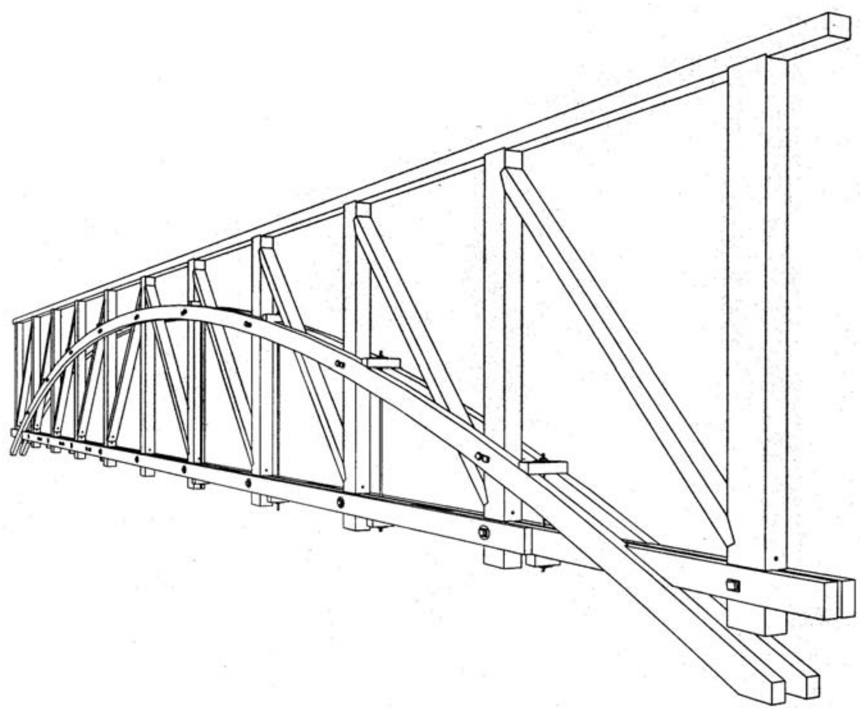

Figure 1. Pictorial View of the Burr arch-truss at Pine Grove.[1]

[1] U.S. Department of the Interior, Historic American Engineering Record (HAER) No. PA-586, Architectural Drawings: "Pine Grove Bridge," 2002. Prints and Photographs Division, Library of Congress, Washington, D.C. (Drawing on the 'Conclusion' page also from this source).

HISTORICAL CONTEXT

Theodore Burr patented his arch-reinforced truss in 1817 (Figure 2). However, the idea of arch reinforcement of a truss was not new, as J. G. James points out in his 1982 article, “The Evolution of Wooden Bridge Trusses to 1850.”[2] Prior to Burr’s patent, wooden bridges were often given additional stiffness through the addition of an arch. Indeed, Palladio, in the 16th century, published diagrams of wooden arched bridges, probably derived from those that had existed in central Europe long before his time. Switzerland and Germany, being rich in timber resources, produced many of the earliest wooden covered bridges. France and England also contributed to the development of wooden bridges, although the stone building legacy of the Romans continued to dominate. James notes that in 1764 a wooden bridge was constructed in Switzerland with “trusses [consisting] of rectangular frames supported by massive arched ribs.”[3] A description that sounds theoretically much like Burr’s design. Whether or not any of these bridges actually originated the arch-truss, it is Burr who receives popular credit for the design to this day.

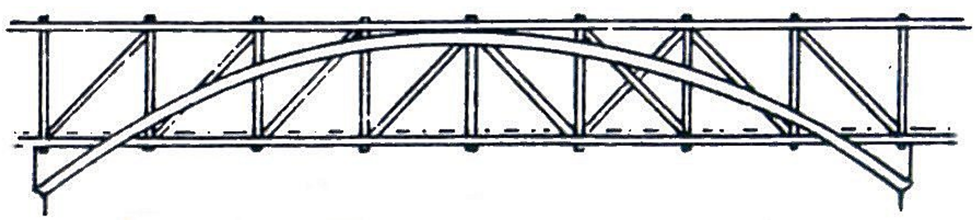

Figure 2. Theodore Burr's Patented Truss (Counter-Brace Optional).[4]

The Roman legacy of the arch form and its value as a structural element was a strong influence on European practice. The concept of each element of the arch being locked in compression under dead load was an easy one for early builders to grasp, although it appeared more suited to stone construction. Applying the arch to wood construction was a more ambiguous endeavor. Since wood existed in longitudinal lengths, which were strong in both tension and compression, it was a completely different material from stone. As a result of its unique properties, the truss system of construction developed over the ages. Based on the unique geometrical rigidity of the triangle (versus the flexibility of other shapes; see Figure 3), trusses were an ideal means of framing wooden spans, such as roofs. When applied to bridges, a truss system, which acted much like a deep beam, often was adequate, however, as strength and stiffness requirements increased for longer spans, designers sought to combine the concepts of the arch and truss into an ideal form that would utilize the strengths of both.

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[2] J. G. James, “The Evolution of Wooden Bridge Trusses to 1850,” *Journal of the Institute of Wood Science* 9 (June 1982): 116-135; (December 1982): 168-193.

[3] James, 124.

[4] Figures 2, 4, 5, and 6 were taken directly from James’ article, 170, 173.

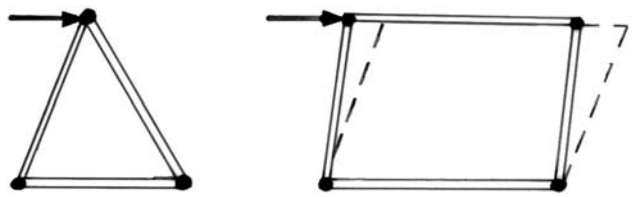

Figure 3. Rigidity of a Triangle Compared to Flexibility of Other Geometries.

Wooden bridge development enjoyed a resurgence with the expansion of the American frontier in the late 18th and early 19th centuries, due to the numerous rivers to be crossed and the abundant forests. Some of the new ideas from this time provide a direct link to the ideas of Burr and his arch-truss. As James describes, in the 1790s Timothy Palmer designed bridges that "did not use arched reinforcing ribs but … made the whole truss into an arch by giving it a generous longitudinal camber."[5] (Figure 4) By the time he built his last bridge in 1806, Palmer had developed a flat deck which ran through the truss and he simply formed the lower chord into an arch (Figure 5), forming a design that visually approached Burr's arch-truss.

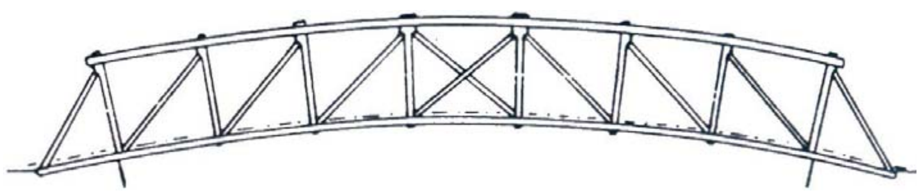

Figure 4. Early Palmer Truss.

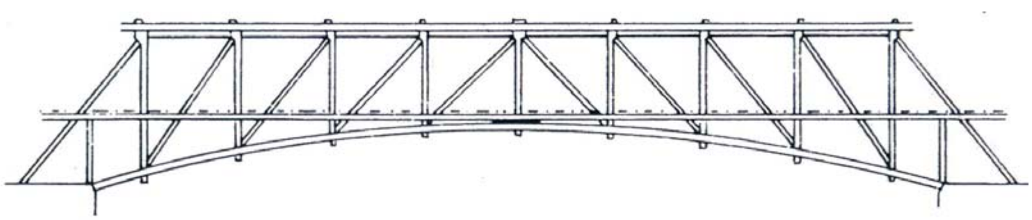

Figure 5. Last Palmer Truss.

When Theodore Burr began his bridge-building career in 1803, he "took the below-deck arch ribs of Palmer's last bridges and carried them up to the top chord in the Swiss manner" which left the truss form horizontal.[6] After several other experiments, it was this basic form, which he patented in 1817, that is now known as the Burr arch-truss,

[5] James, 169.
[6] James, 171.

or simply the Burr truss (Figure 2). The design is an integration of the truss and arch systems—two arch ribs sandwich the truss, providing further bracing for the truss, and the truss simultaneously provides stability for the arch. Which system is the dominant structural system, however, has long been a subject of debate. Nevertheless, the Burr arch-truss proved to be a most reliable and one of the most popular wooden bridge forms. As James describes, "the tried and trusted Burr truss was naturally used on many early railroads."[7]

After Burr's patent, other builders modified his arch-truss design. Lewis Wernwag installed iron ties between the arch and lower chord "for additional support."[8] In addition to increasing stability, these added a safety mechanism should the tension connections of the posts and lower chords fail. Wernwag also increased construction efficiency in his 1829 patent by calling for bolted connections instead of elaborately hewn wooden joinery.[9] Despite this simplification, the Burr arch-truss remained a relatively complicated design.

Another drawback of the Burr arch-truss was that it required large timbers, which were certainly more expensive than the small planks of Ithiel Town's lattice truss (Figure 6)–a competing structural form for wooden bridges of similar span lengths. Patented in 1820, Town's lattice truss gained popularity for its economy and simple construction technique. The design used smaller timbers and simple connections to "[minimize] the use of complicated timber joinery."[10] Thus, it was often a cheap and easy solution for bridge builders.

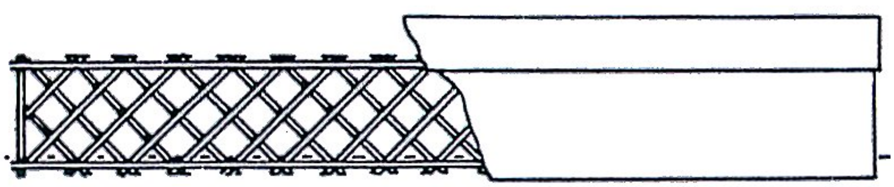

Figure 6: Town's Lattice Truss

While designers such as Stephen Harriman Long, William Howe, and others continued to introduce structural advancements throughout the 1800s, the Burr arch-truss remained a reliable form for wooden bridges that saw use as late as 1922.[11] Of the forty-five wooden covered bridges that today exist in Chester and Lancaster Counties,

[7] James, 175.
[8] James, 171.
[9] James, 172.
[10] Phillip C. Pierce, "Those Intriguing Town Lattice Timber Trusses," *Practice Periodical on Structural Design and Construction* 3 (August 2001): 92-94.
[11] Richard Sanders Allen, *Covered Bridges of the Middle West* (Brattleboro, VT: Stephen Greene Press, 1970), 130.

Pennsylvania, all but one are Burr-arch trusses.[12] This speaks for the strong heritage of the form in this area.

One of the most successful bridge builders in this area was Captain Elias McMellen. McMellen began his bridge building career in 1859, and by 1884, the year Pine Grove Bridge was erected, he was well established in the practice, having built at least twenty-three covered bridges. He predominantly built wooden covered bridges of the Burr arch-truss type, but he also has three stone bridges to his name. (There is, coincidentally, a stone arch in the approach to the Pine Grove Bridge, though it is not now known if it is original or built by McMellen.)[13] The selection of the Burr arch-truss type for Pine Grove was probably a combination of the area's heritage in Burr arch-truss bridges and Captain McMellen's extensive experience building them.

Today the Pine Grove Bridge is equipped with steel ties, which run from the arch to the lower chord (see Figure 1), recalling Wernwag's improvements. However, these steel ties were added to Pine Grove simultaneously with the other Burr arch-trusses of Lancaster County in 1935,[14] so they are not original and do not represent the intentions of McMellen. Since they do now exist, however, their effects are addressed in Section 5.2.

The Pine Grove Bridge and other Burr arch-trusses across the country are the result of centuries of worldwide engineering efforts to find a practical method to span intermediate distances using timber. Certainly, credit must be given to Theodore Burr, who formalized a practical combination of the truss and arch forms in his 1817 patent. Consequently, over sixty years later Captain Elias McMellen, certain of Burr's design through years of practice, decided to apply the design to bridge Octoraro Creek in southeastern Pennsylvania. Time has proven the value of their work.

DESIGN AND CONSTRUCTION PRACTICES IN 1884

When studying historic engineering structures, a question that arises in every engineer's mind is, "how exactly did they do it back then?" In the case of the Pine Grove Bridge, McMellen presumably relied on his extensive experience in Burr arch-truss bridge building to guide his design.

Design based on scientific engineering calculations steadily grew in popularity during the nineteenth century. Claude-Louis Navier developed one of the earliest methods of analyzing truss forms in 1826.[15] The method was based on the analogy of treating a truss as a simple, pin-supported beam. Navier's procedure reliably estimated the stresses in the chords of trusses and began to be used in the United States in the

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[12] Conclusion based on: Allen, 108, 111; "Covered Bridges of Chester County," http://william-king.www. drexel.edu/top/bridge/CBChes.html; "Covered Bridges of Lancaster County," http://www.co.lancaster.pa. us/lanco/cwp/view.asp?a=15&Q=257050.

[13] Elizabeth Gipe Caruthers, "Elias McMellen, Forgotten Man," *Journal of the Lancaster County Historical Society* 85 (1981): 16-29. (All preceding information on McMellen taken from this source)

[14] Report of Chester County Engineer (from Chester County Archives and Records Office, 1935), 2.

[15] D.A. Gasparini and Caterina Provost, "Early Nineteenth Century Developments in Truss Design in Britain, France and the United States," *Construction History—Journal of the Construction History Society* 5 (1989): 22.

1830s. Later, with Squire Whipple and Herman Haupt's publications on truss analysis, in 1847 and 1851 respectively, more advanced methods of analysis became possible.[16] However, Whipple and Haupt's methods were only useful for relatively simple, "statically determinant" structures, in which the internal forces in members depend only on the geometric location of the members, and not on each member's stiffness.[17]

Burr arch-trusses contain an arch, and as a result are not simply trusses, but a more complex system that is "statically indeterminate." Many members are connected to others at more than two places and, thus, "share" forces in a manner that is dependent on both the geometry and stiffness of the connected elements. Accurate methods of analysis for determining the forces in statically indeterminate structures were just being developed in 1864 by James Clerk Maxwell,[18] but the complexity of the Burr arch-truss meant that a thorough analysis of one remained a formidable undertaking.

It is improbable that McMellen bothered with analytical procedures at all. After all, he was a builder, not an engineer.[19] The task of a hand analysis and subsequent sizing of members from this data would have taken much effort, and perhaps been no more accurate, than a conservative choice based on previous examples he had built. To summarize the comments of a trained engineer in 1895, when a skillful carpenter worked with a certain truss over a course of years, he gradually refined the sizing of the members to the precise size suggested by engineering calculation.[20] His reasoning being that timber as a material shows obvious signs of distress when it is overloaded, whereas cast iron, for instance, gives little evidence of distress until it ruptures. From this type of empirical evidence, McMellen would have known which members were in tension, which were in compression, and also which of those members were critical in the design. Member sizing was most likely accomplished, then, from knowledge of past examples of Burr arch-trusses in the area that worked, and perhaps from those that didn't as well.

Another more obvious manner in which designers gained understanding of their bridges was from their short- and long-term deflections and general stiffness. As noted in a previous engineering study of a Burr arch-truss, "The deflections and stiffness of the structure could be studied by a careful observer. One could see deflections and 'feel' the bridge move under live loads."[21]

In addition, there is geometric evidence in McMellen's design to suggest that construction concerns had priority over structural rationalization. For example, while the upper chord of the truss typically carried higher loads than the lower chord because of the arch, the lower chord was composed of two parallel members whose total area was over twice the area of the upper chord. The reason for this incongruity seems to stem from the construction methods. The tension connections that splice members of the lower chord were quite inefficient. As seen in Figure 7, only one of the two, parallel, lower-chord

[16] Stephen P. Timoshenko, *History of Strength of Materials* (New York: Dover, 1953), 185.

[17] Stephen P. Timoshenko, *History of Strength of Materials* (New York: Dover, 1953), 185.

[18] Russell C. Hibbeler, *Structural Analysis* 4th ed. (Upper Saddle River, NJ: Prentice Hall, 1999), 353.

[19] Caruthers, 16.

[20] Jonathan Parker Snow, "Wooden Bridge Construction on the Boston and Maine Railroad," *Journal of the Association of Engineering Societies* (July 1985).

[21] Emory L. Kemp and John Hall, "Case Study of Burr Truss Covered Bridge," *Issues in Engineering, Journal of Professional Activities* 100-101 (July 1975): 410.

members was spliced at any single location so that the extra, continuous member provided a factor of safety against the joint's uncertain capacity.

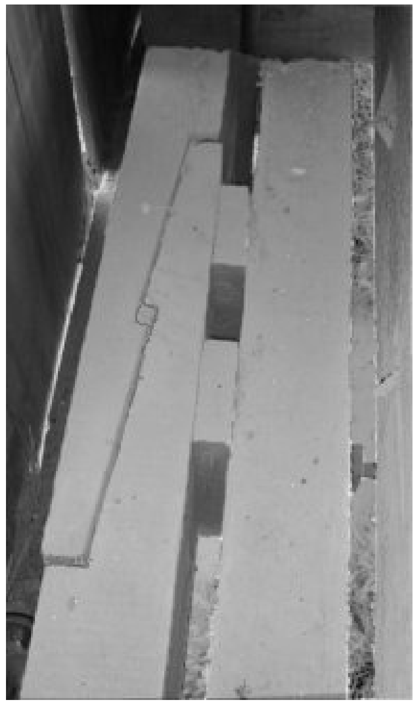

Figure 7. Top View of a Scarf Joint in the Lower Chord of Pine Grove Bridge.

Since the joints between the posts and the bottom chord were tension connections, the gap between the parallel bottom chord members allowed the posts to be notched to fit between the chords and thereby transmit vertical tensile forces directly to the underside of the chords (Figure 8).

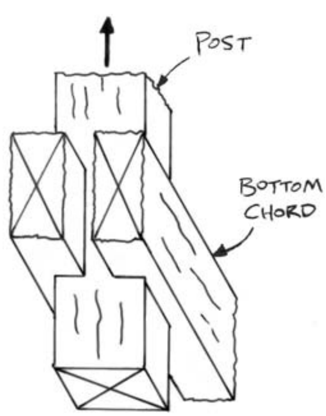

Figure 8. Typical Connection Between Notched Post and Bottom Chords.

Another example of the priority of constructional over structural necessity occurs in the center post. All of the vertical posts are of uniform cross-sectional area except for the center post, which is slightly larger. While the stresses in the center post are typically no larger than those of the other posts, the larger size is necessitated by construction. At this post the diagonals meet at a common point (since the center post is the symmetry line of the span). Since these connections required by notching into the post on both sides, the larger size was needed to maintain adequate material and strength in the critical section adjacent to the notches (Figure 9).

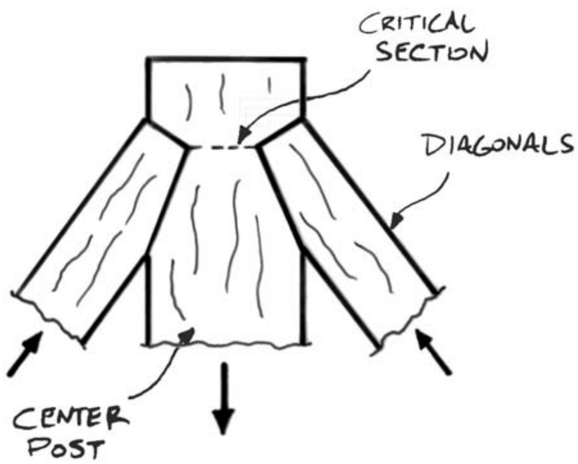

Figure 9. Notching of Center Post from Both Sides.

McMellen used uniform section sizes for all other similar members of the bridge, regardless of the forces the different members carried. This, too, speaks for the priority of constructional efficiency over structural efficiency. (This is still a common practice, as in the uniform sizing of a building or bridge's columns.) One curiosity to the Pine Grove Bridge truss is that the end panels are one-and-a-half inches narrower than the others. This could be due to McMellen's recognition that stresses in the diagonals are often largest in the end panels, and decreasing the panel size would increase the diagonal's stability, but it is more likely a result of some unknown constructional convenience, since one-and-a-half inches of width would not decrease the stress in the diagonals by any significant amount.

When Captain Elias McMellen erected the Pine Grove Bridge in 1884, it seems clear he trusted his years of bridge-building experience far more than any formal structural analysis. Complex as it was, a successful Burr arch-truss proved to be more easily realized through the wisdom of experience than through technical calculations.

STRUCTURAL BEHAVIOR CALCULATIONS

The arch-truss of Pine Grove Bridge, as shown in Figure 10, was modeled and analyzed using MASTAN2, a structural analysis computer program, assuming linear-elastic behavior.[22] The bridge's geometry, using centerlines of the members measured directly from the bridge in its current state, and section and material properties were entered into the program.[23] The labeling system used is shown in Figure 11. Panels were labeled A to E from center to ends, and the panel points were labeled 1 through 6 in the same manner. An "L" or "R," corresponding to the left or right side of the truss, was added where such designation was necessary, primarily in analyses involving quarter-point live loads that generated unsymmetrical forces throughout the bridge.

Figure 10. Arch-Truss Construction of the Pine Grove Bridge.

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[22] MASTAN2, version 1.0, developed by Ronald D. Ziemian and William McGuire, 2000
[23] See Appendix A.

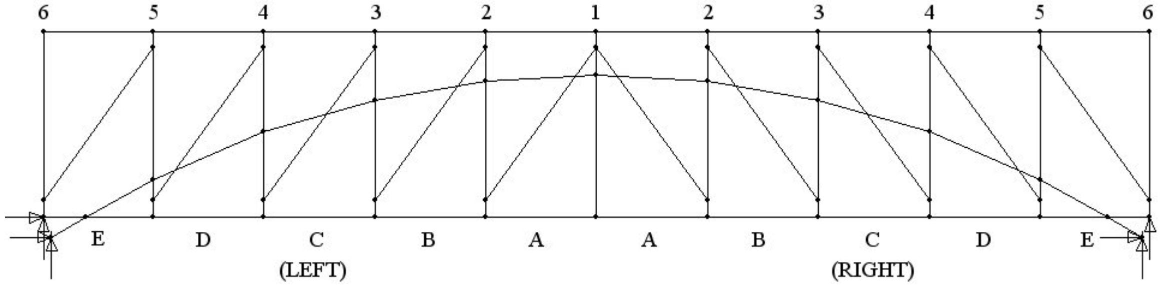
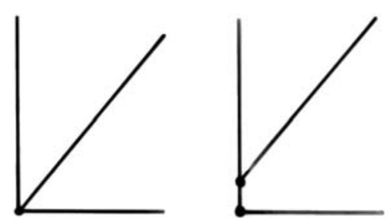


Figure 11. Center-line Model of Arch-Truss with Labeling System.

Of particular note in the model is the location of the ends of the diagonals. While an earlier analysis of a typical Burr arch-truss conducted by Emory L. Kemp and John Hall carried the diagonals to the corners where the posts and chords meet,[24] the diagonals of the Pine Grove Bridge actually meet the posts with their centerlines some distance from the post-chord intersections. This was replicated in the model (though the posts are modeled as continuous through these points) as shown in Figure 12 below, in an effort to achieve more accurate shear and moment values at these locations.



Kemp-Hall Pine Grove

Figure 12. Difference in Diagonal Termination from a Previous Study.

The exact species of wood used for the Pine Grove Bridge is not known. Though rehabilitation work in 1977 used Douglas Fir to replace some truss members,[25] it is unlikely this was originally used since this species does not grow east of the Rocky Mountains. A historical account from 1923 mentions white pine being used in the bridges of the area, though it speaks with unknown authority.[26] Timber specialist, Jan Lewandowski believes it likely that a softwood such as Eastern Hemlock was used, but testing or inspection resources to make a sure determination were not available.

Approximate properties of Eastern Hemlock and Eastern White Pine were obtained from the Forest Products Laboratory.[27] For this model, the most important

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[24] Kemp and Hall, 407.

[25] John Ebersole, Pennsylvania Department of Transportation, interview by author, phone conversation, July 2002.

[26] D.F. Magee, "The Old Wooden Covered Brides of the Octoraro," *Papers Read Before the Lancaster County Historical Society* 27 no. 7 (1923): 126.

[27] Forest Products Laboratory, *Wood Handbook—Wood as an Engineering Material* (Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, 1999), p. 4-12.

property was modulus of elasticity. While this value is highly variable between, and even within the same, species, a value of 1,200 kilopounds per square inch (ksi) was selected for our model, as it lies at the conservative end of the range (lower values yield greater deformation). Unit weights for these woods at 12 percent moisture content are in the neighborhood of 30 pounds per cubic foot (pcf). A conservative value of 35 pcf was selected to account for the various metal connections that could not otherwise be included in the model.

Maximum stress values for both Eastern Hemlock and Eastern White Pine are shown in Table 1. As can be seen there are two conflicting values for each property. The National Design Specification (NDS) values are lower, since these are design values and reflect scatter in the data from using conservative estimates. The values from the Forest Products Laboratory (FPL), however, are based on an average of actual test results without adjustments. While new structures are required to have stresses below those designated as "maximum allowable" by the NDS, stresses in excess of these values are certainly possible, up to the range prescribed by the FPL, and this is often observed in older structures.

Table 1. Maximum Strengths for Suspect Wood Species.*

	NDS Max Allowable Stress[28]			FPL Max Strength[29]	
	Compression, // psi	Shear, // psi	Tension, // psi	Compression, // Psi	Shear, // psi
Eastern Hemlock	1000	80	925	5410	1060
Eastern White Pine	725	65	700	4800	900

* "//" = Strength parallel to the wood grain (Shear strength parallel to grain is the limiting strength, even when loaded transversely)

Due to the complex geometry and the variety of connections, several models were developed to derive a more thorough understanding of how the truss behaved. A primary issue was the behavior at the joints. One of two conditions was assumed to exist: either the end of a member was perfectly free to rotate (pinned), or it was perfectly rigid (fixed). In actuality, the joints exhibited a combination of these ideal conditions, but consideration of these the two extremes yielded a "worst-case" condition for each joint. Two models were created. The first, called the flexible model (Figure 13**),** assumed pin connections at the ends of the diagonals and posts. The chords and arch were assumed continuous across the panels, and all other joints were assumed to be fixed. This was thought to reflect the predominant behavior of the various connections of the bridge and, thus, to be the more accurate model of it. The second model, termed the rigid model,

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[28] American Forest and Paper Association, American Wood Council, *National Design Specification for Wood Construction—Supplement* (1997), 39. Note, values shown are tabulated design values—they do not contain adjustment factors for safety or resistance and therefore are only approximate.

[29] Forest Products Laboratory, p. 4-12. Note, values of tension parallel to grain are available only for a select number of small specimens which are not reliable for large timbers.

assumed all joints to be perfectly fixed. Although this was known not to be realistic by itself, the rigid model exposed areas where stresses were greater than those predicted by the flexible model, thus allowing a more complete examination of load and stiffness sharing between the truss and the arch.

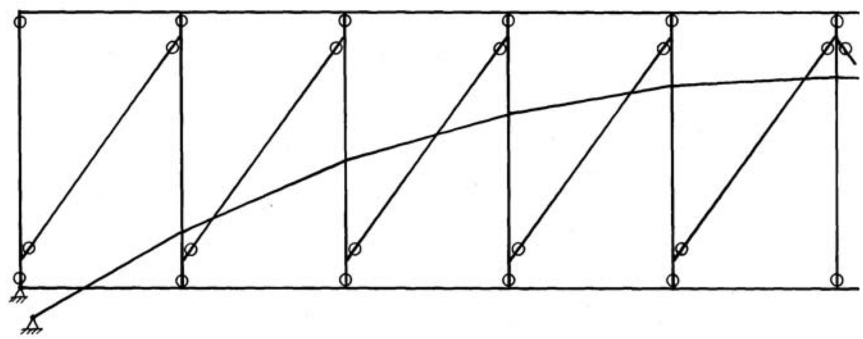


Figure 13. Diagram of the Flexible Model (left side only). Circles denote pin connections.

The supports for the truss have been modeled as pinned at the left end, and roller supported (resisting only vertical movement) at the other. This elicits the greatest forces in the lower chord. The arch, however, is pinned at both ends; which is believed to be the original design. Today, some ends of the arches are cast in concrete, suggesting a fixed connection. It appears, however, that this concrete was a later addition, and the arch originally rested in corners of the stone abutments and piers.

While the Pine Grove Bridge consists of two spans, they are similar in design, so only one was modeled. This assumed each span to be completely independent of the other. Although the roof is continuous between the spans, its structural contribution was considered negligible, and insufficient to warrant consideration of the two-spans as a single, continuous span.

Dead loads were approximated by measuring the bridge’s truss members, roofing, siding, etc. in situ. Member volumes were then calculated, multiplied by the assumed unit weight of 35 pcf, and applied to upper and lower chord panel points in a manner that approximates the actual loading condition (see Appendix B).

Live load was arbitrarily modeled as a 5-ton concentrated load centered between the two trusses, resulting in 2½ tons on each truss. This load was first applied at mid-span of the lower chord and then at the approximate quarter point of the truss (two panel points from the end). Quarter-point loading was selected since this was well known to be the worst-case live load for any arch. The live load of five tons was selected because that this is the maximum-posted weight limit for a wooden covered bridge in Lancaster County. Though the Pine Grove Bridge was posted for a 4-ton maximum, the 5-ton value may, in fact, be a more-realistic figure. At the very least, it gave some indication of the bridge’s safety factor.

For reference, a summary of the maximum forces and stresses for each type of element is provided at the end of this report. Note that every axial stress in this report is the maximum value calculated for the member. Where bending moments are present, this maximum will not usually be along the member’s centerline.

TRUSS BEHAVIOR

The Pine Grove Bridge was first considered with its arch removed in order to obtain an independent analysis of its truss. In this form it would be termed a multiple-king-post, or Howe, truss. Figure 14 depicts this truss under the dead load of the full bridge (including the weight of the arch), with shaded line widths representing the axial forces in each member. Thickness is an indication of the magnitude of the force in that portion of the member. It must be emphasized that this diagram shows only how the forces of the dead load are carried through the structure. It indicates nothing about the stresses in individual members.[30] Members carrying the greatest forces are not necessarily under the greatest stresses, since the various members have different cross-sectional areas.

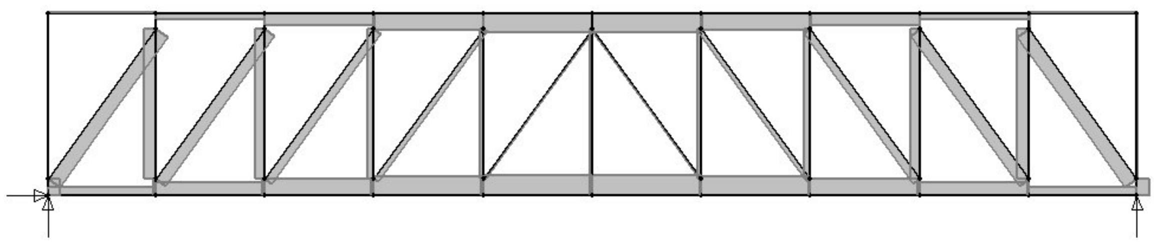


Figure 14. Axial Forces in Truss due to Dead Load.

DEAD LOAD – FLEXIBLE MODEL

Under a uniform dead load, the top chord acts in compression (shading below the element correspond to compression), and the bottom chord acts in tension (shading above the element correspond to tension). The diagonals are in compression, and the posts are in tension (for these elements, left-hand shading indicates tension, and right-hand shading shows compression). It should be noted that chord forces are greatest in the center, and diagonal and post forces are greatest toward the ends.

A common manner of conceptualizing the structural behavior of a truss is to think of an analogous beam. Indeed, one of the earliest means of approximating the chord forces in a statically indeterminate truss, developed by Navier in 1826, was based on just such an analogy. Through statics, one can calculate the shear forces and bending moments in a beam under various loadings. For example, Figure 15 displays the shear and moment diagrams for a beam placed under uniform dead load, represented by the series of arrows pointing down.

[30] Stress is a force over a given area; therefore a pound of force exerted on a toothpick yields a much greater stress than a pound exerted on, say, a pencil. The toothpick would be under a greater stress since it has a smaller area over which to carry the force.

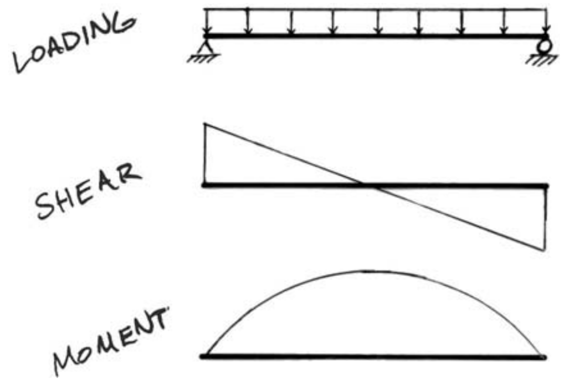


Figure 15. Shear and Bending Moment Diagram for a Beam Under Uniform Load.

It is apparent that bending moment is greatest at mid-span (referred to as the "global bending moment" in reference to the truss) and shear has the greatest magnitude at the ends ("global shear"). This is represented in the truss by greater chord forces at mid-span, corresponding to the global moment, and greater forces in the diagonals and posts at the ends, which corresponds to the global shear. It should be noted that the last post (at position 6) and the end of the upper chord (at panel E) are structurally non-functional. They carry only the load directly applied to them, but have no significant loads transferred to them by other members. For this reason they do not exhibit the global behavior just mentioned.

Table 2 contains the maximum stress and deflection values.[31] The end post's stress of –1207 psi, while probably overstressed by today's standards, is acceptable, and indeed possible considering the FPL limits.

Table 2. Maximum Stress and Deflection in Flexible Truss due to Dead Load.

Max Compressive Stress (psi)	-1207	Post 6, below diagonal
Max Tensile Stress (psi)	1038	Post 5
Max Deflection (in)	-0.96	Mid-span

In Figure 16, the local shear forces in the truss elements are displayed. Since the posts are the only members under transverse loading by the diagonals, they are the only members bearing significant shear. Just as the forces in the diagonals increase toward the ends of the span, so do the local shear stresses in the posts. The greatest shear stress is 117 psi, and it occurs near the bottom of the end posts as shown. Since the diagonal-post connection was made by notching the post (Figure 17), the reduced section area makes this shear stress in the post even more critical (although the notched section was not accounted for in the determination of shear stress). This stress also is in excess of allowable NDS values.

[31] Axial stresses in this and all such following tables are for the extreme fiber of the member and include the effects of moment.

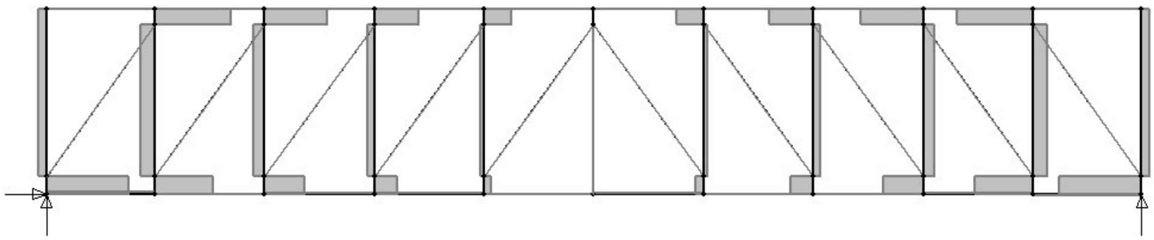


Figure 16. Shear Forces in Truss due to Dead Load.

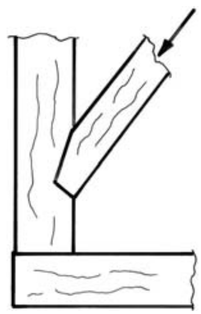


Figure 17. Notch in Post at Diagonal Connection.

Figure 18 shows the local bending moments on the truss elements. Again, only the posts contain significant flexure due to their transverse loads. The maximum moment of 17,786 ft-lbs occurs near the bottom of post 6 as shown. Due to the large moment, this is also the location of the greatest axial stress (which will be along the outboard edge, not along the centerline).

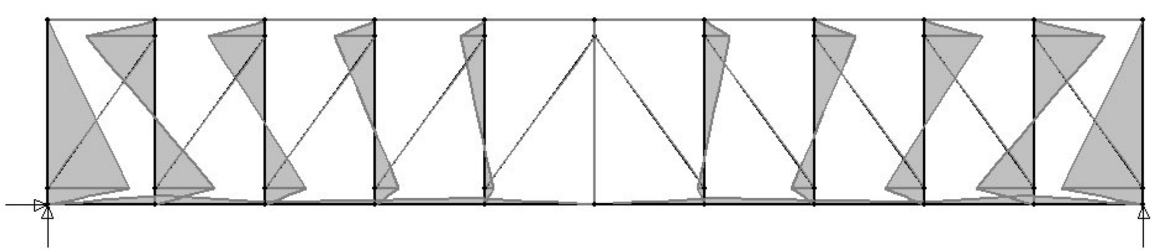


Figure 18. Bending Moments on Truss due to Dead Load.

Since the joint of post 6 is overstressed by current design code for both axial and shear stress, it is possible that this truss form *without the arch reinforcement* would not be adequate in the case of the Pine Grove Bridge, especially since these already critical values would only increase with the addition of live loads such as vehicles and snow.

DEAD LOAD – RIGID MODEL

Although several stresses increase in the rigid model, the maximum stresses are lower than in the flexible model (Table 3). Larger moments act on the chords since the posts now contribute a bending action to them. For example, the shear and bending moment at the ends of the lower chord are much greater in the rigid model since the lower chord must remain at a perfect right angle to the post (Figure 19, right). Thus the bending moment is transferred from the post to the lower chord, inducing large bending moment and shear force into the end of the lower chord. In the flexible (pinned) model the post is free to rotate with respect to the chord, and thus does not transmit any rotational forces to the chord. As expected, due to the lesser stiffness of pinned connections, the mid-span deflection is 30 percent greater in the flexible model than in the rigid model.

Table 3. Maximum Stress and Deflection in Rigid Truss due to Dead Load.

Max Compressive Stress (psi)	-905	Post 6, below diagonal
Max Tensile Stress (psi)	697	Lower Chord, Panel E
Max Deflection (in)	-0.74	Mid-span

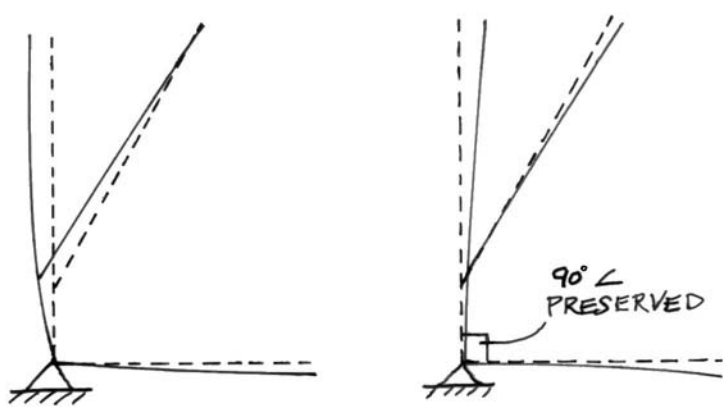


Figure 19. Deflection of End Post-Lower Chord Connection for Flexible System (left) and Rigid System (right). Unloaded configuration is shown by dashed lines.

In reality, the connection of the post to the lower chord acts somewhere between these two ideal behaviors. Because of the inherently flexible nature of wooden joinery, the actual behavior would likely be closer to that predicted by the flexible model than by the rigid one.

MID-SPAN LIVE LOAD

Considering only the 2½-ton live load at mid-span (neglecting effects of dead load), the behavior of the truss is again congruous with the simply supported beam analogy. Figure 20 shows that the stresses in the diagonals and posts, for example, are fairly uniform throughout. As seen in Figure 21, this corresponds to the global shear produced by a mid-span point load, which has a constant magnitude along the length of the beam. Also, the chord forces correspond to the global bending moment, which is greatest at mid-span. Table 4 contains the maximum values calculated for this loading condition.

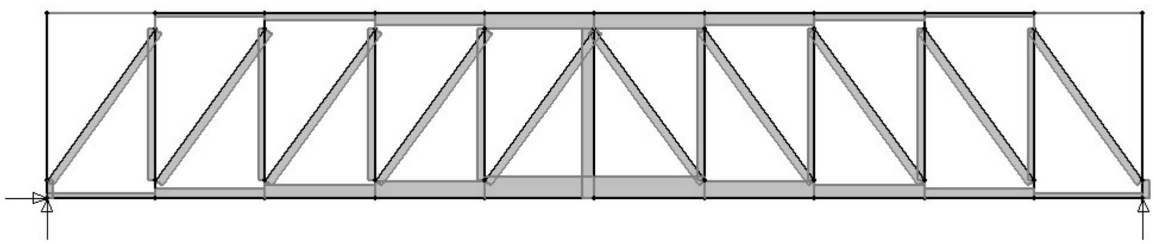


Figure 20. Axial Forces in Truss due to Mid-Span Live Load.

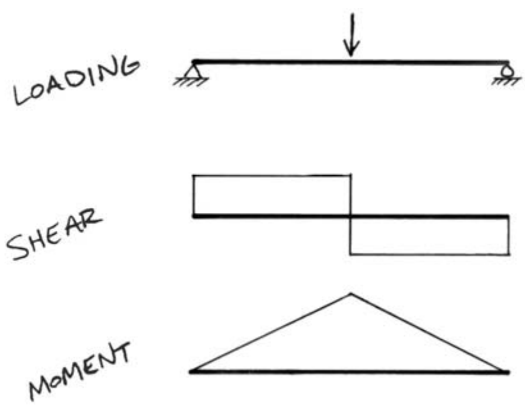


Figure 21. Shear and Bending Moment Diagram for a Beam Under Mid-Span Concentrated Load.

Table 4. Maximum Stress and Deflection in Truss due to Mid-Span Live Load.

Max Compressive Stress (psi)	-152	Post 6, below diagonal
Max Tensile Stress (psi)	143	Post 5
Max Deflection (in)	-0.19	Mid-span

DEAD LOAD PLUS MID-SPAN LIVE LOAD

When combining the dead and mid-span live load, the dead load behavior dominates, since it is almost *ten times* the live load. Because this is a linear analysis, the reactions to a combined loading are simply a sum of the reactions to individual loadings. For instance, the maximum compressive force in the upper chord for the combined loading is exactly equal to the sum of the forces for dead load alone and live load alone. The same is the case for deflection. Table 5 displays the maximum values for this loading.

Table 5. Maximum Stress and Deflection in Truss due to Dead Load Plus Mid-Span Live Load

Max Compressive Stress (psi)	-1359	Post 6, below diagonal
Max Tensile Stress (psi)	1168	Post 5
Max Deflection (in)	-1.16	Mid-span

QUARTER-POINT LIVE LOAD

The truss behavior under live loading at the quarter point (Figure 22) also follows the global shear and moment diagrams, which are shown in Figure 23. The global shear is, as expected, largest to the left of the point of loading and uniform, but considerably less, to its right. The forces in the diagonals and posts represent this with the chords picking up large axial forces due to the moment demands and the diagonals picking up large axial forces due to the shear demands. Maximum values for this loading are contained in Table 6.

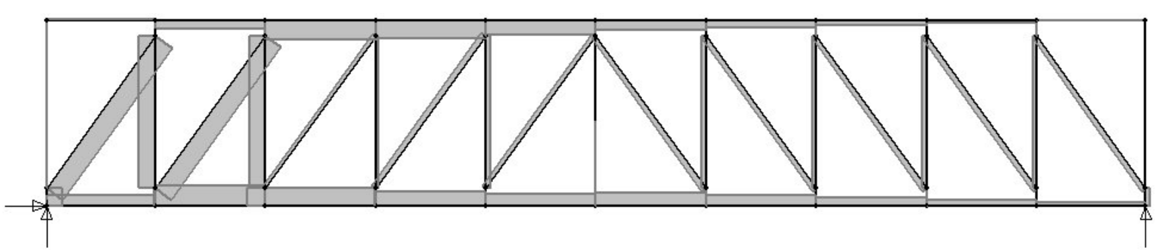


Figure 22. Axial Forces of Truss due to Quarter Point Live Load.

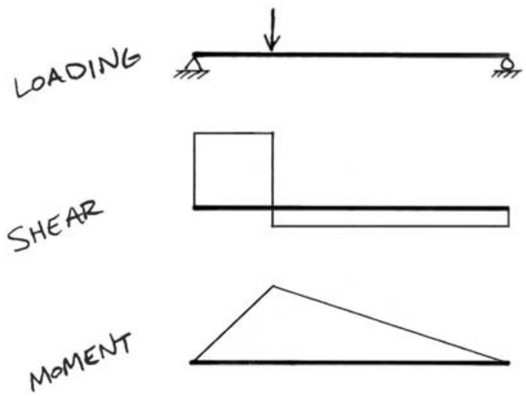


Figure 23. Shear and Bending Moment Diagram for a Beam Under Quarter-Point Concentrated Load.

Table 6. Maximum Stress and Deflection in Truss due to Quarter Point Live Load.

Max Compressive Stress (psi)	-247	Post 6, below diagonal
Max Tensile Stress (psi)	99	Post 4, below diagonal
Max Deflection (in)	-0.13	Panel Point 4

DEAD LOAD PLUS QUARTER-POINT LIVE LOAD

In this combined loading case, the dead load dominates, and the stresses are a linear sum of the results from dead load and quarter-point live load. Comparing this load combination with the combination of dead and mid-span live load, reveals that, as the global shear and moment diagrams suggest, diagonal and post stresses (global shear) are greater in the quarter-point loading case, and chord stresses (global moment) are greater in the mid-span loading case. Table 7 contains maximum values for this loading.

Table 7. Maximum Stress and Deflection in Truss due to Dead Load Plus Quarter-Point Live Load.

Max Compressive Stress (psi)	-1455	Post 6, below diagonal
Max Tensile Stress (psi)	1251	Post 5
Max Deflection (in)	-1.06	Mid-Span

ARCH BEHAVIOR

As was done for the truss, the arch was isolated and analyzed by itself in order to gain a better understanding of its behavior. The Pine Grove Bridge's arch was modeled as twelve continuous beam elements, one for each panel, plus one between the lower chord and abutment at each end. Since this was a two-dimensional analysis, the arch was assumed to be laterally supported along its length, so that out-of-plane buckling could not be a failure mode. In actuality, the roof, deck, and truss structures furnish this lateral support, so the assumption was reasonable.

DEAD LOAD

The arch was loaded with the full dead load of the entire bridge (including the weight of the truss), as was done for the truss alone. The maximum deflection was found to be 0.91 inch, versus 0.96 inch for the truss-only configuration. The axial force in each member increases from mid-span to the ends; however the largest stress occurs at mid-span, due to the local bending moment there. This maximum stress is –583 psi, less than half of the –1207 psi in the truss at. Therefore, the arch has approximately the same stiffness as the truss, but the arch's shape allows it to carry the uniform, unchanging dead load far more efficiently than the truss. The nature of an arch also tends to produce large outward horizontal forces at its abutments, in this case 40,400 lb.

Table 8. Maximum Stress and Deflection in Arch due to Dead Load.

Max Compressive Stress (psi)	-583	Mid-Span
Max Compressive Stress (psi)	-582	Panel D
Max Deflection (in)	-0.91	Mid-Span

MID-SPAN LIVE LOAD

The application of a 2½-ton load at mid-span without the uniformly distributed dead load reveals much about the nature of an arch. The strength of an arch lies in the fact that its shape matches its loading in such a way that the axial compression dominates and little bending occurs. For stone arches, their large weight (dead load) dominated all other live loading, and it held the arch's shape while the radial joints kept individual stones from sliding out. When this large uniform dead load is taken away and only a mid-span load applied, an arch will deflect downward in the middle and bow outward on each side, as seen in Figure 24. Consequently, the model of this arch with only mid-span live load results in stresses and deflection over twice that seen under dead loading. The stresses generated are also in excess of the maximum design stresses for the suspected wood species (Table 9). Note that this live load amounts to only one-tenth of the dead load, but the point, instead of distributed, application generates a very different response.

An elastic buckling analysis revealed that in-plane buckling was not a problem, as it would take over eight times this loading before elastic buckling occurs.

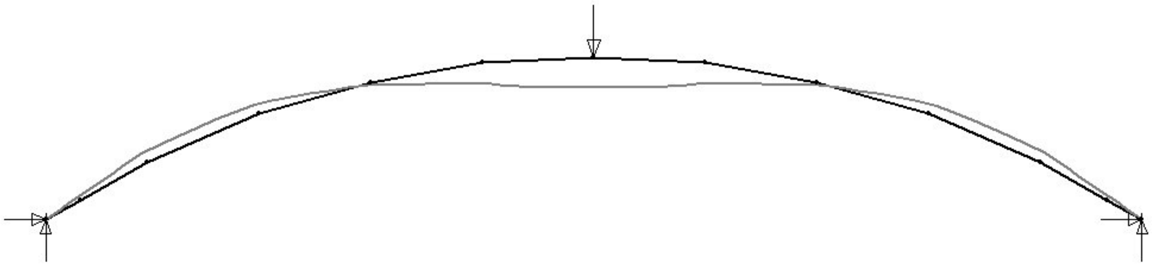


Figure 24. Deflected Shape of Arch due to Mid-Span Live Load.

Table 9. Maximum Stress and Deflection in Arch due to Mid-Span Live Load.

Max Compressive Stress (psi)	-1249	Mid-Span
Max Deflection (in)	-2.01	Mid-Span

DEAD LOAD PLUS MID-SPAN LIVE LOAD

The combined effect of dead load and mid-span live load is a linear sum of the two. The maximum deflection at mid-span is almost three inches, compared to just over one inch of deflection for the same loading in the truss system. At –1830 psi, the maximum stress in the arch is also significantly greater than in the truss, at –1359 psi. The weakness of an arch in carrying concentrated loads is clearly apparent.

Table 10. Maximum Stress and Deflection in Arch due to Dead Load plus Mid-Span Live Load.

Max Compressive Stress (psi)	-1832	Mid-Span
Max Deflection (in)	-2.92	Mid-Span

QUARTER-POINT LIVE LOAD

Application of the 2½-ton live load at the quarter-point generates a large, asymmetrical deflection, although the forces are smaller than for mid-span live load (Table 11). The bending moment, however, is significant, and it causes a large deformation, as seen in Figure 25. For this reason, quarter-point loading tends to be the worst case of loading for an arch. Again, in-plane elastic critical buckling is not the limiting case, as it would not theoretically occur until the live load reached over 32 tons—long after crushing of the wood would occur.

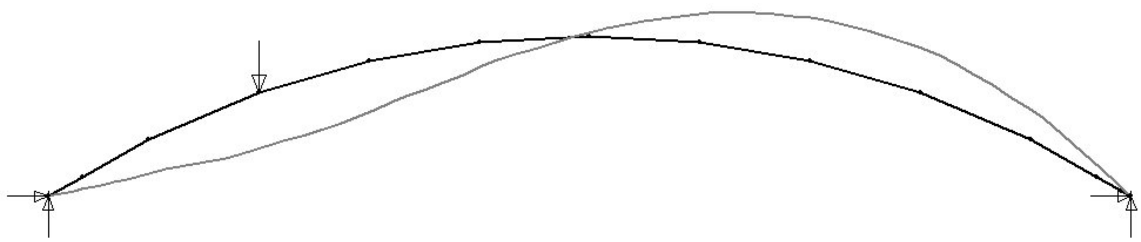


Figure 25. Deflected Shape of Arch due to Quarter-Point Live Load.

Table 11. Maximum Stress and Deflection in Arch due to Quarter-Point Live Load.

Max Compressive Stress (psi)	-1788	Panel D, side of loading
Max Deflection (in)	-4.76	Panel Point 4 (Quarter Point)

DEAD LOAD PLUS QUARTER-POINT LIVE LOAD

The maximum stress from the quarter point live load is only slightly increased by the addition of the dead load (Table 12), and the stress at mid-span is actually *decreased* substantially, because the dead load counteracts the outward deformation from the live load. The largest deflection is still almost four times that in the truss system, which again demonstrates the low stiffness of an arch under concentrated loading. While the stress is reduced at mid-span, the maximum stress—at a different location—remains slightly greater than that experienced in the arch under mid-span live load plus dead load (-1855 psi versus -1832 psi), and 28 percent greater than the maximum stress (-1860 psi versus -1455 psi) in the truss under the same loading.

Table 12: Maximum Stress and Deflection in Arch due to Dead Load plus Quarter-Point Live Load

Max Compressive Stress (psi)	-1855	Panel D, side of loading
Max Deflection (in)	-4.73	Panel Point 4 (Quarter Point)

COMBINED ARCH-TRUSS BEHAVIOR

DEAD LOAD – FLEXIBLE MODEL

Perhaps the most intriguing aspect of the combined arch-truss behavior is the deflection of the combined structure as compared to that of its component parts. Recall that the truss alone deflected 0.96 inch under dead load, and the arch a similar 0.91 inch. The combined system deflects only 0.25 inch under the same dead load, indicating a strong synergistic effect from linking the two structural forms. This effect is best understood by examining the concept of stiffness.

All materials and structures are elastic to some degree. Within their elastic range, a structure can be analyzed like a spring, which has a spring constant "k" equal to the force exerted on the spring (or structure) divided by its resulting deflection. For example, if a given spring elongates 1 inch under 10 pounds of load, dividing 10 by 1 gives a spring constant (or stiffness coefficient) of 10 pounds per inch. If two springs are combined in parallel as shown in Figure 26, the resulting stiffness coefficient is a linear sum of the two:

$$k_r = k_1 + k_2 \ .$$

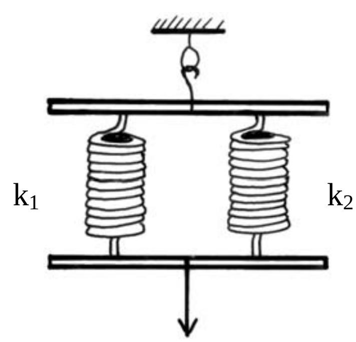


Figure 26. Example of a Parallel Combination of Springs.

The stiffness coefficient (total dead load divided by the maximum deflection) of the truss alone under dead load, k_1 = 50,250 lb/in. Similarly, for the arch alone, k_2 = 53,380 lb/in. Combining k_1 and k_2 yields k_r = 103,630 lb/in, a value 47 percent less than the actual stiffness coefficient for the combined system, k = 195,910 lb/in. Clearly, the arch-truss system involves more than the independent, parallel performance of the two structures, but rather an interaction between them that works for the betterment of both to produce a structure of higher stiffness than might be expected.

The additional element is how these two structures are interconnected. As can be seen in Figures 1 and 10, all of the posts except for the end posts are bolted (considered to be pinned in this analysis) to the arch where they intersect. The posts are sandwiched between the two arch ribs, and the two structural forms augment one other. In particular, the truss serves to stiffen the arch against excessive deformation and to transmit the truss's dead load, as well as live loads from the deck and roof, to the arch in a distributed fashion. In turn, the arch takes loads from the truss at several points and reduces the stress in its members.

The axial-force diagram of the arch-truss (Figure 27) reveals the distribution of these forces in the combined system. The arch members carry significantly greater axial forces than do the truss members (the largest arch force is 350 percent greater than the largest truss force), suggesting that the arch is structurally dominant under dead load. Compared to each system by itself, the maximum stress in the arch decreases by only 33 percent while the maximum truss stress decreases by about 77 percent. Further, the vertical force component at the supports is twice as large for the arch than for the truss. The lower chord of the truss carries little force due to its connection with the arch, which restricts it from developing any significant tension. All of this confirms the observation that the arch, albeit stabilized by the truss, carries most of the dead-load forces.

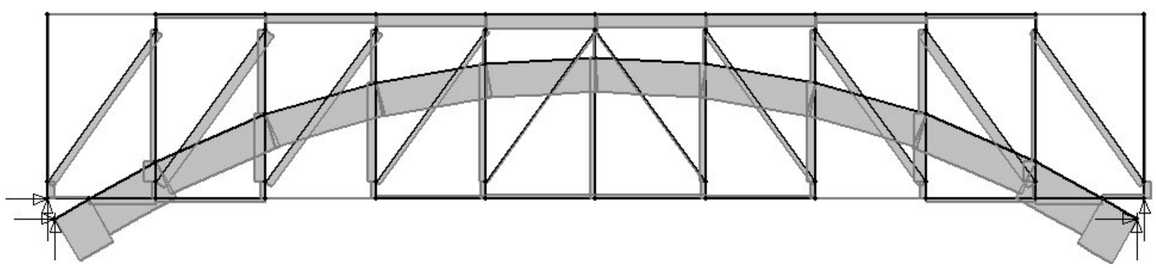


Figure 27. Axial Forces in Arch-Truss due to Dead Load.

The maximum axial stress in the arch-truss, as seen in Table 13, is well below the NDS maximum allowable stresses for the suspected wood species. This suggests that the members were sized in a conservative nature for dead load and that failure would more likely occur in a connection rather than a member. Additionally, the low stresses on the members suggest that serviceability issues, such as deflection and vibrations, played a larger role in the actual member sizing than strength. While strongly evident, the degree to which McMellen incorporated these serviceability issues into his design is, unfortunately, impossible to accurately determine through calculation at this remove.

Table 13. Maximum Stress and Deflection in Arch-Truss due to Dead Load.

Arch Max Compressive Stress (psi)	-391	Ends
Truss Max Compressive Stress (psi)	-272	Post 4, just above diagonal
Max Tensile Stress (psi)	261	Post 5, just below arch
Max Deflection (in)	-0.25	Mid-Span

Figure 28 and Figure 29 show the local shear and bending moment diagrams of the arch-truss elements under dead load. The trend for the truss is approximately the same with or without the arch. Note that the largest magnitudes of both shear and moment are found at the ends of the span, where the global shear is greatest.

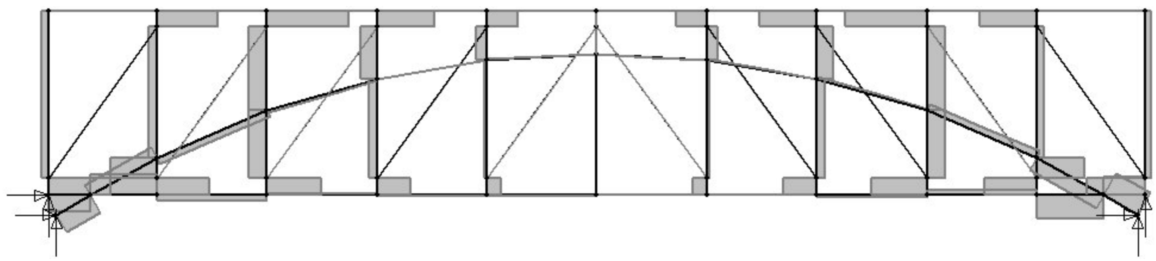


Figure 28. Shear Forces in Arch-Truss due to Dead Load.

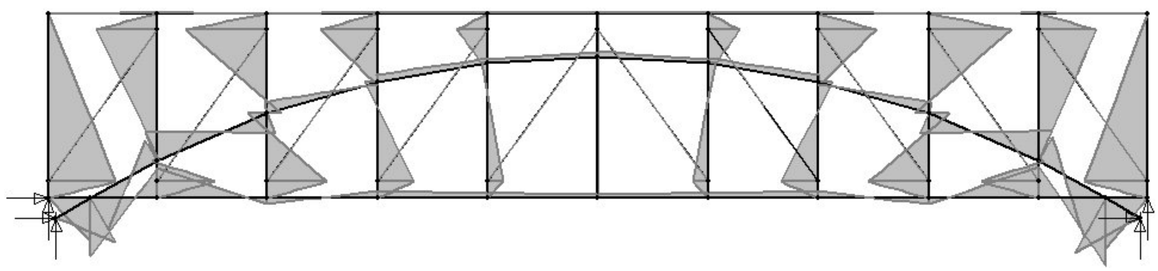


Figure 29. Bending Moments on Arch-Truss due to Dead Load.

DEAD LOAD – RIGID MODEL

Making the joints of Figure 13 rigid does not drastically change the behavior of the combined arch-truss structure. Larger moments occur that increase some stresses, but overall, stresses remain well below the maximum allowable range. As with the above analysis of the truss alone, the actual behavior of the arch-truss is safely bounded between these two, i.e., flexible and rigid, limiting conditions.

Interestingly, in the stiffer, rigid model, the vertical support reactions to the arch decreased by 17 percent, while the truss reaction increased by 35 percent, compared to the flexible model. This makes sense, since the arch itself was fixed in both models. In the rigid model, only the stiffness of the truss actually changed, causing it to carry a greater load, thus reducing the load on the arch.

Table 14. Maximum Stress and Deflection in Rigid Arch-Truss due to Dead Load.

Arch Max Compressive Stress (psi)	-404	Ends
Truss Max Compressive Stress (psi)	-279	Post 6, below diagonal
Max Tensile Stress (psi)	163	Post 5
Max Deflection (in)	-0.23	Mid-Span

MID-SPAN LIVE LOAD

A look at mid-span live loading without dead load effects provides insight into the arch-truss interaction. The axial-force diagram (Figure 30**)** reveals large forces at mid-span in the top and bottom chords that are not seen in the dead-load case. The forces here form a couple that resists a large global moment. Large global moments occurred here before under dead load, but they did not generate such a large tensile force in the bottom chord, so what caused the change?

With the arch's inherent weakness under concentrated loading, it generates a significant moment at mid-span, which, as in the arch-alone case, results in a large deflection. In this case, however, the couple from the top and bottom chords counters the bending moment, which explains why the moment at the center of the arch decreases from 304,000 inch-pounds in the arch alone to 10,000 inch-pounds in the combined arch-truss. In this case, the truss stiffens the arch dramatically.

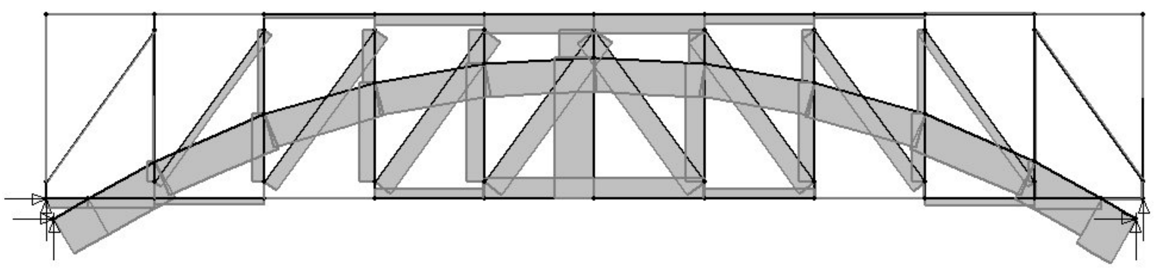


Figure 30. Axial Forces in Arch-Truss due to Mid-Span Live Load.

Table 15. Maximum Stress and Deflection in Arch-Truss due to Mid-Span Live Load.

Arch Max Compressive Stress (psi)	-68	Ends
Truss Max Compressive Stress (psi)	-58	Upper Chord, panel A
Max Tensile Stress (psi)	119	Post 2, just below arch
Max Deflection (in)	-0.07	Mid-Span

The structure's global shear behavior is also interesting. The shear produced by the mid-span loading is first carried by the diagonals of the "A" panels, then transmitted to the posts, and then partially to the arch. This cycle continues until at the ends nearly all shear is carried in the arch, and the diagonal and post stresses are negligible. At mid-span where the arch is roughly horizontal, its capacity for global shear is low, but as it curves toward the vertical at the ends it takes on an increasing amount of the global shear forces.

DEAD LOAD PLUS MID-SPAN LIVE LOAD

With the addition of the dead load, the mid-span live load behavior of the truss is not as prominent. Since the dead load is about ten times the live load, the axial force diagram looks much the same as for dead load alone (Figure 31). This loading condition elicits the greatest deflection in the bridge at a mere -0.32 inch, which is quite small for such a long timber span. For example, the Timber Construction Manual recommends a deflection limit of *L*/300 for highway bridge stringers, where *L* equals the span length.[32] In this case, the calculated deflection is much less—equal to *L*/3300. Additionally, the greatest stress, at the ends of the arch, is still well below the NDS maximum allowable stress.

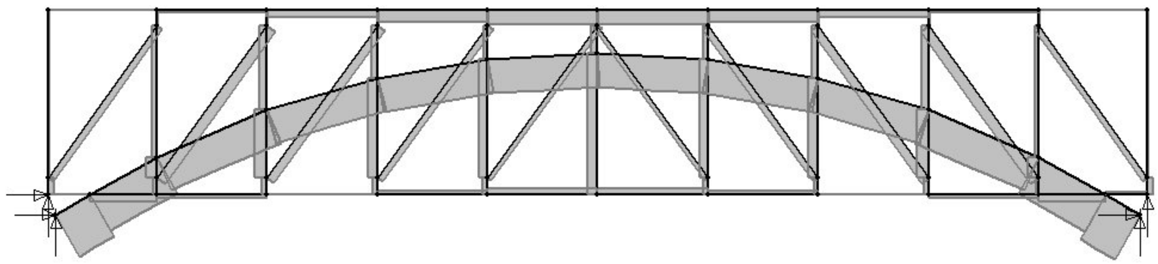


Figure 31. Axial Forces in Arch-Truss due to Dead Load plus Mid-Span Live Load.

Table 16. Maximum Stress and Deflection in Arch-Truss due to Dead Load plus Mid-Span Live Load.

Arch Max Compressive Stress (psi)	-429	Ends
Truss Max Compressive Stress (psi)	-305	Post 4, above diagonal
Max Tensile Stress (psi)	296	Post 5, just below arch
Max Deflection (in)	-0.32	Mid-Span

QUARTER-POINT LIVE LOAD

Figure 32 displays the axial forces for live load at the quarter point (mid-way between posts 4L and 5L). Plus and minus signs are shown in some of the graphs for clarification of tension and compression, respectively. The effects of global moment, which is greatest at the point of loading, are apparent in the chord forces. Just as for mid-span loading, the chords of the truss counter the moment produced in the arch. The shear is carried primarily by the diagonals and posts until, on the right side, the arch achieves enough of an angle that it can efficiently carry the shear at the ends.

Note the significant tensile forces calculated in the diagonals just to the right of the loading that decrease toward mid-span. Since the diagonal/post connection is designed for bearing in compression only, tensile loads in the diagonals are not possible and must be disregarded. This condition was not anticipated, and the model was not

[32] Donald E. Breyer, Kenneth J. Fridley, Kelly E. Cobeen, *Design of Wood Structures, ASD*. 4th ed. (New York: McGraw-Hill, 1999) p. 2.21.

designed to handle “compression-only” joints. It is, however, only a theoretical case in that it does not include the dominant dead load. This may not be a problem when considering the total dead-plus-live-load condition, however. As long as the *net* forces in the diagonal members are compressive, the model will remain useful.

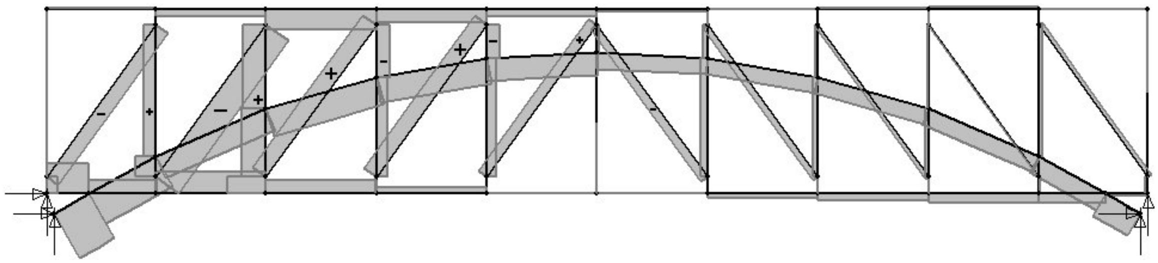


Figure 32. Axial Forces in Arch-Truss due to Quarter Point Live Load.

Table 17. Maximum Values of Arch-Truss due to Quarter Point Live Load.

Arch Max Compressive Stress (psi)	-98	Ends
Truss Max Compressive Stress (psi)	-67	Diagonal, panel D, left
Max Tensile Stress (psi)	122	Post 4, above diagonal, left
Max Deflection (in)	-0.06	Post 4, left

DEAD LOAD PLUS QUARTER-POINT LIVE LOAD

The forces shown in Figure 33 are the linear combination of the quarter-point live load and dead load results, with the dead load reaction dominating. This loading produces the greatest stresses of any case considered (Table 18). The largest stress, -489 psi, occurs at the left end of the arch, but this is well below current maximum design values. The force at this location is also 375 percent greater than the largest force in the truss, which again speaks for the arch’s structural dominance. The greatest shear stress also occurs under this loading case, but it, too, is safely below allowable limits.

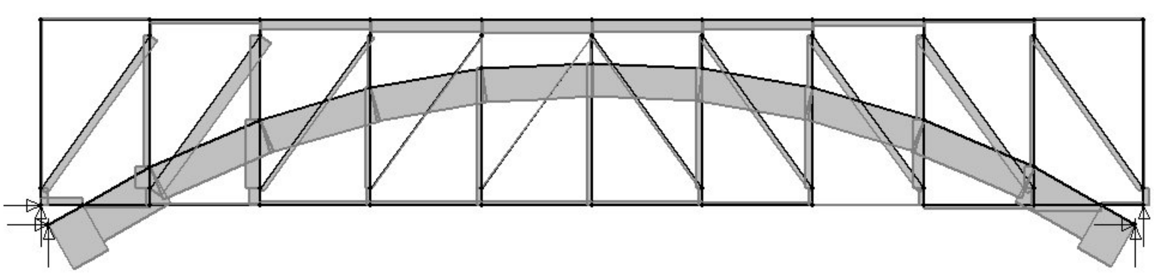


Figure 33. Axial Forces in Arch-Truss due to Dead Load plus Quarter-Point Live Load.

Table 18. Maximum Stress and Deflection in Arch-Truss due to Dead Load plus Quarter-Point Live Load.

Arch Max Compressive Stress (psi)	-489	Left End
Truss Max Compressive Stress (psi)	-394	Post 4, above diagonal, left
Max Tensile Stress (psi)	458	Post 4, above arch, left
Max Deflection (in)	-0.27	Post 2, left

A surprising result is that a small, 150-pound, tensile force remains in the diagonal of panel A on the left side. In the actual structure, the diagonal simply butts into a notch in the post, with a "toenail" spike driven in to hold it in place. This toenail likely could not adequately transmit this tension over the life of the structure, so some movement of this diagonal over time would probably be evident if tension did in fact occur regularly. Phillip C. Pierce, an engineer with experience in wooden covered bridge analysis, suggests that bridges built with "substantial initial camber" tend not to have these loading reversal problems in the diagonals.[33] The Pine Grove Bridge does have some camber to it, and its effect will be discussed later in this report.

This analysis, then, has shown that, while stresses and deflections for the arch and truss separately are often above allowable limits, the combined arch-truss system at the Pine Grove Bridge has both stresses and deflections that are safely within acceptable limits. The large degree to which the stresses are below maximum allowable values suggests that serviceability issues such as deflection and vibration, not stress limits, governed its design.

What does this indicate about the relative structural contributions of the truss and the arch? The general attitude, both of the original builders and of contemporary bridge enthusiasts, holds to the view that the arch "stiffened" the truss. For example, many sagging trusses, or those covering longer spans, had arches added to them to reduce deflections. Although this study only considered immediate deflections, in a previous study Kemp and Hall mention that long-term deflections could be substantially greater, due to creep, shrinkage, and a general loosening of joints from repeated loading and climatic effects. They mention that, "these problems are largely eliminated with the addition of the arch, which transmits its loads by compression in a very direct manner to the abutments." The creep of the arch contributes very little to the overall vertical deflection of the system, and the inability of its compression connections to loosen also allows little, if any, deflection.[34] Compared to the truss, the arch is very resistant to long term deflections, which seems to support the idea of it as a stiffening element.

This study has revealed something different. For realistic loading conditions, the arch carries over *three times* the load carried by the truss, making it the dominant structural element. The arch does, however, depend heavily on the truss to counteract the large bending moments and shear forces that would otherwise severely distort it under

[33] Phillip C. Pierce, "Covered Bridges," Chapter 15 of *Timber Construction for Architects and Builders*, by Eliot W. Goldstein (New York: McGraw-Hill, 1999), p. 15.11.

[34] Kemp and Hall, 410.

concentrated live loads. The truss both stiffens the arch and serves to distribute concentrated live loads so that they can be transmitted to the arch at several locations.

While this analysis suggests that designers of Burr arch-truss bridges like McMellen sized their members primarily on their experience in dealing with serviceability issues, the separate consideration of the truss and arch elements shows that the members of both are much smaller and lighter than would have been necessary if either element had been used by itself in this bridge. Whether or not they fully understood why, Theodore Burr and his protégés nevertheless achieved a decided synergy and structural efficiency in their bridges.

FURTHER CONSIDERATIONS

CAMBER

Camber, an initial upward curvature, has traditionally been built into a bridge to counteract expected live-load deflections as well as sag over the life of the structure, as well as to provide for rainwater run-off. The Pine Grove Bridge currently has a nine-inch camber at mid-span, although it was probably somewhat greater when constructed, before the effects of creep and general joint loosening occurred. While the above analyses neglected this camber and assumed perfectly horizontal chords, the effects of its presence on stress distribution should be understood. Figure 34 shows the centerline model with the nine-inch camber added. It is assumed that the camber itself generates no pre-stressing in the members, i.e., that the bridge was built in the cambered state.

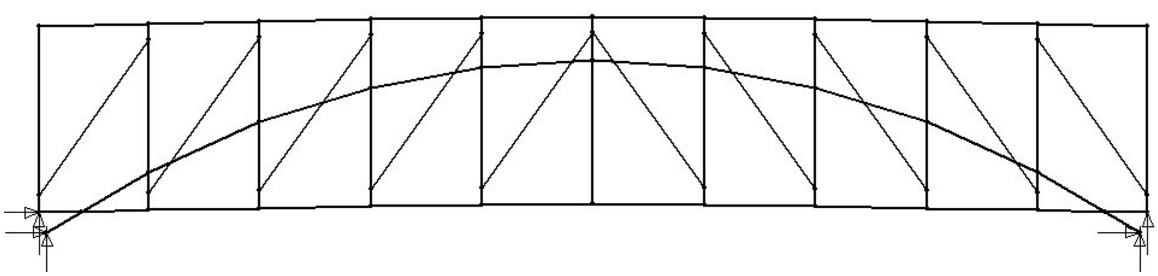


Figure 34. Centerline Model with Camber.

The camber is based on field measurements of the bottom chord of a single truss of one span. As seen in Table 19, analysis of this system under dead load resulted in values comparable to the previous horizontal-chord dead-load analysis.

Table 19. Maximum Axial Stresses due to Dead Load with and without Camber.

	Without Camber (psi)	With Camber (psi)	% Change
Upper Chord	-167	-160	-4
Lower Chord	-184	-202	10
Diagonal	-142	-140	-1
Post	-272	-266	-2
Arch	-391	-416	6
Deflection (in)[35]	-0.247	-0.227	-8

When camber is included, stresses are slightly lessened in most of the truss members and increased in the arch. The greatest change is a stress increase in the compressive segment of the lower chord, due to the cambered lower chord beginning to act like a shallow arch. As expected, deflection is decreased in the cambered system. The reduced stresses in truss members alone would probably justify the increased constructional efforts to add camber. For example, with nine inches of camber, the maximum tension at mid-span was found to be just 23 psi.

The consequences of camber in the Burr arch-truss system as seen in the Pine Grove Bridge are mixed. The original camber has successfully counteracted long-term sag, which is at least beneficial in the visual sense. Camber also reduced stresses in the truss, other than the compression in the lower chord. On the other hand, the arch and lower chord experience greater compressive forces because to the camber, thus increasing what was already the greatest stress in the bridge at the arch ends.

From a serviceability standpoint the camber appears effective, but from a strength standpoint the advantages are unclear. The strength issue depends on the first failure mode of the bridge. If the first failure involves an interior connection, such as a diagonal-post joint, then the camber tends to decrease the force demands upon it and its likelihood of failure. If, however, the first failure would be the crushing of the arch ends or the failure of an abutment at the arch support, then the camber has detrimental effects.

The analysis of the dead-load-plus-quarter-point-live-load case produced a 150-pound tensile force in the left diagonal of Panel A. While Pierce suggests that this loading reversal would be avoided with the presence of substantial camber, this was found not to be the case for the Pine Grove Bridge. Adding camber to this bridge, under the same loading, indicated that the tensile force actually *increased* to 600 pounds. The toenailed spike would certainly work loose with repeated applications of this much tensile load. This suggests that either this model is in error, or the connections of the diagonals can withstand greater tension than believed.

Field inspection of the bridge revealed a metal reinforcing patch at the mid-span intersection of the diagonals and post near the top of the post (Figure 35). This patch provides evidence of possible problems with these diagonals shifting, a likely scenario for an unloaded butt joint. In the case of the Pine Grove Bridge, its camber may have

[35] Deflection measured from cambered position.

been beneficial for serviceability, but it could not guarantee optimal joint performance under all loading conditions.

Figure 35. Photo of Patch over Diagonal and Post Intersection at Mid-Span.

STEEL TIES

Figures 1 and 10 clearly show vertical steel ties between the arch and lower chord. Their addition to this and other Burr arch-trusses is a source of curiosity. Perhaps it was thought that their presence would relieve the posts of some of their tensile forces. This could be advantageous considering the nature of the post-lower chord connections. As seen in Figure 8, the post-lower chord connection transmits forces through bearing surfaces cut into the post. Tension forces in the post must be borne by the inverted “tee” below the notch at the bottom of the post. In other bridges, this bearing area has been observed to “shear off” in a vertical plane due to overstressing. The bottom ends of posts also are susceptible to damage from ice and flood-borne debris, either of which can fatally weaken the post-lower chord connections. The result of this occurring to several posts would be catastrophic to the bridge. It could make sense, then, to install these ties to provide a redundant load path in the event of such a failure, especially if such damage actually occurred.

An analysis of the undamaged Pine Grove Bridge with these ties, under dead load plus mid-span live load resulted in the axial forces shown in Figure 36. In this condition, the ties receive little force compared to the posts.

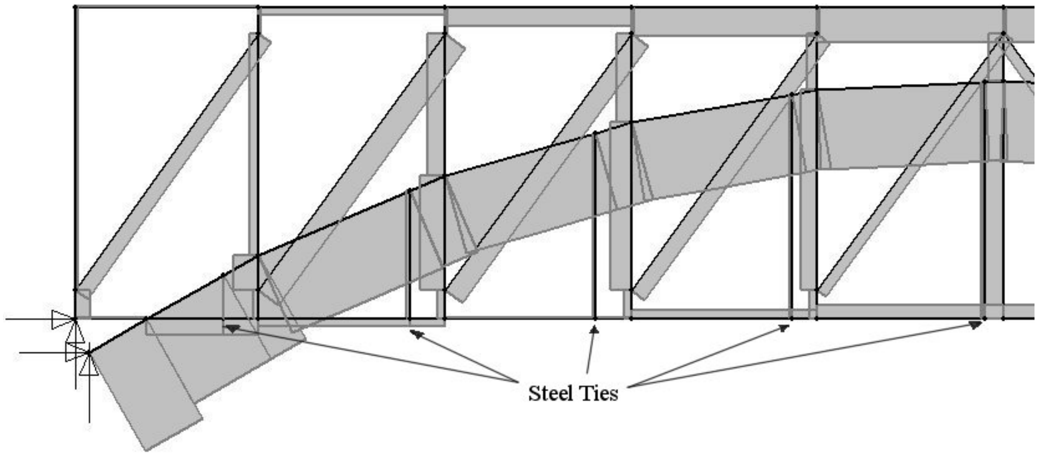


Figure 36. Axial Forces in Arch-Truss with Steel Ties due to Dead plus Mid-Span Live Load.

To evaluate the value of the ties in the event of a post failure, an analysis was done for the same conditions, except with a single ruptured post/lower chord connection. The axial force diagram of the system, with the bottom of Post 3 removed to simulate such a rupture, is shown in Figure 37. As can be seen, the result was a redistribution of forces such that the adjacent tie carried most of the post’s former load, and the force in the Panel B diagonal was partially redistributed, primarily to the Panel C diagonal.

Table 20 compares the maximum stresses in this area of the modified system (with ties) to the original bridge configuration (without ties) after a Post 3 failure. Although the stress in Tie 3 may seem high at first glance, the tie is steel, not wood, and this stress is well within the allowable limit for mild steel.

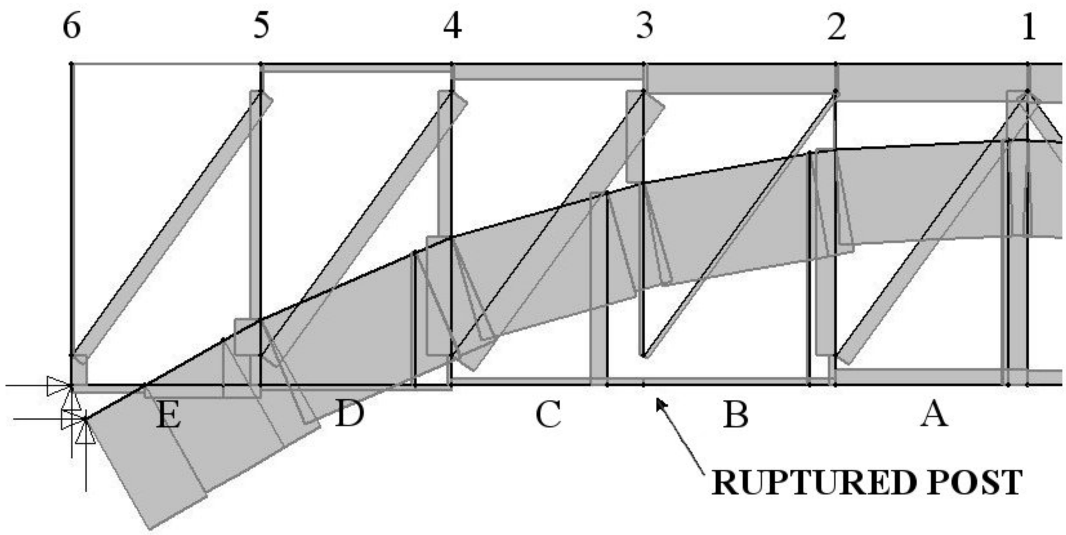


Figure 37. Axial Forces in Damaged Arch-Truss with Steel Ties due to Dead Load plus Mid-Span Live Load.

Table 20. Key Maximum Axial Stresses after Post 3 Rupture.

	Without Ties (psi)	With Ties (psi)	% Change
Post 3	504	363	-28
Tie 3	---	5272	---
Post 2	253	250	-1
Post 4	261	346	33
Diagonal B	-34	-25	-28
Diagonal C	-86	-149	74

The maximum stress in Post 3 is decreased by the presence of the tie. The neighboring Post 2 carries approximately the same amount of stress, however, it is surprising to find that Post 4 experiences a greater stress *with* the presence of the ties after a rupture. Post 4 receives less axial force, but it receives a greater bending moment, which accounts for the increased axial stress. The effects of the ties' presence on the neighboring diagonals varies—less in Panel B, but significantly greater in Panel C.

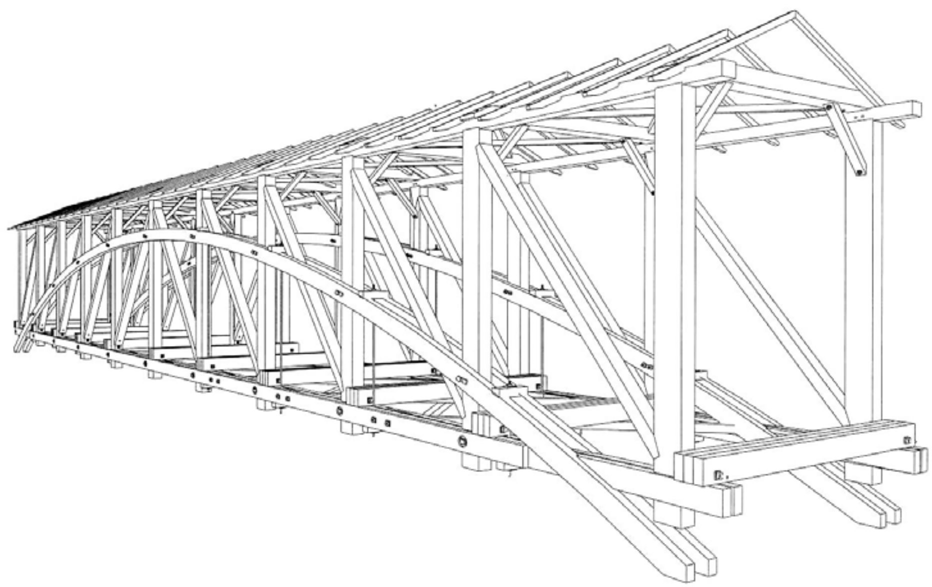
The ties have a negligible effect on an undamaged truss, but the redundancy they provide would be beneficial in the event of a truss-lower chord connection failure. A more conclusive judgment of the ties' value, particularly if multiple failures are contemplated, would require a more thorough analysis of loading conditions and failure locations.

CONCLUSIONS

The arch-truss combination as formalized by Theodore Burr in his 1817 patent is an excellent combination of two structural forms, the truss and the arch. Although wooden truss technology continued to develop and diversify, the Burr arch-truss was utilized over a century after its conceptualization. Captain Elias McMellen respected the design enough to use it throughout his career of over three decades.

The details of design and construction of the Pine Grove Bridge were determined by McMellen's extensive experience. Evidence suggests his structural knowledge was derived from his experience as a builder, rather than through theoretical calculations. When given the choice, his design and construction methods seem to favor constructional efficiency over structural efficiency—which arguably may have been both cheaper and safer for him. Certainly, the Pine Grove Bridge's record of over 120 years of service makes a strong case for the wisdom of his choices, regardless of method.

The structural findings of this report reveal that the Pine Grove Bridge was well designed, even by today's standards. For example, the maximum stress of the bridge, which occurs at the arch ends, was found to be well below current design standards. The structural behavior of the Burr arch-truss system found in the Pine Grove Bridge is rich and complex. Although it is traditionally thought that the arch was added to the truss as reinforcement, something observable deflection behavior seems to support, analysis reveals that the arch actually carries a substantially greater load than the truss, clearly making it the dominant structural element. However, the arch's contributions are only made possible by the truss, since without the truss, the arch would undergo such large deformations under live loading as to render it useless. The arch provides a direct route to carry loads to the abutments, and the truss provides the moment capacity of its chords and the shear capacity (especially toward mid-span where the arch is nearly horizontal) of its diagonals and posts. Contrary to the popular belief that the arch stiffens the truss, it seems more appropriate to say the truss stiffens the arch. While both forms can and have worked separately in other designs, the collaboration of the two produces a true synergy—a structure with greater strength and stiffness than the sum of its parts.



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SECTION AND MATERIAL PROPERTIES

SECTION PROPERTIES[36]

Member	Height (in)	Depth (in)	Area (in^2)	2nd Moment of Area I_{zz}, (in^4)
Top Chord	6.5	9.5	61.75	217.41
Bottom Chord[37]	11.75	11.5	135.13	1554.64
Arch	11.75	11.5	135.13	1554.64
Post, normal	11.5	9.5	109.25	1204.03
Post, center	13.5	9.5	128.25	1947.80
Diagonals	5.5	9.5	52.25	131.71
Steel Ties	1.1 (diameter)		0.950	0.072

MATERIAL PROPERTIES[38]

Material	Modulus of Elasticity (psi)	Unit Weight (pcf)
Timber	1,200,000	35
Steel	29,000,000	(not used)

[36] Given dimensions are a result of subtracting 1/8" from each face of the timber as measured in the field. Nominal section sizes (those actually measured) were used in dead load computation. The iron ties were measured at a point free of surface roughness.

[37] The bottom chord and arch actually consist of two parallel 6x12 in. members, with a gap between. Properties listed are equivalent.

[38] Modulus of Elasticity based on NDS and FPL values. Unit weight based on FPL data.

DEAD LOAD COMPUTATION

FOR EACH TRUSS

CONCENTRATED LOADS AT UPPER CHORD PANEL POINTS

	Volume in^3	Unit Weight lbf/ft^3	Weight lbf
Top Chord	7147	35	145
Roofing	30994	35	628
Lateral Bracing	8590	35	174
		Total:	950

CONCENTRATED LOADS AT LOWER CHORD PANEL POINTS

	Volume in^3	Unit Weight lb/ft^3	Weight lbf
Posts	20813	35	422
Diagonals	11436	35	232
Bottom Chord	15638	35	317
Arch	17018	35	345
Siding	29970	35	607
Lower Bracing	19656	35	398
Decking	56187	35	1138
		Total:	3460

TOTAL BRIDGE DEAD LOAD PER TRUSS = (946.5 + 3457.9) * 11 = 48,400 lbf

TOTAL BRIDGE DEAD LOAD = 96,900 lbf

ARCH-ONLY LOADS

CONCENTRATED LOADS APPLIED AT NODES
(NO LOAD APPLIED AT ENDS. HALF LOAD APPLIED AT 1st INSIDE NODES)

48450 lb / 10 nodes = 4850 lb   APPLIED AT INTERIOR NODES
4850 lb / 2 = 2425 lb   APPLIED AT 1st INSIDE NODES

ANALYSIS DATA SUMMARY

Calculations based on:

Axial Stress, $\sigma = \frac{F}{A} \pm \frac{M \cdot y}{I}$ where F = axial force, A = cross-sectional area, M = moment, y = distance from neutral axis, and I = second moment of area.

Shear Stress, $\tau = \frac{V}{A}$ where V = shear force.

TRUSS--DEAD LOAD

Element[39]	Location	Axial Force, lbf (F)	Shear Force, lbf (V)	Max Moment, in lbf (M)	Axial Stress, psi (σ)	Shear Stress, psi (τ)
Upper Chord	A	-31200	24	2714	-546	0
Lower Chord	A	32941	168	-18196	316	1
Diagonal	E	-24098	0	0	-461	0
Post	6, below diag	-20563	12742	213430	-1207	117
"	5	18673	2233	181540	1038	20
Support Reaction, F_y (lbf)	6	24260				
Deflection (in)	1	-0.96				

RIGID TRUSS--DEAD LOAD

Element	Location	F, lbf	V, lbf	M, in lbf	σ, psi	τ, psi
Upper Chord	A	-31304	123	-7743	-623	2
Lower Chord	A	32871	62	-11529	289	0
"	E	11919	2384	154250	697	18
Diagonal	E	-20854	136	13153	-674	3
Post	6, below diag	-18411	11919	154250	-905	109
"	5	15761	819	85688	553	8
Support Reaction, F_y (lbf)	6	24260				
Deflection (in)	1	-0.74				

[39] Stresses occurring due to the largest force in each element are listed initially. If effects of moment, shear, or reverse loading (tension) also result in significant stresses they are listed and denoted with a ditto (").

TRUSS--MID-SPAN LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-5801	12	1392	-115	0
Lower Chord	A	7530	107	11066	99	1
Diagonal	B	-3110	0	0	-60	0
Post	6, below diag	-2483	1614	27040	-152	15
"	1	4786	0	0	44	0
"	5	2484	319	25249	143	3
Support Reaction, F_y (lbf)	6	2500				
Deflection (in)	1	-0.19				

TRUSS--DEAD LOAD PLUS MID-SPAN
LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-37001	11	2777	-641	0
Lower Chord	A	40471	61	-17651	369	0
Diagonal	E	-27152	0	0	-520	0
Post	6, below diag	-23046	14357	240470	-1359	131
"	5	21157	2552	203980	1168	23
Support Reaction, F_y (lbf)	6	26760				
Deflection (in)	1	-1.16				

TRUSS--QUARTER-POINT LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	C, L	-4684	29	2566	-114	0
Lower Chord	D, L	4867	210	19212	112	2
Diagonal	E, L	-4975	0	0	-95	0
Post	6, below diag	-4038	2630	44059	-247	24
"	4, below diag	4558	716	11994	99	7
Support Reaction, F_y (lbf)	6	4004				
Deflection (in)	4	-0.13				

TRUSS--DEAD LOAD PLUS QUARTER-POINT LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A, L	-34814	27	3072	-610	0
Lower Chord	A, R	35971	169	18206	338	1
Diagonal	E, L	-29073	0	0	-556	0
Post	6, below diag	-24600	15373	257490	-1455	141
"	5	22682	2730	218450	1251	25
Support Reaction, F_y (lbf)	6	28260				
Deflection (in)	1	-1.06				

ARCH--DEAD LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Arch	E, below chord	-47111	1177	45383	-528	9
"	A	-40481	423	71775	-583	3
"	D	-43830	342	-65350	-582	3
Support Reaction, F_x (lbf)	6	40410				
Support Reaction, F_y (lbf)	6	24250				
Deflection (in)	1	-0.91				

ARCH--MID-SPAN LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Arch	E, below chord	-6900	1036	-39923	-209	8
"	A	-6630	2175	304240	-1249	16
"	D	-6972	272	-150280	-644	2
Support Reaction, F_x (lbf)	6	6514				
Support Reaction, F_y (lbf)	6	2500				
Deflection (in)	1	-2.01				

ARCH--DEAD LOAD PLUS MID-SPAN
LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Arch	E, below chord	-54011	142	5460	-421	1
"	A	-47111	2597	376010	-1832	19
"	D	-50800	615	-215600	-1226	5
Support Reaction, F_x (lbf)	6	46920				
Support Reaction, F_y (lbf)	6	26750				
Deflection (in)	1	-2.92				

ARCH--QUARTER-POINT LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Arch	E, below chord, L	-5310	1620	62442	-286	12
"	A	-3769	1161	-84623	-362	9
"	D, L	-5100	2193	443710	-1788	16
Support Reaction, F_x (lbf)	6, L	3821				
Support Reaction, F_y (lbf)	6, L	4027				
Deflection (in)	4, L	-4.76				
Positive Deflection (in)	4, R	3.70				

ARCH--DEAD LOAD PLUS QUARTER-POINT
LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Arch	E, below chord, L	-52420	2797	107830	-813	21
"	A	-44249	739	-67482	-61	5
"	D, L	-48929	1850	378350	-1855	14
Support Reaction, F_x (lbf)	6, L	44230				
Support Reaction, F_y (lbf)	6, L	28270				
Deflection (in)	4, L	-4.73				
Positive Deflection (in)	4, R	3.73				

ARCH/TRUSS--DEAD LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-9845	0	-505	-167	0
Lower Chord	A	2723	6	-3423	34	0
"	E, inside arch	-4081	904	39081	-184	7
Diagonal	D	-7439	0	0	-142	0
Post	6, below diag	-5163	2738	45853	-266	25
"	5, just below arch	8356	2061	38696	261	19
"	4, top	-960	3590	-55210	-272	33
Arch	E, below chord	-34649	886	-34141	-391	7
Truss Support, F_x (lbf)	6	1126				
Truss Support, F_y (lbf)	6	7925				
Arch Support, F_x (lbf)	6	30580				
Arch Support, F_y (lbf)	6	16310				
Deflection (in)	1	-0.25				

RIGID ARCH/TRUSS--DEAD LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-11392	24	-1449	-206	0
Lower Chord	A	4040	31	3755	45	0
"	E, inside arch	-4945	1291	43450	-208	10
Diagonal	D	-7258	14	2017	-181	0
Post	6, below diag	-6517	3777	46021	-279	35
"	5, above arch	4348	192	25761	163	2
"	4, top	-874	3822	-33665	-169	35
Arch	E, below chord	-29563	1218	-46953	-404	9
Truss Support, F_x (lbf)	6	4099				
Truss Support, F_y (lbf)	6	10660				
Arch Support, F_x (lbf)	6	26320				
Arch Support, F_y (lbf)	6	13510				
Deflection (in)	1	-0.23				

ARCH/TRUSS--MID-SPAN LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-2342	15	-1369	-58	0
Lower Chord	A	2428	128	11029	61	1
"	E, outside arch	-1087	9	373	-10	0
Diagonal	A	-2500	0	0	-48	0
Post	1	4743	1	-142	44	0
"	2, just below diag	157	1240	-20769	101	11
"	2, just below arch	2179	229	20769	119	2
Arch	A	-3973	114	9671	-68	1
Truss Support, F_x (lbf)	6	1160				
Truss Support, F_y (lbf)	6	120				
Arch Support, F_x (lbf)	6	4185				
Arch Support, F_y (lbf)	6	2357				
Deflection (in)	1	-0.07				

ARCH/TRUSS--DEAD LOAD PLUS MID-SPAN LIVE LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-12188	15	-1874	-225	0
Lower Chord	A	5151	123	13824	93	1
"	E, inside arch	-5138	899	39767	-195	7
Diagonal	D	-8369	0	0	-160	0
Post	6, below diag	-5274	2810	47074	-273	26
"	5, just below arch	9114	2264	44458	296	21
"	4, top	-962	4039	-62125	-305	37
Arch	E, below chord	-39452	898	-34616	-429	7
Truss Support, F_x (lbf)	6	2286				
Truss Support, F_y (lbf)	6	8045				
Arch Support, F_x (lbf)	6	34770				
Arch Support, F_y (lbf)	6	18660				
Deflection (in)	1	-0.32				

ARCH/TRUSS--QUARTER-POINT LIVE LOAD

Element	Location	F, lbf	V, lbf	M, in lbf	σ, psi	τ, psi
Upper Chord	C, L	-2525	11	1110	-57	0
Lower Chord	E, outside arch, L	3818	301	-12494	78	2
"	E, outside arch, R	-260	71	2929	-13	1
Diagonal	D, L	-3497	0	0	-67	0
Post	4, below diag, L	4858	963	16122	121	9
"	4, top, L	22	1654	-25439	122	15
Arch	E, below chord, L	-6440	330	-12733	-98	2
Truss Support, F_x (lbf)	6, L	-2855				
Truss Support, F_y (lbf)	6, L	1178				
Arch Support, F_x (lbf)	6, L	5766				
Arch Support, F_y (lbf)	6, L	2887				
Deflection (in)	4, L	-0.05				

ARCH/TRUSS--DEAD LOAD PLUS QUARTER-POINT
LIVE LOAD

Element	Location	F, lbf	V, lbf	M, in lbf	σ, psi	τ, psi
Upper Chord	A, L	-10806	0	-438	-182	0
Lower Chord	E, outside arch, L	5430	999	-41478	204	7
"	E, inside arch, R	-5346	1113	-50965	-241	8
Diagonal	D, L	-10935	0	0	-209	0
Post	6, below diag, L	-6642	3701	61987	-357	34
"	4, just above arch, L	7911	1182	80649	458	11
"	4, top, L	-938	5244	-80649	-394	48
Arch	E, below chord, L	-41088	1216	-46875	-489	9
Truss Support, F_x (lbf)	6, L	-1729				
Truss Support, F_y (lbf)	6, L	9103				
Arch Support, F_x (lbf)	6, L	36350				
Arch Support, F_y (lbf)	6, L	19190				
Deflection (in)	2, L	-0.26				

CAMBERED ARCH/TRUSS--DEAD LOAD

Element	Location	*F*, lbf	*V*, lbf	*M*, in lbf	σ, psi	τ, psi
Upper Chord	A	-9279	4	-619	-160	0
Lower Chord	A	989	26	-4002	23	0
"	E, inside arch	-5386	919	-41102	-202	7
Diagonal	D	-7301	0	0	-140	0
Post	6, below diag	-5030	2615	43805	-255	24
"	5, just below arch	8233	1970	37428	**254**	18
"	4, top	-1029	3492	53699	-266	**32**
Arch	E, below chord	-35792	993	38271	**-416**	7

Truss Support, F_x (lbf)	6	1275
Truss Support, F_y (lbf)	6	7904
Arch Support, F_x (lbf)	6	31360
Arch Support, F_y (lbf)	6	16780
Deflection (in)	1	-0.23