

1.1

The Decimal Place-Value System

1.1 OBJECTIVES

1. Write numbers in expanded form
2. Determine the place value of a digit
3. Write a number in words
4. Write a number, given its word name

Overcoming Math Anxiety

Throughout this text, we will present you with a series of class-tested techniques that are designed to improve your performance in this math class.

Hint #1 Become familiar with your textbook.

Perform each of the following tasks.

1. Use the Table of Contents to find the title of Section 5.1.
2. Use the Index to find the earliest reference to the term *mean*. (By the way, this term has nothing to do with the personality of either your instructor or the textbook authors!)
3. Find the answer to the first Check Yourself exercise in Section 1.1.
4. Find the answers to the pre-test for Chapter 1.
5. Find the answers to the odd-numbered exercises in Section 1.1.
6. In the margin notes for Section 1.1, find the origin for the term *digit*.

Now you know where some of the most important features of the text are. When you have a moment of confusion, think about using one of these features to help you clear up that confusion.

Number systems have been developed throughout human history. Starting with simple tally systems used to count and keep track of possessions, more and more complex systems were developed. The Egyptians used a set of picturelike symbols called **hieroglyphics** to represent numbers. The Romans and Greeks had their own systems of numeration. We see the Roman system today in the form of Roman numerals. Some examples of these systems are shown in the table.

Numerals	Egyptian	Greek	Roman
1	I	I	I
10	∩	Δ	X
100	Ϟ	H	C

NOTE The prefix *deci* means 10. Our word *digit* comes from the Latin word *digitus*, which means finger.

NOTE Any number, no matter how large, can be represented using the 10 digits of our system.

Any number system provides a way of naming numbers. The system we use is described as a **decimal place-value system**. This system is based on the number 10 and uses symbols called **digits**. (Other numbers have also been used as bases. The Mayans used 20, and the Babylonians used 60.)

The basic symbols of our system are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

These basic symbols, or digits, were first used in India and then adopted by the Arabs. For this reason, our system is called the Hindu-Arabic numeration system.

Numbers may consist of one or more *digits*.

3, 45, 567, and 2359 are examples of the **standard form** for numbers. We say that 45 is a two-digit number, 567 is a three-digit number, and so on.

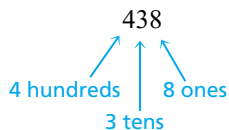
As we said, our decimal system uses a *place-value* concept based on the number 10. Understanding how this system works will help you see the reasons for the rules and methods of arithmetic that we will be introducing.

Example 1

Writing a Number in Expanded Form

Look at the number 438.

We call 8 the *ones digit*. Moving to the left, the digit 3 is the *tens digit*. Again moving to the left, 4 is the *hundreds digit*.



If we rewrite a number such that each digit is written with its units, we have used the **expanded form** for the number.

In expanded form, we write 438 as

$$400 + 30 + 8 \text{ or}$$

$$(4 \times 100) + (3 \times 10) + (8 \times 1)$$

NOTE Each digit in a number has its own place value.

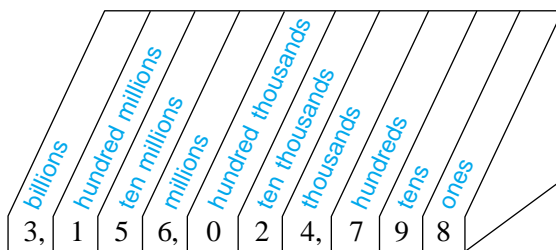
NOTE Here the parentheses are used for emphasis. (4×100) means 4 is multiplied by 100. (3×10) means 3 is multiplied by 10. (8×1) means 8 is multiplied by 1.



CHECK YOURSELF 1

Write 593 in expanded form.

The following place-value diagram shows the place value of digits as we write larger numbers. For the number 3,156,024,798, we have



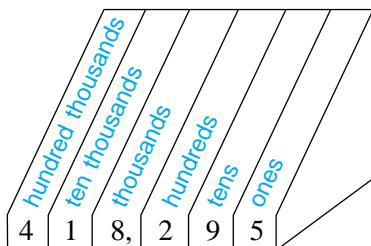
Of course, the naming of place values continues for larger and larger numbers beyond the chart

For the number 3,156,024,798, the place value of 4 is thousands. As we move to the left, each place value is 10 times the value of the previous place. The place value of 2 is ten thousands, the place value of 0 is hundred thousands, and so on.

Example 2

Identifying Place Value

Identify the place value of each digit in the number 418,295.



**CHECK YOURSELF 2**

Use a place-value diagram to answer the following questions for the number 6,831,425,097.

- (a) What is the place value of 2? (b) What is the place value of 4?
(c) What is the place value of 3? (d) What is the place value of 6?

Understanding place value will help you read or write numbers in word form. Look at the number

7 2, 3 5 8, 6 9 4
Millions Thousands Ones

NOTE A four-digit number, such as 3456, can be written with or without a comma. We have chosen to omit the comma in these materials.

Commas are used to set off groups of three digits in the number. The name of each group—millions, thousands, ones, and so on—is then used as we write the number in words. To write a word name for a number, we work from left to right, writing the numbers in each group, followed by the group name. The following chart summarizes the group names.

Billions Group			Millions Group			Thousands Group			Ones Group		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Example 3**Writing Numbers in Words**

NOTE Notice that the commas in the word statements are in the same place as the commas in the number.

27,345 is written in words as twenty-seven *thousand*, three hundred forty-five.
2,305,273 is two *million*, three hundred five *thousand*, two hundred seventy-three.

Note: We do *not* write the name of the ones group. Also, “and” is not used when a number is written in words. It will have a special meaning later.

**CHECK YOURSELF 3**

Write the word name for each of the following numbers.

- (a) 658,942 (b) 2305

We reverse the process to write the standard form for numbers given in word form. Consider the following.

Translating Words into Numbers

Forty-eight thousand, five hundred seventy-nine in standard form is
48,579

Five hundred three thousand, two hundred thirty-eight in standard form is

503,238

Note the use of 0 as a placeholder
in writing the number.



CHECK YOURSELF 4

Write twenty-three thousand, seven hundred nine in standard form.

CHECK YOURSELF ANSWERS

1. $(5 \times 100) + (9 \times 10) + (3 \times 1)$ 2. **(a)** Ten thousands; **(b)** hundred thousands;
(c) ten millions; **(d)** billions 3. **(a)** Six hundred fifty-eight thousand, nine hundred
forty-two; **(b)** two thousand three hundred five 4. 23,709



Exercises

Name _____

Section _____ Date _____

Write each number in expanded form.

1. 456

2. 637

3. 5073

4. 20,721

Give the place values for the indicated digits.

5. 4 in the number 416

6. 3 in the number 38,615

7. 6 in the number 56,489

8. 4 in the number 427,083

9. In the number 43,729,

(a) What digit tells the number of thousands?

(b) What digit tells the number of tens?

10. In the number 456,719,

(a) What digit tells the number of ten thousands?

(b) What digit tells the number of hundreds?

11. In the number 1,403,602,

(a) What digit tells the number of hundred thousands?

(b) What digit tells the number of ones?

12. In the number 324,678,903,

(a) What digit tells the number of millions?

(b) What digit tells the number of ten thousands?

Write the word name for each of the following.

13. 5618

14. 21,812

15. 200,304

16. 103,900

Write each of the following in the standard form of a number.

17. Two hundred fifty-three thousand, four hundred eighty-three

18. Three hundred fifty thousand, three hundred fifty-nine

19. Five hundred two million, seventy-eight thousand

20. Four billion, two hundred thirty million

ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

17. _____

18. _____

19. _____

20. _____

ANSWERS

21.

22.

23.

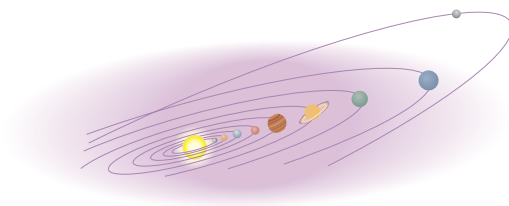
24.

Write the whole number in each sentence in standard form.

21. The first place finisher in the 1999 U.S. Senior Open won three hundred fifteen thousand dollars.

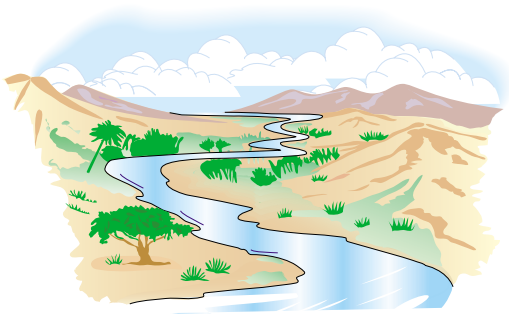


22. Scientific speculation is that the universe originated in the explosion of a primordial fireball approximately twenty billion years ago.



23. The population of Kansas City, Missouri, in 1990 was approximately four hundred thirty-three thousand, seven hundred.

24. The Nile river in Egypt is about four thousand, one hundred forty-five miles long.



Sometimes numbers found in charts and tables are abbreviated. The following table represents the population of the 10 largest cities in the 1990 U.S. census. Note that the numbers represent thousands. Thus Phoenix had a population of 983 thousand or 983,000.

Name	Rank	Population (thousands)
New York, NY	1	7,323
Los Angeles, CA	2	3,485
Chicago, IL	3	2,784
Houston, TX	4	1,631
Philadelphia, PA	5	1,586
San Diego, CA	6	1,111
Detroit, MI	7	1,028
Dallas, TX	8	1,007
Phoenix, AZ	9	983
San Antonio, TX	10	936

In exercises 25 to 28, write your answers in standard form using the given table.

25. What was the population of Detroit in 1990?
26. What was the population of Chicago in 1990?
27. What was the population of Philadelphia in 1990?
28. What was the population of Dallas in 1990?

Assume that you have alphabetized the word names for every number from one to one thousand.



29. Which number would appear first in the list?
30. Which number would appear last?

Determine the number represented by the scrambled place values.

- | | |
|-----------------|-----------------|
| 31. 4 thousands | 32. 7 hundreds |
| 1 tens | 4 ten-thousands |
| 3 ten-thousands | 9 ones |
| 5 ones | 8 tens |
| 2 hundreds | 6 thousands |



33. Inci had to write a check for \$2565. There is a space on the check to write out the amount of the check in words. What should she write in this space?



25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____
32. _____
33. _____

ANSWERS



34.

35.

36.

37.



38.



39.



40.



41.

34. In addition to personal checks, name two other places where writing amounts in words is necessary.



35. In a rental agreement, the amount of the initial deposit required is two thousand, five hundred forty-five dollars. Write this amount as a number.

36. How many zeros are in the number for one billion?

37. Write the largest five-digit number that can be made using the digits 6, 3, and 9 if each digit is to be used at least once.



38. Several early numeration systems did not use place values. Do some research, and determine at least two of these systems. Describe the system that they used. What were the disadvantages?



39. What are the advantages of a place-value system of numeration?



40. The number 0 was not used initially by the Hindus in our number system (about 250 B.C.). Go to your library (or “surf the net”), and determine when a symbol for zero was introduced. What do you think is the importance of the role of 0 in a numeration system?



41. A *googol* is a very large number. Do some research to find out how big it is. Also try to find out where the name of this number comes from.



Answers

We will provide the answers (with some worked out in detail) for the odd-numbered exercises at the end of each exercise set. The answers for the even-numbered exercises are provided in the instructor’s resource manual.

1. $(4 \times 100) + (5 \times 10) + (6 \times 1)$ 3. $(5 \times 1000) + (7 \times 10) + (3 \times 1)$
 5. Hundreds 7. Thousands 9. (a) 3; (b) 2 11. (a) 4; (b) 2
 13. Five thousand, six hundred eighteen
 15. Two hundred thousand, three hundred four 17. 253,483 19. 502,078,000
 21. \$315,000 23. 433,700 25. 1,028,000 27. 1,586,000 29. Eight
 31. 34,215 33. Two thousand, five hundred sixty-five 35. \$2545
 37. 99,963 39.

