MATHEMATICS

SUPPORT CENTRE

Title: Algebraic fractions and operations.

Target: On completion of this worksheet you should be able to add, subtract, multiply and divide algebraic fractions.

When we **multiply** fractions we multiply the numerators together and the denominators together.
Examples.

$$\frac{2}{3} \times \frac{6}{7} = \frac{12}{21}.$$

$$\frac{5}{6} \times 4 = \frac{5}{6} \times \frac{4}{1} = \frac{20}{6} = \frac{10}{3}.$$

We do exactly the same with algebraic fractions. We should always simplify our answers.
Examples.

1. $\dfrac{a}{s} \times \dfrac{3b}{d} = \dfrac{3ab}{sd}.$

2. $\dfrac{12a}{b^2} \times \dfrac{b}{18a^3} = \dfrac{12ab}{18a^3 b^2} = \dfrac{2}{3a^2 b}.$

3. $\dfrac{2(x+3)}{6x} \times \dfrac{xy}{(x+3)(x+4)} = \dfrac{2xy(x+3)}{6x(x+3)(x+4)}$
$$= \dfrac{y}{3(x+4)}$$

Exercise.
Simplify the following:

1. $\dfrac{12x^2}{6y^2} \times \dfrac{9y^3}{x^4}.$

2. $\dfrac{as}{4b^3} \times \dfrac{b^2}{3ad}.$

3. $4 \times \dfrac{hs}{2p+c}.$

4. $\dfrac{x+2}{y+3} \times \dfrac{4(y+3)z}{5(x+2)p}.$

(Answers: $\dfrac{18y}{x^2}, \dfrac{s}{12bd}, \dfrac{4hs}{2p+c}, \dfrac{4z}{5p}.$)

When we **divide** fractions we invert the second fraction and then multiply the numerators together and the denominators together.
Examples.

$$\frac{6}{7} \div \frac{3}{4} = \frac{6}{7} \times \frac{4}{3} = \frac{24}{21} = \frac{8}{7}.$$

$$\frac{5}{6} \div 4 = \frac{5}{6} \div \frac{4}{1} = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}.$$

We do exactly the same with algebraic fractions. We should always simplify our answers.
Examples.

1. $\dfrac{a}{s} \div \dfrac{3b}{d} = \dfrac{a}{s} \times \dfrac{d}{3b} = \dfrac{ad}{3sb}.$

2. $\dfrac{12a}{b^2} \div \dfrac{3ba}{ab^3} = \dfrac{12a}{b^2} \times \dfrac{ab^3}{3ba} = \dfrac{12a^2 b^3}{3ab^3} = 4a.$

3. $\dfrac{2(x+3)}{6x} \div \dfrac{(x+5)(x+3)}{3x}$
$$= \dfrac{2(x+3)}{6x} \times \dfrac{3x}{(x+5)(x+3)}$$
$$= \dfrac{6x(x+3)}{6x(x+3)(x+5)}$$
$$= \dfrac{1}{(x+5)}.$$

Exercise.
Simplify the following:

1. $\dfrac{12x^2}{5y^2} \div \dfrac{9x^3}{y^2}$.

2. $\dfrac{5cd^4}{6e^2} \div \dfrac{10d^3}{e^5}$.

3. $\dfrac{16xyz^3}{48y^2} \div \dfrac{24yz^5}{8x^2}$.

4. $\dfrac{18r^2}{6p^2} \div \dfrac{9r^3}{q^4}$.

5. $\dfrac{8pr^4}{5q^2} \div \dfrac{15pr^5}{40q^8}$.

6. $\dfrac{3rp^2}{2q} \div 2p$.

7. $\dfrac{3(p+q)q^2}{(r+s)} \div \dfrac{6q}{4(r+s)s}$.

8. $\dfrac{p^2 - q^2}{r^2 + 2rs + s^2} \div \dfrac{2(p+q)}{3(r+s)s}$.

(Answers:

$\dfrac{4}{15x}, \dfrac{cde^3}{12}, \dfrac{x^3}{9y^2z^2}, \dfrac{q^4}{3p^2r}, \dfrac{64q^6}{15r}, \dfrac{3rp}{4q},$

$2(p+q)qs, \dfrac{3s(p-q)}{2(r+s)}$.)

When we **add** or **subtract** fractions we find a common denominator, express each of the fractions as equivalent fractions with this common denominator and then add the numerators.

Example.

$$\frac{3}{7} + \frac{2}{3} = \frac{9}{21} + \frac{14}{21} = \frac{23}{21}.$$

If you have difficulty with this refer to the number sheet on fractions.

We do the same with algebraic fractions. First we need to be able to find a common denominator. The easiest way to do this is to multiply the denominators together.

Example.

A common denominator of $\dfrac{s}{a+3}$ and

$\dfrac{(p+s)}{qp(r+s)}$ is $(a+3)(qp(r+s))$.

Notice that putting all the denominators in brackets avoids mistakes.

Exercise. Find the common denominator of

$$\frac{a}{d}, \frac{(p+q)}{(r+s)} \text{ and } \frac{r+s}{a+b}.$$

(Answer: $(d)(r+s)(a+b)$.)

We then need to be able to express each fraction as an equivalent fraction whose denominator is the common denominator.

Remember that equivalent fractions are obtained by multiplying the top and the bottom of the fraction by the same thing.

Example.

Multiply by $(r+s)(a+b)$

$$\frac{a}{d} = \frac{a(r+s)(a+b)}{d(r+s)(a+b)}.$$

Multiply by $(r+s)(a+b)$

To add or subtract algebraic fractions we must:

- Find a common denominator.
- Express each fraction as an equivalent fraction with this denominator.
- Add or subtract the numerators.
- Simplify

Example.

$$\frac{a}{b} - \frac{2c}{r+s} = \frac{a(r+s)}{b(r+s)} - \frac{2c(b)}{b(r+s)} = \frac{a(r+s) - 2cb}{b(r+s)}.$$

Exercise. Simplify the following:

1. $\dfrac{a}{2} + \dfrac{a}{3}$.

2. $\dfrac{ab}{4} - \dfrac{2ba}{5}$.

3. $\dfrac{s}{d} + \dfrac{b}{c}$.

4. $\dfrac{2a}{c} + \dfrac{c}{a} - f$.

5. $\dfrac{2q}{p+q} - \dfrac{q}{p}$.

6. $\dfrac{x-y}{x+y} + \dfrac{x+y}{x-y} - 2$.

(Answers:

$\dfrac{5a}{6}, \dfrac{-3ab}{20}, \dfrac{sc+db}{dc}, \dfrac{2a^2 + c^2 - fca}{ca}, \dfrac{q(p-q)}{p(q+p)},$

$\dfrac{4y^2}{(x+y)(x-y)}$.)