Safe Portfolio Withdrawal Rates in Retirement

Comparing Results from Four Monte Carlo Models

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Drawing Income from Your Investment Portfolio

One of the most basic questions in retirement planning is to determine how much money you can safely withdraw from an investment portfolio with little or no risk of running out of money in the long term. Let's say that you are 60 years old and you have $1 Million in a retirement account. How much could you safely withdraw from that account and be fairly confident that you won't run out of money before you die? I have recently seen an estimate in educational materials from a 401(k) plan provider that people will need ten times their annual salary in an investment account when they retire. I am not sure where this comes from—and the literature provided no explanation. Where does one start in such a calculation?

To begin, you must account for investment risk. If you have your money invested in anything other than cash, there is some possibility that your account will drop in value over an extended period of time. The impact of investment risk, when you are drawing income from an investment portfolio, is that you can run down your funds far faster than you might think. There are now many discussions that detail this phenomenon, but most people continue to use retirement planning calculators that do not take risk into account. In such calculators, you simply enter some average rate of return that you think you will get, your annual contribution, and years to retirement and you get out some estimate of how much income you can draw. These calculators will either estimate your age at death or have you estimate how long you think you need your retirement to last. **If you do not account for investment risk, you will estimate that your money will last considerably longer than is likely to be the case.** If this idea is not immediately clear, have a look at this good article on the T. Rowe Prices website:

http://www3.troweprice.com/ric/RIC/eprim_TutorialRight.html#buildingblocks

There are a number of ways to estimate a safe withdrawal rate from your retirement funds, and the most popular and flexible of these is what is called Monte Carlo simulation. Rather than calculating a single value of your retirement portfolio's value for

each future date, Monte Carlo tools simulate out hundreds of possible futures, with the odds of each one calculated based on the historical volatility in stock market returns. This approach means that you can account for the risk of the range of possible outcomes.

The ***Quantext Retirement Planner*** (http://www.quantext.com) is a Monte Carlo calculator that can be used to estimate reasonable withdrawal rates, while accounting for risk. There are other Monte Carlo calculators devoted to this problem and we will show a range of results and explain similarities and differences. Monte Carlo models are rapidly becoming established as the standard of practice for people to use in planning their savings rates and investment allocations so as to fund retirement income. The purpose of writing this paper is to examine the consistency of the results generated from four Monte Carlo models (including the ***Quantext Retirement Planner***) that calculate the safe withdrawal rates from an investment portfolio.

The Basic Numbers

There are several numbers that you need to understand with regard to the overall returns and risk that are available from stock and bond portfolios, and indeed from any risky investment. The first variable is **average annual return**. The renowned economist Dr. Robert Schiller, of Yale University, has created an historical reference series for returns on the S&P500 composite index and on a short-term bond index (web link to the data is given at the end of this paper). These data, or similar data from other sources, are often used to provide the basic inputs to retirement planning models.

S&P500: 60%		Short Bond: 40%	
Years	Period	Average Annual Return	Standard Deviation in Annual Return
100	1905-2004	6.68%	11.29%
90	1915-2005	7.02%	11.50%
80	1925-2004	7.17%	11.52%
70	1935-2004	7.12%	10.54%
60	1945-2004	7.35%	9.87%
50	1955-2004	7.09%	9.40%
40	1965-2004	6.73%	9.70%
30	1975-2004	8.53%	9.48%
20	1985-2004	8.62%	10.10%
10	1995-2004	8.14%	12.16%

Raw Data: Website of Robert Schiller at Yale (http://www.econ.yale.edu/~shiller/data.htm)

Let's start by looking at the historical performance of a portfolio that is made up of 60% stocks (invested in the market index) and 40% short-term bonds (as above). What level of historical annual return has been obtained in the past? Over the last 30 years (through 2004), a portfolio of 60% stocks and 40% bonds has had an average annual return of 8.53%. Clearly the amount of time that you include in your history has a fairly large impact here. Assuming that we are interested in stock and bond performance that is generally representative, we might decide that 7.5% in annual return is a fairly reasonable long-term estimate for this portfolio mix.

The variability in return on an asset or mix of assets from year to year is typically characterized by a measure called the Standard Deviation in Annual Return. This variable is a measure of the normal level of variations in return around the average. From the table above, we might say that over the last 30 years we have seen a lot of returns in the range of 8.53% plus or minus 9.48%. A Standard Deviation in Annual Return of around 10% or so for the 60/40 portfolio is a fair assumption.

Surviving Retirement

Let's assume that we are looking at a portfolio with 60% stock and 40% bonds, and we want to know how much income a given portfolio can provide. As we suggested, one of the easiest and best ways to examine this problem is to look at a Monte Carlo calculator. In fact, this exact case has been examined by a number of interested parties.

William Bernstein, an economist and asset management professional, published a short paper on his website examining this situation using a Monte Carlo model called ***MCRetire*** (*The Retirement Calculator from Hell, Part III*, citation at the end of this paper). He assumed that the 60/40 portfolio (as above) would reasonably be expected to generate 4.5% per year in **real** return (i.e. if inflation is 3% per year then this portfolio will generate 7.5% per year), with a 10% standard deviation. If we are going to assume a 3% inflation rate per year (pretty reasonable), we are looking at the long-term average return of 7.5% per year that we suggested from our analysis of historical data. Bernstein then ran the Monte Carlo calculator and examined the probability of running out of money over a 40 year retirement period. It is useful to look at the entire paper, but his results are quite straightforward. If you withdraw 4% of your portfolio at retirement per year (i.e. you draw $40,000 per year if you retire with $1,000,000), and increase that draw with inflation, you have an 80% chance of having your money survive 40 years of retirement. If you withdraw 3% of your portfolio at retirement per year (i.e. you draw $30,000 per year), you have a 97% chance of having your money last 40 years. The paper goes on to suggest that trying to plan for a 40-year retirement with better than an 80% success rate is likely to be somewhat counterproductive because there are so many other factors than can impact your financial well being over that time horizon.

In a related online paper, (*Where do these guys get a 2% withdrawal rate?* by John Greaney, 2005), the author looks at a range of approaches to estimating this sort of survival rate for a retirement portfolio allocated with 60% stock and 40% bonds. This paper (link provided at the end of this paper) is well worth a read. In this analysis, the author compares an historical analysis of survival rates to a Monte Carlo model (written

for EXCEL by Prof. Peter Ponzo). Looking at the 60/40 portfolio with a withdrawal rate of 4% (i.e. $40,000 per year for a portfolio worth $1,000,000) for a retirement period of 30 years, the pure historical approach gives a survival rate of 100%. The Monte Carlo model gives a survival probability of 89%. The Monte Carlo model used in this study yields an average portfolio annual return of 9.1% with an annual standard deviation of 12.1% and an average rate of inflation equal to 4%. The Monte Carlo inputs in this study are derived directly from historical data. The title of the paper is due to the fact that all three approaches agree that you can achieve a 99+% survival rate if you withdraw only 2%. The 60/40 diversified portfolio with 4% draw is estimated to have a survival rate of 89% (using Monte Carlo), which means that you have an 11% chance of running out of money if you attempt to draw this income level from your retirement portfolio for 30 years.

There is also a nice simple online Monte Carlo retirement planning model developed by T. Rowe Price called the *Retirement Income Model* (or RIC for short). The RIC is available at no cost and requires no registration, so it is fast and easy to go in and give the model a spin. Links to the model and to a good basic tutorial on Monte Carlo models in general are given in the web resources section at the end of this paper. In the RIC, you can select from a range of different portfolio mixes (stocks, bonds, etc.) and then select your portfolio value at retirement and your monthly draw and the model will calculate your probable success rate for a retirement of a specified duration. The RIC does not include an explicit 60/40 split but it does have a case with 60% stocks, 30% bonds and 10% short term securities. Given all of the other uncertainties in these calculations, this should give generally similar results to our previous cases. The RIC gives an 80% success rate for a 40-year retirement with an income draw of $3200 per month, or $38,400 per year—very close to the $40,000 per year that we estimated from the analysis by Bernstein:

Screenshot from T. Rowe Price's RIC Monte Carlo model for 40-year retirement

Given the various uncertainties in all of the other variables, it would be fair to say that this is a high level of agreement. We can also then look at the survival rate for a 30-year retirement, the case examined by John Greaney using the Prof. Peter Ponzo's Monte Carlo model, which gives a 10% chance of failure for a 30 year retirement with a 4% withdrawal rate:

Screenshot from T. Rowe Price's RIC Monte Carlo model for 30-year retirement

The T. Rowe Price model yields a 10% failure rate (90% success rate) for a 30-year retirement (above) for a draw of $3300 per month (i.e. $39,600 per year), once again very consistent with the results from the Monte Carlo analysis by Greaney using Prof. Ponzo's Monte Carlo model. It is also interesting that the RIC model gives essentially identical results for the 80% stock / 20% bond case.

Pulling All this Together

We have compared the results from these three models to our own ***Quantext Retirement Planner***, which is also a Monte Carlo model. This Monte Carlo model can be used to examine fairly simple cases, such as those cited from the earlier studies, and more complex cases in which you specify a portfolio by individual components (stocks or

funds) with varying amounts of time until retirement, changing allocation in time, etc. The ***Quantext Retirement Planner*** is explained in detail at www.quantext.com in several online papers, including a 78 page mini-text that explains key concepts of portfolio allocation and statistical tools for portfolio planning. ***The Retirement Planner*** generates a 7-page standard report, and the results that we will be showing are taken directly from that report.

To begin, we will simply reproduce the results from the earlier studies. Let's assume that Jane Doe is 60 years old and is retiring this year. She invests in a 60/40 portfolio as above and we are going to assume an annual return of 7.5%, a standard deviation in return of 10% and an annual inflation rate of 3%. This should give results consistent with the analysis by Bernstein using ***MCRetire***. We first input the baseline market return variables (top right hand corner below) to reflect these conditions. These boxes are normally set up for input parameters for the S&P500, but we have lowered the standard deviation and average annual return to yield an index that is similar to that assumed in Bernstein's analysis:

Retirement Planning Report

Prepared For: **Jane Doe**

Preparation Date: **5/31/2005**

Page 1: Basic Input and Projections

www.quantext.com

Current Age	60
Date of Retirement	2005
Age at Retirement	60
Annual Contribution (2005 Dollars)	$0
Current Portfolio Value	$1,000,000
Inflate Contributions at inflation?	Yes
Inflate Income Draw?	Yes
Income in Retirement (2005 Dollars)	$40,000

Assumed Inflation Rate (Annual)	Annual Standard Deviation of Market Return (% of normal)
3.00%	66.00%
Annual Standard Deviation	9.94%
Delta Return	Average Annual Return of Market
-2.80%	7.50%

Note: Delta Return is your estimate of the difference between annual return in the future and historical annual return from the S&P500

Median date at which you are worth $1 Million: 2005

Portfolio value at retirement
(Median): $1,075,189
Average: $1,075,059

Annual Draw (2005 $)	$40,000	$36,000
Probability of Running Out of Money	**Age**	**Age**
10%	91	97
15%	95	102
20%	99	109
25%	102	117
30%	109	Not Found
35%	115	Not Found
40%	122	Not Found
45%	Not Found	Not Found
50%	Not Found	Not Found

Annual Withdrawal Rate as % of Portfolio at Retirement: 3.72%

Age to Exhaust Funds if Cashed Out at Retirement:

80th Percentile:	81
Median:	80
20th Percentile:	79

Calculator by Quantext

We have assumed a portfolio value of $1,000,000 but the calculations for Portfolio Value at Retirement are scaled to the end of the year and we are assuming that you do not draw in the retirement year so we see an additional 7.5% in average value at year's end. For the annual draw of $40,000 per year, the probabilities of running out of funds are shown in the table in the lower left-hand corner of this report. Jane has a 20% chance of running out of money during her 99th year (i.e. 39 years of retirement), only one year different from the result from Bernstein's analysis using ***MCRetire***. The results also show a 10% chance of running out of money at age 91, which implies a 90% success rate for a 31-year

retirement, consistent with the Monte Carlo model run by John Greaney and cited in the previous section.

If we modify the inputs to the Retirement Planner to match those in the Monte Carlo results by Greaney, we get the following result:

Retirement Planning Report

Prepared For: **Jane Doe**

Preparation Date: **5/31/2005**

Page 1: Basic Input and Projections

www.quantext.com

Current Age	60
Date of Retirement	2005
Age at Retirement	60
Annual Contribution (2005 Dollars)	$0
Current Portfolio Value	$1,000,000
Inflate Contributions at inflation?	Yes
Inflate Income Draw?	Yes
Income in Retirement (2005 Dollars)	$40,000

Assumed Inflation Rate (Annual)	4.00%
Annual Standard Deviation of Market Return (% of normal)	80.00%
Annual Standard Deviation	12.05%
Delta Return	-1.20%
Average Annual Return of Market	9.10%

Note: Delta Return is your estimate of the difference between annual return in the future and historical annual return from the S&P500

Median date at which you are worth $1 Million: 2005

Portfolio value at retirement
(Median): $1,091,234
Average: $1,091,077

Annual Draw (2005 $)	$40,000	$36,000
Probability of Running Out of Money	**Age**	**Age**
10%	90	96
15%	94	102
20%	100	109
25%	104	122
30%	113	Not Found
35%	124	Not Found
40%	Not Found	Not Found
45%	Not Found	Not Found
50%	Not Found	Not Found

Annual Withdrawal Rate as % of Portfolio at Retirement: 3.67%

Age to Exhaust Funds if Cashed Out at Retirement:

80th Percentile:	80
Median:	79
20th Percentile:	77

Calculator by Quantext

We now have an average return from the portfolio of 9.1%, an annual standard deviation of 12% and an inflation rate of 4% per year (see top right hand corner of the figure

above). The real rate of return (the average return beyond inflation is 9.1% - 4% = 5.1%) is now higher than the assumption used by Bernstein using ***MCRetire***, but the results are effectively the same, with a 10% chance of running our of money for a 30-year retirement and a 20% chance of running out of money for a 40-year retirement.

The results from using the ***Quantext Retirement Planner*** are also obviously similar to the analysis generated using the T. Rowe Price model. The portfolio from T. Rowe Price with 60% stock / 30% bonds /10% short-term securities gives a 40-year lifespan for the retirement portfolio with a confidence of 80% (i.e. a 20% failure rate) and a 30-year lifespan with a confidence of 90% (i.e. a 10% failure rate). This is very close to the results from the ***Quantext Retirement Planner*** shown above. In an earlier section, we also mentioned that the T. Rowe Price model gave virtually identical results for a portfolio with 80% stock and 20% bonds. When we set up a portfolio using Schiller's historical data to provide the average return and standard deviation of return for an 80/20 portfolio, we can also get very similar results to our 60/40 case (see next figure). We end up with a 10% chance of running out of money by age 89 (29 years of retirement) and a 20% chance at age 99. There is a problem with taking these examples too far, of course, and that is that we really need to get more specific about real portfolio allocation.

Retirement Planning Report

Prepared For: **Jane Doe**

Preparation Date: **6/1/2005**

Page 1: Basic Input and Projections

www.quantext.com

Current Age	60
Date of Retirement	2005
Age at Retirement	60
Annual Contribution (2005 Dollars)	$0
Current Portfolio Value	$1,000,000
Inflate Contributions at inflation?	Yes
Inflate Income Draw?	Yes
Income in Retirement (2005 Dollars)	$40,000

Assumed Inflation Rate (Annual)	Annual Standard Deviation of Market Return (% of normal)
3.00%	87.00%
Annual Standard Deviation	13.11%
Delta Return	Average Annual Return of Market
-2.00%	8.30%

Note: Delta Return is your estimate of the difference between annual return in the future and historical annual return from the S&P500

Median date at which you are worth $1 Million: 2005

Portfolio value at retirement
(Median): $1,083,257
Average: $1,083,086

Annual Draw (2005 $)	$40,000	$36,000
Probability of Running Out of Money	**Age**	**Age**
10%	89	94
15%	93	100
20%	99	107
25%	104	119
30%	114	Not Found
35%	Not Found	Not Found
40%	Not Found	Not Found
45%	Not Found	Not Found
50%	Not Found	Not Found

Annual Withdrawal Rate as % of Portfolio at Retirement: 3.69%

Age to Exhaust Funds if Cashed Out at Retirement:

80th Percentile:	82
Median:	80
20th Percentile:	79

The basic calculations for retirement planning with a treatment of risk using a Monte Carlo model are sufficiently standard that it is not surprising that their results are similar. Still, given that these results were generated by four different Monte Carlo models, it is certainly compelling to see how closely they agree. These results show why it is so important to account for risk. If we take the results from the ***Quantext Retirement Planner*** from the previous chart and simply ignore risk by setting the average annual standard deviation to zero, we find that it appears that we can draw a great deal more money from the retirement account:

Retirement Planning Report

Prepared For: **Jane Doe**

Preparation Date: **5/31/2005**

Page 1: Basic Input and Projections

www.quantext.com

Current Age	60
Date of Retirement	2005
Age at Retirement	60
Annual Contribution (2005 Dollars)	$0
Current Portfolio Value	$1,000,000
Inflate Contributions at inflation?	Yes
Inflate Income Draw?	Yes
Income in Retirement (2005 Dollars)	$57,450

Assumed Inflation Rate (Annual)	4.00%
Annual Standard Deviation of Market Return (% of normal)	0.00%
Annual Standard Deviation	0.00%
Delta Return	-1.20%
Average Annual Return of Market	9.10%

Note: Delta Return is your estimate of the difference between annual return in the future and historical annual return from the S&P500

Median date at which you are worth $1 Million: 2005

Portfolio value at retirement
(Median): $1,090,976
Average: $1,090,976

Annual Draw (2005 $)	$57,450	$51,705
Probability of Running Out of Money	**Age**	**Age**
10%	100	121
15%	100	121
20%	100	121
25%	100	121
30%	100	121
35%	100	121
40%	100	121
45%	100	121
50%	100	121

Annual Withdrawal Rate as % of Portfolio at Retirement: 5.27%

Age to Exhaust Funds if Cashed Out at Retirement:

80th Percentile:	74
Median:	74
20th Percentile:	74

Because there is no risk, we can draw $57,450 from the portfolio each year and the portfolio will last 40 years in retirement. This means that by ignoring risk, it appears that you can draw substantially more in income per year, with no risk of running down the portfolio before 40 years. If we run with this amount of draw but with the standard deviation turned back on (i.e. accounting for risk), we get the following:

Retirement Planning Report

Prepared For: **Jane Doe**

Preparation Date: **5/31/2005**

Page 1: Basic Input and Projections

www.quantext.com

Current Age	60
Date of Retirement	2005
Age at Retirement	60
Annual Contribution (2005 Dollars)	$0
Current Portfolio Value	$1,000,000
Inflate Contributions at inflation?	Yes
Inflate Income Draw?	Yes
Income in Retirement (2005 Dollars)	$57,450

Assumed Inflation Rate (Annual)	Annual Standard Deviation of Market Return (% of normal)
4.00%	80.00%
Annual Standard Deviation	12.05%
Delta Return	Average Annual Return of Market
-1.20%	9.10%

Note: Delta Return is your estimate of the difference between annual return in the future and historical annual return from the S&P500

Median date at which you are worth $1 Million: 2005

Portfolio value at retirement
(Median): $1,091,234
Average: $1,091,077

Annual Draw (2005 $)	$57,450	$51,705
Probability of Running Out of Money	**Age**	**Age**
10%	77	80
15%	80	83
20%	82	85
25%	83	88
30%	84	89
35%	86	91
40%	88	95
45%	92	100
50%	95	104

Annual Withdrawal Rate as % of Portfolio at Retirement: 5.26%

Age to Exhaust Funds if Cashed Out at Retirement:

80th Percentile:	75
Median:	74
20th Percentile:	73

When we include risk, you now have more than a 40% chance of running out of money before you get to thirty years in retirement—quite a change. If we decide to go with Bernstein's suggestion of planning for the 80% success rate (a 20% probability of running out of funds), including the effects of risk reduces our confident horizon for retirement from 40 years (age 100 from the no risk case) down to 22 years (age 82 above). **Accounting for risk has lowered the calculated retirement lifetime by 18 years!**

Some Further Thoughts

We have compared results from four Monte Carlo analyses of retirement withdrawal rates and the estimated survival probabilities. There are many permutations on such an analysis. Obviously, you can vary the allocation to determine how things change. A varying mix of bonds and stocks will provide different levels of expected return and different levels of risk. The ***Quantext Retirement Planner*** also allows you to take the next step and model your portfolio using real stock and bond portfolios in a mix and see the effect of different assumptions about future market conditions on the returns from the different components of the portfolio. Detailed discussion of this process is provided in the online text describing the ***Quantext Retirement Planner***. This functionality allows you to look, for example, at the impact of having part of your portfolio in your company's stock in a Monte Carlo framework. It is also important to understand that the analyses shown here assume that you know exactly how much you will have in your portfolio at the time of retirement. If you are substantially younger than your retirement age, and you are investing in order to reach your retirement target portfolio, there is additional investment risk because you do not know how much return you will actually generate between now and when you retire. The ***Quantext Retirement Planner*** allows you to start the simulation at a specified number of years before your desired retirement—an important feature. Links to our papers on these and related issues are given in the Web Resources section.

Given these caveats, and the obvious fact that there is uncertainty in all estimates of future market returns and standard deviations, the four Monte Carlo models used in this study still generate some useful and remarkably consistent results that can help in planning for retirement. A four percent withdrawal rate, given a portfolio which is allocated with 60% in diversified stock funds and 40% in short or intermediate term bond funds, looks quite reasonable in light of the risk and return associated with this mix. Further, the fact that the models agree so well is important in the process of establishing Monte Carlo models as the standard of practice for personal financial planning.

Web Resources

Quantext Retirement Planner:
http://www.quantext.com/gpage3.html

Quantext Retirement Planner Online Textbook
http://www.quantext.com/Retirement%20Planner.pdf

Dr. Robert Schiller's Historical Data:
http://www.econ.yale.edu/~shiller/data/ie_data.htm

The Retirement Calculator from Hell, Part III:
http://www.efficientfrontier.com/ef/901/hell3.htm

Where do these guys get a 2% withdrawal rate?
http://www.retireearlyhomepage.com/twoperc.html

T. Rowe Price tutorial on Monte Carlo models
http://www3.troweprice.com/ric/RIC/eprim_TutorialRight.html#buildingblocks

T. Rowe Price Retirement Income Calculator
http://www3.troweprice.com/ric/RIC/