

## Chapter 16

# Dots-and-Boxes

Come, children, let us shut up the box.  
William Makepeace Thackeray, *Vanity Fair*, Ch. 67.

I could never make out what those damned dots meant.  
Lord Randolph Churchill.

**Dots-and-Boxes** is a familiar paper and pencil game for two players and has other names in various parts of the world. Two players start from a rectangular array of dots and take turns to join two horizontally or vertically adjacent dots. If a player completes the fourth side of a unit square (**box**) he initials that box and must then draw another line (so that completing a box is a complimenting move). When all the boxes have been completed the game ends and whoever has initialled more boxes is declared the winner.

A player who *can* complete a box is not obliged to do so if he has something else he prefers to do. Play would become significantly simpler were this obligation imposed; see the article by Holladay mentioned in the references.

Figure 1 shows Arthur's and Bertha's first game, in which Arthur started. Nothing was given away in the fairly typical opening until Arthur was forced to make the unlucky thirteenth move, releasing 2 boxes for Bertha. Her last bonus move enabled Arthur to take the bottom 3 boxes, but he then had to surrender the last 4.

This is how most children play, but Bertha is brighter than most. She started the return match with the opening that Arthur had used. He was happy to copy Bertha's replies from that game, and was delighted to see her follow it even as far as that unlucky thirteenth move, which had proved his undoing (Fig. 2). He grabbed those 2 boxes and happily surrendered the bottom 3, expecting 4 in return. But Bertha astounded him by giving him back 2. He pounced on these, but when he came to make his bonus move, realized he was doubled-crossed!

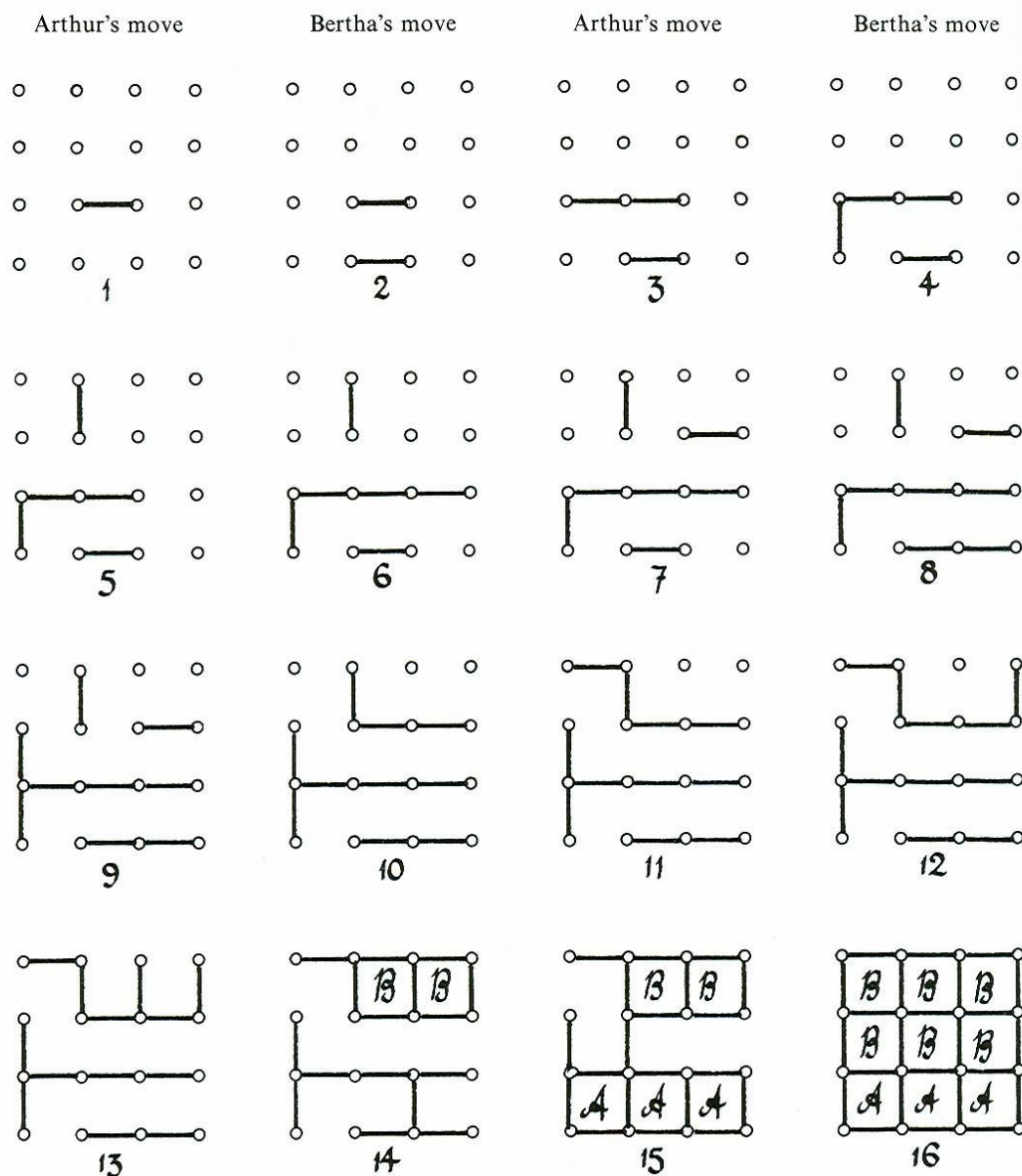


Figure 1. Arthur's and Bertha's First Game.

Bertha beats all her friends in this double-dealing way. Most children play at random unless they've looked quite hard and found that every move opens up some chain of boxes. Then they give the shortest chain away and get back the next shortest in return, and so on.

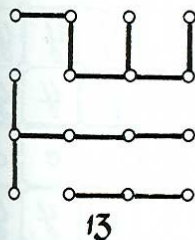
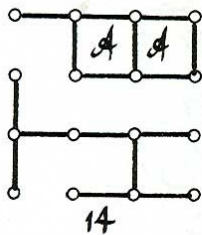
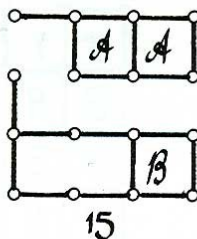
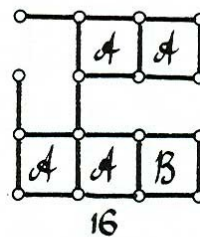
Bertha's  
Unlucky(?) MoveArthur's  
Delighted!Bertha's  
Brilliant MoveArthur's  
Double-crossed!

Figure 2. Bertha's Brilliance Astounds Arthur.

But when you open a long chain for Bertha, she may close it off with a double-dealing move which gives you the last 2 boxes but forces you to open the next chain for her (Fig. 3). In this way *she* keeps control right to the end of the game.

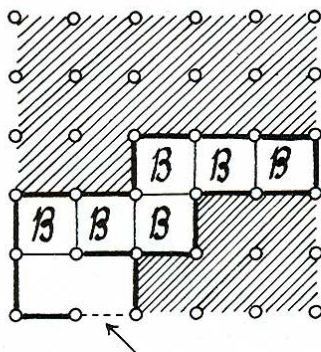
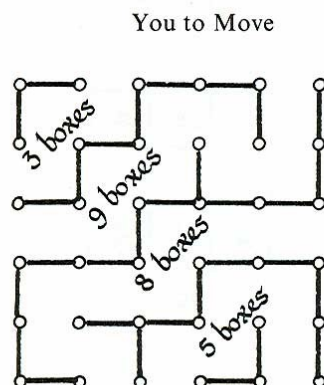


Figure 3. Bertha's Double-Dealing Move.

You can see in Fig. 4 just how effective this strategy can be. By politely rejecting two cakes on every plate but the last you offer her, Bertha helps herself to a resounding 19 to 6 victory. In the same position you'd have defeated the ordinary child 14 to 11.

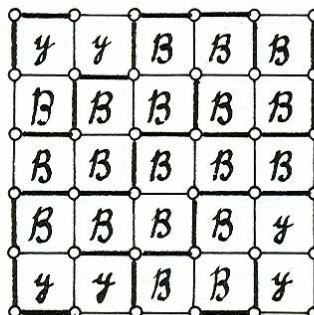
## DOUBLE-DEALING LEADS TO DOUBLE-CROSSES

Each double-dealing move is followed, usually immediately, by a move in which two boxes are completed with a single stroke of the pen (Fig. 5). These moves are very important in the theory. We'll call them **doublecrossed** moves, because whoever makes them usually has been!



3-chain  
5-chain  
8-chain  
9-chain

Bertha (B) beats you (y)



B	y
1	2
3	2
6	2
9	
<hr/>	
19	6

You (Y) beat the ordinary child (c)



c	Y
3	
	5
8	
	9
<hr/>	
11	14

Figure 4. Double-Dealing Pays Off!

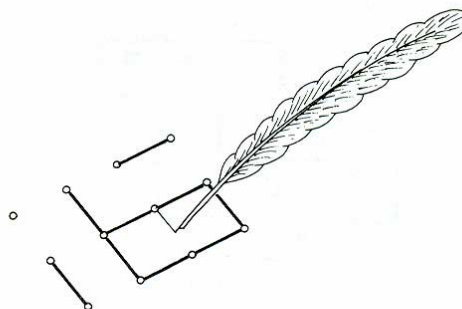


Figure 5. A Doublecross—Two Boxes at a Single Stroke.

Now Bertha's strategy suggests the following policy:

Make sure there are long chains about  
and try to force your opponent to be  
the first to open one.

Try To Get Control ...



We'll say that whoever can force her opponent to open a long chain has **control** of the game. Then:

When you have control, make sure you keep it by politely declining 2 boxes of every long chain except the last.

... And Then Keep It.

The player who has control usually wins decisively when there are several long chains.

So the fight is really about control. How can you make sure of acquiring this valuable commodity? It depends on whether you're playing the odd- or even-numbered turns ...



Figure 6. Which is Dodie and Which is Evie?

Arthur and Bertha live next to the Parr family, in which there are two little sisters called Dodie and Evie (Bertha often teases them by calling them the Parrotty Girls!). You can see them playing the 4-box game in Fig. 6. Dodie's a year younger than Evie and so always has first turn in any game they play. They've got so used to playing like this that even when they're playing somebody else, Dodie always insists on taking the odd-numbered moves while Evie will only take the even-numbered ones:

Dodie Parr: odd parity,  
Evie Parr: even parity.

The rule that helps them take control is:



*Dodie*  
tries to make the number of  
initial dots + doublecrossed moves  
*odd.*  
*Evie*  
tries to make this number  
*even.*

Be SELFish about Dots + Doublecrosses!

In simple games the number of doublecrosses will be one less than the number of long chains and this rule becomes:

#### THE LONG CHAIN RULE

Try to make the number of  
initial dots + eventual long chains  
*even* if your opponent is *Evie*,  
*odd* if your opponent is *Dodie*.

The OPPOsite for Dots + Long Chains!

The reason for these rules is that whatever shape board you have on your paper, you'll find that:

$$\begin{array}{r} \text{Number of dots you start with} \\ + \text{Number of doublecrosses} \\ \hline = \text{Total number of turns in the game.} \end{array}$$

We'll show this in the Extras.

#### HOW LONG IS "LONG"?

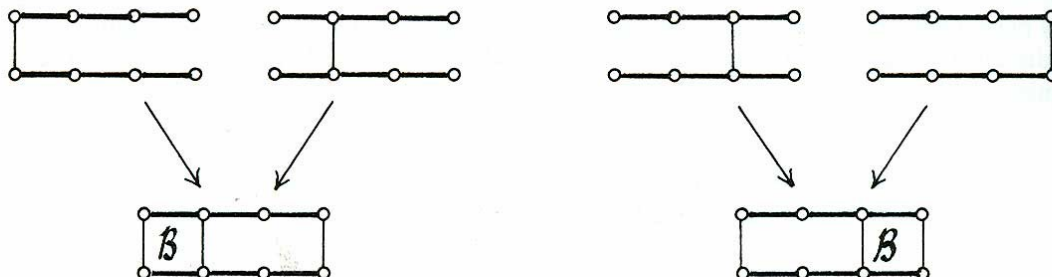


Figure 7. Bertha's Endgame Technique.

We can find the proper definition of long by thinking about Bertha's endgame technique. A **long chain** is one which contains 3 or more squares. This is because whichever edge Arthur draws in such a chain, Bertha can take all but 2 of the boxes in it, and complete her turn by drawing an edge which does not complete a box. Figure 7 shows this for the 3-square chain. A chain of 2 squares is *short* because our opponent might insert the *middle* edge, leaving us with no way of finishing our turn in the same chain. This is called (Fig. 8(a)) the **hard-hearted handout**.

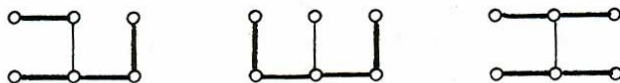


Figure 8(a) Hard-Hearted Handouts.

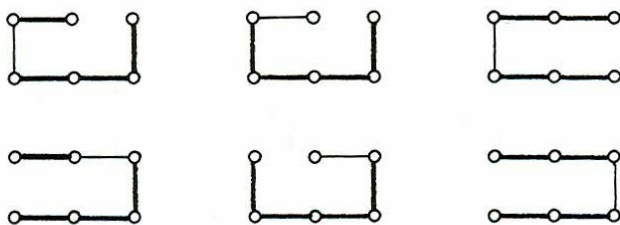


Figure 8(b). Half-Hearted Handouts.

When you think you are winning, but are forced to give away a pair of boxes, you should always make a hard-hearted handout, so that your opponent has no option but to accept. If you use a **half-hearted** one (Fig. 8(b)) he might reply with a double-dealing move and regain control. But if you're losing, you might try a half-hearted handout on the Enough Rope Principle (Chapter 1 Extras). Officially this is a bad move, since your opponent, if he has any sense, will grab both squares. But boys by billions, being bemused by Bertha's brilliance, blindly blunder both boxes back.

## THE 4-BOX GAME

When Dodie was *very* young, the girls often played the 4-box game and offset Dodie's first move advantage by calling it a win for Evie (the second player) when they each got 2 boxes:

TWO TWOS IS A WIN TO TWO

At first Dodie would never give away a box if she could see something else to do, and Evie, who you can see is a very symmetrical player, would always win by copying Dodie's moves on the opposite side of the board. But after watching Bertha playing Evie, Dodie found how to counter this strategy by making a Greek gift on her 7th move. Evie can still win if Dodie dares to stray from the Path of Righteousness but must resist her temptation to make *every* move a symmetrical one (Fig. 9).



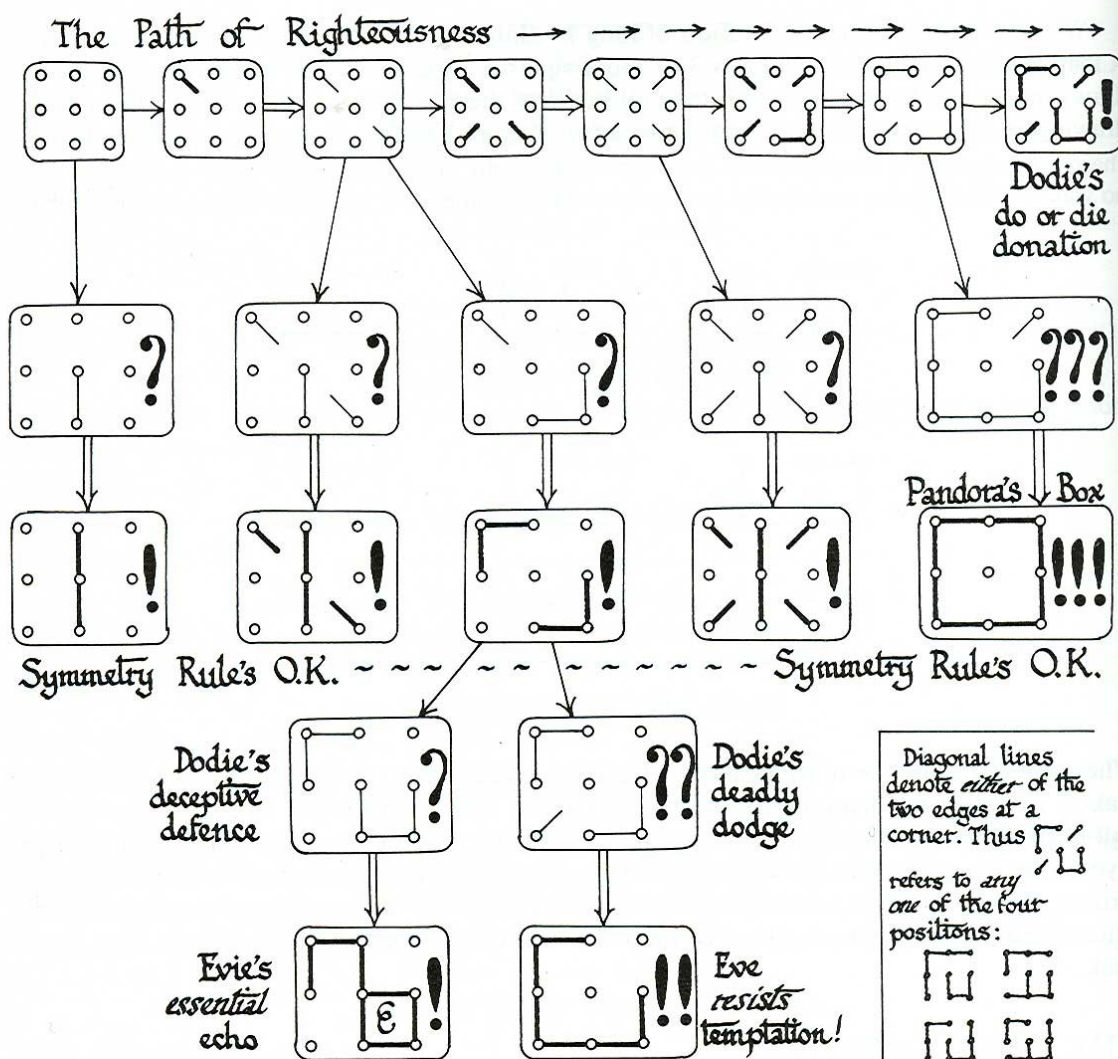


Figure 9. Evie Envisaging Every Eventuality.

Even though Dodie has the win, it's much harder to write out in full her best plays against sufficiently cunning opponents. In Fig. 34 of the Extras we give an adequate strategy for Dodie and in Fig. 35 a complete list of  $\mathcal{P}$ -positions for both players. This little game is full of traps for the unwary, and those of you who have written to us for advice on becoming Professional Boxers will find these tables very useful in the bruising preliminary contests on the 4-box board.

If the chain lengths are

4                      or                      loop of 4  
3 + 1                      2 + 2  
                                 1 + 1 + 1 + 1

the winner will usually be

Dodie                      or                      Evie



in agreement with the Long Chain Rule, but on this small board, Dodie should often defy the rule and win by splitting the chains as  $2 + 1 + 1$ .

## THE 9-BOX GAME

Surprisingly, the Long Chain Rule makes the 9-box game seem easier than the 4-box one. This time Evie wins, and her basic strategy is to draw 4 spokes as in Fig. 10, forcing every long chain to go through the centre. Against most children this wins for Evie by at least 6–3, but Dodie can hold her down to 5–4, perhaps by sacrificing the centre square, after which Evie should abandon her spoke strategy. Of course, Evie's real aim is to arrange that there's just one long chain, and she often improves her score by forming this chain in some other way.

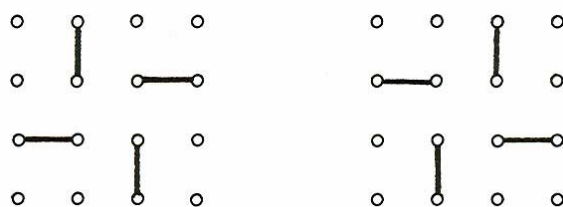


Figure 10. Lucky Charms Ward Off More Than One Long Chain; Evie Puts Spokes in Dodie's Wheel.

Evie usually prefers to put her spokes in squares where another side is already drawn, and she's careful to draw spokes in only *one* of the two swastika patterns of Fig. 10. There usually aren't any double-crossed moves, so that Evie wins at the  $(16 + 0 =)$  16th turn.

Dodie tries to arrange *her* moves so that some spoke can only be inserted as a sacrifice, and *either* cuts up the chains as much as possible (maybe with a centre sacrifice) *or* forms *two* long chains when Evie isn't thinking. Every now and then a half-hearted handout has saved the game for her just when she thought that all was lost.

## THE 16-BOX GAME

We don't know who wins on the  $4 \times 4$  box board, which makes a very interesting game to play. Evie tries to make the number of long chains 2, while Dodie tries to cut it down to 1 or force

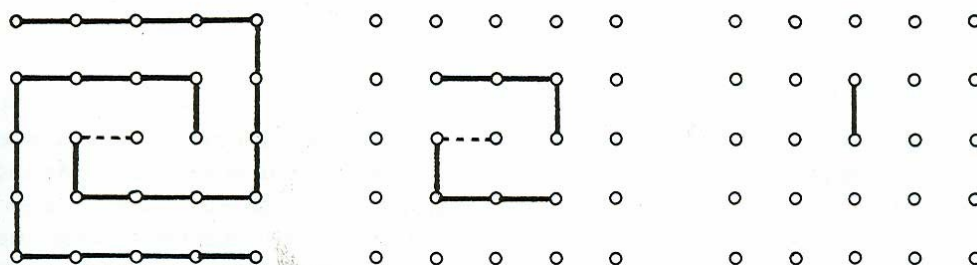


Figure 11. "Come into my symmetrical parlor!"



it up to 3. Evie beats many children with her symmetry strategy, but Dodie remembers her trick from the 4-box game. If she thinks her opponent will mimic her every move, she can lure him into the spider's web of Fig. 11(a), but when he's less predictable she finds it safer just to use the middle of the web (Fig. 11(b)). Dodie doesn't usually open with Fig. 11(c), because she finds the symmetry strategy very hard to beat.

## OTHER SHAPES OF BOARD

To beat all your friends on larger square and rectangular boards you'll really need the Long Chain Rule. Remember to count a closed loop of 4 or more cells as *two* long chains and that each doublecross, no matter who makes it, changes the number of long chains you want. (Think of a doublecross as a long chain that's already been filled in.) It's good tactics to make the long chains as long as possible and avoid closed loops when you can, because you forfeit *four* boxes when declining a loop. These rules work for all large boards and even for triangular Dots-and-Boxes boards, like that in Fig. 12.

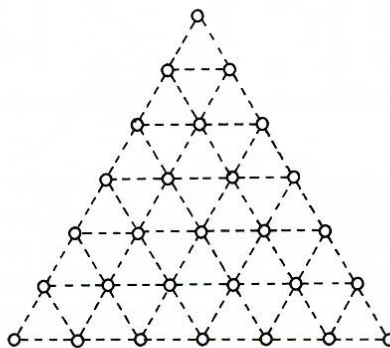


Figure 12. A Board with 28 Dots and 36 Triangular Cells.

Of course, if your opponent is also using the Long Chain Rule, the fight for control might be quite hard. The game of Nimstring, discussed in the rest of this chapter, is what control is all about. There's a piece in the Extras that describes some of the rare occasions when you might find it wise to lose control.

## DOTS-AND-BOXES AND STRINGS-AND-COINS

You can play a dual form of Dots-and-Boxes, called **Strings-and-Coins**, with strings, coins and scissors. The ends of each piece of string are glued to two different coins or to a coin and the ground (each string has at most one end glued to the ground) and each player in turn cuts a new string. If your cut completely detaches a coin, you pocket it and must then cut another string (if there's one still uncut). The game ends when all coins are detached and the player who pockets the greater number is the winner.