Probability and Tree Diagrams

NY CCSS Lesson 6 & 7

Today's Objectives:

By the end of today's class, you should:

1. Be able to make a tree diagram to represent the outcomes in a sample space.

3. Be able to use a weighted tree diagram to calculate the probability of compound events

A tree diagram is an important way of organizing and visualizing outcomes.

A tree diagram is a particularly useful device when the experim can be thought of as occurring in stages.

When the information about probabilities associated with each branch is included, a weighted tree diagram supports the computation of the probabilities of the different possible outcome

Family Game Night

A family decides to play a game each night. They all agree to use a tetrahedral die (i.e., a four-sided pyramidal die where each of four possible outcomes (1, 2, 3 or 4) is equally likely) each night to randomly determine if they will play a board game ($\boldsymbol{B}$) or a card game ($\boldsymbol{C}$). They've decided that an odd number represents a board game and an even number represents a card game.

We'll develop the tree diagram mapping the possible overall outcomes for tow nights, Monday and Tuesday, on the next page.

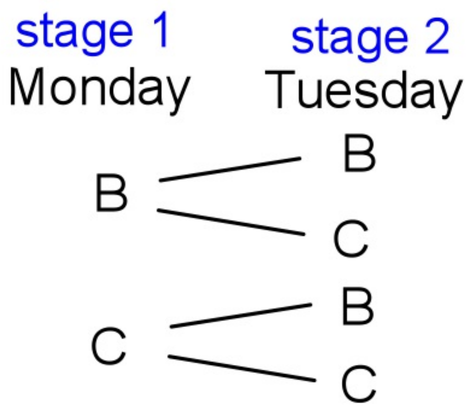
☐ To make a tree diagram, first present all possibilities for the first stage. (In this case, Monday.) We'll use B for board games and C for card gam

stage 1 stage 2

☐ Then, from each branch of the first stage, attach all possibilities for the second stage.

If the situation has more than two stages, this proces would be repeated until all stages had been presentec

☐ List all possible outcomes and calculate probability. outcome & probability



outcome & probability	
BB	.25
BC	.25
CB	.25
CC	.25

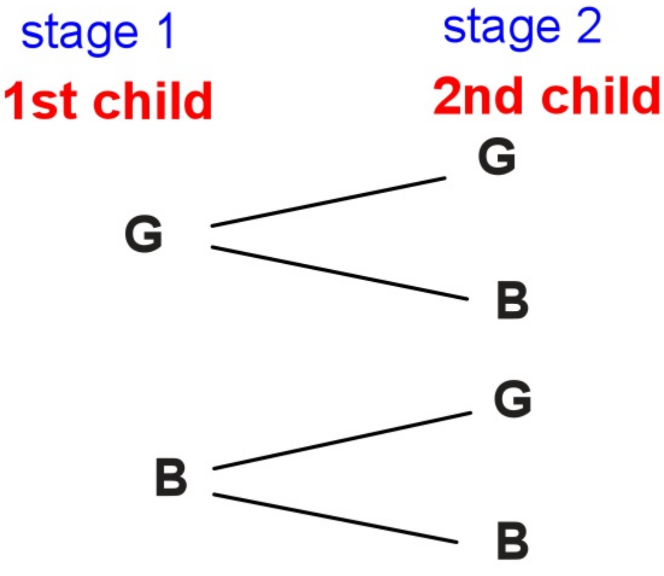
If “$\boldsymbol{BB}$” represents two straight nights of board games, what does “$\boldsymbol{CB}$” represent‘

List the outcomes where exactly one board game is played over two days. How r
ıtcomes were there?

How would our tree diagram change if the family decided that they liked board gá
ıore than card games and so changed the representation of board games to 1, 2 &
ard games to 4?

Activity 1: Make a tree diagram to represent the situation below.

Two friends meet at a grocery store and remark that a neighboring f
just welcomed their second child. It turns out that both children in thi
family are girls, and they are not twins. One of the friends is curious
what the chances are of having 2 girls in a family's first two births.
Assume that for each birth the probability of a "boy" birth is 0.5 and
probability of a "girl" birth is also 0.5.



Outcomes	Probability
G G	$\frac{1}{4}$
G B	$\frac{1}{4}$
B G	$\frac{1}{4}$
B B	$\frac{1}{4}$
	$\frac{4}{4} = 1$

Activity 2: A baker can make yellow or white cakes with a choice of chocolate, strawberry, or vanilla icing. Make a tree diagram show all of the possible combinations of cakes.

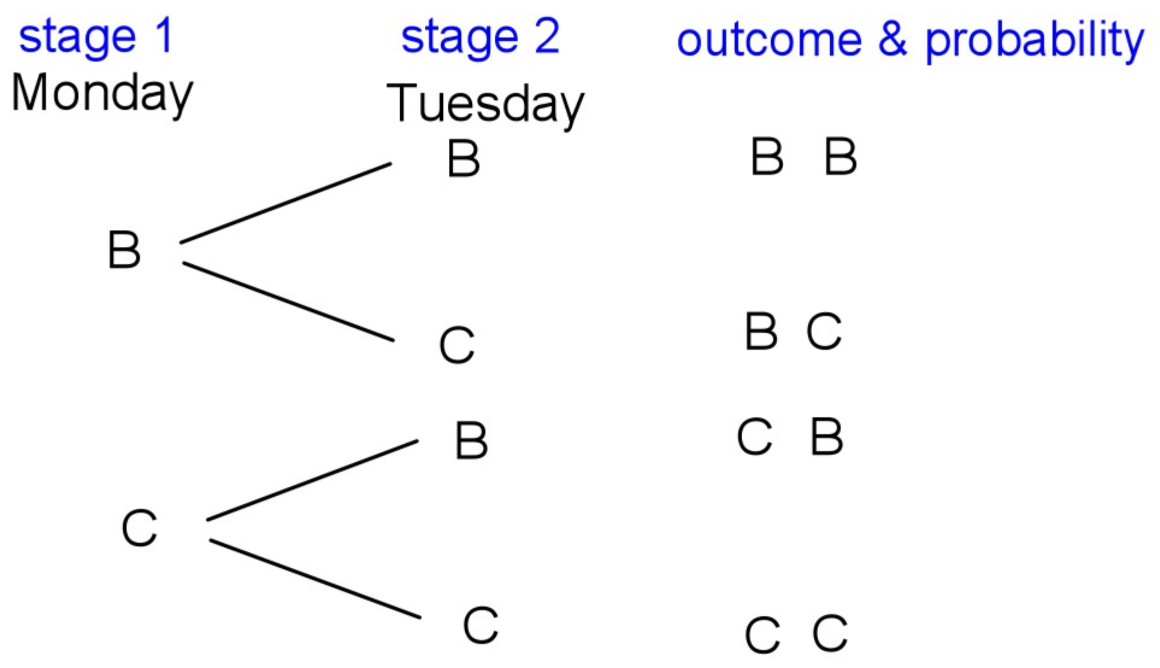
stage 1 **Cake**	stage 2 **Icing**	**Outcomes**	**Probability**
Yellow	**Chocolate**	**YC**	$\frac{1}{6}$
	Strawberry	**YS**	$\frac{1}{6}$
	Vanilla	**YV**	$\frac{1}{6}$
White	**Chocolate**	**WC**	$\frac{1}{6}$
	Strawberry	**WS**	$\frac{1}{6}$
	Vanilla	**WV**	$\frac{1}{6}$

$$\frac{6}{6} = 1$$

Using a Weighted Tree Diagram for Compound Events that are Independent

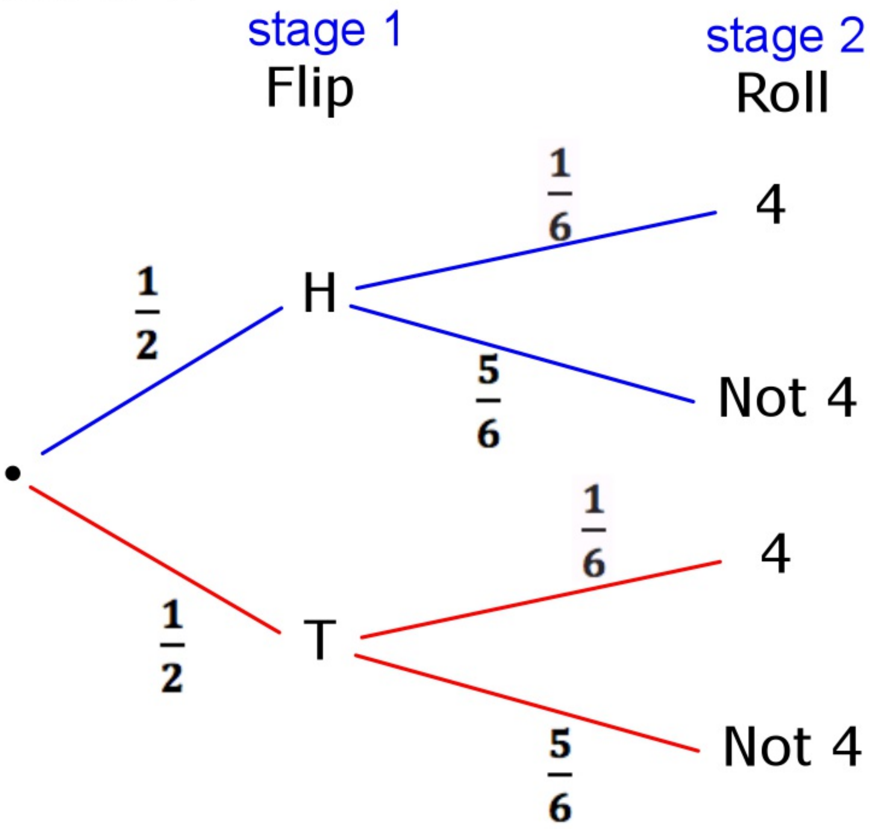
t's go back to our family game night situation and see how a weighted ee diagram can easily answer the question you considered earlier.

How would our tree diagram change if the family decided that they liked board games more than card games and so changed the representation of board games to 1, 2 & 3 and card games to 4?



Using a Weighted Tree Diagram for Compound Events that are Independ

Let's take a look at an example using two different events, flipping a coi and then rolling a die. We might want to know the probability of getting Head and a 4.



Probability

$P(H,4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$P(H, \text{not } 4) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

$P(T, 4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$P(T, \text{not } 4) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

$\frac{12}{12} = 1$

The probability of getting a head and a 4 is $\frac{1}{12}$

Let's take a look again at flipping a coin and then rolling a die.

But this time we want to find the probability of getting a H or a 4.

This might seem to be a "mutually exclusive" situation since it's an OR

But is

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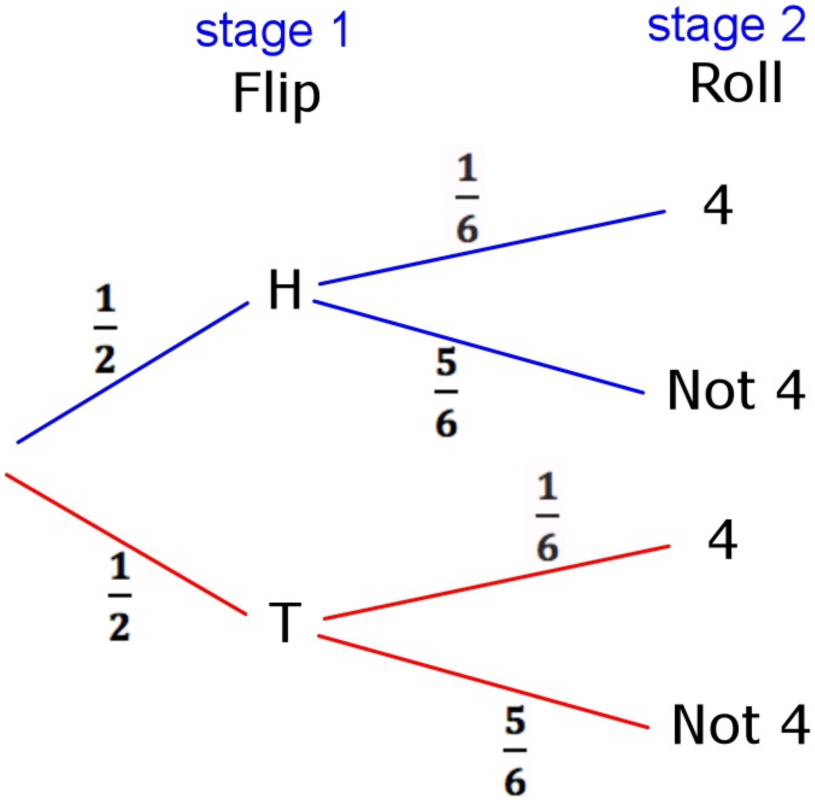


Probability

$P(H,4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

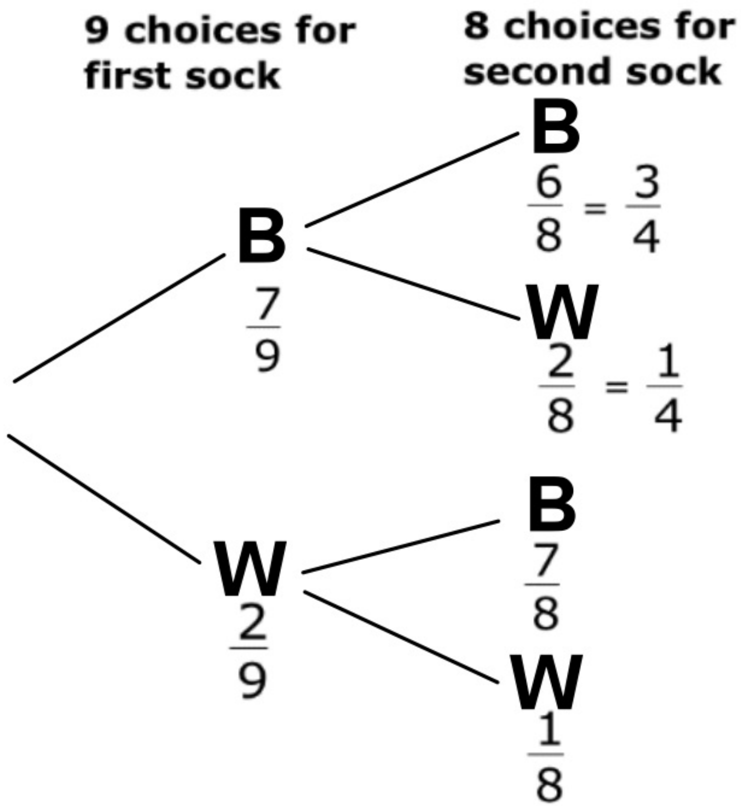
$P(H, \text{not } 4) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

$P(T, 4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$P(T, \text{not } 4) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

Using a Weighted Tree Diagram for Dependent Events

There are 7 black socks and 2 white socks in a drawer. Two socks are removed. What is the probability that the colors match? A weighted tree diagram can be used to find the probability that the colors match.

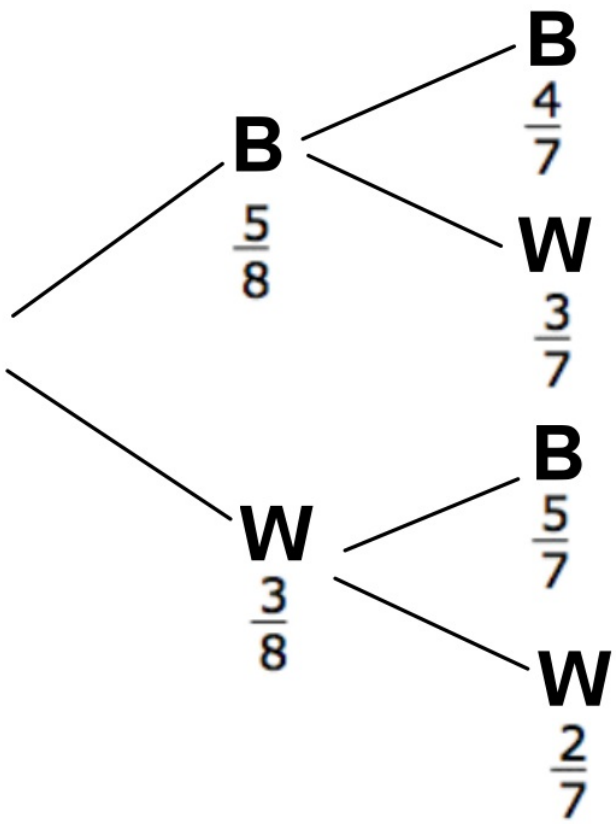


Outcomes	Multiply to find the probablity of each outcome
B B	$\frac{7}{9} \bullet \frac{3}{4} = \frac{21}{36}$
B W	$\frac{7}{9} \bullet \frac{1}{4} = \frac{7}{36}$
W B	$\frac{2}{9} \cdot \frac{7}{8} = \frac{7}{36}$
W W	$\frac{2}{9} \cdot \frac{1}{8} = \frac{1}{36}$

probability of drawing a matching pair is P(b, b or w, w) = P(b, b) + P(w, w)

$$\frac{7}{12} + \frac{1}{36} = \frac{11}{18}$$

Activity 3: There are 5 black socks and 3 white socks in a drawer. Two socks are removed. Make a weighted Tree Diagram showi the probability of each outcome and use it to find the probability that the colors match.



B B $\frac{5}{8} \bullet \frac{4}{7} = \frac{20}{56} \; or \; \frac{5}{14}$

B W $\frac{5}{8} \bullet \frac{3}{7} = \frac{15}{56}$

W B $\frac{3}{8} \bullet \frac{5}{7} = \frac{15}{56}$

W W $\frac{3}{8} \bullet \frac{2}{7} = \frac{6}{56} \; or \; \frac{3}{28}$

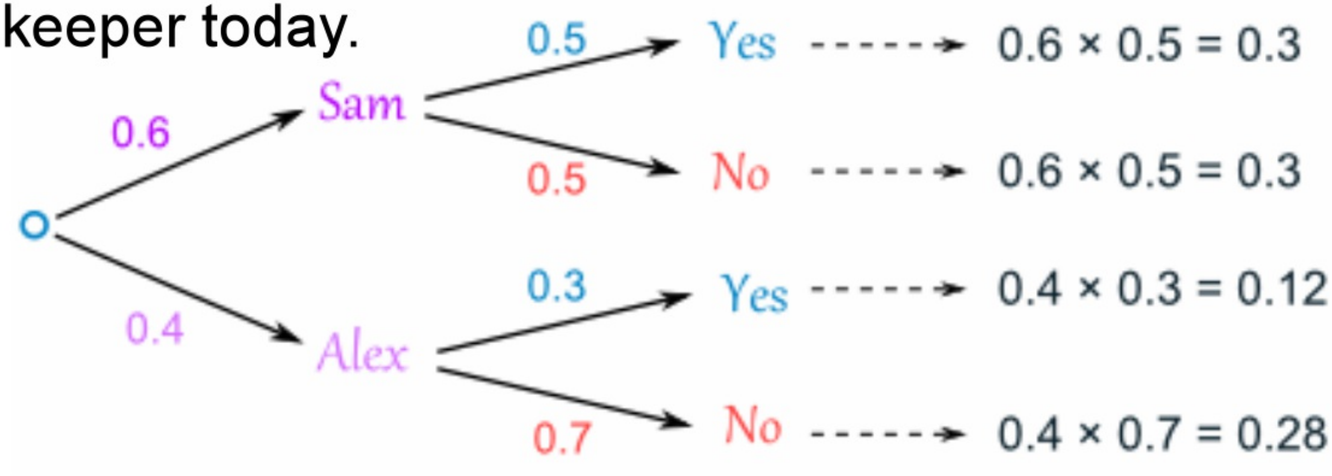
The probability of drawing a matching pair is $\frac{5}{14} + \frac{3}{28} = \frac{13}{28} \approx 0.46.$

ctivity 4: Soccer Game

You are off to soccer, and love being the Goalkeeper, but that depends who is the Coach that day.

- Sam is the coach for about 6 out of every 10 games
- with Coach Sam the probability of being Goalkeeper is 0.
- with Coach Alex the probability of being Goalkeeper is 0

onstruct a weighted tree diagram to find the probability that you will a Goalkeeper today.



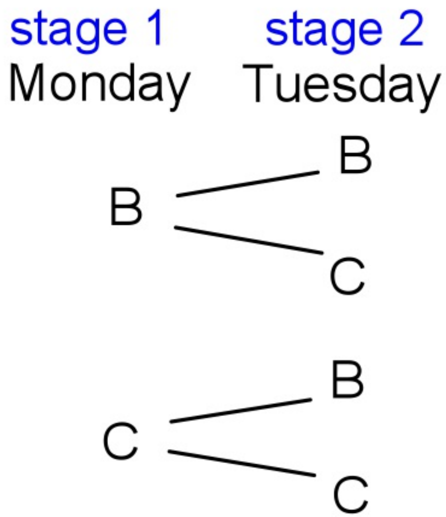
0.3 + 0.12 = **0.42 probability** of being a Goalkeeper today

ctivity 5: Discuss and do the following.

What if there are more than two stages in a compound event.

1) Can we still use a tree diagram to represent outcomes and calculate probability? Why or Why not?

2) Go back to our family game night. What could we do if we wai to show outcomes over three nights, rather than the two we did earlier?



3) If "BBB" represents three straight nights of board games, what does "CBB" represent?

4) List all outcomes where exactly two board games were played over three days. How many outcomes were there?

Activity 6: In your groups, discuss and answer the questions on the handout you've been given.

HW# 57:

1) **10.6 #4-5 & 9-10 p. 566**
2) **Ch. 10 NY CCSS Lesson 6 & 7 HW handout**

Remember Ch. 10 Test corrections due Wednesday