GRAPHING WITH LOGARITHMIC PAPER

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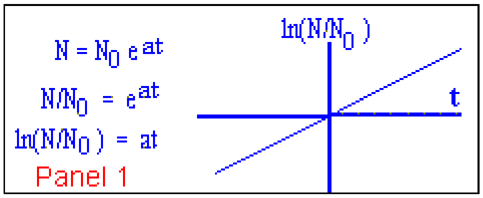
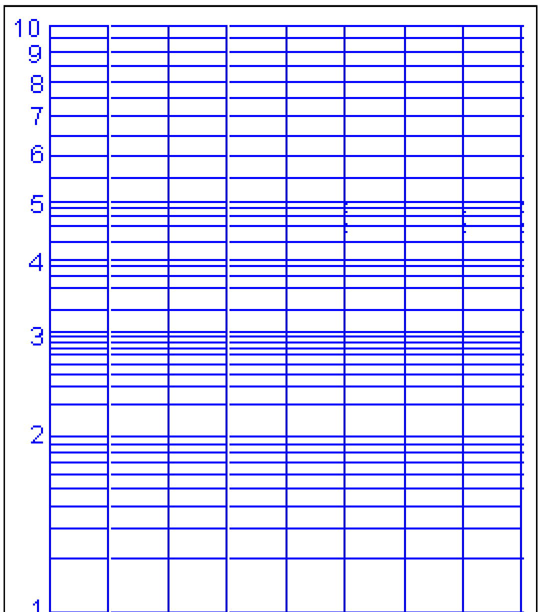


Panel 1

At the end of the tutorial on Graphing Simple Functions, you saw how to produce a linear graph of the exponential function $N = N_0\, e^{at}$ as shown in panel 1. This was done by taking the natural logarithm of both sides of the equation and plotting $\ln(N/N_0)$ vs t to get a straight line of slope a.



Panel 2

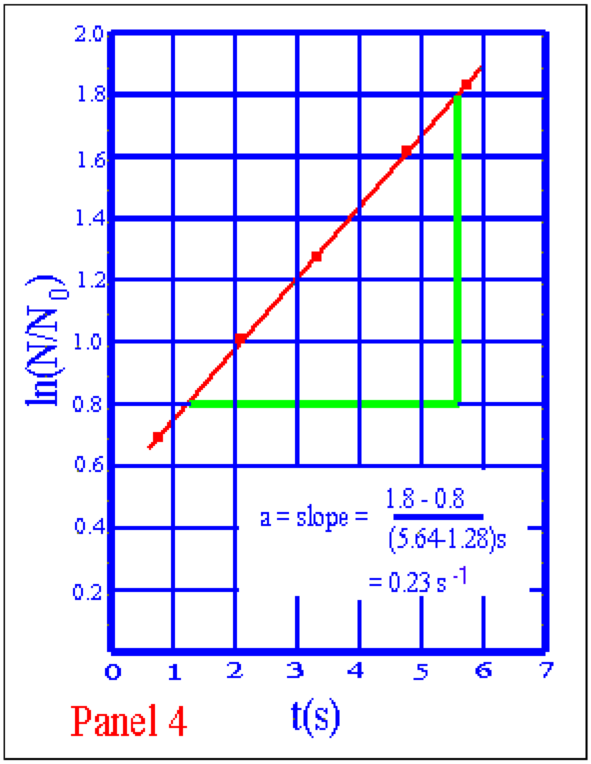
Sometimes it's a nuisance to look up a bunch of logarithms of values of N/N_0 so we make use of a special type of graph paper which does this automatically. A sheet of this paper is shown in panel 2. Notice that it has a linear scale horizontally but a logarithmic scale vertically. It's called *"semilogarithmic paper"*. Notice that the vertical scale goes from 1 to 10. This paper is called *"one-cycle semi-logarithmic paper"*. The significance of this name will become apparent in a little while.

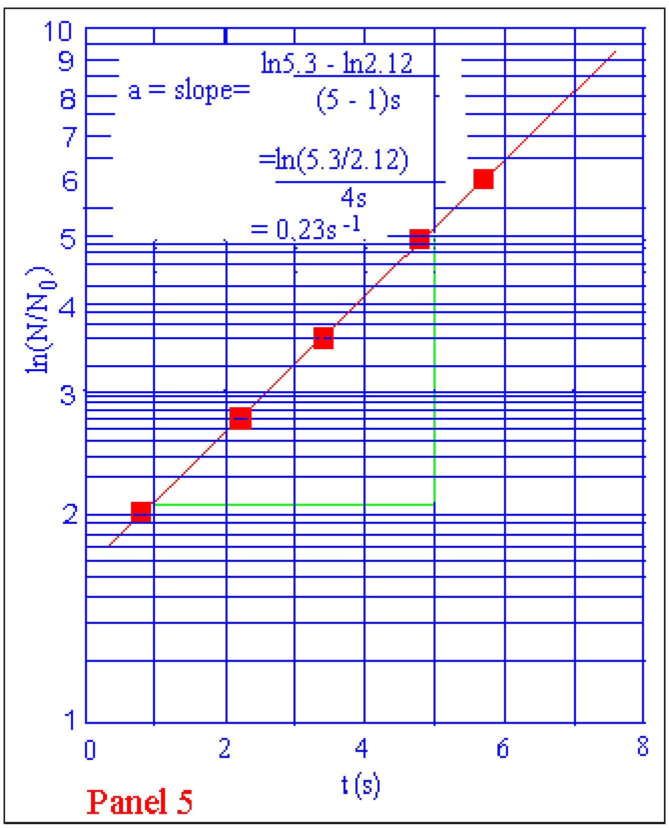
N/N_0	t(s)	$\ln(N/N_0)$
2.02	0.8	0.70
2.78	2.2	1.02
3.58	3.3	1.28
5.05	4.8	1.62
6.35	5.8	1.85

Panel 3

In panel 3, there's a table of values of N/N_0 which obey an exponential relationship. In the right-hand column, I've looked up the natural logarithms of N/N_0

The graph shown in panel 4 is a plot of these values vs t. Notice that this graph is on normal graph paper, not semi-log paper. We'll use semi-log paper in a moment. As you see, the graph is a straight line and its slope, and thus the constant a, can be found. Pause for a moment and check the calculation of a.



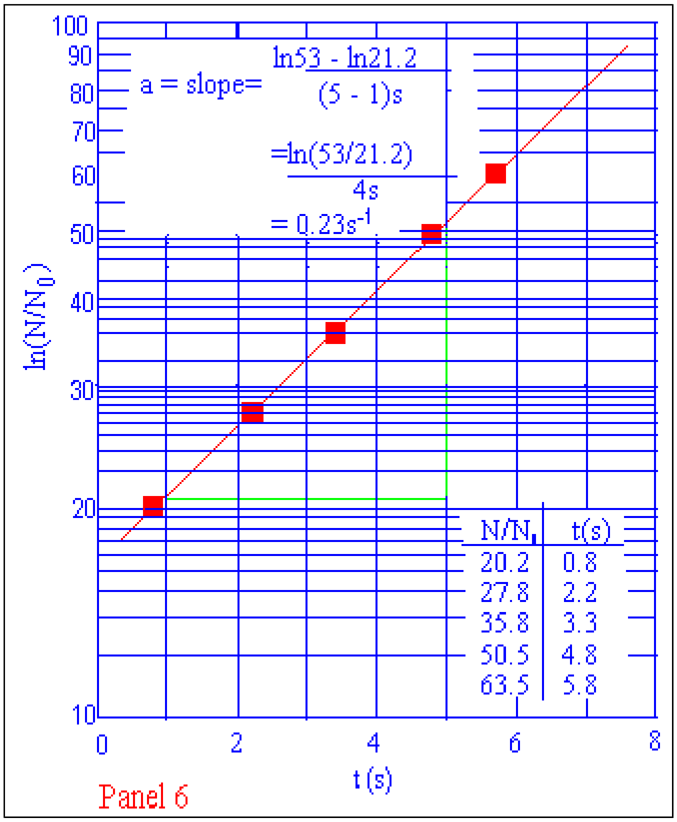


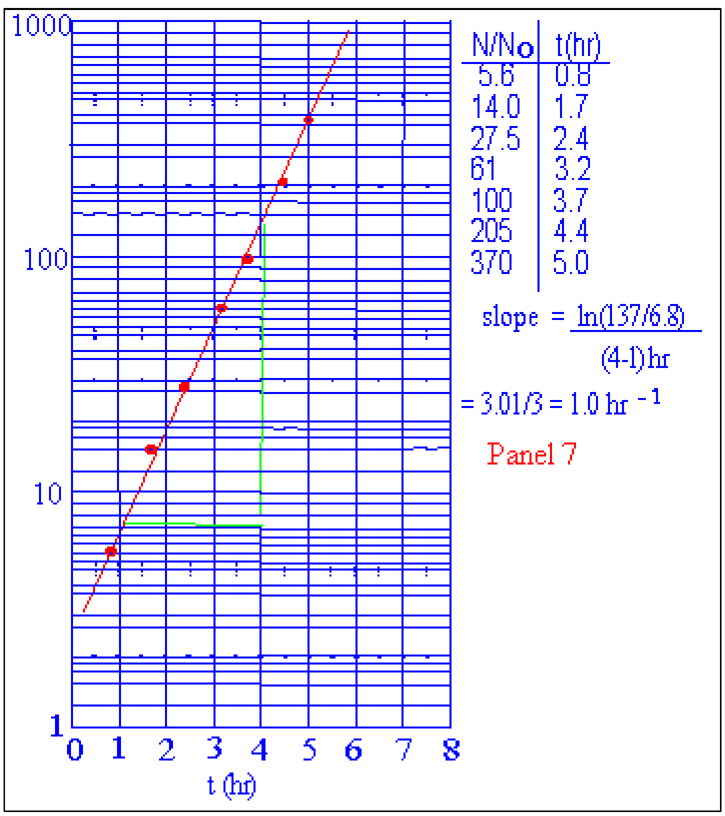
Now let's see how the semi-log paper simplifies all this as shown in panel 5. The same data as in panel 3 is used here and, since our N/N_0 data is all between 1 and 10, we can use the numbers on the left-hand edge of the graph paper just as they stand. All we have to do is plot the numbers as given. We don't have to find logarithms, the paper does it for us. That's the beauty of semi-log paper. You have to watch out how the paper is sub-divided, though. In this example, it's sub-divided in 0.1, from 1 to 3, but in divisions of 0.2, from 3 to 5 and 0.5 from 5 to 10. Pause here and see how this graph is plotted.

Now let's find a. Again, we must find the slope and this will involve finding logarithms but only at two points on whatever triangle we use to determine the slope. Pause again and check the calculation of the slope in panel 5.

Notice, in fact, we had to look up only one logarithm in the slope calculation when we remembered that the difference of two logs is the log of the quotient. Of course, we got the same value as before, $a = 0.23\ s^{-1}$ but with a lot less work.

Suppose, however, our data had been as shown in the table of panel 6. Now the values of t are the same but the values of N/N_0 are 10 times larger. What do we do now? The answer is that the decade over which the vertical axis runs is quite arbitrary. It can be 1 to 10 as previously, or it can be 10 to 100 which is what we need now, or 100 to 1000, or 0.1 to 1, and so on. Pause and see that you understand how this graph in panel 6 was plotted.

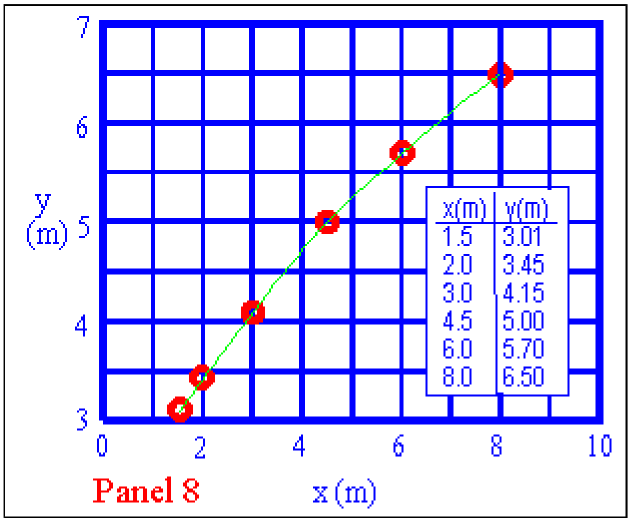




Now let's suppose that you have the data given in the table on panel 7. None of the semi-log paper you have seen up to this point will work. You could plot the first number, or the 2nd to 5th, or the 5th to 7th, but you couldn't plot them all. Your one-cycle paper will go only from 1 to 10, or 10 to 100, or 100 to 1000, in other words, one decade. But now N/N_0 goes over parts of 3 decades, that is 1 to 1000. For this you need *three-cycle* semi-log paper which has been used here to plot this data. Pause and check over the plot and calculation on panel 7.

As you can see from this, you choose the number of cycles in the graph paper you use to match the span of data which you have; semi-log paper comes in one, two, three, seven cycles etc.

Let's now turn to a new problem. Suppose you were presented with the set of data shown in panel 8. A graph of y vs x is also shown in panel 8, and you can see it's a smooth curve. But other than that, it's not very informative. Suppose, however, in addition, there were theoretical grounds for believing that this data obeyed a power-law, $y = ax^b$. How could we



find if this were true and, if it were, evaluate the constants a and b?

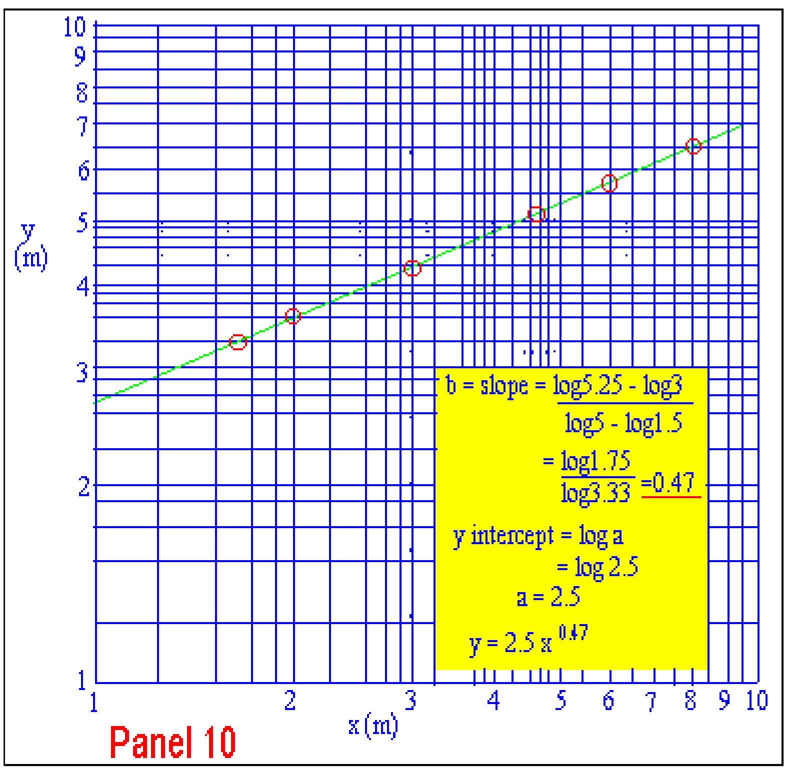
$$y = a\,x^{b}$$

$$\log y = \log a + b \log x$$

$$(\text{compare with } y = b + mx)$$

Panel 9

Let's take logarithms of both sides of the equation as in panel 9. You can see that, if $y = ax^b$, then a graph of log y vs log x yields a straight line of slope b and y intercept log a. Conversely, if a graph of log y vs log x for a set of data is a straight line, then the data does indeed follow the relation $y = ax^b$. Now we could look up a table of values of log x and log y and plot it but I won't bother to do it because, just as with the exponential law, there's a simpler way.



Since we must plot log y vs log x, we need graph paper divided logarithmically along both axes. It's called *"log-log paper"* and a 1 x 1 cycle sample is shown in panel 10 where our data of panel 8 is plotted. Pause and see that you understand how the points were plotted.

The graph is a straight line so the data does obey $y = ax^b$. Now let's find a and b. The constant b is given by the slope. Study panel 11 and see that you understand how the value was obtained. In calculating the slope, you may use either logs to the base 10 or logs to the base e as long as you are consistent. Notice that since logs have no units, then the slope has no units.

The value of log a is the same as the value of the y intercept. To obtain this, we look on the graph for the point where the horizontal variable is 0. Remember that since the horizontal axis is logarithmic, the horizontal variable is actually log x, not just x, so we want the point where log x = 0. In order for log x to be 0, x must be 1. The y intercept can be read off the graph along the vertical line where x = 1.

In this case, the y intercept is log 2.5. Remember that it is not just 2.5 since the vertical axis is logarithmic, so we now have log a = log 2.5, so a must be 2.5. Therefore, we can write that this data fits the equation $y = 2.5x^{0.47}$. You should really include the proper units with the value of a. To find them, simply rearrange the equation $y = a\,x^{0.47}$ to solve for a, in other words, $a = y/x^{0.47}$ and note that the units for both y and x were given as metres. So the units for a must be metres$^{0.53}$. One final point about this graph. Suppose the horizontal axis didn't start at 1, and there's no need that it should. After all, your values of x might have been between 10 and 100, so you would have started your horizontal axis at 10. In that case, you couldn't read the y intercept right off the graph. It must be read where log x = 0, in other words, where x = 1. To find the value of a, in this case, you just use the equation $y = a\ x^b$, substitute values for x, y and b, and solve for a.

If values of x and y extend over more than one decade, then more cycles must be used. Log-log paper comes in many combinations, such as 2 x 1, 2 x 3 and 5 x 3.

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