Control Chart

Control Chart - history

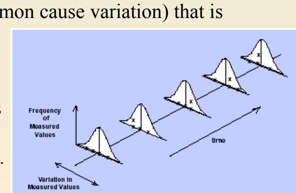
- Developed in 1920’s
- By Dr. Walter A. Shewhart

“A phenomenon is said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future.”

Walter A. Shewhart, 1931

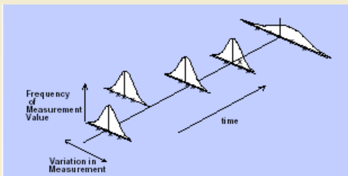
Process in control

A production process is said to be **in control** when the quality characteristics of a product are subject only to **random variation** (or common cause variation) that is variation in process performance due to normal or inherent interaction among process components (people, machine, material, environment, and methods).



Process out-of-control

A production process is said to be **out-of-control** when the quality characteristic of a product are subject also to variation due to **assignable causes** that is variation in process performance due to events that are not part of the normal process and represents sudden or persistent abnormal changes to one or more of the process components.



Control Chart

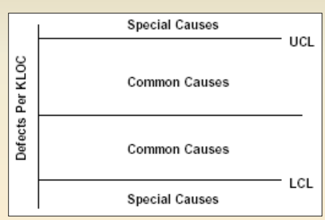
Control charts are useful to establish when a process has endured a meaningful modification; control charts separate the two types of variation in a product quality characteristic.

Control Chart

All control charts have three basic components:

- **a centerline** that represents the mean value for the in-control process
- **two horizontal lines**, called the upper control limit (UCL) and the lower control limit (LCL) that define the limits of common variation causes.
- **performance data** plotted over time.

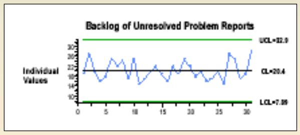
Control Chart



Common Cause ⟶ **In** the process (normal noise)
Special Cause ⟶ **Outside** the process (extraordinary)

Control Chart

Control charts let you know what your processes can do: **you can set achievable goals.**

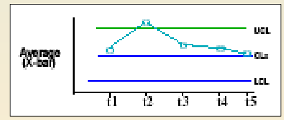


They represent the "**voice of the process**".

Control charts provide the evidence of stability that justifies predicting process performance.

Control Chart

Control charts help to separate signal from noise, so that you can recognize a process change when it occurs.



Control charts identify unusual events. They pinpoint fixable problems and potential process improvements.

Control Chart - assumptions

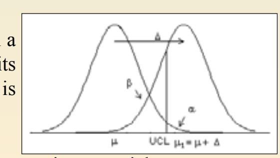
The two important assumptions are:

- The measurement-function (e.g. the mean), that is used to monitor the process parameter, follows a **normal distribution**. In practice, if your data seem very far from meeting this assumption, try to transform them.
- Measurements are independent of each other.

Control Chart – 3 sigma

Control limits on a control chart are commonly drawn at 3-sigma from the center line because 3-sigma limits are a good balance point between two types of errors:

•**Type I** or alpha errors occur when a point falls outside the control limits even though no special cause is operating.

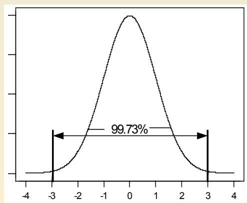


•**Type II** or beta errors occur when you miss a special cause because the chart isn't sensitive enough to detect it.

Control Chart – 3 sigma

All process control is vulnerable to these two types of errors. The reason that 3-sigma control limits balance the risk of error is that, for normally distributed data, data points will fall inside 3-sigma limits **99.73%** of the time when a process is in control.

The limits are chosen so that it is likely that unusual causes of variation will be detected.



Instabilities and Out-of-Control Situations

To test for instabilities in processes, we examine control charts for instances and patterns that signal non-random behavior.

Values falling outside the control limits and unusual patterns within the running record suggest that assignable causes exist.

"**in control**" implies that all points are between the control limits and they form a random pattern.

Instabilities and Out-of-Control Situations

Test 1: A single point falls outside the 3-sigma control limits.

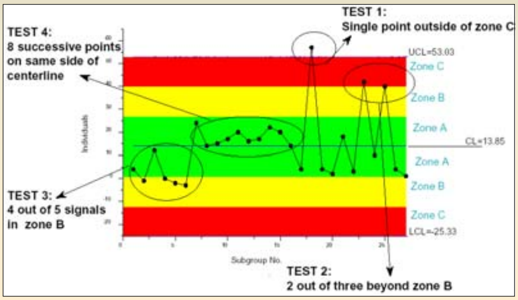
Test 2: At least two of three successive values fall on the same side of, and more than two sigma units away from, the center line.

Instabilities and Out-of-Control Situations

Test 3: At least four out of five successive values fall on the same side of, and more than one sigma unit away from, the center line.

Test 4: At least eight successive values fall on the same side of the center line.

Instabilities and Out-of-Control Situations



Control Chart

Is there a Best Control Chart?

Variable and attribute data

Two broad classes of control charts:

•**variable data**, which is continuous
•**attribute data**, which is discrete

Choice of what control chart to use should be based on knowing the right assumptions!

Use the correct formulas for the kind of control chart selected!

Variable and attribute data

It is important to understand the distinction between variables data and attributes data

Because control limits for attributes data are often computed in ways quite different from control limits for variables data.
Unless you have a clear understanding of the distinctions between the two kinds of data, you can easily fall victim to inappropriate control charting methods.

Variable and attribute data

Variables data (sometimes called measurement data) are usually measurements of continuous phenomena.

Examples: measurements of length, weight, volume and speed.

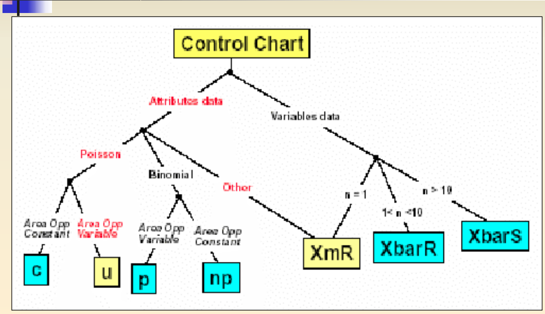
Software examples: elapsed time, effort expended, years of experience, memory utilization and cost of rework.

Variable and attribute data

Attributes data occur when information is recorded only about whether an item conforms or fails to conform to a specified criterion or set of criteria. Attributes data almost always originate as counts.

Examples: the number of defects found, the number of source statements of a given type, the number of lines of comments in a module of n lines, the number of people with certain skills or experience on a project or team, and the percent of projects using formal code inspections.

Control Chart selection



Types of Variable Control Chart

- X-bar chart
- R chart
- s chart
- Individual chart
- Moving Range chart

Types of Variable Control Chart

- **X-bar chart**: based on the **average** of a subgroup. Subgroups of 2 to 30 samples may be used when computing the control limits for the X-bar chart when based on the range.

- **R chart**: takes into account the **range** of a subgroup. Subgroup sizes may be as small as 2 or as large as 30.

- **S chart**: takes into account the **standard deviation** of a subgroup. There is no limit to the subgroup size.

Types of Variable Control Chart

- **Individual chart**: displays **each value**. A subgroup size is used to compute the limits, with value of 2 being most common, although the subgroup size may be as large as 30.

- **Moving Range chart**: takes into account the **moving range** of a process. It is used to control variability of processes which do not form natural subgroups.

Notation for Variable Control charts

n: size of the sample (collection of observations, sometimes called a subgroup) chosen at a point in time

m: number of samples selected

$\bar{x}_i$ = average of the observations in the i-th sample (where i = 1, 2, ..., m)

$\bar{\bar{x}}$ = grand average or "average of the averages (this value is used as the center line of the control chart)

Notation and Values

R_i= range of the values in the i-th sample

$$R_i = |\max(X_i) - \min(X_i)|$$

$\bar{R}$= average range for all m samples

μ is the true process mean, usually unknown but it can be estimated by averaging a large number (for example 20) of samples mean obtained when the process is in control

σ is the true process standard deviation, usually unknown but it can be estimated from a large sample of data collected while the process is in control

X-bar charts

"The process mean is changed during

the observation period?"

X-bar charts

• Let $X = \{X_1, \ldots\ldots, X_m\}$ be the set of observations divided into samples.

•For each sample $X_i = \{x_{i1}, \ldots\ldots, x_{in}\}$ compute the average

$$\overline{X}_i = \frac{x_{i1} + x_{i2} + \cdots + x_{in}}{n}$$

and the mean average

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \cdots + \overline{X}_m}{m}$$

X-bar charts

$\overline{X}_i$ is normally distributed with mean, μ, and standard deviation, $\sigma_{\bar{x}} = \sigma / \sqrt{n}$. (Central Limit Theorem)

Lower Control Limit: $\overline{\overline{X}} - 3\sigma_x \cong \overline{\overline{X}} - A_2\overline{R}$

Center Line: $\overline{\overline{X}}$

Upper Control Limit: $\overline{\overline{X}} + 3\sigma_x \cong \overline{\overline{X}} + A_2\overline{R}$

A_2 is a constant based on the subgroup size.

R charts

"Is the dispersion of the values observed in the

samples due to the presence of exceptional

causes?"

R charts

- Let $X = \{X_1, \ldots\ldots, X_m\}$ be the set of observation divided into samples.
- For each sample X_i the range is

$$R_i = |\max(X_i) - \min(X_i)|$$

and the range average is

$$\overline{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$$

R charts

Lower Control Limit: $\overline{R} - 3\sigma_R \cong D_3\overline{R}$

Center Line: $\overline{R}$

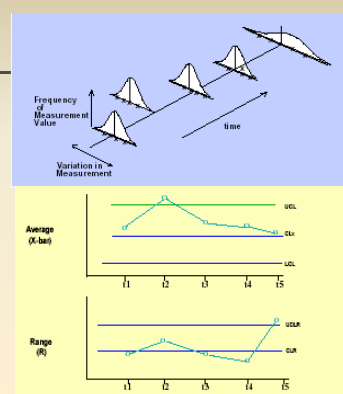
Upper Control Limit: $\overline{R} + 3\sigma_R \cong D_4\overline{R}$

D_3 and D_4 are constants based on the subgroup size.

X-bar charts and R charts

X-bar chart is typically used in conjunction with R chart.
In fact, since the sample range is used to construct the X-bar chart, it is essential to examine an R chart first (to be sure that the process variation is stable).

X-bar charts and R charts for a process out-of-control



X-bar charts and R charts

It is important to construct and interpret an R chart before the X-bar chart.

When the R chart indicates that process variation is in control, analyze the X-bar chart otherwise X-bar chart are not meaningful

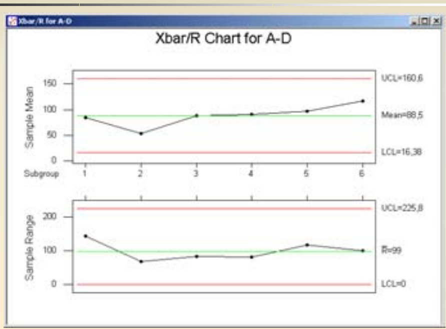
X-bar charts and R charts: example

As an example, suppose you misure those 4 software module sizes each month for 6 months. Our example collected data looks like the following:

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6
	month	A	B	C	D	
1	1	104	12	156	68	
2	2	89	21	26	77	
3	3	124	41	103	83	
4	4	139	57	76	92	
5	5	168	62	51	108	
6	6	168	68	112	119	
7						
8						

X-bar charts and R charts: example



s charts

Process variability can be controlled by either a R chart or a Standard Deviation chart (s chart) depending on how the population standard deviation is estimated.

S chart is used to determine whether the standard deviation has changed.

s charts

- Let $X = \{X_1, \ldots\ldots, X_m\}$ be the set of observations divided into samples.

- For each sample X_i the standard deviation is

$$S_i^2 = \frac{\sum_{j=1}^{n}(x_{ij} - \bar{x}_i)^2}{n-1}$$

and the average is

$$\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_i$$

X-bar charts and s charts

Paremeters for X-bar char

Lower Control Limit: $\bar{\bar{X}} - 3\sigma_x \cong \bar{\bar{X}} - A_3\bar{S}$

Center Line: $\bar{\bar{X}}$

Upper Control Limit: $\bar{\bar{X}} + 3\sigma_x \cong \bar{\bar{X}} + A_3\bar{S}$

Paremeters for s char

Lower Control Limit: $\bar{S} - 3\sigma_S \cong B_3\bar{S}$

Center Line: $\bar{S}$

Upper Control Limit: $\bar{S} + 3\sigma_S \cong B_4\bar{S}$

XmR: Individual and Moving Range charts

An individual chart is equivalent to X-bar but reported to single observation, not to samples.

Used when the nature of the process is such that it is difficult or impossible to group measurements into subgroups

This occurs frequently in low volume production and in situations in which continuously varying quantities within the process are process-related variables.

XmR chart

The solution is to artificially create subgroups from the data and then calculate the range of each subgroup.

This is done by creating rolling groups (most often pairs) of data through time and using the pairs to determine the range R.

The resulting ranges are called *moving ranges*.

XmR chart

The moving range is defined as $MR_i = |x_i - x_{i-1}|$

which is the absolute value of the delta between two consecutive data points.

XmR chart

Paremeters for Individual char

$$\begin{cases} \text{Lower Control Limit: } \overline{X} - 3\sigma_x \cong \overline{X} - E_2\overline{MR} \\ \text{Center Line: } \overline{X} \\ \text{Upper Control Limit: } \overline{X} + 3\sigma_x \cong \overline{X} + E_2\overline{MR} \end{cases}$$

Paremeters for Moving Range char

$$\begin{cases} \text{Lower Control Limit: max } \left[0, \overline{MR} - 3\sigma_{MR}\right] = \text{max } \left[0, D_3\overline{MR}\right] \\ \text{Center Line: } \overline{MR} \\ \text{Upper Control Limit: } \overline{MR} + 3\sigma_{MR} \cong D_4\overline{MR} \end{cases}$$

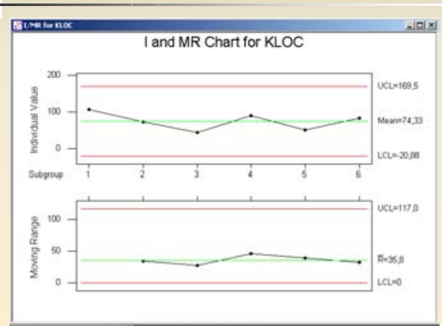
XmR chart: example

The following table contains a set of sample data, for the KLOC generated each month for one software module.

Worksheet 1 ***

↓	C1	C2	C3	C4
	month	KLOC		
1	1	106		
2	2	72		
3	3	44		
4	4	90		
5	5	51		
6	6	83		
7				
8				

XmR chart: example



Attribute Control Chart

Attribute control charts arise when items are compared with some standard and then they are classified as to whether they meet the standard or not.

The control chart is used to determine if the rate of non-conforming products is stable and detect when a deviation from stability has occurred.

Attribute Control Chart

The argument can be made that a LCL should not exist, since rates of nonconforming product outside the LCL is in fact a good thing; we WANT low rates of nonconforming product.

However, if we treat these LCL violations as simply another search for an assignable cause, we may learn from the drop in nonconformities rate and possibly permanently improve the process.

Types of Attribute Control Chart

- p chart
- np chart
- c chart
- u chart

Types of Attribute Control Chart

- **p chart**: a chart of the **percent defective** in each sample set. The sample size may vary.
- **np** chart: a chart of the **number defective** in each sample set. The samples have the same size.
- **c** chart: a chart of the **number of defects** per unit in each sample set
- **u** chart: a chart of the **average number of defects** in each sample set

p charts

Suppose y is the number of defective units in a random sample of size n.

We assume that y is a binomial random variable with unknown parameter p.

Let k be the number of samples.

p charts

The fraction: $\hat{p}_i = y_i / n$ for each sample is plotted on the chart.

The mean sample proportion is $\bar{p} = \dfrac{\sum_{j=1}^{k} \hat{p}_j}{k}$

The variance of the statistic p is $\sqrt{\dfrac{p(1-p)}{n}}$

p charts

Using $\bar{p}$ to estimate the process proportion defective p the center line and upper and lower control limits for the p chart are:

Lower Control Limit: $\bar{p} - 3\sigma_p = \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

Center Line: $\bar{p}$

Upper Control Limit: $\bar{p} + 3\sigma_p = \bar{p} + 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

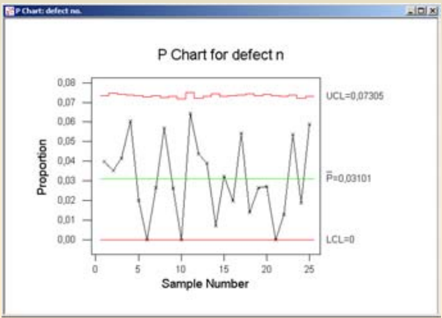
p charts: example

The variable **defect no.** contains the number of defective units, and the variable **lot sample size** contains the lot sample size.

Worksheet 1 ***

↓	C1	C2	C3	C4
	lot no.	lot sample size	defect no.	
1	1	151	6	
2	2	142	5	
3	3	145	6	
4	4	149	9	
5	5	150	3	
6	6	156	0	
7	7	150	4	
8	8	158	9	

p charts: example



np charts

It can be used when the sample are of equale size, n.

In the same way of the p chart: $n\overline{p} = \frac{\sum_{j=1}^{k} \hat{p}_j}{k}$ and

Lower Control Limit: $n\overline{p} - 3\sigma_p = n\overline{p} - 3\sqrt{\frac{n\overline{p}(1-\overline{p})}{n}}$

Center Line: $n\overline{p}$

Upper Control Limit: $n\overline{p} + 3\sigma_p = n\overline{p} + 3\sqrt{\frac{n\overline{p}(1-\overline{p})}{n}}$ or 0 if UCL<0

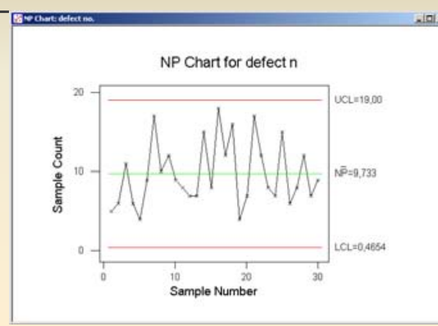
np charts: example

The variable **defect no.** contains the number of defective units, and the lot sample size is 500.

Worksheet 1 ***

↓	C1	C2	C3
	lot no.	defect no.	
1	1	5	
2	2	6	
3	3	11	
4	4	6	
5	5	4	
6	6	9	
7	7	17	
8	8	10	
9	9	12	

np charts: example



c charts

c charts are used to chart count of defects where the area of opportunity for a defect is constant.

Es: defects per 1000 feet of base material in a roll of plastic film produced. The area of opportunity could be a physical area (defect/1000ft), a product such as scratches per monitor, an amount of time such as broken spindles per day or any combination of these area of opportunities.

c charts

The Poisson probability distribution provides a good model for the probability distribution for the number c of defects.

If c possesses a Poisson probability distribution with parameter λ, then $E(c) = \lambda$ and $\sigma_c = \sqrt{\lambda}$.

Observe c over a reasonably large number, k, of equally spaced points in time and use $\bar{c}$, the average value c of to estimate λ.

c charts

The average value of c is: $\bar{c} = \dfrac{\sum_{i=1}^{k} c_i}{k}$

So the center line and upper and lower control limits for the c chart are:

Lower Control Limit: $\bar{c} - 3\sigma_c = \bar{c} - 3\sqrt{\bar{c}}$

Center Line: $\bar{c}$

Upper Control Limit: $\bar{c} + 3\sigma_c = \bar{c} + 3\sqrt{\bar{c}}$

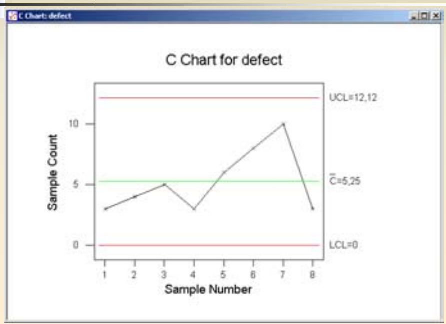
c charts: example

Assume that the following table contains defect data for one of the system design document.

Worksheet 1 ***

↓	C1	C2	C3	C4
	event no.	defect		
1	1	3		
2	2	4		
3	3	5		
4	4	3		
5	5	6		
6	6	8		
7	7	7		
8	8	3		

c charts: example



u charts

If the area of opportunity is not constant, use the u chart instead of the c chart.

Let be u_i the number of defect over a_i the i-th area of opportunity then the average number of defect is

$$\overline{u} = \frac{\sum_{i=1}^{k} u_i}{\sum_{i=1}^{k} a_i}$$

u charts

The center line and upper and lower control limits for the u chart are:

Lower Control Limit: $\overline{u} - 3\sqrt{\overline{u} / a_i}$

Center Line: $\overline{u}$

Upper Control Limit: $\overline{u} + 3\sqrt{\overline{u} / a_i}$

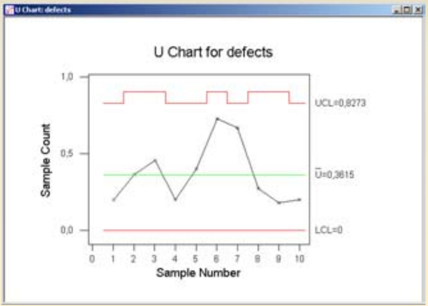
u charts: example

Assume that the following table contains defect data for the system design documents of 8 sofware applications

Worksheet 1 ***

↓	C1	C2	C3	C4	C5
	event no.	sample size	defects		
1	1	15	3		
2	2	11	4		
3	3	11	5		
4	4	15	3		
5	5	15	6		
6	6	11	8		
7	7	15	10		
8	8	11	3		

u charts: example



Control Chart - conclusions

First Step: Determine what type of data you are working with.

Second Step: Determine what type of control chart can be used with your data set.

Third Step: Calculate the average and the control limits.

Fourth Step: Detect Instabilities and Out-of-Control Situations