

# Use of Dot Paper as an Aid to Enhance Learning of Mathematics for Classes I - VIII

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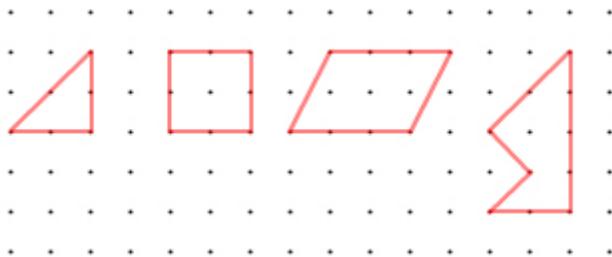
## Geometry

Heiles (1958) proposed that there are 5 levels of thought at which we can discuss, learn and teach geometry. At level 0 the students perceive figures by their appearance as a whole. When they call a figure a square, they react to the total figure and not to its specific properties such as all its angles are right angles and all its sides have equal lengths. At level 1, students are able to analyse figures and see such properties. They are also able to identify figures by their properties. That is a figure that has all its sides of equal lengths and all its angles right angles is a square. They still do not see relations between figures e.g. a square is a rectangle. At level 2 the students begin to see relationship between figures e.g. a rectangle is a parallelogram. At level 3, they can follow proof of theorems and they can develop sequence of statements to deduce one statement from another. Level 4 is the most rigorous level at which students have deep understanding of mathematical structures and are able to justify statement made about structures. Instruction in Geometry to be effective should be based on student's level of thinking. It besides enriching their thinking at that level should move them towards next level. In this paper we suggest some activities using dot or Geo paper that can be used for enrichment at levels 1 and 2 and would help in preparing students for high school geometry. A  $5 \times 5$  dot paper given in Appendix 1, a larger one in Appendix 2, circular dot paper is given in Appendix 3 and an isometric or triangle dot paper is given in Appendix 4. These can be photocopied for use in class. These will also enhance understanding of concepts, communication and reasoning in geometry. Some opportunities are provided for exploration of ideas. We have restricted activities to 2005 syllabus of NCERT for classes I -VIII. Teachers can choose activities appropriate for their class.

## Shapes

Draw and identify shapes on a Dot paper.

Identify and copy the shapes given below:



Using a  $5 \times 5$  Dot paper, make triangles and share your solutions with other classmates. How many dots does the triangle touch?

Make triangles which touch three dots, four dots, or five dots. Can you make a triangle so that the triangle touches more dots? How many corners and sides does it have? Repeat it for a square, rectangle, rhombus, parallelogram and trapezium.

### Lines

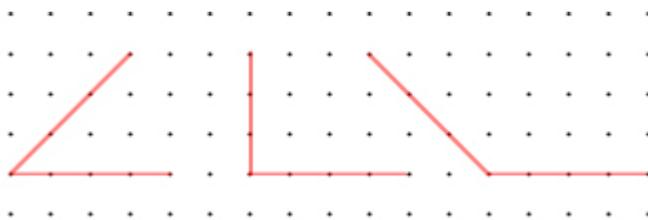
1. Draw horizontal, vertical and slant lines
2. Mark mid point of lines given below:



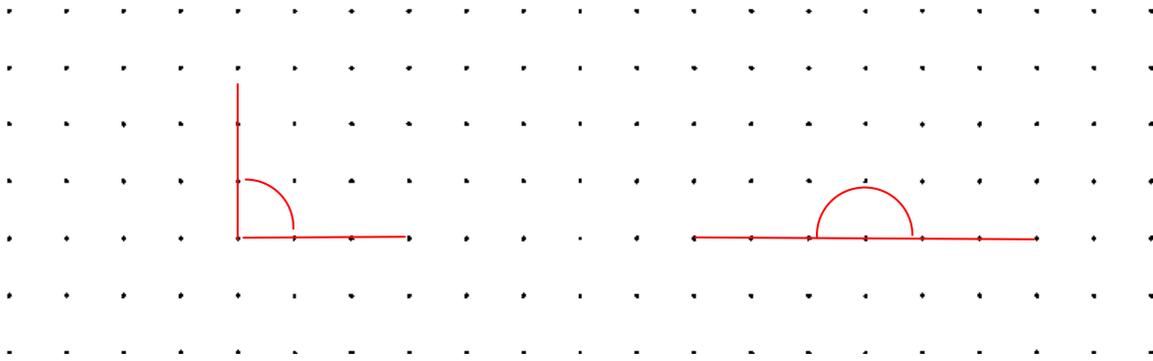
3. On a  $5 \times 5$  dot paper draw
  - Two parallel lines
  - Two parallel line segments that touch a total of 9 dots
  - Two perpendicular lines
  - Two perpendicular line segments that touch a total of 9 dots
  - Two intersecting line segments that touch a total of 9 dots, but are not perpendicular
4. On a  $5 \times 5$  dot paper
  - Draw two equal line segments that touch a total of 9 dots.
  - Draw two equal line segments that touch a total of 8 dots.
  - Compare your lines with others. If they differ how are they different?
5. Draw a horizontal line segment that touches 3 dots. If the shortest distance between two dots is taken as a unit, what is the length of this line segment?
6. Draw another line segment that touches 3 dots, but with different length. Is the length of this new line segment less than or greater than the previous one?

### Angles

Identifying and copy the angles given below:



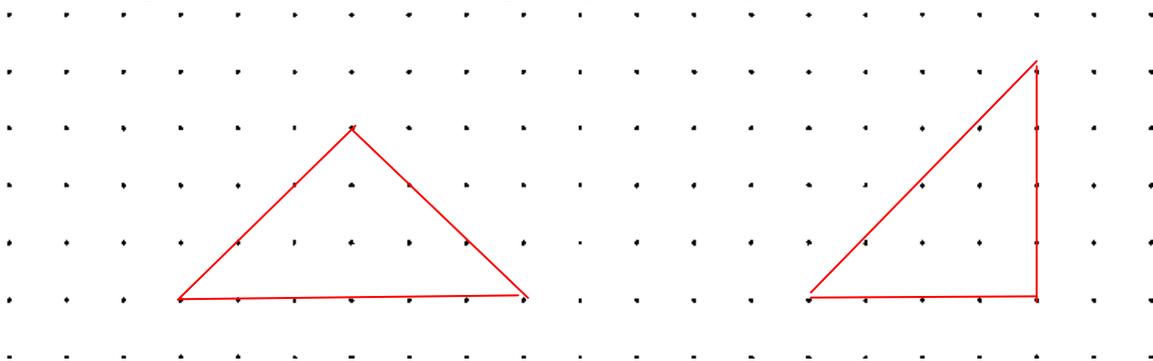
Draw acute, obtuse and right angles on a dot paper.  
Draw bisector of angles given below:



**Draw shapes with given properties**

**Triangles**

1. Draw right, acute and obtuse angled triangles.
2. Draw isosceles and scalene triangles on a dot paper.
3. Draw right bisectors of the following triangles. Are they concurrent?



4. Draw a right triangle having two equal sides
5. Draw a right triangle that is not isosceles.

**Quadrilaterals**

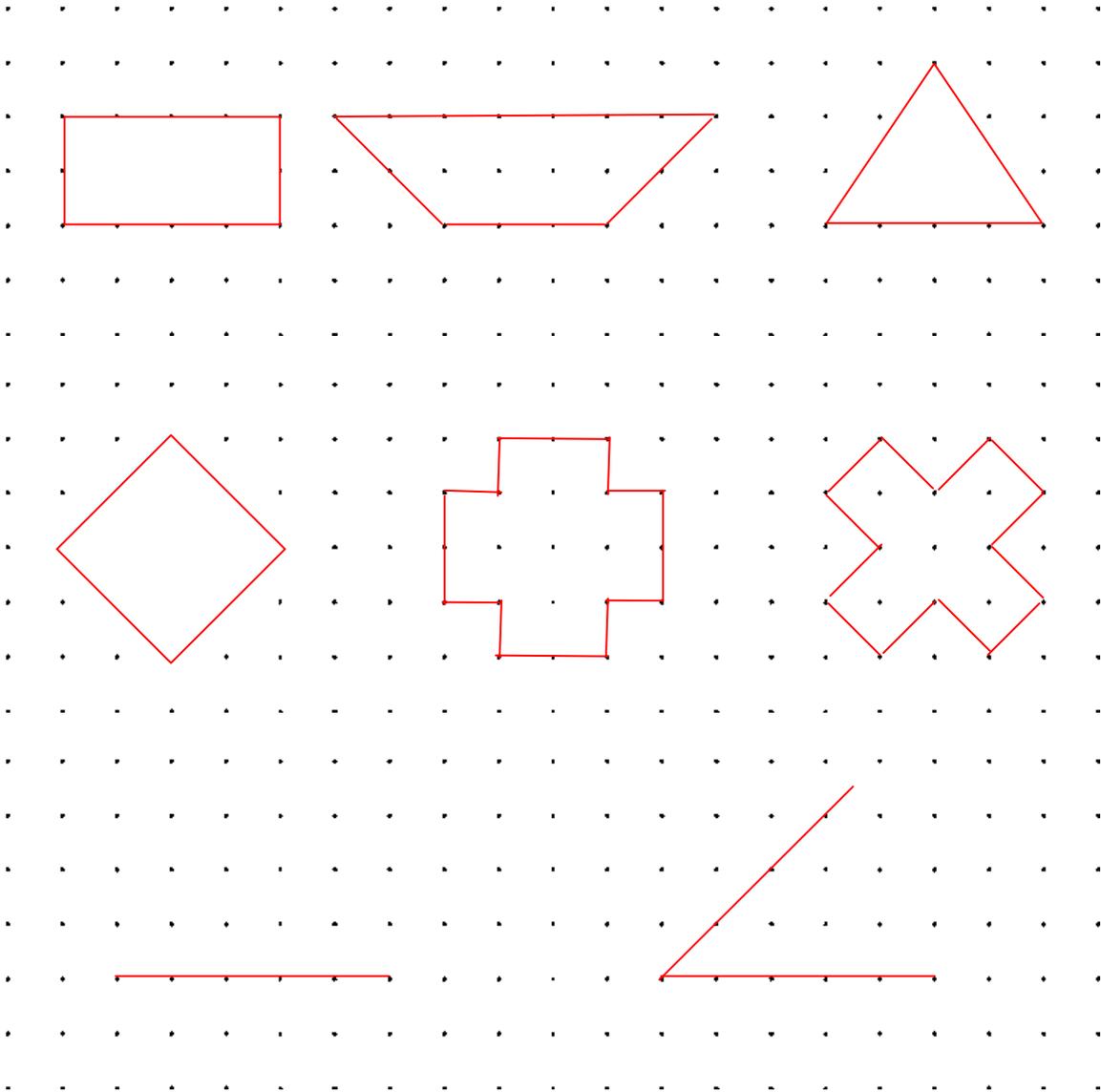
1. Draw a square, rectangle, parallelogram, rhombus and trapezium on a dot paper.
2. Draw all quadrilaterals with the following properties and name them:
  - Both sets of opposite lines are parallel
  - One set of opposite lines are parallel
  - All angles are right angles.
3. Draw all squares, rectangles, parallelograms on a  $5 \times 5$  dot paper that has a base 2.

**Symmetry**

Draw figures e.g. a square, rectangle, triangle, trapezium, parallelogram etc and draw all lines of symmetry in them. How many lines of symmetry you could draw for these quadrilaterals?

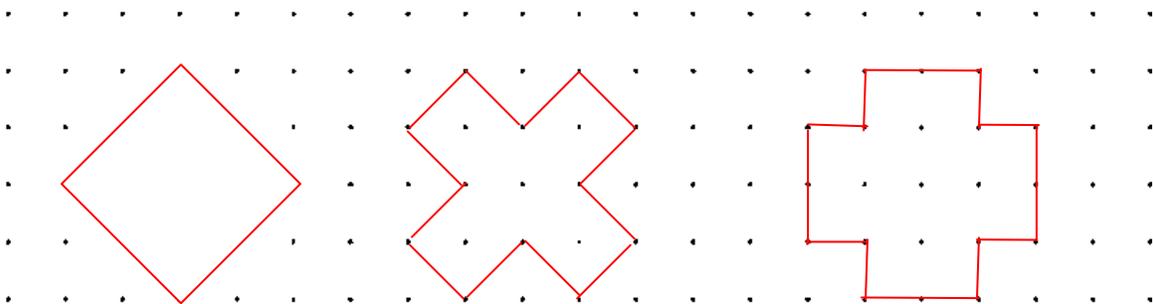
Complete the following figures so that the dashed line is a line of symmetry:





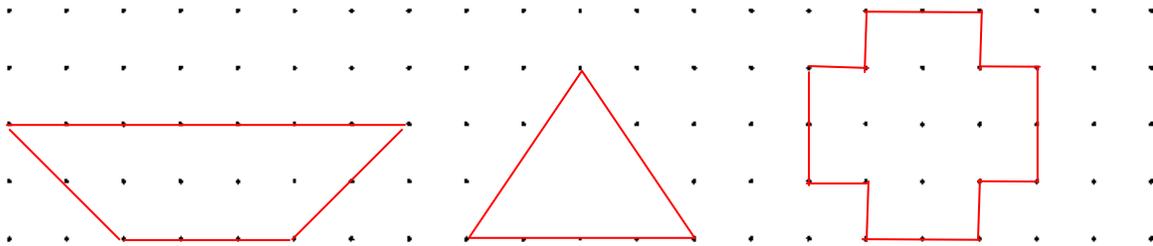
**Transformations**

Copy these figures and draw horizontal and vertical slides of figures given below:

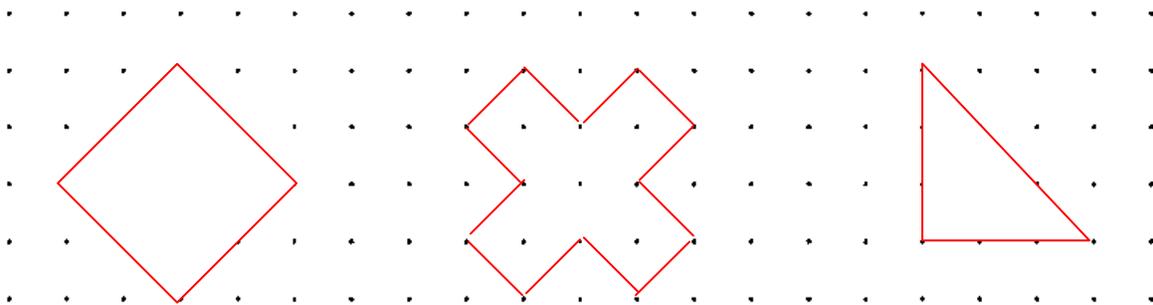


Compare it with others, if the figures differ, how do they differ?

Draw mirror images of figures given below when flipped along a side



Compare it with others, if the figures differ, how do they differ?  
Trace, cut and draw rotation of figures given below in your notebook.



Trace, cut and draw rotation of figures given below in your notebook.  
Compare it with others, if the figures differ, how do they differ?

**Similarity**

Draw figure similar to given figures.  
Are congruent figures always similar? If no, give a counter example.  
Are similar figures always congruent? If no, give a counter example.  
Which figures are always similar?  
Enlarge regular and irregular figures with a specific similarity ratio.  
Find the perimeter and area of enlarged figures.  
How are the perimeters of figures and their enlargement related?  
How are the areas of figures and their enlargement related?

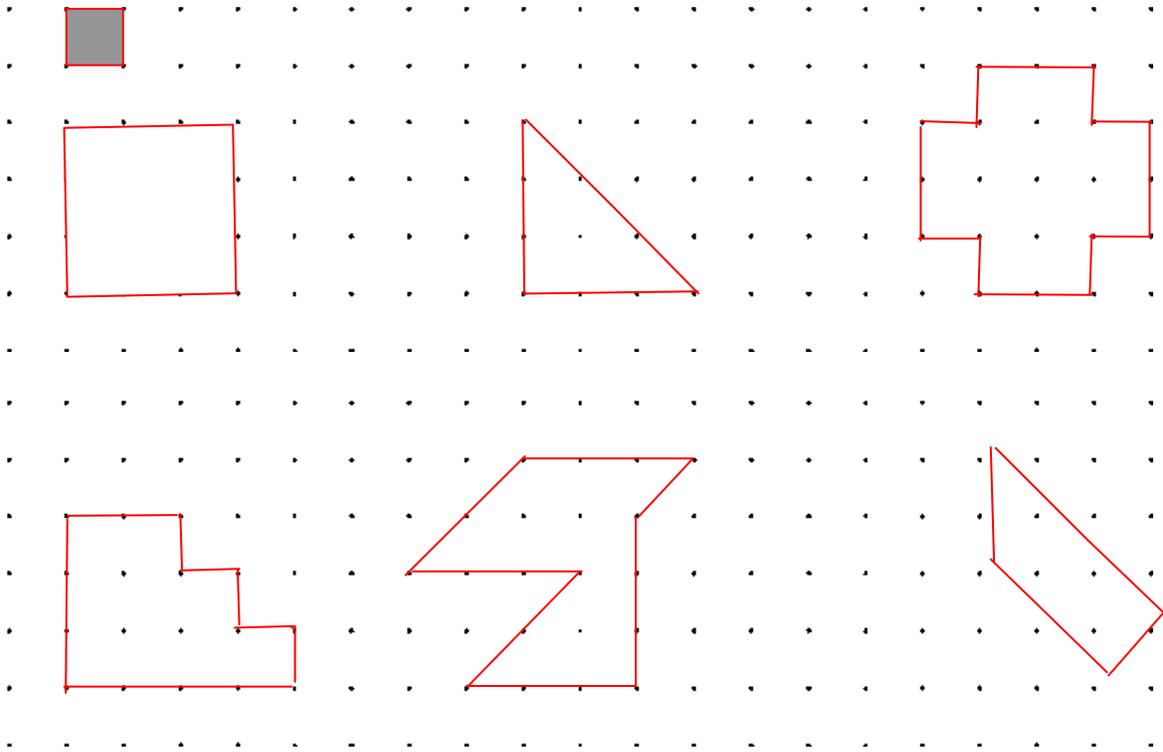
**Measurement**

**Measurement of perimeter**

If the shortest distance between two dots is taken as a unit of measurement, find the perimeter of given figures.  
Draw a square whose perimeter is 9 units.  
Draw all non congruent rectangles whose perimeter is 12 units.

**Measurement of area**

1. If the grey square represents a unit of area, find the areas of figures given below:



2. Draw squares whose area is a specific number of units e.g. 4, 9, 16 units
3. Draw all rectangles whose area is a specific number of units e.g. 6, 8, 12 units.
4. Draw right triangles whose area is a specific number of units e.g. 1,  $1\frac{1}{2}$ , 2 and  $2\frac{1}{2}$  units.
5. Do all rectangles with the same perimeter have the same area?
6. Make a rectangle with base 2 units on Dot paper. Without removing the rectangle, make a parallelogram with the same base and height.  
What is the area of the rectangle? What is the area of the parallelogram? How do you know?

#### **Activities with circular dot paper**

A circular dot paper is given in Appendix 3.

Draw a circle on the circular dot paper.

Draw a radius, a chord, a diameter and an arc in it.

Shade a sector and a segment in it.

Draw an angle and draw another angle adjacent to it.

Draw an angle and draw an adjacent angle so that the two angles are complementary.

Draw an angle and an adjacent angle so that the two angles are supplementary.

Make shapes on a circular dot paper and describe them so that others can make them.

#### **Activities with Isometric dot paper**

An Isometric dot paper given in Appendix 4 is to be used in all the activities in this section using the following units of measurement:



Unit of length

Unit of area

### Angle

Draw an acute, obtuse and acute angle.

Draw an angle of  $60^\circ$ ,  $120^\circ$ ,  $180^\circ$ ,  $240^\circ$  and  $300^\circ$ .

### Triangle

Draw acute, obtuse and right angled triangles.

Draw equilateral, isosceles and scalene triangles.

If the side of the small triangle is taken as a linear unit of measurement, find the perimeters of a given equilateral triangle.

Draw equilateral triangles with specific perimeters e.g. 3, 9 and 15.

Find the area of a given equilateral triangle.

Draw equilateral triangles with specific areas e.g. 4, 9 and 25.

Draw an equilateral triangle similar to a given equilateral triangle with a specific similarity ratio.

### Polygons

Draw a rhombus.

If the side of the small triangle is taken as a linear unit of measurement, find the perimeter of the rhombus.

Draw rhombuses with specific perimeters.

If the area of the small triangle is taken as a unit of area, find the area of rhombus.

Draw rhombuses with specific areas.

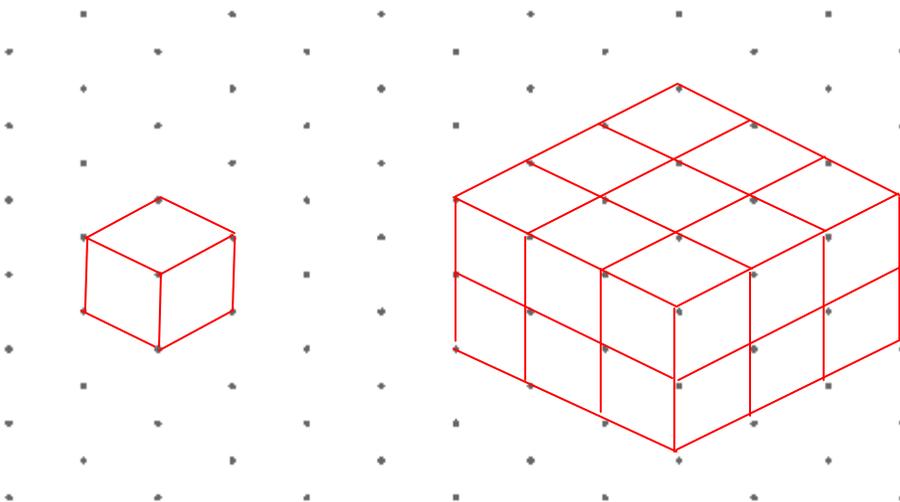
Draw a rhombus similar to a given rhombus with a specific similarity ratio.

Repeat all of these for a parallelogram, trapezium and a hexagon.

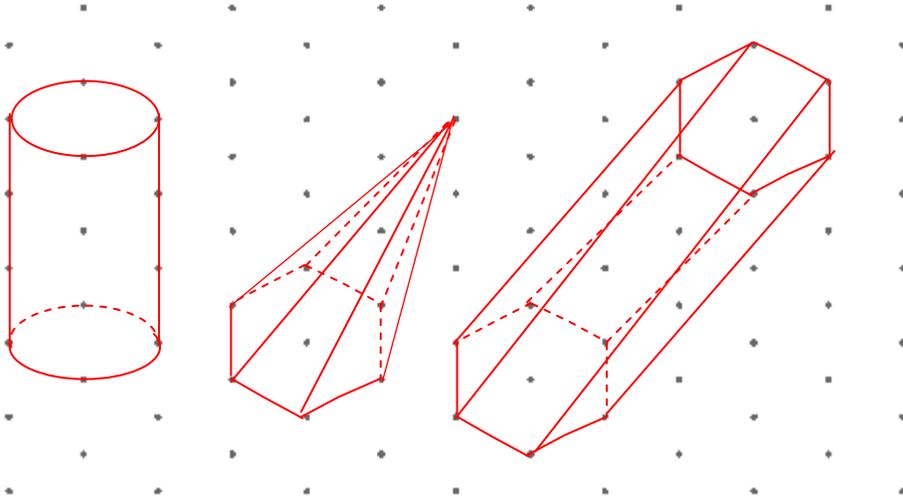
### Drawing 3-D Shapes in 2-D

We can draw 3-D shapes on an isometric dot paper given in Appendix 4.

The figure given below shows a cube and cuboid made with blocks drawn on it.



We would need curved portions to draw some figures e.g. cylinder, for others the lines may not fall right on the dots e.g. hexagonal pyramid and hexagonal prism. The dotted lines show the hidden faces.



Draw a cuboid, how many faces, edges and corners does it have? Are the faces plain or curved? What are the shapes of the faces?

Repeat it for a triangular prism.

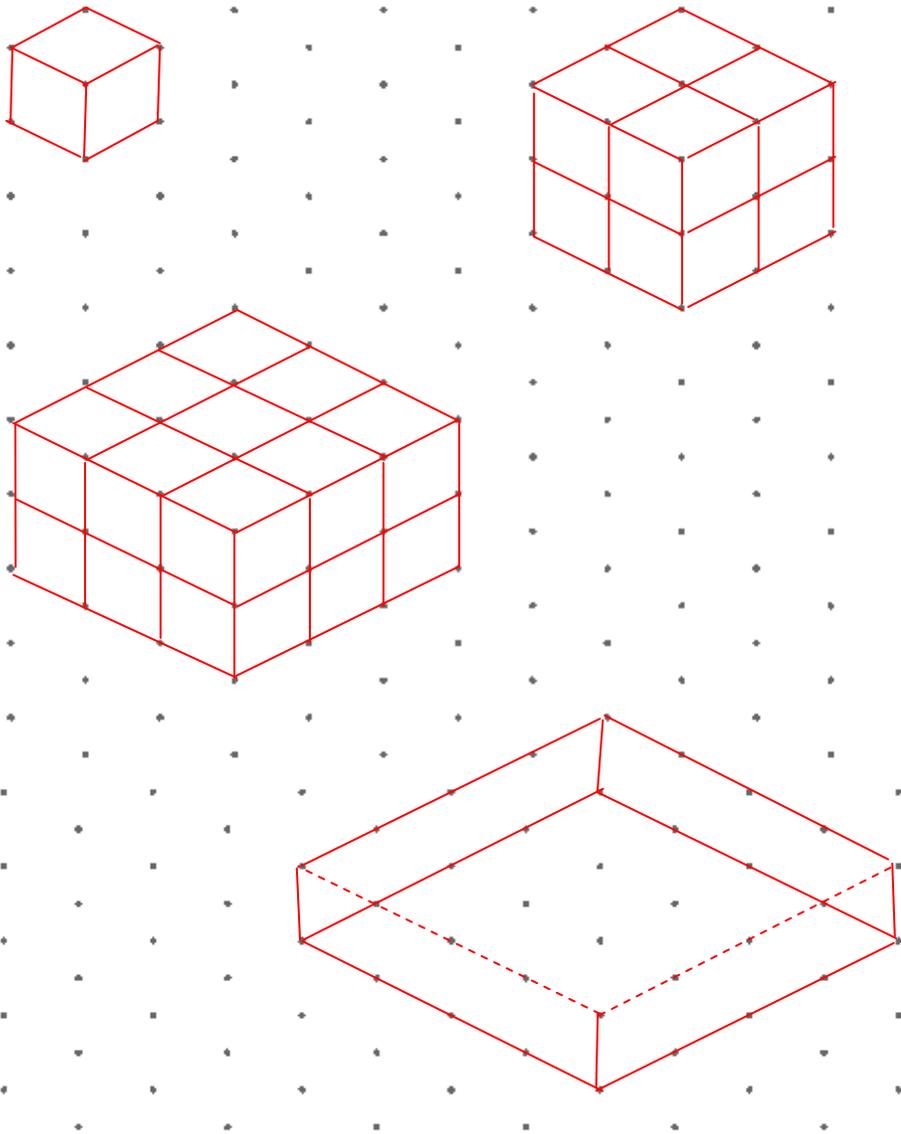
Repeat it for a cone and a triangular pyramid.

Make 3-D structures using blocks and draw 2-D figures of those on an isometric graph paper.

Make 2-D figures of structures made by others on an isometric graph paper.

**Volume**

If the small cube is taken as a unit, find the volume of cubes and cuboids given below:



Make cubes whose volume is given.

Make cuboids whose volume is given.

### **Irrational Numbers**

If the horizontal and vertical distance on a dot paper from one dot to the other is 1 unit.

1. Draw line segments with the following lengths and label with their lengths.

$\sqrt{2}$ ,  $\sqrt{5}$ ,  $\sqrt{8}$ , and  $\sqrt{13}$  (Use Pythagoras Theorem)

2. Choose 3 other lengths of irrational numbers that you can draw by connecting dots.  
Draw and label.

3. Give 3 other irrational-number lengths that you can *not* draw connecting dot to dot.

4. Are all the diagonals irrational? If no, give a counter-example. (Diagonal of a rectangle whose sides are 3 and 4 units long.)

### **Patterns**

### Line segments

If the shortest distance between two adjacent dots is one unit. How many segments one unit long you can construct on Dot papers of sizes  $2 \times 2$ ,  $3 \times 3$ ,  $4 \times 4$  and  $5 \times 5$ ,  $n \times n$ .

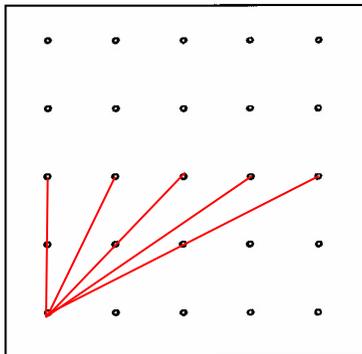
Dot paper size	$2 \times 2$	$3 \times 3$	$4 \times 4$	$5 \times 5$	$n \times n$
Number of segments					

Make a segment that is as long as the diagonal of a  $1 \times 1$  square. How many segments can be constructed with this length on dot papers of sizes  $2 \times 2$ ,  $3 \times 3$ ,  $4 \times 4$ ,  $5 \times 5$  and  $n \times n$ ? Write the answers in a table.

Table 1

Dot paper size	$2 \times 2$	$3 \times 3$	$4 \times 4$	$5 \times 5$	$n \times n$
Number of segments					

- Find on a  $5 \times 5$  Dot paper five line segments with length greater than 4 units.
- Find three line segments with length less than 3 units. Are there others?
- Continue the pattern given below on a  $7 \times 7$  dot paper:



- Make patterns and Continue patterns made by others on a dot paper.
- Find all possible line segments of different length on a  $5 \times 5$  dot paper. (There are 14 such line segments.)

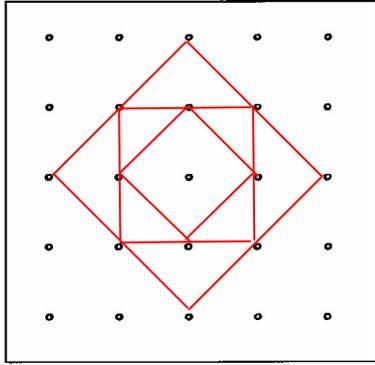
### Square

If the shortest distance between two dots is taken as a unit, Make squares whose sides are 1, 2, 3, 4, 5 units and fill in the table given below

Table 2

Length of Side of square	1	2	3	4	5	n
Perimeter	4	8				
Area	1	4				

Continue the pattern given below on a  $7 \times 7$  dot paper



Make patterns and Continue patterns made by others on a dot paper  
 Find all possible squares on a 5 x 5 dot paper. (There are 8 such squares.)

**Rectangle**

1. Find all possible rectangles of different area on a 5 x 5 dot paper. (There are 16 such rectangles, half of which are squares.)
2. Order each of the rectangles in the main activity by area.
3. Draw and continue patterns made by others on a dot paper.

**Parallelogram**

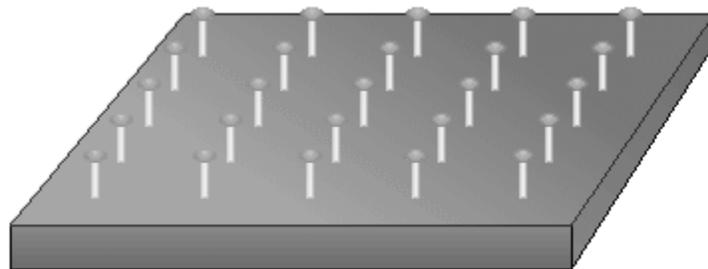
Repeat activities with right triangle and parallelogram similar to that of a rectangle  
 (There are a total of 42 parallelograms, 16 of which are rectangles and there are 17 right triangles on a 5x 5 Dot paper)

For more activities on patterns go to the websites

<http://homepage.mac.com/efithian/Geometry/Activity-03.html>  
[http://mathforum.org/trscavo/dot\\_papers/geobd5.html](http://mathforum.org/trscavo/dot_papers/geobd5.html)

We have discussed all the activities using Dot paper. All these activities can also be performed with a **Geoboard**. A simple **Geoboard** can be made by drawing 5 vertical lines and 5 horizontal lines one cm. apart on a square piece of wood. Nails are placed at the intersections of the lines so that they extend about one centimeter.

Similarly a circular and a triangular Geo Board can be made by placing nails on a wooden board as in the dot papers given in Appendices 3 and 4.

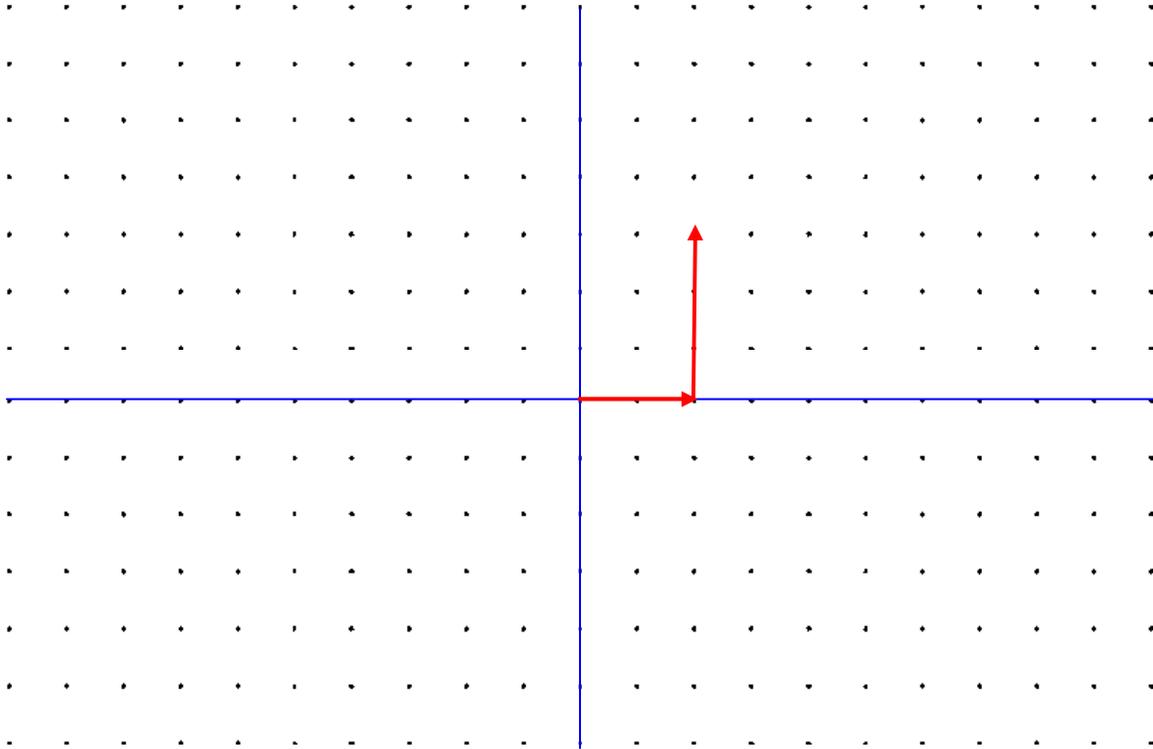


Figures are made on the Dot paper by stretching rubber bands from one nail to another until the desired shape is formed. Line segments can be shown by connecting only two nails. It would be desirable to use it in lower grades. It should be used with dot paper in higher grades so as to keep the record.

**Plotting points and reading coordinates of points**

Draw two lines at right angles to each other. The intersection of lines point o is called the **origin**. The horizontal direction is named as x axis and vertical direction is named as y

axis. We can plot a point  $(x, y)$  on it by moving  $x$  units along  $x$  axis from  $o$  and then  $y$  units along  $y$  axis from  $o$ ; the end point is the point  $(x, y)$ . The distances  $x$  and  $y$  are called the coordinates of a point  $(x, y)$ . We can plot a point  $A(2, 3)$ , taking the shortest distance between two dots as a unit of length, by moving 2 units along  $x$  axis from  $o$  and then 3 units along  $y$  axis from  $o$ , the end point is the point  $(2, 3)$



The region that is to the right of  $o$  and above  $o$  is called the **first quadrant**.

The region that is to the left of  $o$  and above  $o$  is called the **second quadrant**.

The region that is to the left of  $o$  and below  $o$  is called the **third quadrant**.

The region that is to the right of  $o$  and below  $o$  is called the **fourth quadrant**.

The distance along  $x$  to the right is taken as positive and to the left as negative as in a number line. Thus  $x$  is positive in first and fourth quadrant and negative in the second and third quadrant.

The distance along  $y$  above is taken as positive and that below as negative. Thus  $y$  is positive in first and second quadrant and negative in third and fourth quadrant.

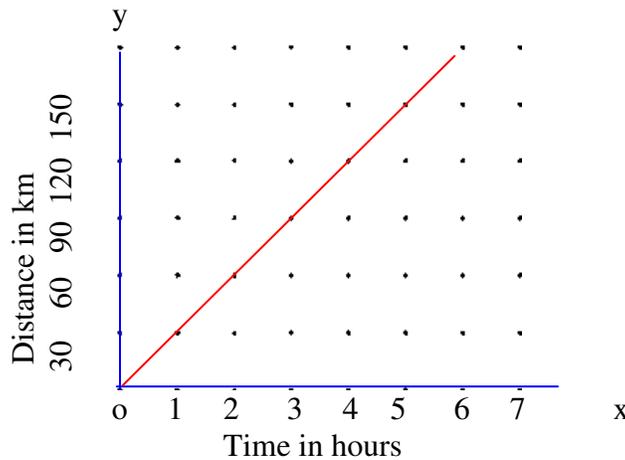
### **Graphs**

The figure obtained by joining many points  $(x, y)$  is called a **graph**.

### **Reading linear graphs and distance versus time graphs.**

The figure obtained by joining many points  $(x, y)$  is called a **graph**.

The graph given below presents the relation between time and distance covered by a bus going from one city to another.



The graph shows that after one hour, the car had covered a distance of 30km, after 2 hours a distance of 60 km and after 3 hours a distance of 90 km and after 4 hours a distance of 120 km. that is the car is moving at an average speed of 30 km an hour.

At this speed how much distance would it cover in 5 hours?

Would the graph differ if smaller intervals were used? If yes how?

Find a time graph in the newspaper or some magazine and write a report about it what does the data show and present it to your class.

### Drawing graphs

Draw equilateral triangles of different side lengths on a triangle graph and fill in the table given below:

Length of the side	1	2	3	4
Perimeter	3			
Area	1			

Draw a graph taking length along x axis and perimeter along y axis.

What does the graph show?

What is the shape of the graph?

If you extend the graph would the points corresponding to larger side lengths say 6 and their perimeters fall on this line? Verify it.

If  $s$  denotes the length of side of a triangle, what would be its perimeter?

Draw another graph taking length along x axis and area along y axis.

What does the graph show?

What is the shape of the graph?

If you extend the graph would the points corresponding to larger side and their areas fall on this graph? Verify it for the length of the side equal to 5 units.

If  $s$  denotes the length of side of a triangle, what would be its area?

Repeat it for a square on a square graph paper.

Collect data for a week on maximum and minimum temperature from a news paper and draw and interpret the graph of that. (Which day was hottest, coolest, had highest, lowest difference between maximum and minimum temperatures etc.)

Draw a graph of simple and compound interest (Compounded annually) that would be earned at the rate of 10% at the end of 1, 2, and 3 years.

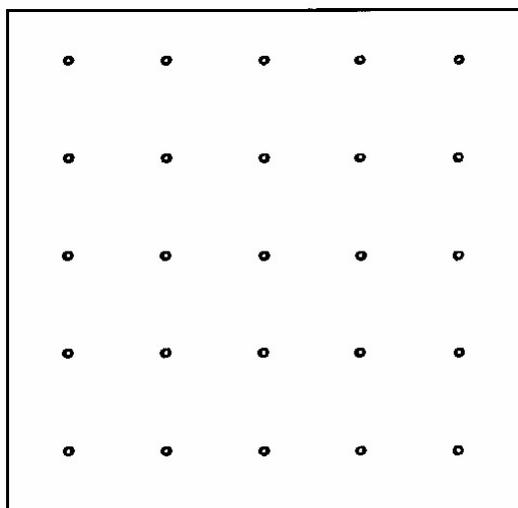
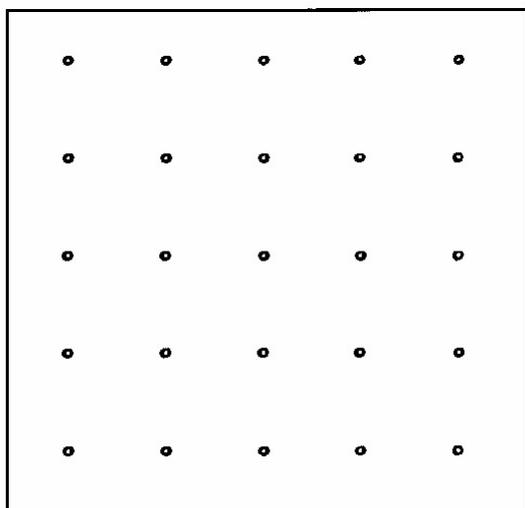
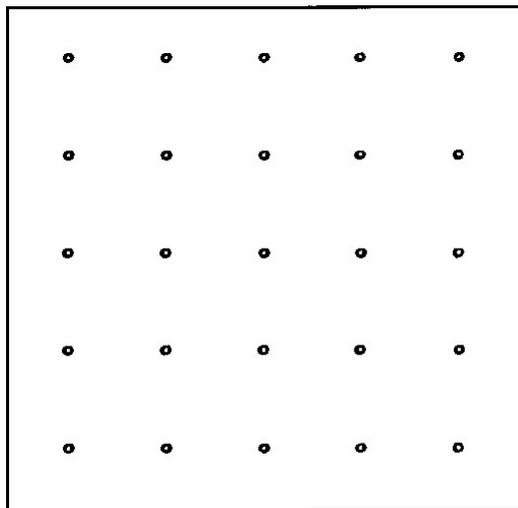
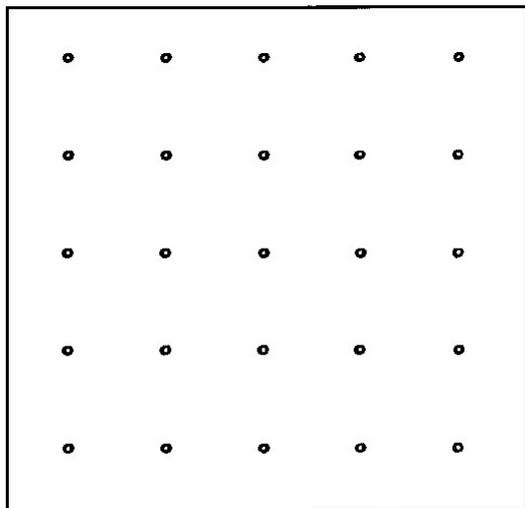
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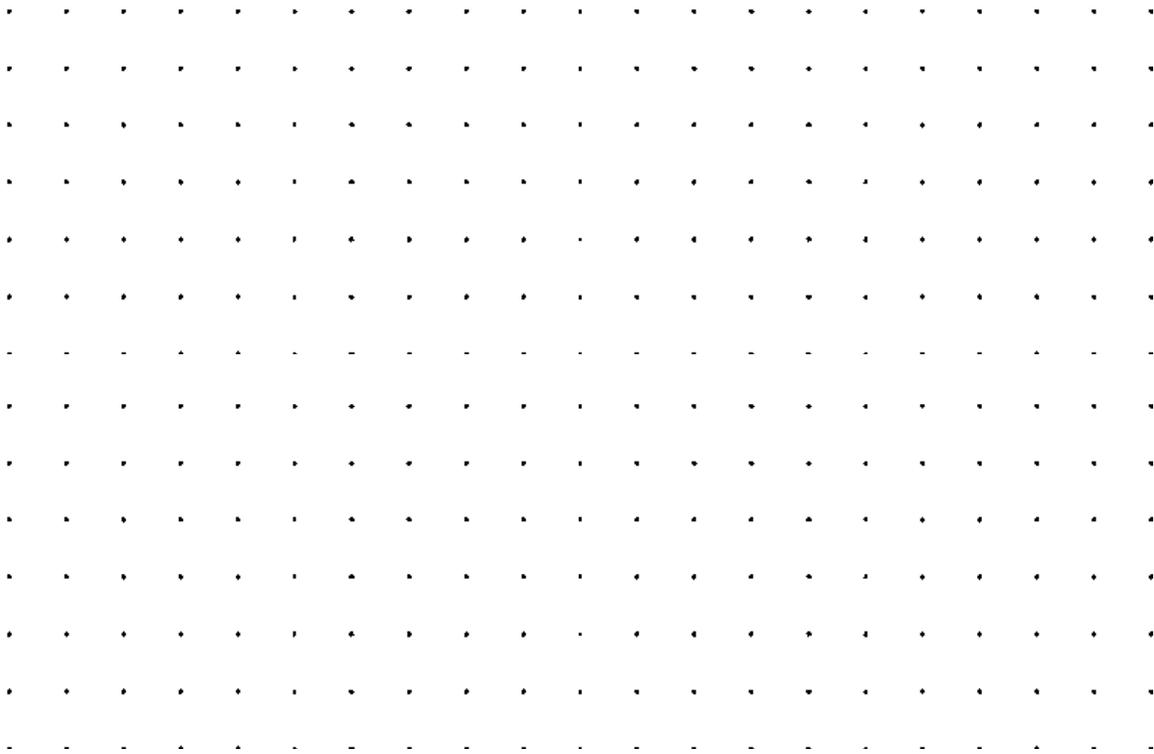
Freuenthal, H. (Ed.) 1958 *Report on methods of unit iteration in geomety*.

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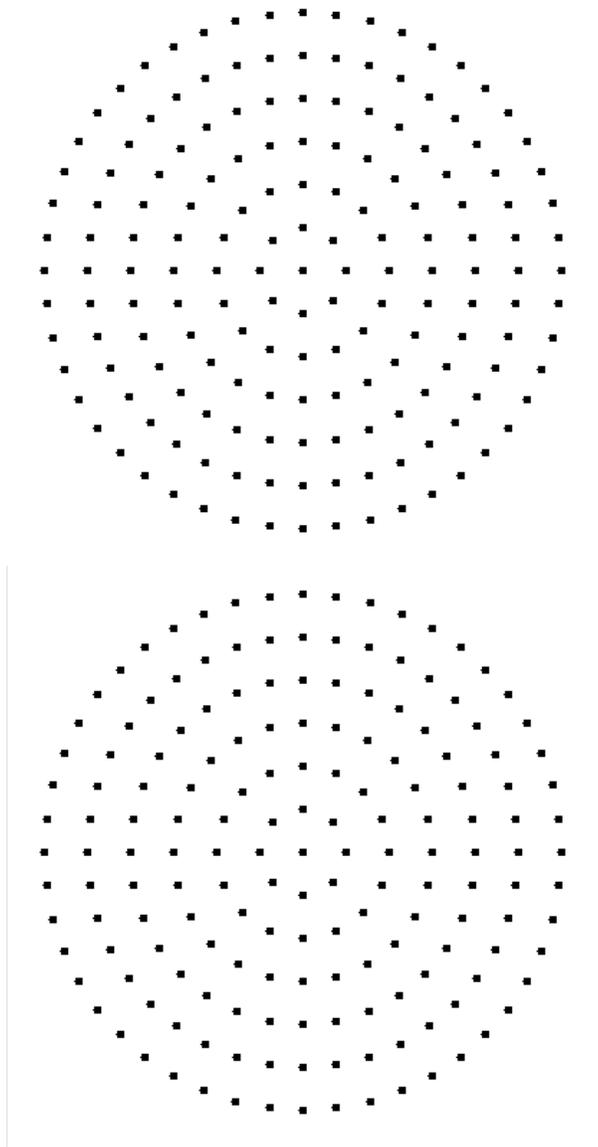
**Appendix 1**  
**5 × 5 Dot Paper**



**Appendix 2**  
**Dot paper**



**Appendix 3**  
**Circular Dot Paper**



## Appendix 4

### Isometric Dot Paper

